TRANSACTIONS

OF THE

AMERICAN INSTITUTE

OF

ELECTRICAL ENGINEERS

JUNE 30, TO DECEMBER 31, 1908

VOL. XXVII, PART II

PUBLISHED BY THE
AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
33 WEST THIRTY-NINTH STREET
NEW YORK, N. Y., U. S A.
1909

Press of McIlroy & Emmet, 22 Thames Street, New York

OFFICERS AND BOARD OF DIRECTORS.

1907-1908.

PRESIDENT.

HENRY GORDON STOTT.

Term expires 1908.

JUNIOR PAST-PRESIDENTS.

SCHUYLER S. WHEELER. SAMUEL SHELDON.

VICE-PRESIDENTS.

Term expires 1908.	Term expires 1909.
H. H. HUMPHREY.	JAMES G. WHITE.
A. H. ARMSTRONG.	W. C. L. EGLIN.
FRANK G. BAUM.	L. A. FERGUSON.

MANAGERS.

Term expires 1908.	Term expires 1909.	Term expires 1910.
CUMMINGS C. CHESNEY.	JOHN J. CARTY.	MORGAN BROOKS.
BANCROFT GHERARDI.	A. M. SCHOEN.	HAROLD W. BUCK.
CHARLES L. EDGAR.	PAUL M. LINCOLN.	PERCY H. THOMAS.
CALVERT TOWNLEY.	PAUL SPENCER.	BENJAMIN G. LAMME.

TREASURER.

GEORGE A. HAMILTON.
New York, N. Y.

Term expires 1908.

SECRETARY.

RALPH W. POPE.
New York, N. Y.

PAST-PRESIDENTS.

NORVIN GREEN, 1884-5-6.
FRANKLIN L. POPE, 1886-7.
T. COMMERFORD MARTIN, 1887-8.
EDWARD WESTON, 1888-9.
ELIHU THOMSON, 1889-90.
W. A. ANTHONY, 1890-91.
ALEX. GRAHAM BELL, 1891-2.
FRANK J. SPRAGUE, 1892-3.
EDWIN J. HOUSTON, 1893-4-5.
LOUIS DUNCAN, 1895-6-7.
FRANCIS B. CROCKER, 1897-8.
A. E. KENNELLY, 1898-1900.
CARL HERING, 1900-1901.
CHARLES P. STEINMETZ, 1901-2.
CHARLES F. SCOTT, 1902-3.
BION J. ARNOLD, 1903-4.
JOHN W. LIEB, JR., 1904-5.
SCHUYLER S. WHEELER, 1905-6.
SAMUEL SHELDON, 1906-7.

STANDING COMMITTEES.

Executive Committee.
H. G. STOTT, Chairman.
C. C. CHESNEY.
C. L. EDGAR.
GEO. A. HAMILTON.
RALPH W. POPE.
CALVERT TOWNLEY.
JOHN J. CARTY.

Library Committee.
EDW. CALDWELL, Chairman.
MORGAN BROOKS.
MAX LOEWENTHAL.
F. A. SCHEFFLER.
PHILIP TORCHIO.

Sections Committee.
PAUL SPENCER, Chairman.
WILLIAM ESTY.
H. H. NORRIS.
D. B. RUSHMORE.
P. B. WOODWORTH.

Law Committee.
C. O. MAILLOUX, Chairman.
A. H. ARMSTRONG.
H. A. LARDNER.
E. A. SPERRY.
C. A. TERRY.

Board of Examiners.
HAROLD W. BUCK, Chairman.
H. H. BARNES, Jr.
MAURICE COSTER.
W. C. L. EGLIN.
GEORGE F. SEVER.

Editing Committee.
GEO. F. SEVER, Chairman.
W. H. BROWNE, Jr.
PERCY H. THOMAS.

Standards Committee.
SAMUEL SHELDON, Chairman.
B. A. BEHREND.
A. E. KENNELLY.
D. C. JACKSON.
CHAS. F. SCOTT.
C. H. SHARP.
C. P. STEINMETZ.
L. B. STILLWELL.
S. W. STRATTON.

Meetings and Papers Committee.
PERCY H. THOMAS, Chairman.
A. H. ARMSTRONG.
PAUL M. LINCOLN.
RALPH D. MERSHON.
H. H. NORRIS.

Code Committee.
F. A. C. PERRINE, Chairman.
J. C. FORSYTH.
A. M. SCHOEN.
GEORGE F. SEVER.
H. A. SINCLAIR.
H. S. WARREN.
ARTHUR WILLIAMS.

Finance Committee.
CALVERT TOWNLEY, Chairman.
H. W. BUCK.
BANCROFT GHERARDI.

SPECIAL COMMITTEES.

High-Tension Transmission Committee.
(Sub-com. of Meetings and Papers Committee.)
RALPH D. MERSHON, Chairman.
F. G. BAUM.
C. C. CHESNEY.
B. G. LAMME.
PAUL M. LINCOLN.
W. S. MOODY.
W. N. RYERSON.

Code of Ethics Committee.
SCHUYLER S. WHEELER, Chairman.
BION J. ARNOLD.
H. W. BUCK.
J. J. CARTY.
L. A. FERGUSON.
B. G. LAMME.
J. W. LIEB, Jr.
C. F. SCOTT.
SAMUEL SHELDON.
C. P. STEINMETZ.
L. B. STILLWELL.

Committee on Increase of Membership.
D. B. RUSHMORE, Chairman.
A. W. BERRESFORD.
S. S. DICKENSON.
GANO DUNN.
BANCROFT GHERARDI.
F. L. HUTCHINSON.
S. M. KINTNER.
A. P. PECK.
W. L. WATERS.

Building Fund Committee.
T. C. MARTIN, Chairman.
J. C. BARCLAY.
J. J. CARTY.
C. L. EDGAR.
F. W. JONES.
J. W. LIEB, Jr.
F. J. SPRAGUE.
J. G. WHITE.

Committee on Forest Preservation.
C. H. PORTER, Chairman.
R. G. BLACK.
P. G. GOSSLER.
A. M. SCHOEN.
J. A. THALER.
C. E. WADDELL.

Railway Committee.
(Sub-com of Meetings and Papers Committee.)
A. H. ARMSTRONG, Chairman,
WM. McCLELLAN.
H. ST. C. PUTNAM.
C. W. RICKER.
N. W. STORER.

Educational Committee.
(Sub-com. of Meetings and Papers Committee.)
HENRY H. NORRIS, Chairman.
GANO DUNN.
DUGALD C. JACKSON.
C. O. MAILLOUX.
CHARLES F. SCOTT.

John Fritz Medal Representatives.
HENRY G. STOTT.
SAMUEL SHELDON.
SCHUYLER S. WHEELER.
JOHN W. LIEB, Jr.

The International Electrotechnical Commission. United States National Committee.
ELIHU THOMSON, President.
CHARLES F. SCOTT, 1st Vice-President
SAMUEL SHELDON, 2d Vice-president.
BION J. ARNOLD.
B. A. BEHREND.
A. W. BERRESFORD.
JOHN J. CARTY.
W. C. L. EGLIN.
CARL HERING.
JOHN W. HOWELL.
DUGALD C. JACKSON.
FRANCIS W. JONES.
ARTHUR E. KENNELLY.
BENJAMIN G. LAMME.
W. A. LAYMAN.
JOHN W. LIEB, Jr.
CLAYTON H. SHARP.
CHARLES P. STEINMETZ
LEWIS B. STILLWELL.
HENRY G. STOTT.
SAMUEL W. STRATTON.

Committee on Bibliography.
WM. D. WEAVER.

Section and Branch Secretaries.

ADOLPH SHANE,
Iowa State College,
Ames, Iowa.

H. F. BAXTER,
556 S. State St.,
Ann Arbor, Mich.

G. J. YUNDT,
Southern Bell Tel. & Tel. Co.
Atlanta, Ga.

B. F. KENYON,
University of Texas,
Austin, Texas.

CHAS. G. EDWARDS,
Electrical Commission,
City Hall,
Baltimore, Md.

J. A. CLARKE, Jr.,
Lehigh University,
Bethlehem, Pa.

A. L. PEARSON,
93 Federal St.,
Boston, Mass.

H. S. BUCHANAN,
University of Colorado,
Boulder, Colo.

J. A. THALER,
Montana Agricultural College,
Bozeman, Mont.

J. E. SNOW,
Armour Inst. of Technology.
Chicago, Ill.

P. B. WOODWORTH,
Lewis Institute,
Chicago, Ill.

J. G. WRAY,
Chicago Telephone Co.,
Chicago, Ill.

A. C. LANIER,
University of Cincinnati,
Cincinnati, O.

C. C. BUCHANAN,
University of Cincinnati,
Cincinnati, O.

F. M. HIBBEN,
10207 Wilbur Avenue,
Cleveland, O.

H. D. CARPENTER,
University of Missouri,
Columbia, Mo.

F. C. CALDWELL,
Ohio State University,
Columbus, O.

H. L. BACHMAN,
533 South 3d St.,
Columbus, O.

E. C. WRIGHT,
Oregon Agr. College,
Corvallis, Ore.

C. R. RHODES,
University of Arkansas,
Fayetteville, Ark.

H. H. NORRIS,
Franklin Hall, Cornell University,
Ithaca, N. Y.

H. T. PLUMB,
Electrical Eng. Dept. Purdue University,
Lafayette, Ind.

H. P. BRODERSON,
University of Kansas,
Lawrence, Kan.

L. E. HURTZ,
Lincoln Tel. Co.,
Lincoln, Neb.

J. W. SHUSTER,
University of Wisconsin,
Madison, Wis.

KIRK H. LOGAN,
Kansas State Agr. College,
Manhattan, Kan.

F. D. NIMS,
Mexican Light & Power Co.
City of Mexico.

S. R. INCH,
Univ. of Montana,
Missoula, Mont.

F. W. WALTER,
National Bank of Commerce,
Norfolk, Va.

GUSTAV WITTIG,
University of Maine,
Orono, Maine.

C. D. PRESTON,
Penn. State College,
State College, Pa.

H. F. SANVILLE,
597 Drexel Bldg.,
Philadelphia, Pa.

R. A. L. SNYDER,
1332 Fulton Building,
Pittsburg, Pa.

H. L. SMITH,
Stanley-G. I. Elec. Mfg. Co.
Pittsfield, Mass.

M. K. AKERS,
State College of Washington,
Pullman, Wash.

C. M. DUNCAN,
Washington University,
St. Louis, Mo.

H. I. FINCH,
5228 Kensington Ave.,
St. Louis, Mo.

A. L. ABBOTT.
165 E. 4th St.,
St. Paul, Minn.

G. R. MURPHY,
11 Hawthorne St.,
San Francisco, Cal.

W. C. ANDREWS,
General Electric Co.,
Schenectady, N. Y.

W. S. WHEELER,
3120 East Union Street.,
Seattle, Wash.

M. VESTAL,
Leland Stanford, Jr., University,
Stanford University, Cal.

R. A. PORTER,
Syracuse University,
Syracuse, N. Y.

GEO. E. KIRK,
The Nicholas,
Toledo, Ohio.

W. G. CHACE,
126 Confederation Life Bldg.
Toronto, Ont.

ELLERY B. PAINE,
University of Illinois,
Urbana, Ill.

PHILANDER BETTS,
Metropolitan Bank Bldg.
Washington, D. C.

S. W. FARNSWORTH,
Worcester Poly. Inst.
Worcester, Mass.

Local Honorary Secretaries.

JAMES S. FITZMAURICE,
210 George St., Sydney, N. S. W.

H. F. PARSHALL,
Salisbury House, London Wall E. C., London.

ROBERT B. OWENS,
McGill University, Montreal, P. Q.

WILLIAM B. HALE,
1a Independencia No. 6, City of Mexico.

CLARE F. BEAMES,
for Porto Rico and West Indies,
San Juan, P. R.

W. G. T. GOODMAN,
Adelaide, South Australia.

ROBERT J. SCOTT,
Christ Church, New Zealand.

CONTENTS.

MEETING AT NEW YORK, OCTOBER 9, 1908.

MEETING OF BOSTON SECTION, NOVEMBER 18, 1908.

MEETING OF PITTSBURGH SECTION, NOVEMBER 10, 1908.

MEETING AT NEW YORK, NOVEMBER 13, 1908.

MEETING AT NEW YORK, DECEMBER 11, 1908.

MEETING OF PITTSBURG SECTION, OCTOBER 2, 1907.

APPENDIX.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 30, 1908.

HIGH VOLTAGE MEASUREMENTS AT NIAGARA

BY RALPH D. MERSHON

In the autumn of 1896, the writer of this paper undertook an investigation of the phenomena existing when transmission line conductors are subjected to high alternating voltages. The work was carried on near Telluride, Colorado, and extended over a period of about a year. The results of this work were embodied in a report made by the writer in 1897.*

Through lack of the necessary facilities at Telluride, the work was not carried as far as seemed desirable and, after its discontinuance, I looked forward to taking it up again and obtaining additional data. This opportunity offered in 1903, and in the autumn of 1904, after the necessary apparatus had been obtained, the work was resumed at Niagara Falls, and the observations carried on more or less continuously until the summer of 1907.

In the meanwhile, Professor Harris J. Ryan† read before the Institute his paper bearing on this subject and embodying the results of investigations made by him of some of the points I had intended to cover, and a number of others which my facilities would not admit of closely investigating.

The present paper has mainly to do with the results of the

*The investigation was undertaken for the joint interests of the Telluride Power Transmission Company and the Westinghouse Electric and Manufacturing Company. Part of the matter of the report was embodied in a paper read at the Fifteenth General Meeting of the American Institute of Electrical Engineers, June 30, 1898, by Mr. Chas. F. Scott, entitled "High Voltage Power Transmission."

† See paper entitled "The Conductivity of the Atmosphere at High Voltages," read at the 184th meeting of the American Institute of Electrical Engineers, Feb. 26, 1904.

work carried on at Niagara Falls, but in the treatment of these results, the work at Telluride and that of Professor Ryan will necessarily be referred to and discussed.

The work at Niagara was made possible, in the first instance, by the generosity of three men; Mr. J. E. Aldred, Mr. Frederic Nicholls, and Mr. James Ross. Later, further support to the work was contributed by Mr. George Westinghouse, and by the African Concessions Syndicate of London. The major portion of the expenses of the work at Niagara was defrayed by the above contributors.

I desire to express my appreciation not only of the generosity of these contributors, but also of the completeness with which they entrusted the expenditures to my judgment and the kindly patience with which they have awaited results so long deferred by reason of the tedious, intricate, and often discouraging nature of the work. It is to be wished that engineering investigation might be more encouraged in a like manner and spirit. I hope the results obtained will appear to justify the contributors in this instance.

For convenience of treatment, the matter of this paper is arranged under the following heads: Equipment; Results of Measurements; Discussion of Results; Résumé and Conclusions.

EQUIPMENT

The line experimented upon at Niagara had a total length of 2000 feet, although, generally, only half its length was used. It was supported upon wooden poles, spaced about 140 feet apart. At first, the line wires were supported upon insulators, but it was found that the loss over the insulators was so great, as compared with the air losses, and so variable that if any reliable results were to be obtained, it would be necessary to find some other way of supporting the line conductors. Finally, the line wires were suspended by means of paraffined cords attached to the necks of the insulators. As long as these cords were clean, the loss over them was negligible. As soon as they became dirty, they were replaced by clean cords. A portion of the line with the suspending cords is shown in Fig. 1. This line will hereafter be referred to as the " Experimental Line ".

In addition to the experimental line, use was made of a number of cross-arms equipped with pins and insulators, similar to those used on the experimental line. This miniature line had a total length of only a few feet, so that the air loss between its conductors was negligible, the loss upon it being due to the insulators only. This miniature line will be designated hereafter as the " Dummy Line ". It is shown in Fig. 2.

The following conductors were used.

ALUMINUM CONDUCTORS

Circular mils	Number of strands	Diameter one strand inch	Outside diameter inch
10500	1	0.1025	0.1025
20740	1	0.144	0.144
34600	1	0.186	0.186
51529	1	0.227	0.227
41800	19	0.0469	0.2345
42910	7	0.0783	0.2349
41750	37	0.0336	0.2352
103850	7	0.1219	0.3657
208200	7	0.1728	0.5184

COPPER CONDUCTORS

10420	1	0.1021	0.1021

The various types of insulators with which experiments were made are shown in Fig. 3 and will be referred to hereafter by the letters designating them in the illustration.

Two single-phase, 100,000-volt transformers were used, each having a capacity of 10 kilowatts. The endeavor was made to have the iron of these two transformers as nearly as possible identical as to loss, etc., for reasons which will be apparent from the description of the method of measurement employed. The two transformers were immersed in oil in the same boiler iron tank. They were of the core type, and had ground shields between the high-tension and the low-tension windings. The windings had taps for connecting the transformers, when desired, for polyphase transformation. The transformers had a number of special low-voltage coils, the use of which is explained below.

The power for the measurements was obtained from a surface-wound, single-phase alternator of the old 133-cycle type, belted to an induction motor and driven at about one-half speed. This machine gave very nearly a sine wave under almost all conditions. The intention was to run it at 60 cycles, but instead it was run usually at about 73 cycles.

The wattmeter made use of was one especially constructed

FIG. 1

for this purpose. It is shown in Fig. 4. It consisted of the regular Weston wattmeter movement and shunt resistance enclosed in a suitable wooden box, and an external field coil which could be slid over the wattmeter movement or away from it, so as to give an instrument of considerable range. The field coil was enclosed in a suitable wooden frame. This field coil consisted of two identical windings, the wires being wound side by side, so that the magnetic axes of the two windings would as nearly as possible coincide. In addition there was supplied with the wattmeter an extra field coil exactly like the one used with the wattmeter. This extra field coil was used as an air transformer, as described later on.

The type of barometer, thermometer, and sling psychrometer recommended by the United States Weather Bureau for the measurements of barometric pressure, temperature, and relative

FIG. 2

humidity, respectively, were made use of in observing the corresponding weather quantities.

In addition the various necessary voltmeters, ammeters, etc., as indicated in Fig. 5 were employed. One of the ammeters was used in the high-tension circuit. It was mounted on an insulator and its movement was shielded from electrostatic action by a tin-foil shield inside the case and attached to one terminal.

In addition to the above apparatus, we had for a while, the use of an oscillograph. Voltage curves were obtained by connecting the oscillograph to the D test coil in place of the voltmeter of Fig. 5.

The apparatus was all housed in a cheap building of corrugated iron, the outside of which is shown in Fig. 2.

The method of measurement employed was that devised by me and used at Telluride. In it the iron-loss of one transformer is balanced against the iron-loss of both transformers in such a manner that no iron-loss reading appears on the wattmeter, with the result that the wattmeter records only the losses in the

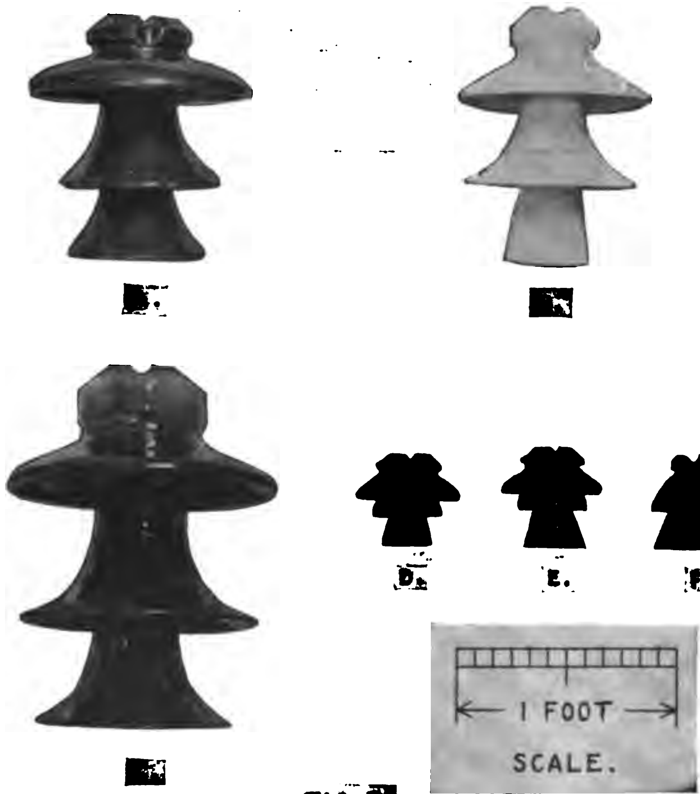


FIG. 3.

high-voltage circuit of the transformer feeding the line. By this method of measurement the only correction which it is necessary to make in the wattmeter reading is to subtract from such reading the $I^2 R$ loss in the high-tension coil of the transformer feeding the line.

The diagram of connections is shown in Fig. 5. One of the transformers, designated as the "Power Transformer" is used to feed the line in the usual way; the other, designated as the "Balancing Transformer" is idle except as to its use for balancing purposes. As will be seen from the diagram each transformer has, in addition to its regular low-tension and high-tension windings, two auxiliary coils, designated as *C* test coil and *D* test coil; from the *D* test coil are brought off a number of leads. The voltmeter and the voltage circuit of the wattmeter are connected to the leads of the *D* test coil of the power transformer. The *D* test coil is so located as to give, as nearly as possible, a voltage reading which will be always proportional to the voltage across the high voltage terminals. In the case of the balancing transformer, the *D* test coil is idle. The *C* test coils of the two transformers

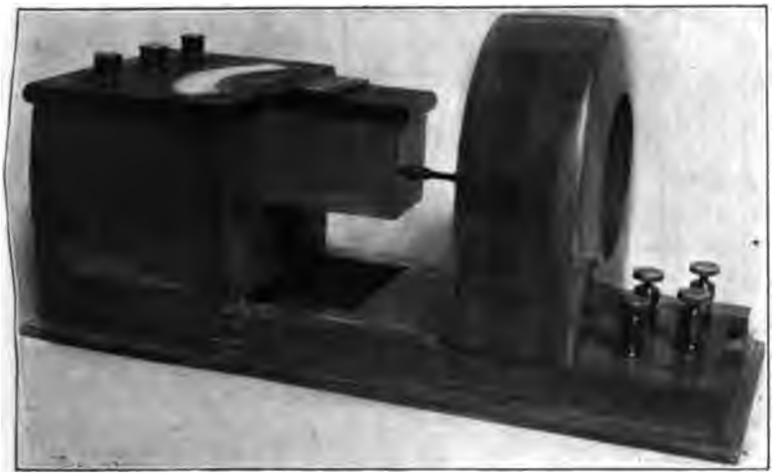


FIG. 4

are connected in series with a small auto transformer between them. With this method of connection, it will be apparent that, provided the auto-transformer does not have an appreciable voltage across its terminals, the magnetization wave of the balancing transformer will, if the iron of the two transformers is the same, be exactly identical with the magnetization wave of the power transformer; for, since the *C* test coil of both of the transformers is close to the iron, the voltage induced in the *C* test coil of the power transformer will be one dependent only upon the flux wave in the iron of this transformer, and will be independent of any reactions which there may be in any of the windings of the transformer. The result is that the *C* test coil of the

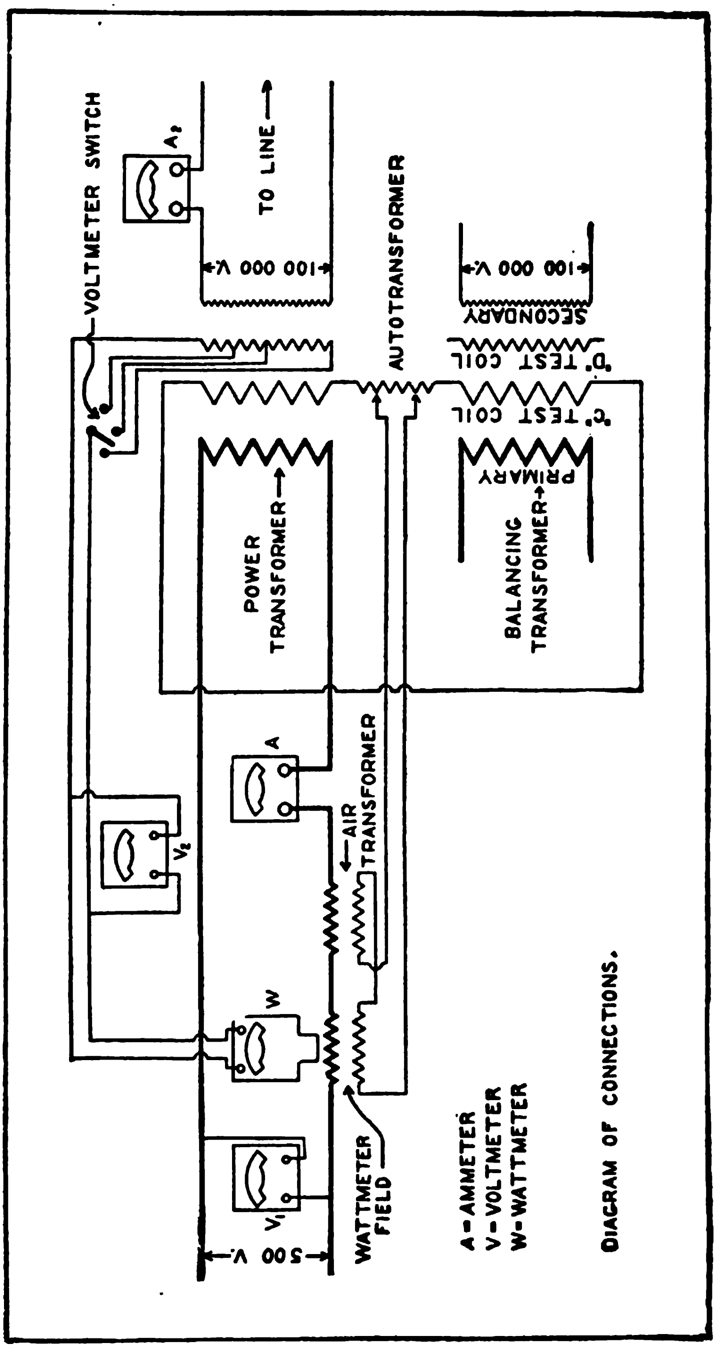

VOLTMETER SWITCH
A₂
TO LINE
100 000 V.
AUTOTRANSFORMER
100 000 V.
SECONDARY
"D" TEST COIL
"C" TEST COIL
PRIMARY
POWER TRANSFORMER
BALANCING TRANSFORMER
A
AIR TRANSFORMER
V₂
W
WATTMETER FIELD
V₁
500 V.
A-AMMETER
V-VOLTMETER
W-WATTMETER
DIAGRAM OF CONNECTIONS.

FIG. 5.

balancing transformer has impressed upon it a voltage exactly corresponding to the iron flux wave of the power transformer, and this condition will hold, no matter what load be put upon the power transformer. This being the case, it is evident that the current flowing in the circuit made up of the two *C* test coils and the auto transformer is proportional to and in step with that component of the main current, *i.e.*, the current supplied by the generator to the low voltage terminals of the power transformer, which corresponds to the iron loss of both transformers. Hence any current taken off of the auto transformer will be proportional to and in step with this iron loss component; and the value of the current in the secondary circuit of the auto transformer relative to the iron-loss component of the main current will depend upon the ratio of transformation employed in the auto-transformer, which ratio will hold through all ranges of voltage and all conditions of load on the power transformer.

As has been previously explained, and as is shown in the diagram, the field coil of the wattmeter is double. Through one of the windings of the wattmeter field coil and through one of the windings of the other coil mentioned above and designated in Fig. 5 as the " Air Transformer ", passes the main current to the low voltage winding of the power transformer. The other two windings, or secondaries, of the wattmeter field coil and the air transformer respectively, are connected in series, but in the reverse sense, so that the equal voltages induced in them oppose and neutralize each other. If the air transformer were not used, the voltage of the secondary of the wattmeter field coil would disturb the adjustment of the circuit including it. The circuit including the secondaries of these two coils is connected to the auto transformer previously mentioned in such a way and with such a ratio that there will pass through this circuit a current of sufficient magnitude and in such a direction as to neutralize in the wattmeter field coil that component of the main current fed to the power transformer which accounts for the iron loss in both transformers. It will be evident that by careful adjustment, not only may the wattmeter be made to read zero when there is no power delivered to the line from the power transformer, but also that it may be made to give no indication of the iron loss in the power transformer when the transformer is delivering power.

The other instruments and connections of Fig. 5 sufficiently explain themselves. The balancing transformer need not necessarily be a transformer, but may be a reactance, the iron of which has the same characteristics as that of the power transformer and which is worked at the same induction as the power transformer.

Results of Measurements

The endeavor was made to get some sort of a resistance for use on 100,000 volts, the value of which would remain practically constant, and which would not have a large charging current, so that by putting it across the terminals of the transformer, taking the wattmeter reading, and at the same time reading in the high voltage circuit the current taken by the resistance. the wattmeter reading might be checked by calculating the $C^2 R$ loss in the resistance. We were not successful, however, in finding any resistance which would answer for this purpose, and had to be content with the check readings described below.

In order to find out what effect, if any, the charging current of the line would have on this combination of wattmeters and transformers in the matter of producing errors in the reading, numerous sets of readings were taken of which the following are fair samples.

A measurement was taken on the experimental line. (The loss in this case is accompanied by a large charging current.) A reading was also taken on the dummy line. (In this case there is practically no charging current.) Then a reading was taken on the experimental line and the dummy line together. Assuming constancy of wave form, absolute accuracy would result in the sum of the separate readings being equal to the reading taken on the two lines together. The following is a typical set of readings, taken on the dummy line and an experimental line, consisting of 42,910 cm. 7-strand* aluminum cable, spaced at 55 in.

Kilovolts	Exper. Line only (A) watts	Dummy Line only (B) watts	Dummy and Exp. Line	(A)+(B)	Error watts	Error per cent.
80	138	54	209	192	+17.	+8.13
70	92	39	140	131	9.	+6.43
60	64	25	97	89	8.	+8.24
70	102	49	152	151	1.	+0.66
70	106	42	158	148	10.	+6.33
70	104	39	158	143	15.	+9.5
			Average error =			+6.55

*Throughout this paper the word "strand" is understood to mean "wire," in accordance with common usage.

The above is considered very satisfactory. It must be remembered that the readings on the line are always more or less unsteady, due to the variation of line loss, especially if the measurement is taken above the critical point. This accounts for the considerable variation in the error. The average error, however, can be accounted for in another way. As will be seen

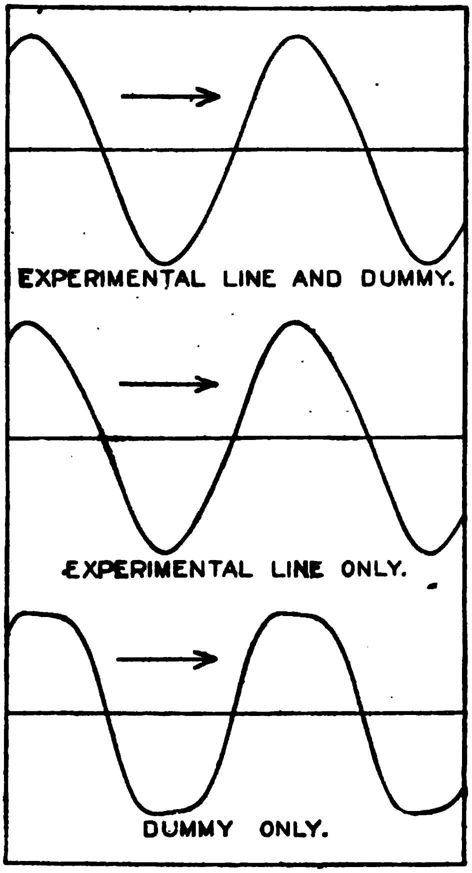


FIG. 6.

later, the loss over the insulators is very sensitive to change of wave form, being greater for sharp wave forms. During the above set of measurements, the wave form was quite different when the experimental line was connected to the transformer from what it was when the experimental line was disconnected. The oscillograph curves in Fig. 6 (corresponding to 70 kilovolts) show this.

The first of them is the wave form with the experimental line and dummy line; the second is the wave form with the experimental line only; the third is the wave form with the dummy line only. There is no difference between the first and second because the wave form is controlled by the charging current and the dummy line has no measurable charging current. As will be seen, the distortion of the third wave form is in the direction to account for the error. That is, the wave form when the dummy line and experimental line are measured together is the same as when the experimental line is measured alone, and is sharper than when the dummy line is measured alone.

Again. the experimental line was connected to the power transformer, and readings taken with the voltmeter (see Fig. 5) disconnected from the D test coil, and with it connected to the D test coil. The difference in the two readings should be equal to the $C^2 R$ loss of the voltmeter calculated from the reading of the voltmeter and its known resistance. The following is a set of such readings at a number of voltages. The line used consisted of 51,529 cir. mil solid aluminum conductors, spaced at 84 in.

Line kilovolts	Watts		Difference = measured *V. M.* loss	Calculated *V. M.* loss	Error watts	Error per cent.
	V. M. on	*V. M.* off				
90	130.5	121.0	9.5	11.6	+2.1	+22.1
80	98.5	89.5	9.0	9.1	+ .1	+ 1.1
70	75.0	68.5	6.5	7.0	+ .5	+ 7.7
60	56.0	51.0	5.0	5.1	+ .1	+ 2.0
50	42.0	38.0	4.0	3.6	— .4	—10.0
				Average error	=	+ 4.6

This set of measurements was a pretty severe test of the method of measurement, and is really unfair to it because we were here measuring by the method of differences, a quantity which is less than 10 per cent. of the two quantities involved, and an error in the two quantities measured of opposite sign and of only 1 per cent. would more than account for any of the errors obtained. It would have been much better to have had the voltmeter loss and the line loss approximately equal; the D test coil had not sufficient capacity for this. But, even leaving these facts out of consideration, the result is not bad. As in the previous case, the variation in the line loss probably accounts for the variation in the amounts of the errors, although to a less extent, inasmuch as the readings could be taken with the voltmeter on and off much more quickly than the changes could be made in the preceding case. It is to be noted that whereas the previous set of readings is only a relative check, the latter set furnishes not only a relative but an absolute check on the accuracy of the method of measurement, since the loss in the voltmeter was accurately known.

It may be added here that at Telluride, where a wire wound resistance of 1,000,000 ohms was available, check readings taken by means of it on the same method of measurement (but with different apparatus) gave very close results.

It is believed that this method of measurement may be made as accurate as is desired by taking the proper precautions, and that the particular apparatus used in these measurements was capable of giving results certainly within two or three per cent., and perhaps closer under favorable conditions.

This method of measurement offers a means of accurately measuring losses in a high voltage circuit by means of instruments in the low voltage circuit, thus eliminating the possibilities of danger, and of error due to electrostatic effects, when instruments are used in the high voltage circuit.

It should be borne in mind that in the line measurements the loss is accompanied by a very large charging current; for instance, in the case of the 60,000-volt reading of the first set of readings, the line current was 0.038 amperes, corresponding to 2280 apparent watts, whereas the loss was only 64 watts, so that the power factor was only 0.028.

Readings were also taken to check up the accuracy of voltage measurements obtained by a voltmeter connected to the *D* test coil of the power transformer (Fig. 5). These check readings were made by simultaneously reading a voltmeter connected as described and a voltmeter connected to the other transformer used as a voltmeter transformer with its high voltage terminals connected across the terminals of the power transformer. These readings show that the reading obtained from the voltmeter connected to the *D* test coil of the power transformer was practically identical with that obtained from the voltmeter transformer whether a line was connected to the power transformer or not.

As an example of the advantage of taking loss measurements by the method used instead of by reading power to the transformer with the line on, power to the transformer without the line, and taking the difference in the readings; and also to illustrate the efficacy of the air transformer, the three curves shown in Fig. 7 were obtained. One of these shows the result obtained by the subtraction method; one by the balancing method without the air transformer; and one by the method employed in this work. The errors in measurement obtained by the subtraction method are due not only to the fact that a difference is being measured and therefore the errors in the two measurements subtracted may be superposed, resulting in a considerable percentage of error in the quantity measured, but also to the

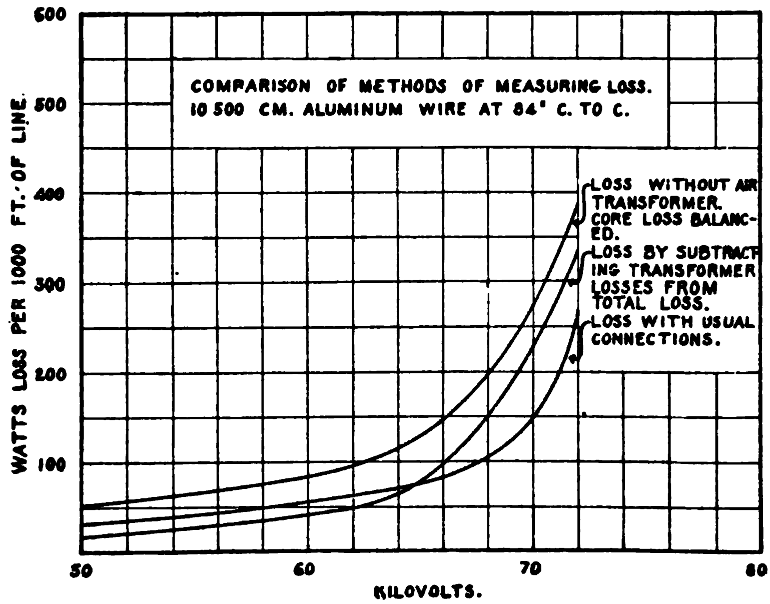


FIG. 7.

fact that the charging current of the line produces more or less distortion in wave form, so that the wave form impressed upon the iron of the transformer when it is feeding the line is different from that when the line is disconnected. The result is that the iron loss on open circuit is not the same as the iron loss when the transformer is delivering current to the line, which greatly increases the subtraction error.

As has been mentioned previously, the measurements on the experimental line were at first made with the line supported on insulators. Measurements were taken in quick succession upon the dummy line and upon the experimental line. By subtracting the loss over the insulators on the dummy line from the total loss on the experimental line, a curve was obtained which, presumably, represented the atmospheric loss. This method of procedure was followed for sometime with apparently concordant results, but after a while very considerable discrepancies began to develop, and it was found that on different days widely differing results would be obtained. Finally, after a great deal of work, we concluded this was due to the variation in the losses over the insulators, brought about by varying weather conditions, and an endeavor was made to find some way of eliminating the insulator loss altogether. This was finally accomplished by suspending the line conductors from cords, the cords being attached to the insulators, which was found to be effective, so long as the cords remained dry and clean. The course finally adopted was that of using ordinary window cord boiled in paraffin for a few minutes, as the saturation with paraffin made the cord less liable to absorb moisture. The cords were frequently tested by placing them all in parallel across 100,000 volts (twice the voltage which they would ordinarily have, since on the line two of them would be in series) and measuring the loss upon them. If they were in good condition, the loss would not be more than three or four watts. If they were not in good condition, they would show a larger loss which would rapidly increase until they began to burn. Defective cords were always thrown away and replaced by new ones, with the result that the losses due to the cords were kept so low as to be altogether negligible.

The measurements with the line supported on insulators were carried on for about eight months before the discrepancies due to insulator losses became evident. The results obtained during this period were worthless and had to be discarded. The length of time which it took to discover these discrepancies and their cause was due, first, to the fact that so long as there were no considerable variations in weather conditions, the discrepancies were small and, secondly, because as the result of the work at Telluride and that done by Professor Ryan, it was thought that weather conditions, except precipitation, would make no difference in loss whether between line conductors, or over insulators, provided the voltage were left long enough on the line to bring the insulator loss to a steady condition.

As soon as the difficulty arising from the insulator loss had been eliminated, results began to be obtained which were more consistent, and it developed that, whereas, the work at Telluride seemed to show that weather conditions, except precipitation, would produce no difference in the loss, the weather conditions had, on the contrary, a great effect upon the loss. At first this result seemed to discredit either the measurements at Telluride or those at Niagara; but, when fuller data had been obtained at Niagara Falls, it was found, as explained later on, that the two results were perfectly concordant. An immense number of readings were taken on different size conductors at different spacings under various weather conditions. It was found that the losses varied considerably, but they seemed to be more affected by humidity conditions than anything else. Such variations in loss as there may have been, due to variations in barometric pressure, were not apparent. If they existed, they were masked by the variations due to humidity conditions. The variations in barometric pressure were small.

An endeavor was made to connect the loss and its variation directly with the relative humidity, but when the loss was plotted against the relative humidity it was not apparent that there was any definite law connecting them. A similar negative result was obtained by plotting loss against absolute humidity. The losses were then plotted against each of the various elements having to do with moisture in the atmosphere and also against combinations of these elements, with the idea of discovering whether any simple relation could be found. As the result of these trials it was found that if the loss were plotted against the product of vapor pressure by relative humidity, what appeared to be a definite relation was obtained. All the readings were then plotted in this way, and curves obtained for different size conductors and different distances between them. For convenience in reference, the product of the vapor pressure by relative humidity will be referred to as the " vapor product ".

In order to give an idea of the closeness with which the results followed the relation found, two of the many target diagrams from which the relation was obtained are shown in Figs. 8 and 9. It will be noted that in some cases the points lie quite close to the curve showing, apparently, a well defined law. In others, the points are more erratic. This is especially true for points obtained above the critical point or bend in the curve connecting loss and voltage (see Figs. 10 to 15 showing loss-voltage curves) because above the critical point, the loss is especially

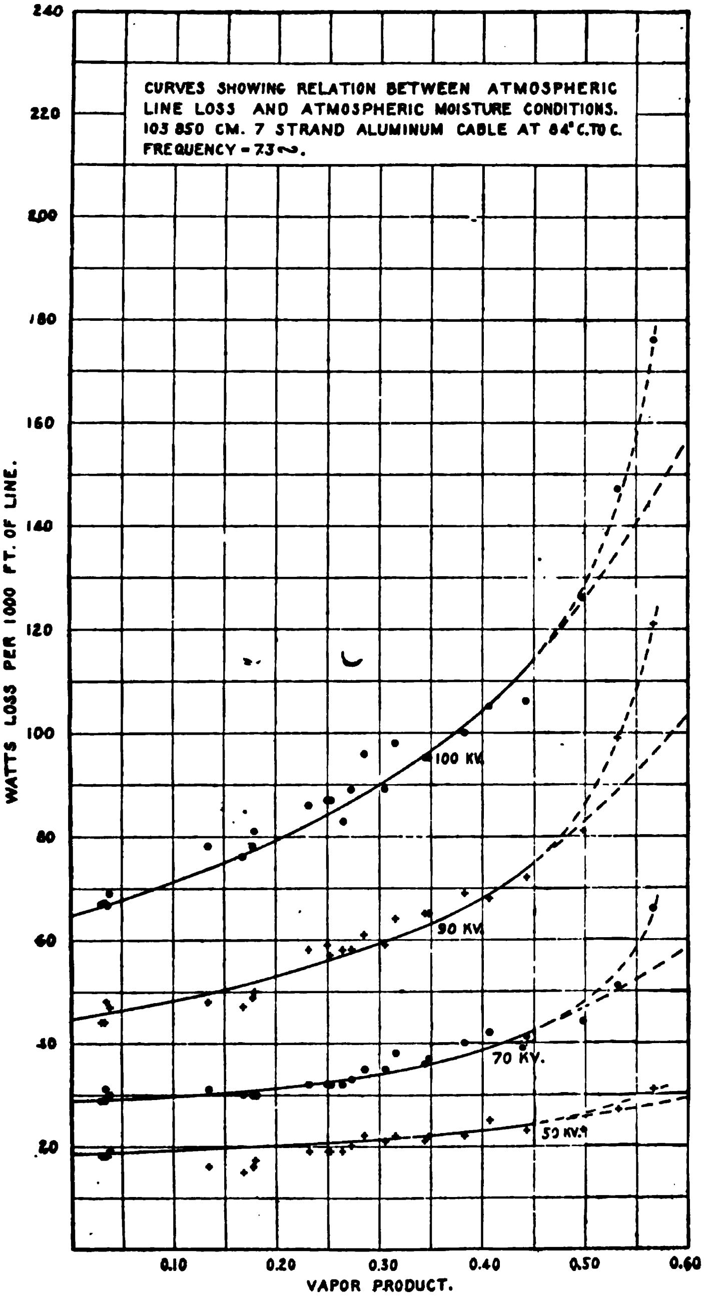

CURVES SHOWING RELATION BETWEEN ATMOSPHERIC LINE LOSS AND ATMOSPHERIC MOISTURE CONDITIONS. 103 850 CM. 7 STRAND ALUMINUM CABLE AT 84°C. TO C. FREQUENCY = 73 ∼.
WATTS LOSS PER 1000 FT. OF LINE.
240
220
200
180
160
140
120
100
80
60
40
20
100 KV.
90 KV.
70 KV.
50 KV.
0.10
0.20
0.30
0.40
0.50
0.60
VAPOR PRODUCT.

FIG. 8.

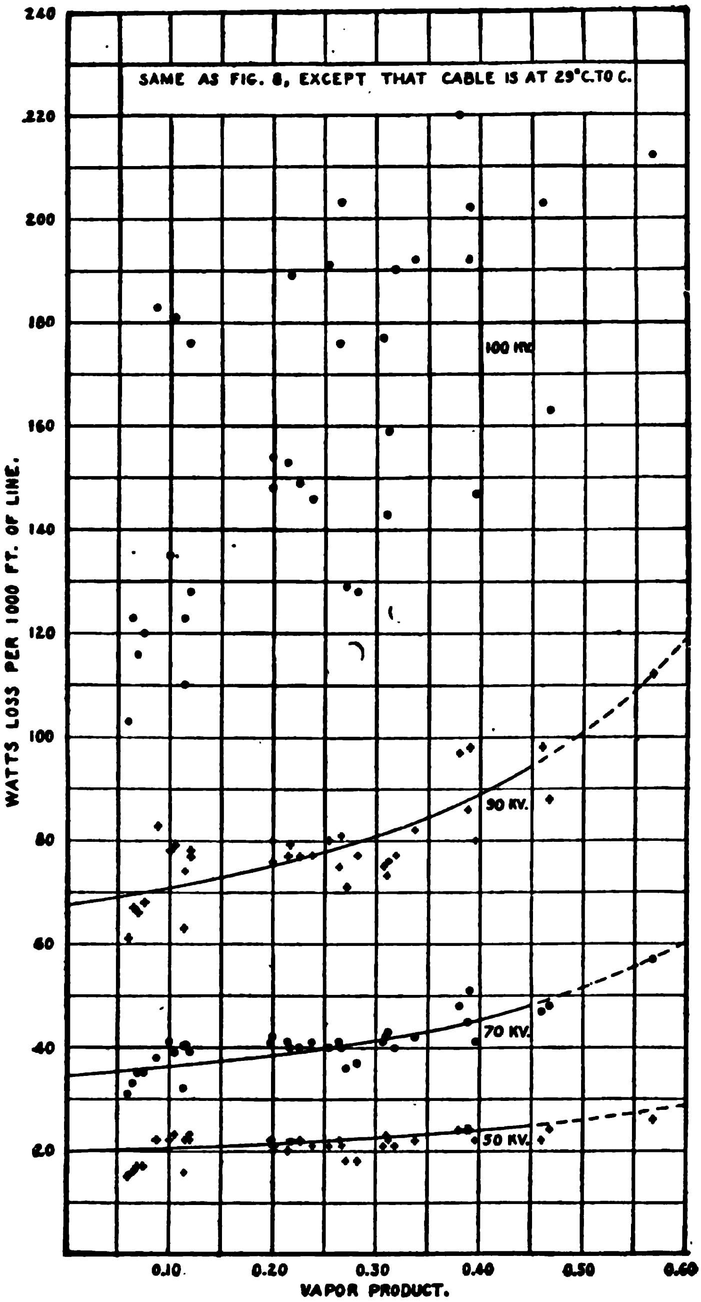

SAME AS FIG. 8, EXCEPT THAT CABLE IS AT 29°C. TO C.
240
220
200
180
160
140
120
100
80
60
40
20
WATTS LOSS PER 1000 FT. OF LINE.
100 KV.
90 KV.
70 KV.
50 KV.
0.10
0.20
0.30
0.40
0.50
0.60
VAPOR PRODUCT.

FIG. 9.

sensitive to any change in the conditions affecting it. The erratic points were almost always obtained either when there was a considerable amount of smoke apparent near the experimental line or when the air temperature was within a few degrees of the dew point, or when both conditions obtained. As both these conditions are unusual ones, the points far from the curve were given less weight in locating the curve. It will be noted also that for high values of vapor product the points are few and somewhat wavering. This part of the curve is not well located by reason of the fact that the vapor product seldom rose above 0.50, and usually was much below this value, so that few opportunities offered for measurements at high values of vapor product. In fact, the average vapor product for Niagara Falls seems to be about 0.20.

There was another condition which tended to introduce an error in the points of these curves. The humidity measurements were made near the ground and at only one point in the line; that is, at one end. They did not, therefore, necessarily represent the *average* condition of the whole line. It would have been better if humidity measurements had been taken simultaneously at two or more points along the line and near to the conductors, but our facilities would not admit of this without unduly prolonging the time and expense of the work.

Vapor product as made use of herein is the value obtained by multiplying together the vapor pressure (in inches of mercury) and the fraction representing the relative humidity (ratio of the vapor pressure at the existing temperature to the pressure of saturation at the same temperature).

In the other figures showing the relation between loss and vapor products, the points themselves have been omitted.

In general, throughout this paper, the points representing the actual readings are omitted, except in cases where it is thought advisable to show the points in order to give an idea as to the accuracy of measurements.

Unless otherwise stated, all values of voltage referred to herein are effective, or square root of mean square, values.

This relation between loss and vapor product is made use of to reduce to the same basis all the loss curves not taken under identical weather conditions and between which comparison is desirable. The value 0.20 was decided upon as being as nearly as could be judged the average value of vapor product which obtained at Niagara Falls, and the curves for comparison, except such as were taken under identical atmospheric conditions (in which case, in general, no correction is made) have been corrected to correspond to that value. On each of the curve sheets is noted either the fact that it has been so corrected, or else the atmospheric conditions under which the curves were actually taken.

As an example of the closeness with which the points fall upon the curves connecting loss and voltage, reference may be made to Figs. 14 and 22.

Figs. 10, 11, 12, 13, 14 and 15 are curves showing for different

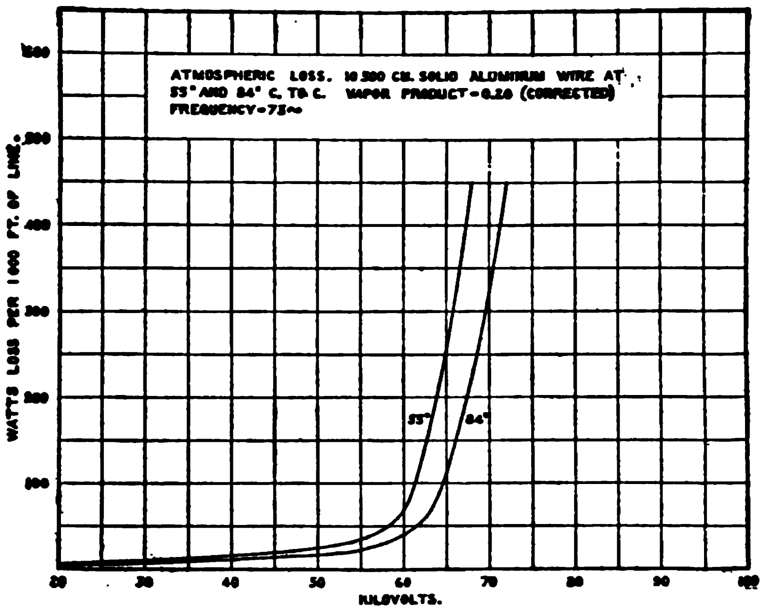


FIG. 10.

sizes of conductors, the variation in loss as the distance between the conductors is varied. In Fig. 14 is also shown the charging current for the 103,850 cir. mil conductor at the various distances.

Fig. 16, 17 and 18 are curves showing the losses for different conductors at the same distances.

Figs. 19, 20 and 21 show the losses for conductors having approximately the same area of cross-section, but different strandings.

Fig. 22 shows the loss in conductors at different distances from the ground.

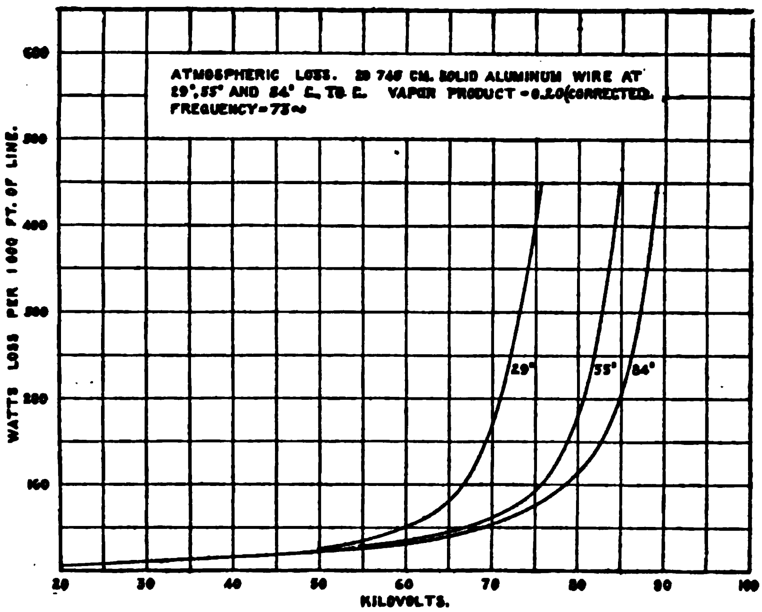

ATMOSPHERIC LOSS. CM. SOLID ALUMINUM WIRE AT
29°, 55° AND 84° C. TO C. VAPOR PRODUCT - 0.20 (CORRECTED.
FREQUENCY - 73 ~
WATTS LOSS PER 1000 FT. OF LINE.
KILOVOLTS.
29°
55°
84°
20
30
40
50
60
70
80
90
100

FIG. 11.

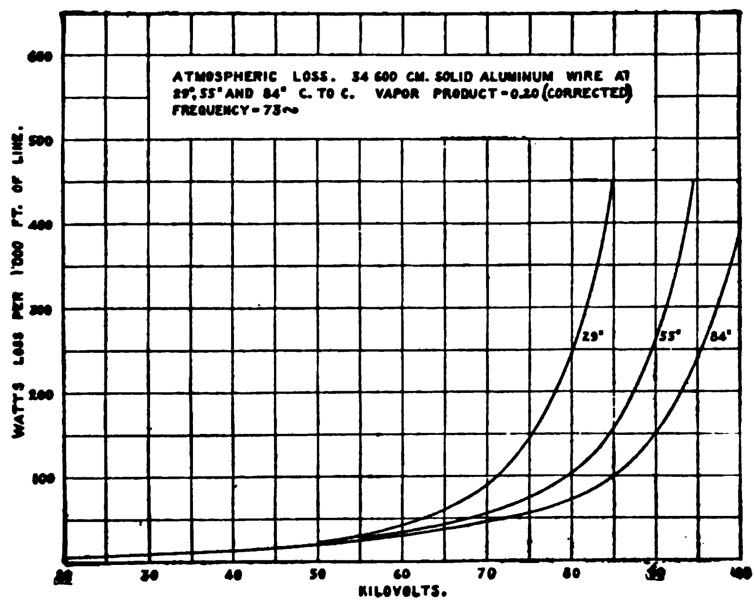

ATMOSPHERIC LOSS. 34 600 CM. SOLID ALUMINUM WIRE AT
29°, 55° AND 84° C. TO C. VAPOR PRODUCT - 0.20 (CORRECTED)
FREQUENCY - 73 ~
WATTS LOSS PER 1000 FT. OF LINE.
KILOVOLTS.
29°
55°
84°
600
500
400
300
200
100
30
40
50
60
70
80
100

FIG. 12

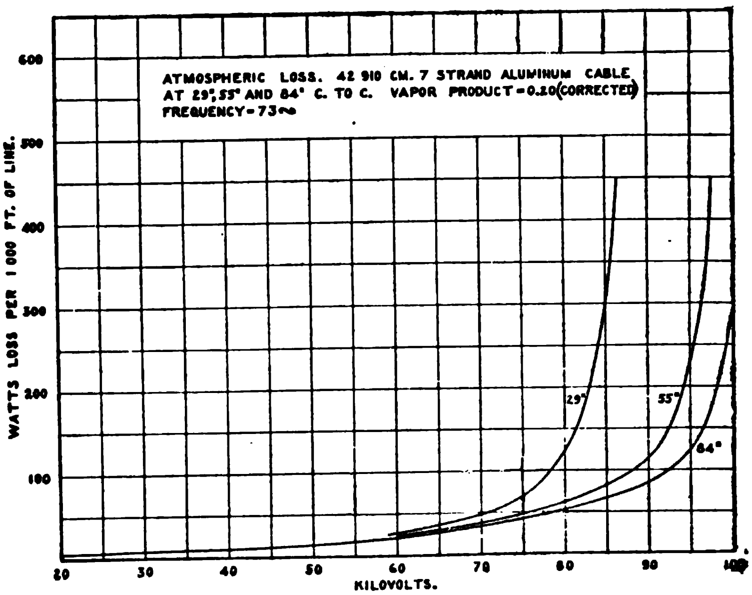

ATMOSPHERIC LOSS. 42 910 CM. 7 STRAND ALUMINUM CABLE AT 29°, 55° AND 84° C. TO C. VAPOR PRODUCT = 0.20 (CORRECTED) FREQUENCY = 73~
WATTS LOSS PER 1 000 FT. OF LINE.
600
500
400
300
200
100
29°
55°
84°
20
30
40
50
60
70
80
90
100
KILOVOLTS.

FIG. 13.

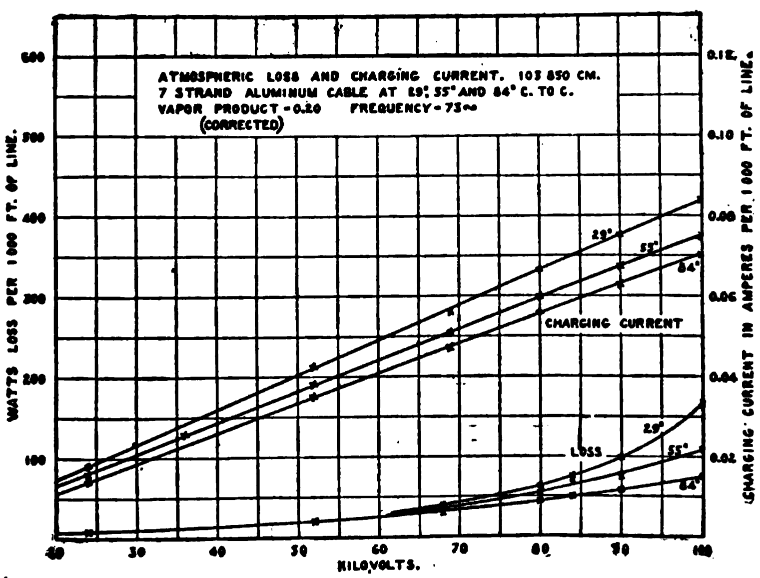

ATMOSPHERIC LOSS AND CHARGING CURRENT. 103 850 CM. 7 STRAND ALUMINUM CABLE AT 29°, 55° AND 84° C. TO C. VAPOR PRODUCT = 0.20 FREQUENCY = 73~ (CORRECTED)
WATTS LOSS PER 1 000 FT. OF LINE.
600
500
400
300
200
100
CHARGING CURRENT IN AMPERES PER 1 000 FT. OF LINE.
0.12
0.10
0.08
0.06
0.04
0.02
29°
55°
84°
CHARGING CURRENT
LOSS
20
30
40
50
60
70
80
90
100
KILOVOLTS.

FIG. 14.

Figs. 23 and 24 show the variation in loss with frequency. These curves are probably in error, as will be pointed out later on. The oscillograph curves in Fig. 25 show the voltage waves at the corresponding points marked X in Fig. 23.

Fig. 26 and 27 show the difference in loss between hard and soft aluminum and between soft aluminum and hard copper.

Measurements on soft aluminum cable, weathered and unweathered, showed the loss to be practically the same in both cases.

In addition to the measurements which were taken bearing

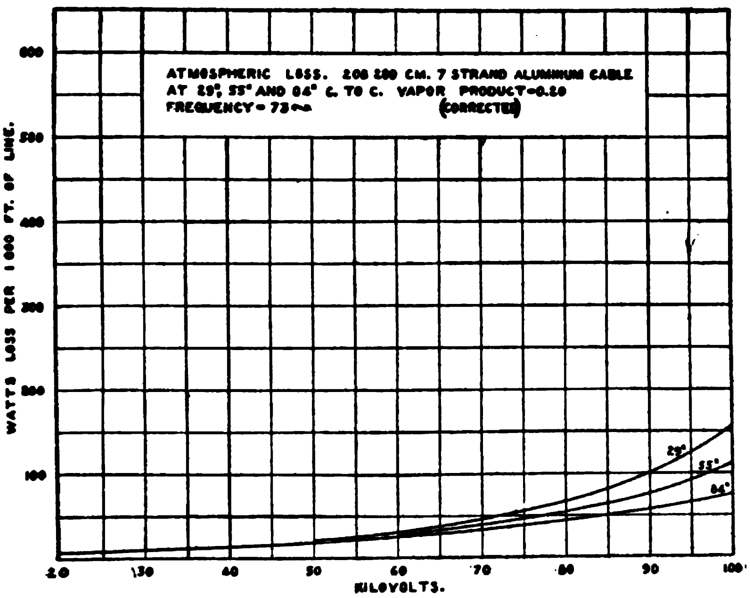


FIG. 15.

on the loss between line wires, an extended study was made on insulators. It was found that the loss over the insulators varied with the weather conditions in the same way as the loss between the line conductors, namely, that there seemed to be a relation between the loss and the vapor product. In the case of the insulators, however, the measurements coincided much more closely with the line embodying the law of variation, probably due to the fact that, the insulators being tested near the ground, there was less chance for smoke in the air to affect them than in the case of the cables some distance above the

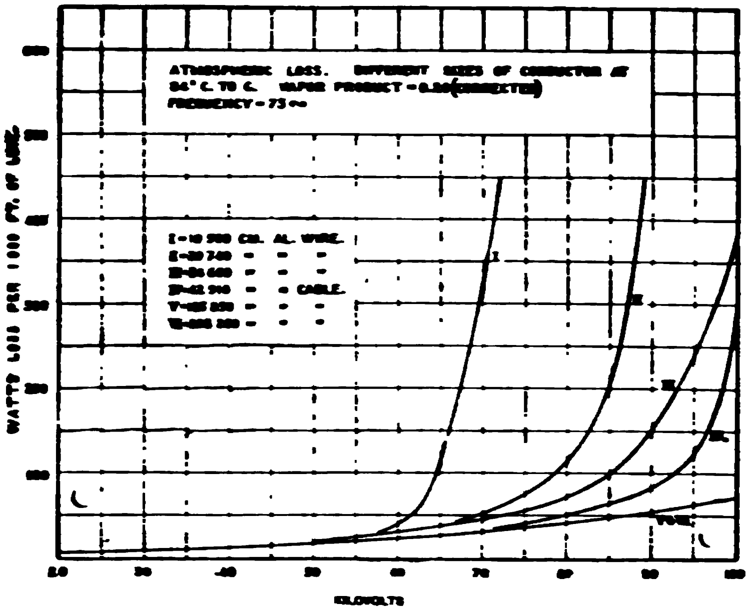
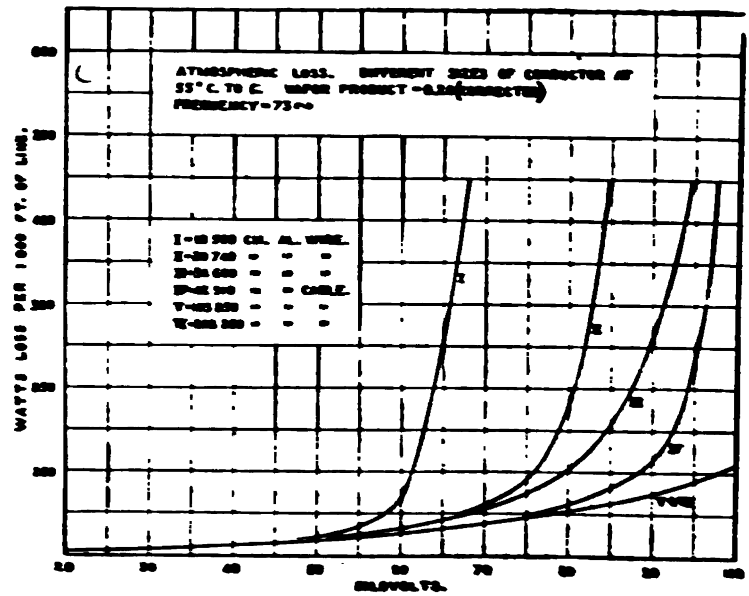

WATTS LOSS PER 1000 FT. OF LINE.

FIG. 16


WATTS LOSS PER 1000 FT. OF LINE.
KILOVOLTS.

ground. This was also due possibly to the fact that the humidity measurements were made beside the insulatoıs, whereas, as previously mentioned, the humidity measurements for the line were made only at one end of the line and near the ground instead of near the line cables. When voltage was first applied to the insulatcrs, thɜ loss was irregular and it was necessary to keep voltage impressed upon them for sometime until the loss steadied down and an accurate reading could be taken. The readings were all taken on five insulators in parallel, the voltage being applied between the necks of the insulators and the metal

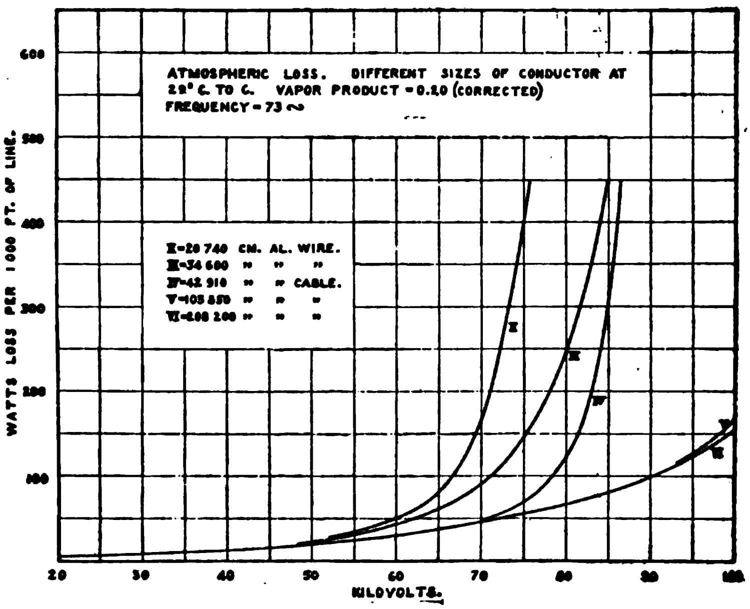


Fig. 18

pins, although the curves are plotted for the loss over a single insulator. The curves çonnecting loss over insulators and vapor product are not carried above a value of the latter quantity equal to 0.20 for the reason that readings could not be obtained above this point without the expenditure of a great deal of time.

Figs. 28 and 29 show for the various insulators, A, B, and C, the relation between the loss and the vapor product. Fig. 28 is a target diagram showing all the points on the curves. In Fig. 29, the points have been omitted. The insulators are designated by letters which correspond to those of the dimension photographs of the insulators previously given in Fig. 3.

The losses over insulators of different sizes were quite different, as is shown by Fig. 30. These go only to 50 kilovolts as it was not safe to go higher on the smaller insulators.

Fig. 31 shows the loss over the larger insulators, up to 100 kilovolts, when there was fog and drizzling rain, as compared with the loss under ordinary conditions.

Fig. 32 shows the loss over an insulator with a metal pin,

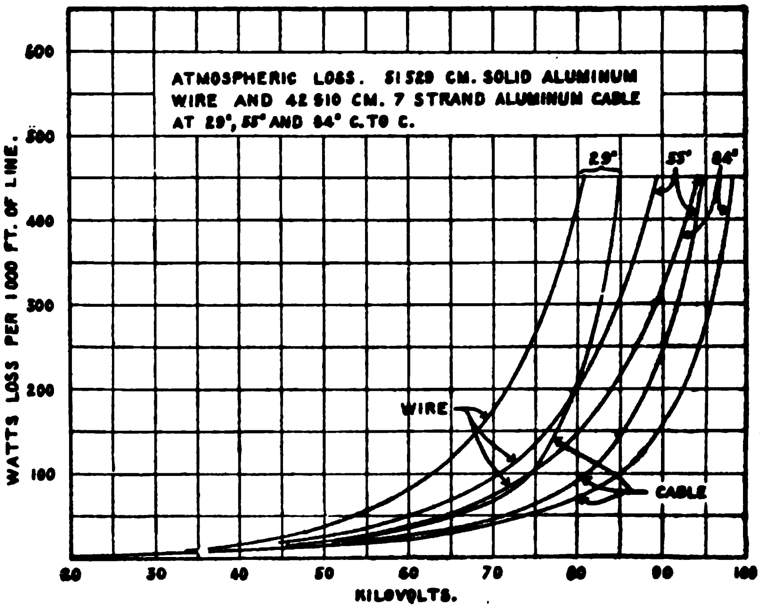


FIG. 18

over the same insulator with a metal pin and a metal plate in contact with the lower petticoat of the insulator, and over the insulator with the metal plate but without the pin, the various arrangements of the insulator being shown in Fig. 33. It will be noted from these loss curves that while the losses over the insulator with the metal pin only and with the metal pin and plate were the same, the loss over the insulator with the metal plate only was considerably less than in the other two cases. This curve was taken as having a bearing on the loss over different size insulators, as will be explained later.

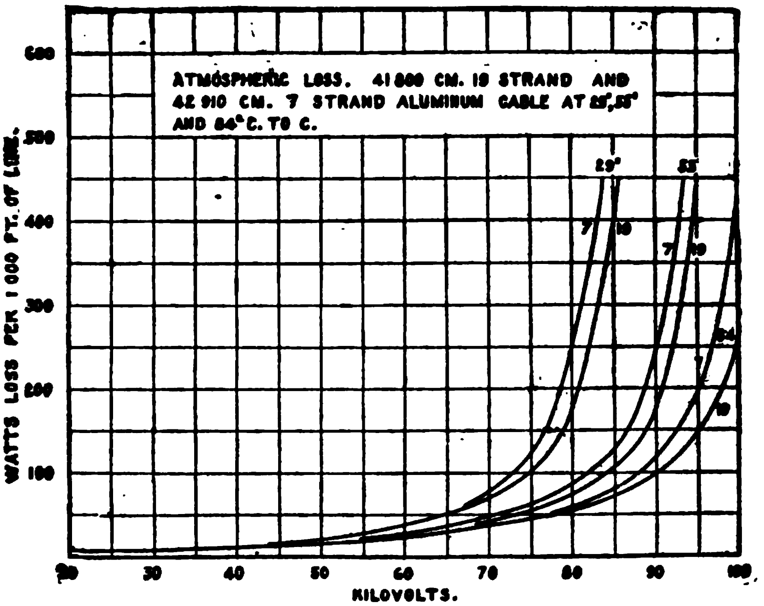

ATMOSPHERIC LOSS. 41 800 CM. 19 STRAND AND
42 910 CM. 7 STRAND ALUMINUM CABLE AT 25°, 55°
AND 84° C. TO C.
WATTS LOSS PER 1 000 FT. OF LINE.
600
500
400
300
200
100
25°
55°
7
19
84
20
30
40
50
60
70
80
90
100
KILOVOLTS.

FIG. 20.

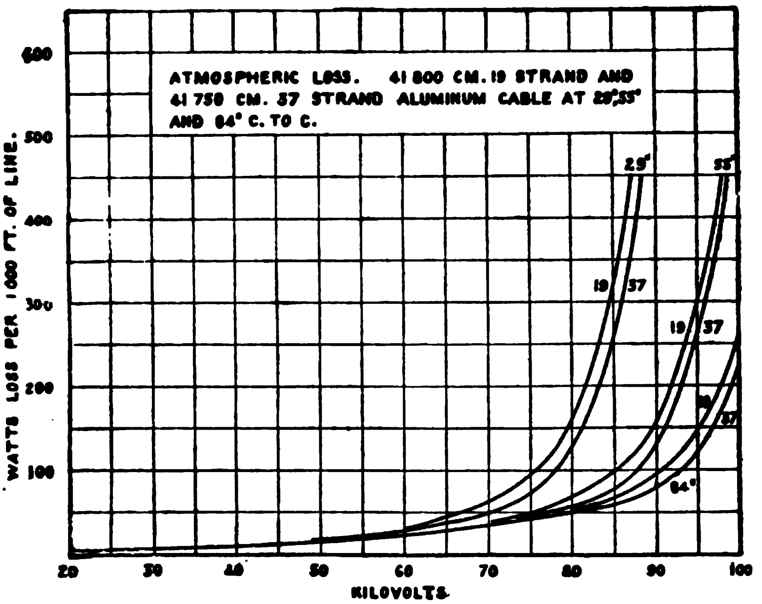

ATMOSPHERIC LOSS. 41 800 CM. 19 STRAND AND
41 750 CM. 37 STRAND ALUMINUM CABLE AT 25°, 55°
AND 84° C. TO C.
WATTS LOSS PER 1 000 FT. OF LINE.
600
500
400
300
200
100
25°
55°
19
37
84°
20
30
40
50
60
70
80
90
100
KILOVOLTS.

FIG. 21.

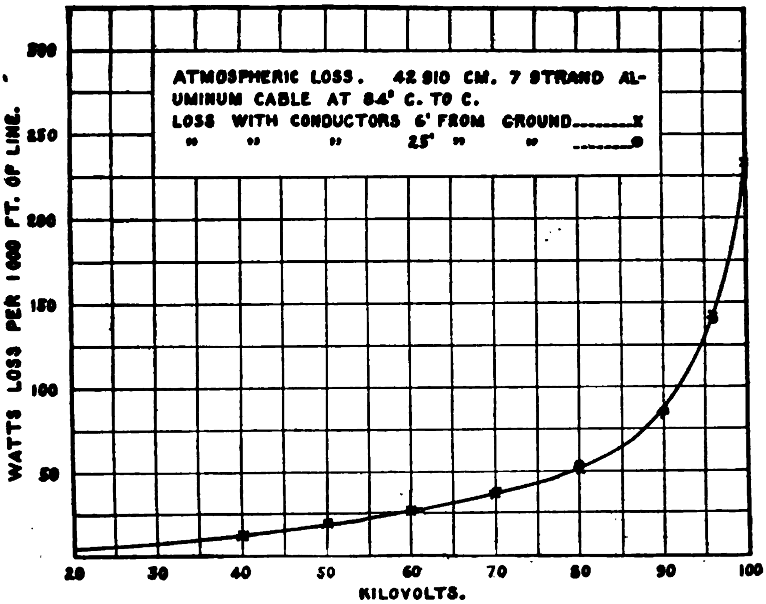

ATMOSPHERIC LOSS. 42 910 CM. 7 STRAND ALUMINUM CABLE AT 24° C. TO C.
LOSS WITH CONDUCTORS 6' FROM GROUND x
" " " 25' " " ....●
WATTS LOSS PER 1 000 FT. OF LINE.
KILOVOLTS.

FIG. 22.

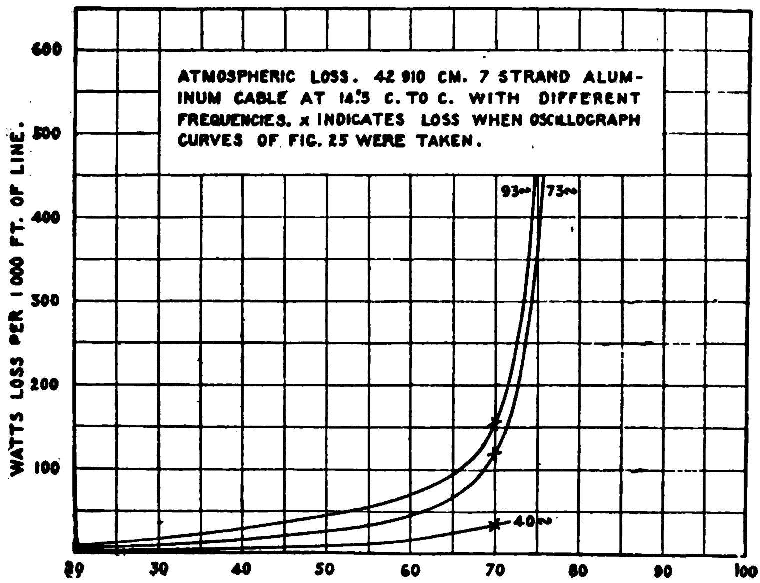

ATMOSPHERIC LOSS. 42 910 CM. 7 STRAND ALUMINUM CABLE AT 14.5 C. TO C. WITH DIFFERENT FREQUENCIES. x INDICATES LOSS WHEN OSCILLOGRAPH CURVES OF FIG. 25 WERE TAKEN.
93~
73~
40~
WATTS LOSS PER 1 000 FT. OF LINE.
KILOVOLTS.

FIG. 23.

Fig. 34 shows the variation in loss over insulators as the frequency was varied, but, as in the case of the corresponding curves for the line conductors, this curve is open to question as will be explained later. The oscillograph curves in Fig. 35 show the voltage waves of the corresponding points, marked X, of Fig. 34.

Fig. 36 shows the loss obtained on insulators with metal and wooden pins. It will be noted in this case that the loss with the wooden pin was greater. In order to demonstrate whether an explanation hit upon for this fact was correct, one of the

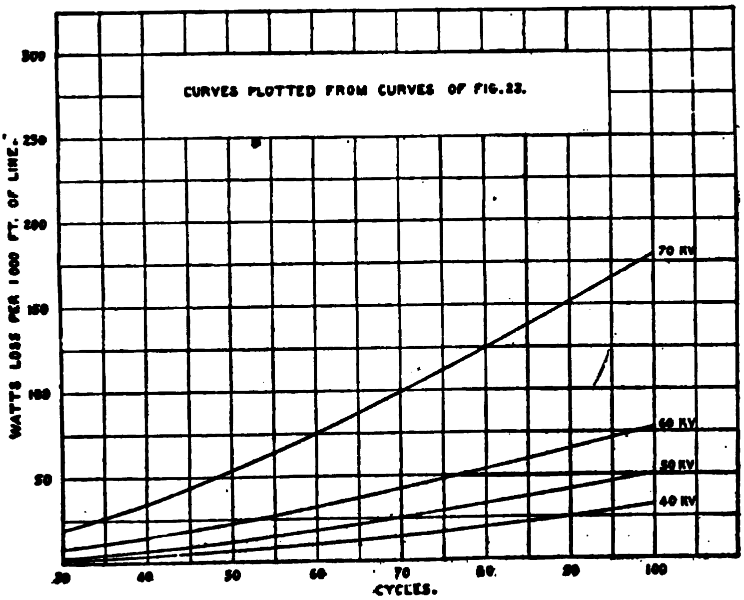


FIG. 34.

insulators, B, was mounted upon a curtain rod, which served as a long wooden pin, and the loss measured over the insulator with different lengths of pin by placing the lower terminal on the curtain rod at different distances from the bottom petticoat of the insulator. Under these conditions, with the rod wet, the curves of Fig. 37 were obtained. It was necessary to wet the wooden pin for the reason that when it was dry, the point of maximum loss fell inside the insulator, so that the loss curve, instead of beginning at a comparatively low value and rising to a maximum and then falling off again, continually fell off as the length of the pin was increased. An explanation of this will be given later.

An endeavor was made to measure the loss over insulators when it was raining, but the loss was so variable, due to variation in the rate of precipitation and gusts of wind, that no

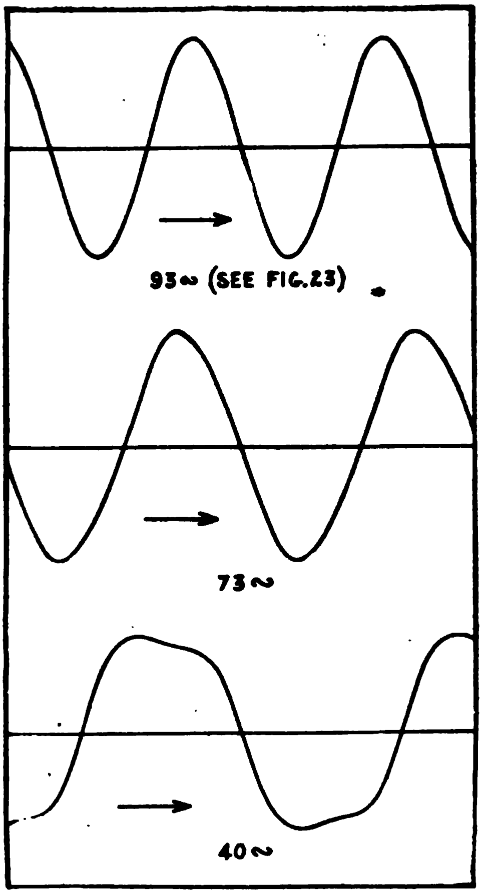


FIG. 25.

satisfactory curve could be obtained. It was determined, however, that the loss with a hard rain was less than in the case of fog and drizzling rain, the curve for which is shown in Fig. 31. The loss over insulators with hoar frost upon them was also less than with fog.

Tests were made on an insulator with increasing and decreasing voltage to see whether or not this made any difference in the loss obtained. It was found that if a high voltage were first applied to the insulator there was no difference in the loss curve with ascending and descending voltage.

Measurements were made on a set of insulators, the metal pins of which were not cemented in the insulators, and on another set, the metal pins of which were cemented, to see if any difference would be found. As long as the cement was wet, there was a difference, but after the cement had set and become dry, there

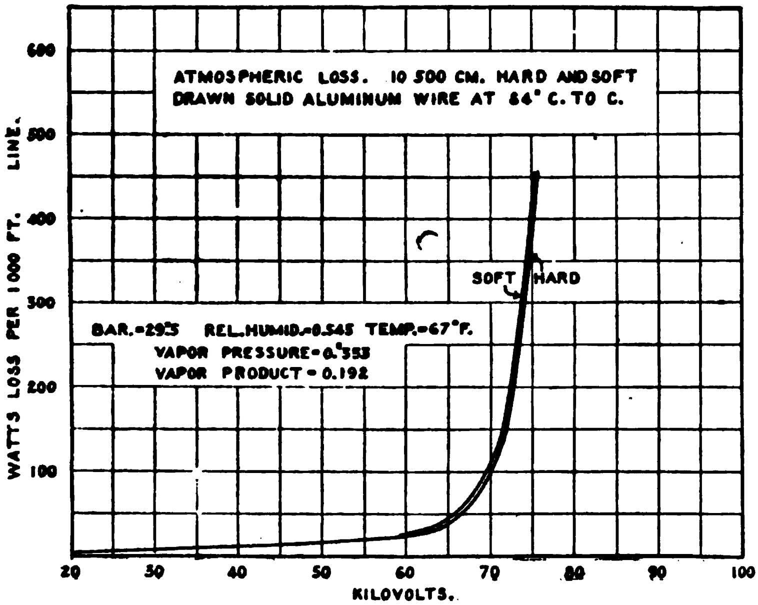


FIG. 26.

was no difference. This being the case, all the subsequent tests were made with the pins uncemented.

A number of readings were taken on broken insulators. These readings were taken by first obtaining a loss reading on the insulator intact, and then gradually breaking the petticoats off, beginning with the inner one. The loss had not materially changed with the two inner petticoats broken away, and the upper petticoat still intact, but immediately ran up when the upper petticoat was broken. These measurements were all made on dry insulators.

Discussion of Results

At Telluride, the measurements were all taken on lines carried upon insulators; measurements were also made on a dummy line having the same number of insulators and cross-arms as the main line, and the atmospheric losses arrived at by subtraction. By this method concordant results were obtained, although such method of measurement did not produce satisfactory results at Niagara. The probable reason for this will be explained later on. The Telluride results apparently demonstrated certain facts and justified certain conclusions which were

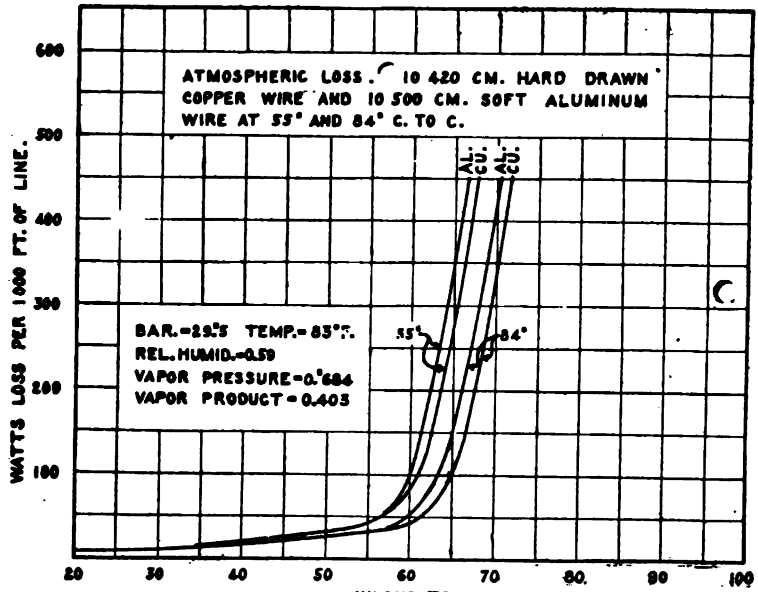


FIG. 27.

brought out in my report on the Telluride work. As the result of the Niagara work, some of these conclusions have had to be abandoned. In order properly to discuss the subject, it is thought advisable to enumerate here the conclusions arrived at as the result of the Telluride work. They are as follows:

(*a*.) That there is a certain critical point in the curve connecting loss and voltage at which the loss begins to increase very rapidly.

(*b*.) That the loss below the critical point is made up entirely of loss over the insulators; and that the loss above the critical point is made up of insulator loss plus a loss through the atmosphere.

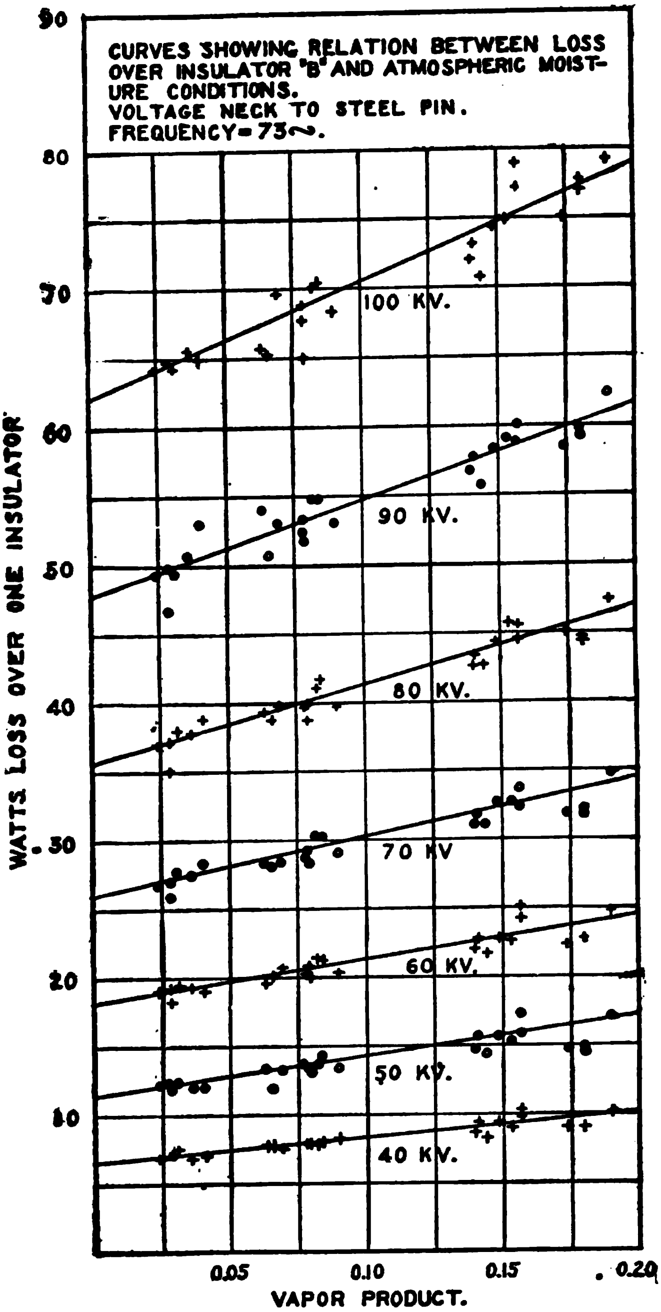
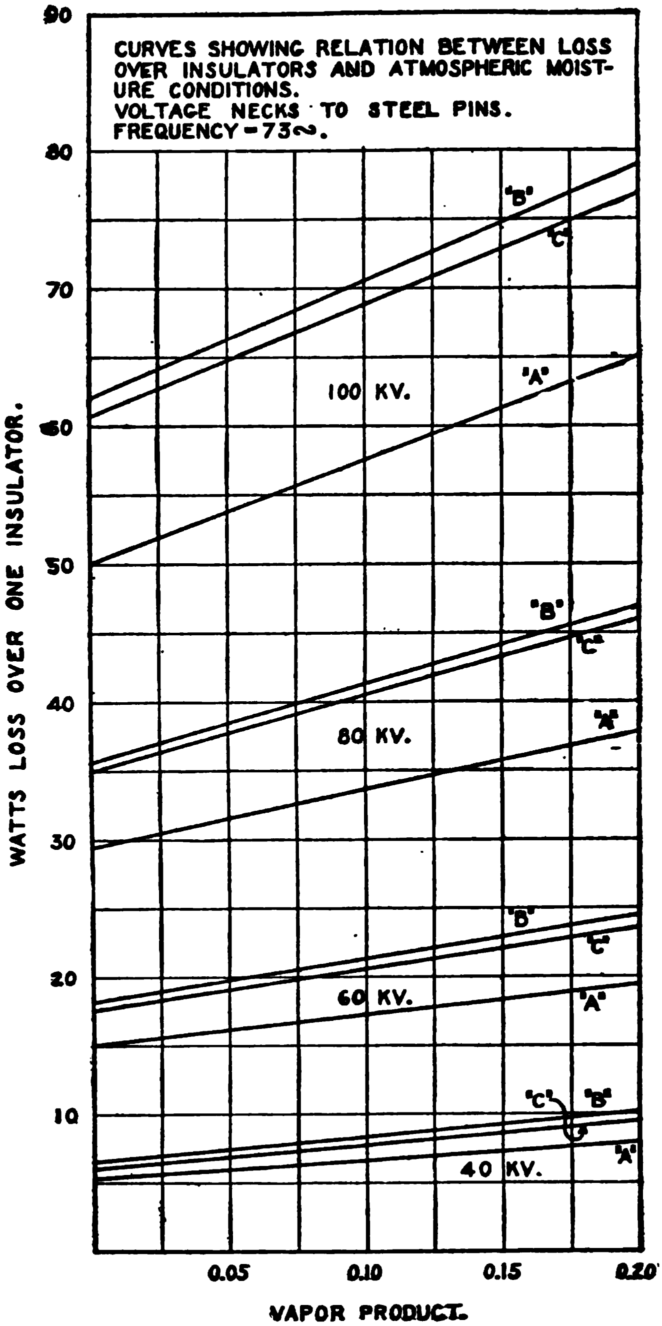

CURVES SHOWING RELATION BETWEEN LOSS OVER INSULATOR "B" AND ATMOSPHERIC MOISTURE CONDITIONS.
VOLTAGE NECK TO STEEL PIN.
FREQUENCY = 73∼.
100 KV.
90 KV.
80 KV.
70 KV
60 KV.
50 KV.
40 KV.
WATTS LOSS OVER ONE INSULATOR
10
20
30
40
50
60
70
80
90
0.05
0.10
0.15
0.20
VAPOR PRODUCT.

FIG. 28.



CURVES SHOWING RELATION BETWEEN LOSS OVER INSULATORS AND ATMOSPHERIC MOISTURE CONDITIONS.
VOLTAGE NECKS TO STEEL PINS.
FREQUENCY = 73~.
WATTS LOSS OVER ONE INSULATOR.
90
80
70
60
50
40
30
20
10
100 KV.
80 KV.
60 KV.
40 KV.
"A"
"B"
"C"
0.05
0.10
0.15
0.20
VAPOR PRODUCT.

FIG. 29.

(*c*.) That the critical point depends upon the maximum value of the electromotive force wave and the distance between conductors. (Facilities were lacking for measurements on different sizes of conductors.)

(*d*.) That the critical point, and therefore, the beginning of atmospheric loss coincides with the voltage at which luminosity and hissing (also probably the formation of atmospheric chemical products) begin, and to a partial breakdown of the dielectric.*

(*e*.) That under conditions which obtained at Telluride, weather conditions made no difference in the loss or the critical point, except when there was actual precipitation.

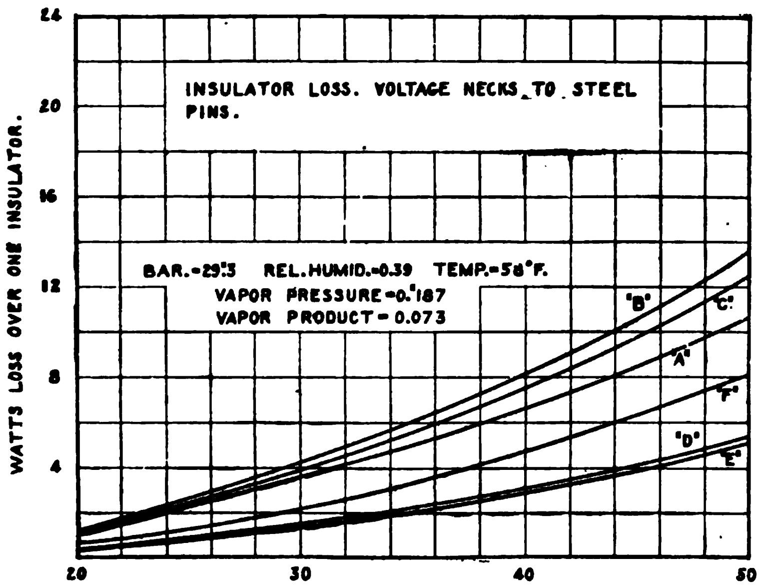


FIG. 30.

(*f*.) That it seemed probable in view of (*e*) and of the range of weather conditions observed at Telluride, that weather conditions (except precipitation) would not affect loss or critical point, except perhaps in case of dense fog, or a considerable amount of foreign matter in the atmosphere.

(*g*.) That the critical point is at a lower voltage with a roughened conductor than with a smooth conductor.

*Professor Ryan gives Mr. Scott credit for this observation, but it was first brought out in my Telluride report, as reference to Mr. Scott's paper will show.

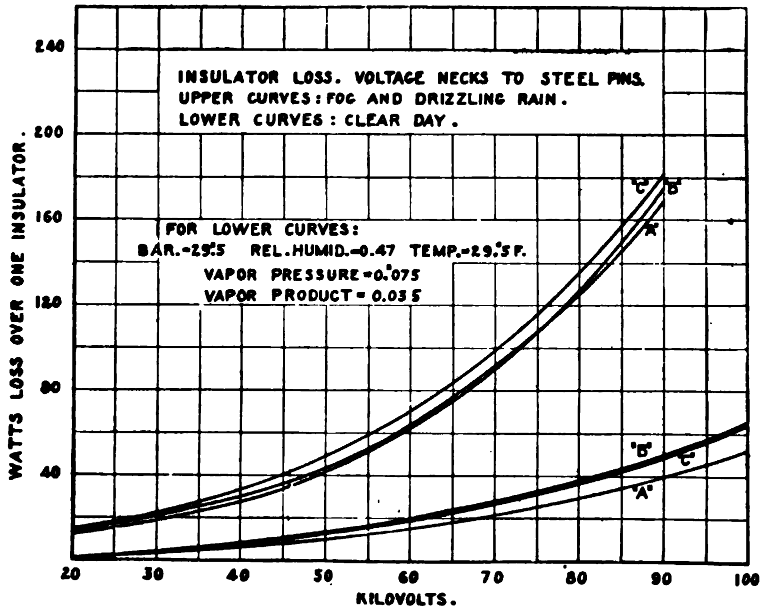

INSULATOR LOSS. VOLTAGE NECKS TO STEEL PINS.
UPPER CURVES: FOG AND DRIZZLING RAIN.
LOWER CURVES: CLEAR DAY.
FOR LOWER CURVES:
BAR.-29.5 REL.HUMID.-0.47 TEMP.-29.5 F.
VAPOR PRESSURE-0.075
VAPOR PRODUCT-0.035
'C' 'B' 'A'
'B' 'C' 'A'
WATTS LOSS OVER ONE INSULATOR.
240
200
160
120
80
40
20 30 40 50 60 70 80 90 100
KILOVOLTS.

FIG. 31.

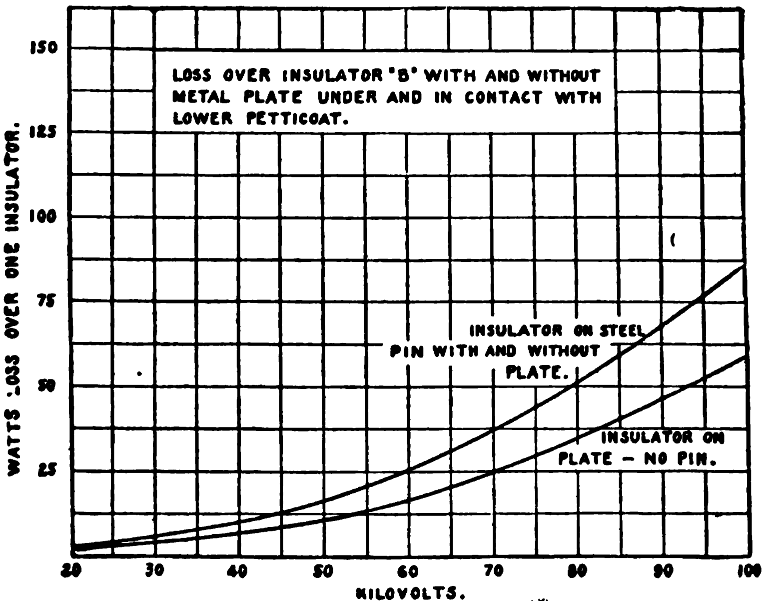

LOSS OVER INSULATOR 'B' WITH AND WITHOUT
METAL PLATE UNDER AND IN CONTACT WITH
LOWER PETTICOAT.
INSULATOR ON STEEL
PIN WITH AND WITHOUT
PLATE.
INSULATOR ON
PLATE - NO PIN.
WATTS LOSS OVER ONE INSULATOR.
150
125
100
75
50
25
20 30 40 50 60 70 80 90 100
KILOVOLTS.

FIG. 32.

(*h*.) Measurements taken at different frequencies showed that the losses were lower and the critical point higher at the lower frequencies, but these results were viewed with suspicion for a

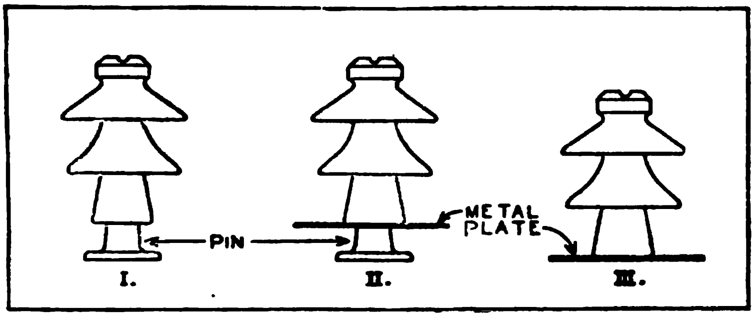


FIG. 33.

number of reasons, amongst which was the fact that at the lower frequency the wave-form became distorted (flattened) in a manner to account for part, if not all, of the decrease in loss.

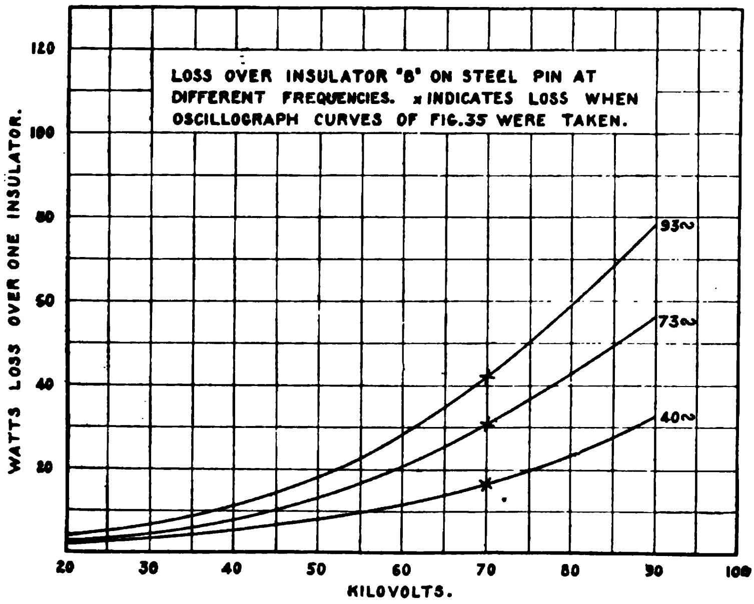


FIG. 34.

Professor Ryan's results, obtained subsequently, confirmed (*a*), (*b*), (*c*), and (*d*). They also confirmed (*f*) so far as it relates to the presence of water vapor, but not so far as it relates to the barometric pressure and temperature. In addition, as the result of his measurements, he arrived at the following conclusions.

(*i.*) That the critical point depends upon the density of the atmosphere, the critical voltage being greater as the atmospheric density increases, the relation being a straight line one.

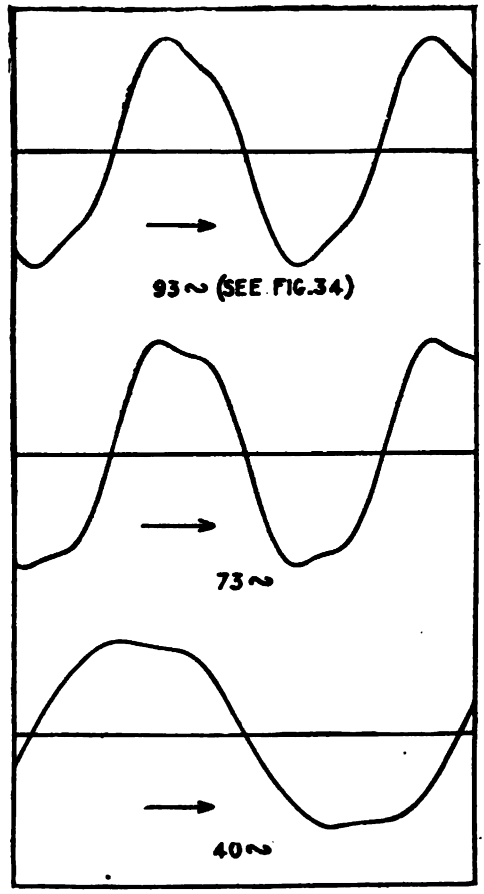


FIG. 35.

(*j.*) That (as had been previously suspected but not definitely known) the critical voltage depends upon the rate of fall of potential, or dielectric stress, near the conductor; that is, for a given atmospheric density, the partial or local breakdown of the dielectric occurs when a certain local rate of fall of potential or dielectric stress, has been reached.

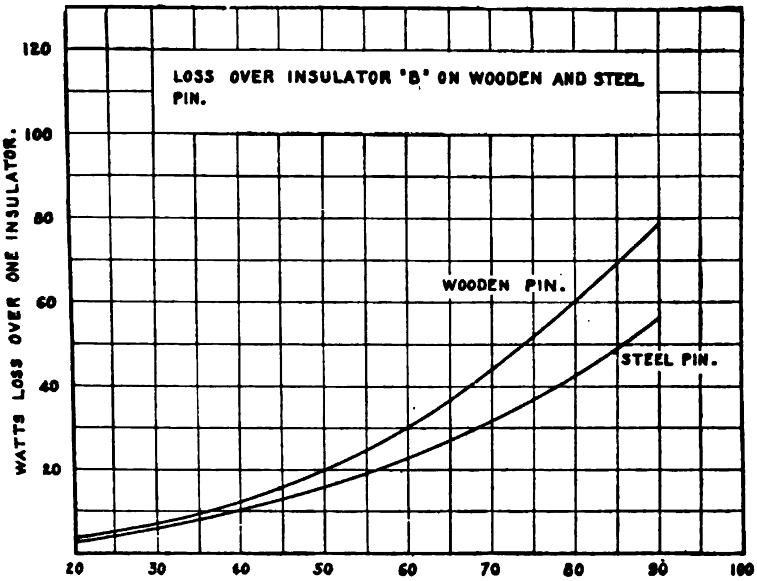

LOSS OVER INSULATOR 'B' ON WOODEN AND STEEL PIN.
WOODEN PIN.
STEEL PIN.
WATTS LOSS OVER ONE INSULATOR.
120
100
80
60
40
20
20
30
40
50
60
70
80
90
100
KILOVOLTS.

FIG. 36.

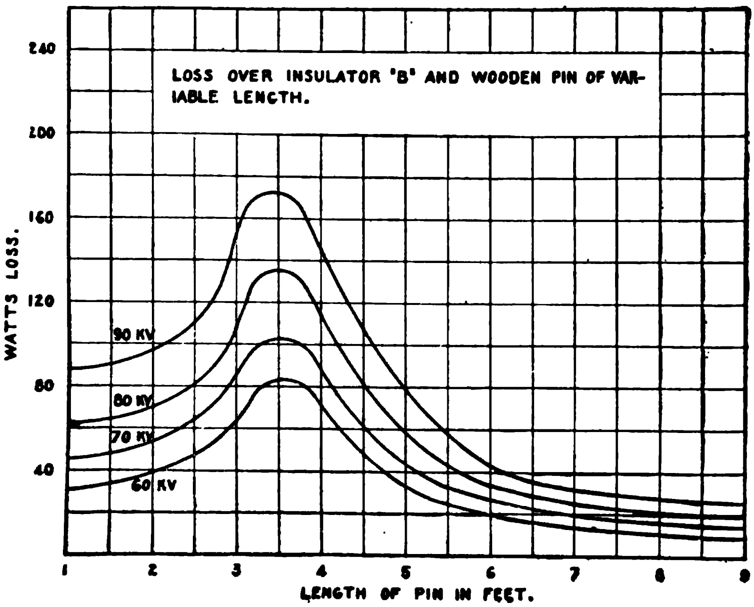

LOSS OVER INSULATOR 'B' AND WOODEN PIN OF VARIABLE LENGTH.
90 KV
80 KV
70 KV
60 KV
WATTS LOSS.
240
200
160
120
80
40
1
2
3
4
5
6
7
8
9
LENGTH OF PIN IN FEET.

FIG. 37.

(*k*.) That the value of the local rate of fall of potential or dielectric stress, D', at which the partial or local breakdown occurs, depends upon the diameter of the conductor being constant for all diameters above 0.25 in., and increasing as the diameter of the conductor decreases below this value.

(*l*.) That the partial breakdown of the dielectric does not occur at the surfaces of the conductors (the point where presumably the rate of fall of potential is greatest) but at a small distance (d) from the surface of the conductor which distance depends upon the diameter of the conductor being constant for all diameters above 0.25 in., but diminishing with the diameter of the conductor below this value.

(*m*.) Ryan combined his results in the following formula for determining the critical point:

$$E_{max} = \frac{17.94\, b}{459 + t} \times 2055 \log_{10} \frac{s}{r} \times D' \times (r+d) \times 10^{10}$$

where

E_{max} = maximum value of the electromotive force wave.
b = barometer in inches.
t = temperature in degrees fahrenheit.
s = distance between centers of conductors in inches.
r = radius of conductors in inches.
D' = dielectric flux density (proportional to local rate of fall of potential).
d = a small distance from the surface of the conductor.

D' and d are not constant. Their values are given by Fig. 45.

(*n*.) That the presence of smoke in the atmosphere causes a loss at all voltages and under conditions which, when the smoke is not present, show no loss.

Items (*a*), (*c*), (*d*), (*e*) and (*g*) of the Telluride results and (*n*) of Ryan's results have all been confirmed by the measurements at Niagara.

The results obtained at Niagara do not, however, agree with (*b*). As is seen by reference to any of the loss curves herein applying to the conductors, there is a loss below the critical point not chargeable to the insulators. This non-agreement may be accounted for in either or both of two ways. In the first place, there was more or less smoke and dust in the atmosphere at Niagara, whereas the atmosphere at Telluride was probably as nearly absolutely free from floating particles as is possible under natural conditions. In the next place, the humidity conditions at Telluride were such as to make the losses due to them very low indeed. It follows, therefore, that the conditions at Telluride were such that the losses below the critical point, other than those due to the insulators, would be very low, and would probably be masked by the insulator losses themselves; whereas at Niagara the conditions were such as to produce appreciable atmospheric losses below the critical point. And, besides, as in the Niagara tests all insulator losses were eliminated, it was more easily possible to detect atmospheric losses below the critical point.

Item (*f*) was not confirmed at Niagara. In fact, quite the contrary was found to obtain. As has been previously mentioned, the results obtained at Niagara, showing that the loss varied with the atmospheric humidity conditions, were at first thought to discredit the results at Telluride, but when fuller data had been obtained at Niagara, it was found that the weather conditions (and the variations of them) which obtained at Telluride would produce so small a variation in the losses that it would have been difficult, if not impossible, to detect them. In other words, the results at Niagara appear to fully confirm (*e*), but were contrary to (*f*), the deduction which was made from (*e*).

As regards (*h*), the measurements at Niagara seem to show that there will be a variation in the loss as the frequency is varied, although at the time of the Telluride measurements, it was thought probable that frequency would make no difference in the loss in spite of the results which had been obtained showing apparently a variation of loss and critical point with frequency.

Referring to Professor Ryan's results:

As regards (*i*), the variations in atmospheric density at both Telluride and Niagara were too slight to obtain any results bearing on this point. At Niagara such variations in loss or critical point as might have been due to variations in atmospheric density would have easily been masked by variations due to other conditions, especially moisture conditions. The barometer was never lower than 29 in. nor higher than 30 in., the average being about 29.5 in. Since the density of the atmosphere varies as the absolute temperature, the variation in density due to temperature would be comparatively small.

As regards (*j*) *k*), (*l*) and (*m*), the results both at Telluride and Niagara show that there is a relation between distance between conductors and the critical point, and the latter measurements show that there is also a relation between diameters of conductors and the critical point. Both these would have a bearing upon the local rate of fall of potential, or dielectric stress. The results obtained at Niagara do not, however, bear out quantitatively those obtained by Ryan. Neither do they seem to bear out his results, qualitatively, as the diameter of the conductor is varied, though they do seem to agree, qualitatively, with his results for variation of critical point as the distance between conductors is varied.

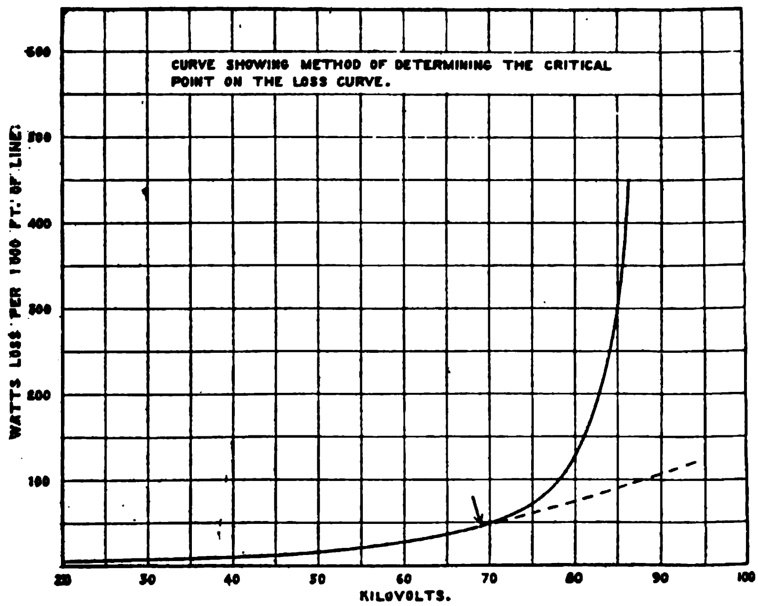


FIG. 38.

A number of the above points are more fully borne upon by what follows:

In Fig. 38 is shown the method of determining the critical point. The lower limb of the loss curve is extended in accordance with what, so far as can be judged, is the law of the lower portion of the curve and the critical point taken as the point where the upper limb leaves the lower limb prolonged. Some writers have determined the critical point by extending the upper limb of the curve down to the horizontal axis. Such course is clearly incorrect.

Fig. 14 shows not only the loss curves for a given conductor at different spacings, but also the corresponding charging current curves. The charging current curves are included mainly to show that there is no change in the charging current due to the critical point. As will be seen from Fig. 14, the line charging current curves are straight lines, and in all the measurements I have ever taken this has been the case, so long as the electromotive force wave has not been distorted. Further, this straight line relation has always held even when the voltage was pushed to a value which took the loss curve very considerably above the critical point.

Figs. 19, 20 and 21 show the comparison of the loss curves of solid and stranded conductors, and of stranded conductors with different numbers of strands. Fig. 19 clearly shows that for a stranded conductor of given circular mil section, the loss is less and the critical point higher than for the corresponding solid conductor. The two conductors compared in this case have not the same circular mil section, the area of the stranded conductor being less than that of the solid conductor; but, even with this disadvantage, the stranded conductor shows a considerable advantage over the solid. Fig. 20 shows a comparison between a 19 strand cable and a 7 strand cable; while Fig. 21 shows a comparison between a 19 strand cable and a 37 strand cable. It is clear from these three curves that not only is a stranded conductor superior to a solid one, so far as loss and critical point are concerned; but, also, that the finer the strands of the stranded conductor, the better, so far as the critical point is concerned. That is to say, the effective diameter of the stranded conductor is greater than for the corresponding solid conductor, and for a given conductive area of cross-section, the greater the number of strands in the stranded conductor, the greater is its effective diameter so far as critical point and loss are concerned. Of course, it is perfectly evident that the actual outside diameter of a stranded conductor is greater than that of the corresponding solid conductor, but there has been some question, heretofore, as to whether the increase of physical diameter would not be offset by the fact that the surface of the stranded conductor is corrugated. These curves, apparently, settle this matter to the effect that the increase of diameter more than offsets any ill effects due to corrugations. As a matter of fact, as will be shown later on, the stranding seems not only to offset the effect of the corrugations but, when the stranding is fine enough, to more than offset them and make the effective diameter of a stranded conductor greater than that of a solid conductor having a diameter equal to the outside diameter of the stranded conductor.

Fig. 22 shows two loss curves, one with conductors 25 feet from the ground and the other with conductors 6 feet from the ground; the size and spacing of the conductors being the same in the two cases. These measurements were taken to determine whether variation in the distance from the earth had any effect on the loss or critical point. They show that such variation has apparently no effect.

Fig. 23 shows measurements taken on a pair of conductors at different frequencies. Apparently the loss changes with the frequency, but the extent of this change as shown by the figure is open to question, as will be seen on referring to Fig. 25 which shows the wave forms corresponding to the 70 kilovolt points of the curves of Fig. 23. The waves for the 93-cycle curve and for the 73-cycle curve are practically identical, but that for the 40-cycle curve is considerably flattened. The 40-cycle loss curve is, therefore, undoubtedly a great deal lower than it would be if the wave form had not been distorted so that the result obtained for 40 cycles is not comparable to those obtained for 73 cycles and 93 cycles respectively. The curves for the latter two frequencies are, however, comparable and seem to show that the loss is less the lower the frequency. I hope, later on, to obtain measurements at different frequencies under conditions which will obviate distortion of the wave form.

Similar remarks will apply to the variation of insulator loss with frequency as shown in Fig. 34, to which Fig. 35, showing the wave forms, applies. Apparently, there is a decrease in insulator loss with frequency, but it is evident that if the wave form were not distorted, the change in loss with frequency would not be as great as is shown by Fig. 34.

Fig. 24 is derived from Fig. 23. In viewing this figure what has been said in regard to distortion of wave form should be borne in mind. It seems evident that if the wave form were not distorted at 40 cycles, the curves shown in Fig. 24 would be higher at the lower frequencies than there shown.

A reference to Figs. 26 and 27 shows that hard drawn wire is somewhat superior to soft drawn wire, because of its smoother surface, but that the difference is not great. These figures, also, show that there is practically no difference between copper and aluminum wire in the matter of loss and critical point. There may be mentioned in this connection a point which has already been referred to, that there is no difference between weathered and unweathered wire as regards loss and critical point.

The results obtained in measuring losses with insulators on wooden and steel pins, as shown by Fig. 36, were rather a surprise. On thinking the matter over, I came to the conclusion that this effect is due to the fact that a large part of the current taken by the insulator is charging current; that is, a current in quadrature with the electromotive force, and that mounting the insulator on a wooden pin is equivalent to inserting a resistance in series with a condenser. As is well known, the value of such a resistance can be considerable, before the voltage impressed upon the condenser is materially reduced, because of the fact that the voltages taken by the resistance and the condenser, respectively, are in quadrature. This being the case, it would be possible to have a considerable resistance, and therefore loss, in the pin, without materially reducing the current taken by the insulator. This would mean considerably more insulator loss with a wooden pin than with a steel pin. In order to test this out, the measurements of Fig. 37 were taken, showing the relation between the loss and the length of the wooden insulator pin. These curves clearly show the correctness of the above explanation since, in the case of a resistance and a capacity in series and a constant voltage impressed upon the combination, the loss would increase as the resistance is increased up to a certain maximum and would thereafter decrease. These measurements go a long way towards explaining some of the difficulties experienced in the burning and deterioration of wooden pins.

In order to get some further light on this matter of charging current in the insulators, the measurements of Fig. 32 were taken. The conditions under which these measurements were taken are shown in Fig. 33. The measurements show that the loss in the case of I and II of Fig. 33 is the same, and greater than III of the same figure. The explanation of this is taken to be that the charging current of the different petticoats of the insulator has to be supplied over the surface film of the insulator and that the greater this charging current is, the greater the loss will be. The charging current would naturally be greater in the case of I and II than in the case of III, since, in addition to the condenser effect between the different petticoats of the insulator, there will be a condenser effect between the different petticoats of the insulator and the metal pin inside them. That this matter of the charging current of the insulator has considerable to do with the insulator losses is also shown by Fig. 30, showing losses on insulators of different sizes. When the measurement of Fig. 30 was taken, it was thought that possibly the loss over an insulator was due, simply, to conductivity through a film on the surface of the insulator. If this were the case, it ought to be possible to compare the losses over different insulators by calculating the relative resistances of their surfaces, assuming the surfaces had the same thickness of conducting film. On making such calculations, however, it was found that the resistances calculated in this way would not account for the different losses. Some of the insulators whose calculated film resistances were highest had also the highest loss; but, in every such case, these insulators were of the type in which the petticoats were more nearly parallel to the metal pin, thus offering a better chance for condenser effects between the petticoats and the pin; whereas, the lower losses, obtained in the case of insulators whose calculated film resistance was lower, were obtained from insulators whose petticoats were more nearly horizontal, that is, whose surfaces rapidly departed from parallelism with the pin, offering less chance for condenser effects.

The relation which has been discovered connecting loss and vapor product is not offered herein as a law, but simply as an empiric relation which seems to exist between the quantities involved and which may be put to practical use in determining what will happen under given atmospheric moisture conditions. As previously explained, the vapor product is a product obtained by multiplying together the vapor pressure exerted by the moisture which exists in space under a given set of conditions, by the fraction representing the relative humidity at that time. The relative humidity is the ratio of the amount of moisture in unit space at a given temperature to the amount of moisture which would be contained in the same space if saturated at the same temperature. This ratio is the same as the ratio of the pressures of the vapor. That is if p = the vapor pressure under any given temperature, t, and if P = the saturated vapor pressure for the same temperature, we have:

$$\text{relative humidity} = \frac{p}{P}$$

or the vapor product as we have defined it is

$$\text{vapor product} = \frac{p^2}{P}$$

The numerator of this expression may be evaluated in terms of the density of the aqueous vapor (*i.e.*, the weight of water vapor in unit space) and the temperature; but the denominator cannot be similarly evaluated. Regnault determined the relation between P and t, but the relation has never been rationalized and, apparently, no satisfactory simple empiric formula has been found connecting these two quantities. I have been unable, therefore, to arrive at any rational explanation for the relation which seems to exist between loss and vapor product and, in the absence of such rationalization, this relation must be considered as purely empirical.

It will be noted that the relation between loss and vapor product is even more clearly defined in the case of the insulator losses than it is in the case of the atmospheric losses between conductors. This, as has been previously explained, is probably due to two things. One, that the insulators being near the ground were probably less affected by smoke or other floating particles in the atmosphere; the other, that the humidity measurements were taken alongside the insulators; whereas those applying to the lines were taken on the ground below the lines and at one end of the 1,000 feet of line used. In the case of the humidity measurements, applying to insulators, therefore, these measurements directly apply to the space in the neighborhood of the insulators, whereas in the case of the line, the humidity measurements apply only to one portion of the line and may or may not have fairly represented the average condition over the whole line.

The curves in the case of the insulators, Figs. 28 and 29, have been drawn in as straight lines, although if the measurements on the insulators had been carried to higher values of vapor product, no doubt the curves would have been drawn in slightly curved as is the case with the lower portions of the curves of Figs. 8 and 9.

The vapor product curves for all the cables measured are not given herein, in order to not unduly extend this paper, but they have been made use of in deriving certain other curves. One of the derived curves is that of Fig. 39, which shows the relation between the critical point and the spacing of the conductors at the value of vapor product = 0.20. One of these curves is drawn in broken. The broken portion is an exterpolation.

There is a curious result of this apparent relation between loss and vapor product. If the voltage loss curve for vapor product = 0.0, be subtracted from the corresponding loss curve for any other vapor product, the residual difference seems to be about the same for all sizes of conductors and all spacings. In Fig. 40 are shown these residual quantities obtained by sub-

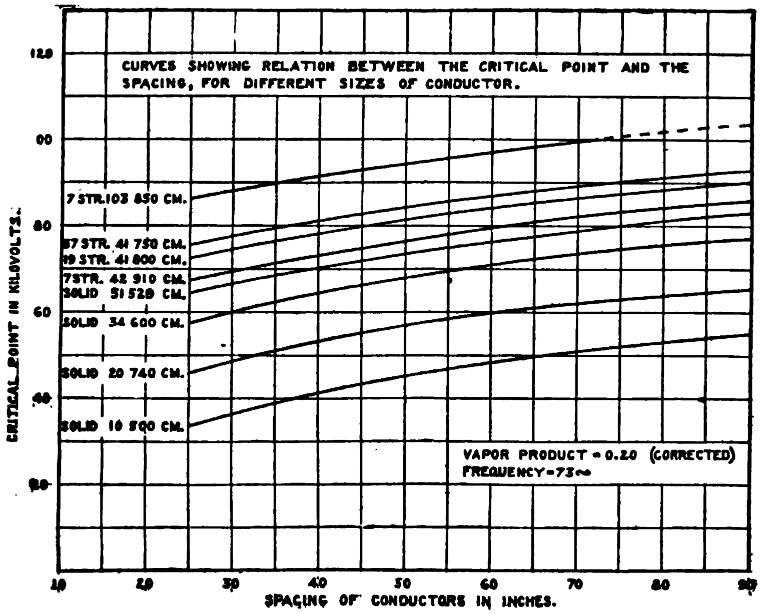


FIG. 39.

tracting from the voltage loss curve for a given size cable at different spacings and at vapor product = 0.20, the corresponding loss curves for vapor product = 0.0. As will be seen the residual quantities are not far apart.

In Fig. 41 corresponding results have been obtained for different sizes of conductors at the same spacing. Here again, the residual differences do not differ widely, except at the high voltages where the loss is very sensitive to any change of condition, such as smoke, etc. On comparison also it will be seen that the residual differences are about the same in the two

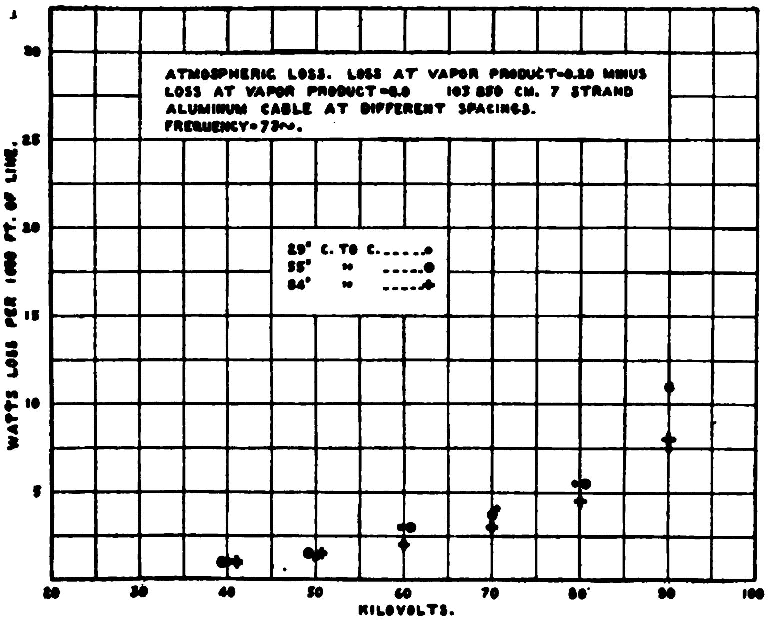

ATMOSPHERIC LOSS. LOSS AT VAPOR PRODUCT=0.20 MINUS LOSS AT VAPOR PRODUCT=0.0 103 850 CM. 7 STRAND ALUMINUM CABLE AT DIFFERENT SPACINGS.
FREQUENCY=73~.
29° C. TO C.
55° "
84° "
WATTS LOSS PER 1000 FT. OF LINE.
30
25
20
15
10
5
20
30
40
50
60
70
80
90
100
KILOVOLTS.

FIG. 40.

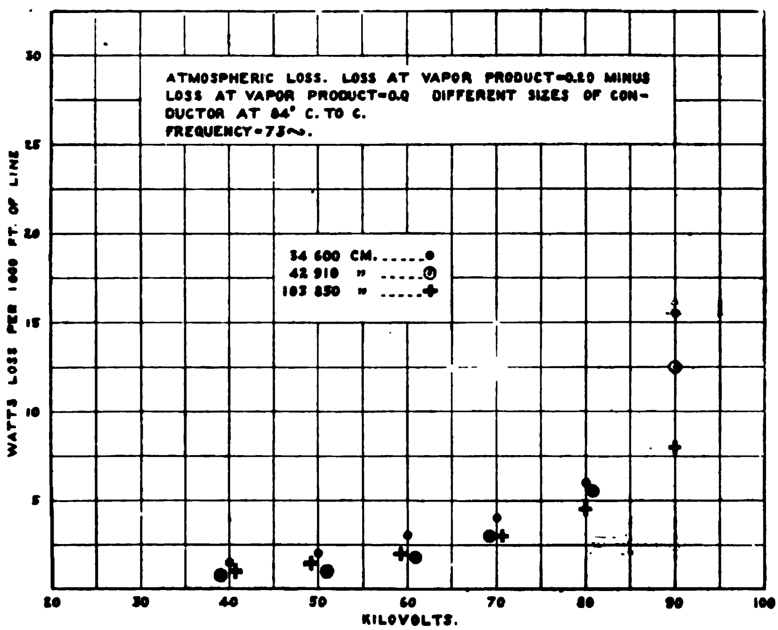

ATMOSPHERIC LOSS. LOSS AT VAPOR PRODUCT=0.20 MINUS LOSS AT VAPOR PRODUCT=0.0 DIFFERENT SIZES OF CONDUCTOR AT 84° C. TO C.
FREQUENCY=73~.
34 600 CM.
42 910 "
103 850 "
WATTS LOSS PER 1000 FT. OF LINE
30
25
20
15
10
5
20
30
40
50
60
70
80
90
100
KILOVOLTS.

FIG. 41.

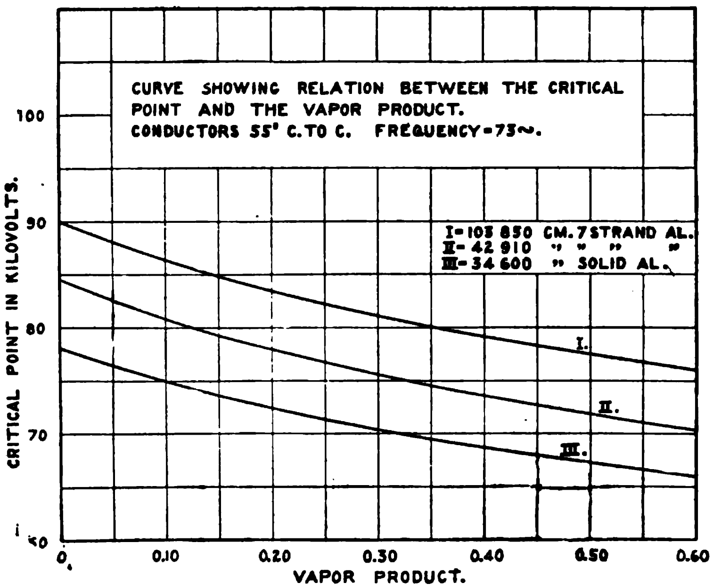

CURVE SHOWING RELATION BETWEEN THE CRITICAL POINT AND THE VAPOR PRODUCT.
CONDUCTORS 55" C. TO C. FREQUENCY = 73~.
I = 103 850 CM. 7 STRAND AL.
II = 42 910 " " " "
III = 34 600 " SOLID AL.
I.
II.
III.
100
90
80
70
60
0
0.10
0.20
0.30
0.40
0.50
0.60
CRITICAL POINT IN KILOVOLTS.
VAPOR PRODUCT.

FIG. 42

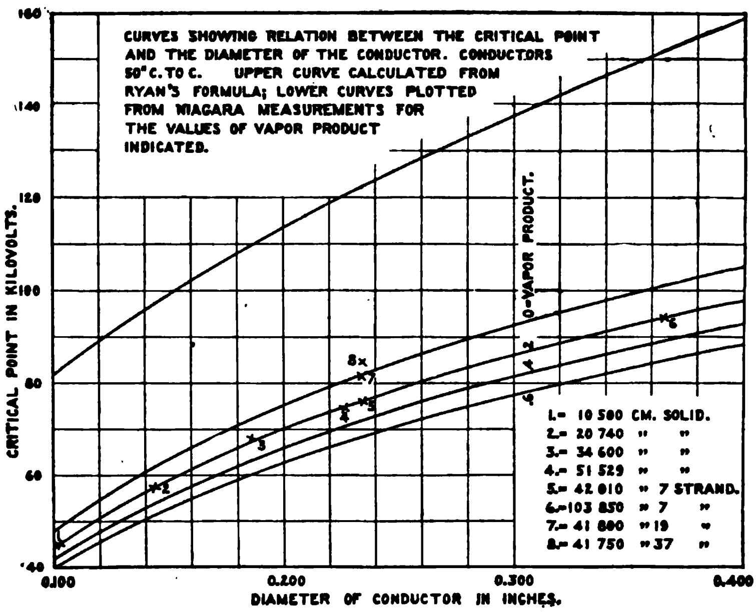

CURVES SHOWING RELATION BETWEEN THE CRITICAL POINT AND THE DIAMETER OF THE CONDUCTOR. CONDUCTORS 50" C. TO C. UPPER CURVE CALCULATED FROM RYAN'S FORMULA; LOWER CURVES PLOTTED FROM NIAGARA MEASUREMENTS FOR THE VALUES OF VAPOR PRODUCT INDICATED.
0-VAPOR PRODUCT.
1.- 10 500 CM. SOLID.
2.- 20 740 " "
3.- 34 600 " "
4.- 51 529 " "
5.- 42 010 " 7 STRAND.
6.-103 850 " 7 "
7.- 41 800 " 19 "
8.- 41 750 " 37 "
160
140
120
100
80
60
40
0.100
0.200
0.300
0.400
CRITICAL POINT IN KILOVOLTS.
DIAMETER OF CONDUCTOR IN INCHES.

FIG. 43.

figures. Apparently, therefore, the increase of loss due to vapor in the atmosphere is the same for the same vapor product no matter what the size of conductor or what the spacing between conductors. Why this should be the case, I am unable to conjecture, any more than I am able to arrive at any rational explanation of the relation between loss and vapor product.

In Fig. 42 is another set of curves derived by means of the vapor product relation. Here is shown the effect of vapor product on the critical points, for different sizes of conductors. As

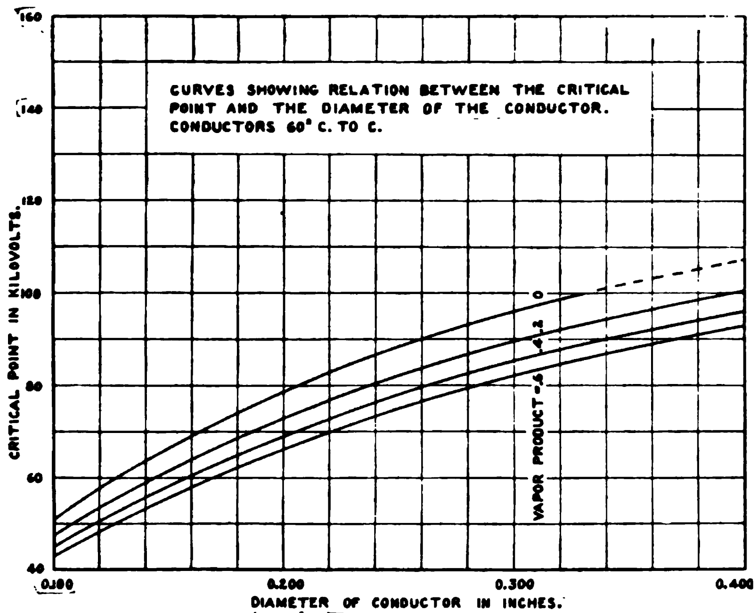


FIG. 44.

will be seen on examining these curves, the percentage reduction in the critical point, due to increase of vapor product, is closely the same for all sizes of conductors; which would seem to be properly the case.

Figs. 43, 44, 45 and 46, applying to distances of 50, 60, 72 and 84 inches between centres of conductors, show the relation between diameter of conductor and critical point for vapor products of 0.0, 0.20, 0.40 and 0.60. Portions of some of these curves are shown broken; the broken portions are exterpolated. The curves show the relation up to a diameter of 0.40 inch.

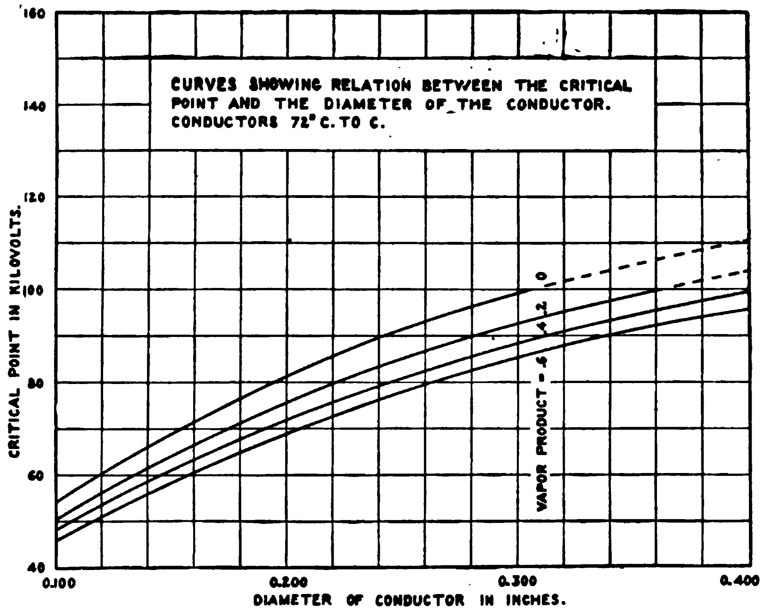

CURVES SHOWING RELATION BETWEEN THE CRITICAL POINT AND THE DIAMETER OF THE CONDUCTOR.
CONDUCTORS 72° C. TO C.
CRITICAL POINT IN KILOVOLTS.
DIAMETER OF CONDUCTOR IN INCHES.
VAPOR PRODUCT = .6 .4 .2 0
160
140
120
100
80
60
40
0.100
0.200
0.300
0.400

FIG. 45.

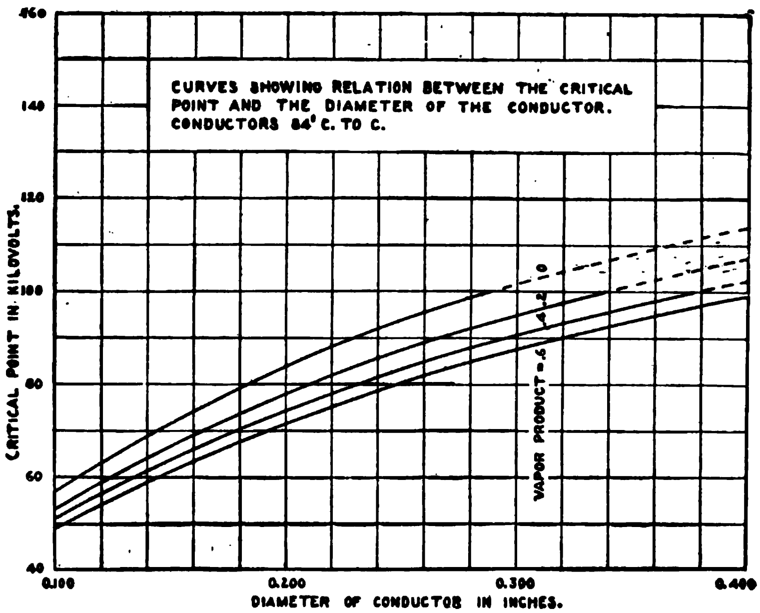

CURVES SHOWING RELATION BETWEEN THE CRITICAL POINT AND THE DIAMETER OF THE CONDUCTOR.
CONDUCTORS 84° C. TO C.
CRITICAL POINT IN KILOVOLTS.
DIAMETER OF CONDUCTOR IN INCHES.
VAPOR PRODUCT = .6 .4 .2 0
160
140
120
100
80
60
40
0.100
0.200
0.300
0.400

FIG. 46.

The maximum diameter of conductor on which measurements were taken was 0.518 inch; but, under all conditions of measurement, the critical point corresponding to this diameter was above the maximum voltage of the transformer, namely, 100,000 volts.

Referring to Fig. 43, particular attention is invited to the curve for vapor product 0.20 which is the only curve whose points are shown. All the points shown in this figure apply to this curve, and are numbered. Beginning at the left hand end of the curve, the first four points are for solid conductors; the next two points (5 and 6) are for stranded conductors of 7 strands each. The values of diameter opposite which these last two points (5 and 6) are plotted are the outside or overall diameter of the stranded conductor. In view of the fact that these last two points fall in well with the curve drawn through the solid conductors, it would appear that, so far as critical point is concerned, the equivalent diameter of a 7 strand cable is equal to the maximum diameter of the cable; that is, the 7 strand cable is equivalent to a solid conductor having a diameter equal to the outside diameter of the cable. Above point, 5 are two other points 7 and 8. These points apply to stranded conductors of 19 and 37 strands respectively, and they, also, are plotted with reference to the outside diameter of the conductor Now, the critical voltages applying to these points correspond to diameters of solid conductors (as shown by the curve 0.20 under consideration) considerably greater than the outside diameter of the stranded conductor. It would seem, therefore, that a stranded conductor of 7 strands is equivalent, as to critical point, to a solid conductor having a diameter equal to the outside diameter of the cable, and that a cable of more than 7 strands is equivalent to a solid conductor whose diameter is *greater* than the outside diameter of the cable.

The upper curve of Fig. 43 is one obtained by means of Ryan's formula (and on the assumption of a sine wave electromotive force) for the same spacing as the lower curves of this figure. As will be seen, the critical points by the formula are much higher than those given by the lower curves even for vapor product = 0.0 (*i.e.*, absolutely dry air). Not only is this the case, but the curve of the formula evidently does not follow the same law as the lower curves. In order to compare the Niagara results with Ryan's formula, the following tables have been prepared, showing the values of critical point from the Niagara measurements, the corresponding values from the formula, and the ratio of the former to the latter.

COMPARISON OF CRITICAL POINTS FROM NIAGARA MEASUREMENTS, AND FROM RYAN'S FORMULA, FOR DIFFERENT SPACINGS OF CONDUCTORS

Spacings of conductors in inches	Critical points from Ryan's formula	Niagara critical points	Ratio $\frac{\text{Niagara}}{\text{Ryan}}$
30	112	69.5	0.62
40	120	73	0.61
50	122	76	0.62
60	125.5	79.5	0.63
70	129	82	0.63
80	131.5	84	0.64
90	134	86	0.64

In making calculations by Ryan's formula, the barometric pressure has been taken as 29.5 inches and the temperature 70° fahr.
The Niagara critical points are for vapor product = 0.20.
The diameter of the conductor was 0.2349 in.
All critical points given in effective kilovolts.

COMPARISON OF NIAGARA CRITICAL POINTS, AT DIFFERENT VAPOR PRODUCTS, AND CRITICAL POINTS CALCULATED FROM RYAN'S FORMULA, FOR DIFFERENT SIZE CONDUCTORS SPACED 50 INCHES CENTER TO CENTER

Area of conductor cir. mils	Diam of cond. inches	Critical points from Ryan's formula	Niagara critical points for the values of vapor product given				Ratio $\frac{\text{Niagara}}{\text{Ryan}}$ for the values of vapor products given			
			0	.2	.4	.6	0	.2	.4	.6
103850	.3657	152	101	94	89	85	0.664	0.618	0.585	0.559
42910	.2349	122	82	76	72	68.5	0.672	0.623	0.590	0.561
34600	.186	110	72	68	63	60	0.655	0.618	0.573	0.545
20740	:144	97	61.5	57	53.5	51.5	0.634	0.588	0.552	0.531
10500	.1025	82.5	49	45	42.5	40.5	0.594	0.545	0.515	0.491

In making calculations by Ryan's formula, the barometric pressure has been taken as 29.5 inches and the temperature 70° fahr.
All critical points given in effective kilovolts.

These tables show that the Niagara values vary from 0.49 to 0.66 of those by the formula; the ratio remaining practically constant for the same size conductor at different spacings but varying for the different sizes of conductor at the same spacing. In other words, qualitatively, the Niagara results agree with the formula as the spacing is varied, but depart somewhat from the formula as the diameter is varied; and quantitatively they do not agree with the formula at all.

In order to make Ryan's formula applicable at different diameters and different vapor products Fig. 47 has been prepared, showing the ratio by which the critical points obtained from Ryan's formula must be multiplied in order to obtain for different diameters and vapor products the critical point corresponding to the Niagara results.

As has been previously mentioned (see page 1066) Ryan's formula contains two quantities, d and D', which are not constant. In making use of his formula, the value of d and D' must be taken from a table, given by him, in which they appear as functions of the diameter of the conductor. The formula is not, therefore, self contained.

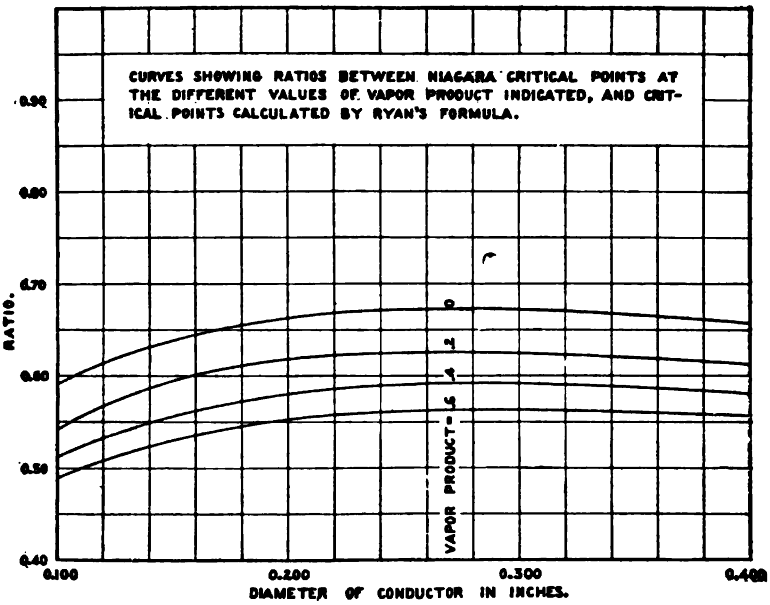


FIG. 47.

On plotting the value of d and D' as functions of the diameter of conductor, the curves of Fig. 48 are obtained. These curves are hardly such as one would expect in connection with physical phenomena. The relations they show would appear to be entirely empirical and of a character to raise some doubt as to the necessity for the assumptions from which they result.

It is possible to represent quite closely the results of the Niagara measurements by a self-contained expression of the form

$$E = k_1 \left(1 + \frac{k_2}{k_3 + r}\right) (r) \log \left(\frac{s}{r}\right)$$

in which E is the e.m.f.; r the radius of conductors; s the distance between conductors; and k_1, k_2, k_3 constants. The last two variable factors, $(r) \log \left(\frac{s}{r}\right)$, of this expression are similar in form to those of the theoretical equation obtained by the usual method of analysis, so that to this extent, at least, the expression is rational. The other variable factor, $\left(1 + \frac{k_2}{k_3 + r}\right)$, may be looked upon as accounting for the existence and variability of the small quantity d, and the variability of the critical dielectric stress D' (if there be any necessity for such).

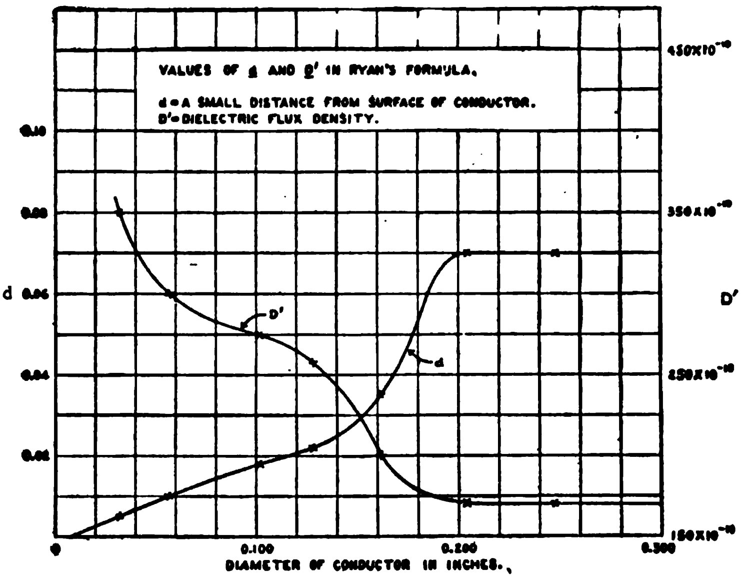


FIG. 48.

Whatever may be the adequacy of the argument in favor of the small quantity d, and its variability, it would seem as though D', the critical dielectric stress, should be constant; that any apparent variation in it is more likely due to errors in the method of calculating electrostatic force (such as the assumption of uniform density of charge) than to an actual variation of the local stress necessary to institute breakdown of the dielectric.

The above form of expression applies for any given vapor product. Variation in vapor product may be quite closely taken into account if there be introduced into the expression an additional empirical factor of the form

$$\frac{1}{1+k_4\,p'}$$

where p' is the vapor product and k_4 a constant.

The above expressions are not offered as final formulæ; I desire to obtain more corroborative data before proposing a final expression. They do, however, quite closely represent the Niagara results and may be advantageously used in adapting or extending them so far as this may seem to be warranted.

It is not known what frequency Ryan used, but it is presumed to have been 60 cycles. The results at Niagara were at 73 cycles. A difference in frequency might account, in part, for the difference in the results obtained.

In order that the Niagara results shall be applicable to different localities, the following table has been prepared by means of data obtained from the United States Weather Bureau. The table applies to two winter and two summer months.

Maximum and Minimum Vapor Products for Four Different Locations in the United States, for January, February, July and August, 1905, as Obtained from the Humidity Measurements of the United States Weather Bureau

		Jan.	Feb.	July	Aug.
Eastport, Me.	Maximum	0.347	0.124	0.465	0.440
76 feet above sea level.					
Mean Barometer about 30 in.	Minimum	0.014	0.014	0.245	0.175
Key West, Fla.	Maximum	0.552	0.566	0.674	0.743
22 feet above sea level.					
Mean Barometer about 30 in.	Minimum	0.0816	0.194	0.493	0.517
St. Paul, Minn.	Maximum	0.141	0.173	0.527	0.555
837 ft. above sea level.					
Mean Barometer about 29 in.	Minimum	0.0187	0.0106	0.157	0.179
Denver, Colo.	Maximum	0.199	0.072	0.413	0.418
5291 ft. above sea level.					
Mean Barometer about 25 in.	Minimum	0.025	0.0112	0.199	0.0224

Resume and Conclusions

It is believed that as the result of the work at Telluride, the work of Professor Ryan, and the results at Niagara, the following conclusions are justified. Such of the following items as were originally due to Professor Ryan are so designated.

1. That with a given conductor at a given spacing and under given atmospheric conditions, there is a certain voltage or "critical point" at which a very appreciable loss begins to occur through the atmosphere.

2. That there may or may not be an appreciable loss exist-

ing below this critical point, depending upon the atmospheric conditions.

3. That the presence of floating particles in the atmosphere may produce a loss below the critical point (Ryan).

4. That the presence of moisture in the atmosphere may produce a loss below the critical point.

5. That the presence of moisture in the atmosphere affects the loss both above and below the critical point.

6. That the presence of moisture or floating particles in the atmosphere affects the position of the critical point (*i.e.*, the value of the critical voltage).

7. That the critical point corresponds to a partial breakdown of the dielectric.

8. That the critical point coincides with the voltage at which luminosity or hissing (or both) of the conductors begins.

9. That the critical point depends upon the maximum value of the electromotive force wave and the distance between the conductors.

10. That the critical point depends upon the local rate of fall of potential or dielectric stress at some point in the atmosphere and, therefore, depends not only upon the maximum value of the electromotive force wave and the distance between the conductors, but also upon the diameter of the conductors. (Ryan.)

11. That there is a loss over insulators which is affected by the moisture conditions of the atmosphere.

12. That the variation of the atmospheric loss between conductors, the variation of the loss over insulators, and the variation of the critical point due to the moisture conditions of the atmosphere bear to the vapor product (*i.e.*, the product obtained by multiplying the vapor pressure by the relative humidity) a definite relation which, so far as is at present known, is an empiric one.

13. That the loss over insulators in a fog is very much higher than the loss in dry air, and somewhat higher than that in a heavy rain.

14. That the smoother the surface of the conductor, the less the loss and the higher the critical point.

15. That the stranding of the line conductors reduces the loss and raises the critical point due to the increase of the equivalent diameter of the conductor.

16. That the increase of the equivalent diameter of the conductor is greater the greater the number of strands.

17. That the weathering of conductors (or at any rate the aluminum conductors and probably copper also) does not appreciably increase the loss or lower the critical point.

18. That the loss and critical point are the same for copper and aluminum under the same conditions.

19. That anything which increases the charging current of an insulator increases the loss over the insulator.

20. That the loss over an insulator on a wooden pin is greater than that over an insulator on a metal pin, because the resistance of the wooden pin is in series with the charging current of the insulator.

21. That the atmospheric loss between conductors and the loss over insulators decreases with the frequency. The law of the decrease is not at present accurately known.

22. That neither the critical point nor the loss between cables is affected by variation in the distance of the cables from the ground.

The effect of atmospheric density upon the critical point has been omitted from the above enumeration. This has not been done because of any doubt as to the accuracy of Ryan's results relating thereto, but simply because neither the results at Telluride nor at Niagara were such as to throw any light, corroborative or otherwise, upon this matter.*†

* Acknowledgment is due to those of my assistants who, under my directions, took the measurements at Niagara. The work was originally intrusted to Mr. Harry L. St. George, assisted in the beginning by Mr. George K. McDougall, and then by Mr. Harry L. Shepherd. Later, Mr. Shepherd was in charge of the work, assisted at first by Mr. McDougall and then by Mr. Walter I. Tamlyn.

† Acknowledgment is also due the following for assistance rendered.

To the Niagara Falls Hydraulic Power and Manufacturing Company for the use of the land on which the work was carried out; for power supplied without charge; for construction work done at cost; and for many other conveniences without which the work would have been greatly hampered. To the Aluminum Company of America for the loan of wire and cable. To the Weston Electrical Instrument Company for the donation of special instruments. To the Westinghouse Electric and Manufacturing Company for the loan of a motor and instruments. To the Niagara Falls Power Company for the use of their laboratory in the calibration of instruments. To Mr. W. S. Moody of the General Electric Company for the pains which he took in designing the transformers to meet the rather severe requirements of this work.

DISCUSSION ON " HIGH-VOLTAGE MEASUREMENTS AT NIAGARA."
ATLANTIC CITY, N. J., JUNE 30, 1908

Henry L. Doherty: This paper is a most valuable contribution to a subject of great importance. I will not attempt to discuss any of the details of the paper for fear it will distract from the broader matters which are worthy of consideration.

As a preface to my remarks I want to say, first, that the American Institute of Electrical Engineers is now a body of great importance, and it is in a position to exercise a pronounced influence upon the educational, industrial and commercial progress of this country; and second, that few, if any, other organizations can boast of greater aggregate intelligence or influence than can this organization. Its members are in touch with nearly every class of people, and their individual work and opinions are universally respected.

This paper deals with the researches made to secure a greater knowledge of the problems which underlie the methods for transmitting electrical energy.

Had my time permitted, I would have offered to the Institute a paper at a much earlier date, covering the various methods for the transportation of energy in its various forms, and would have attempted to show the relative cost of transporting energy, or its material, by various means, such as: (a) electrical energy by transmission at high voltage; (b) natural gas by means of pipe-lines; (c) coal and oil by rail; (d) oil by means of pipe-lines; (e) steam and hot water by means of pipe-lines.

I think it can readily be demonstrated that, where a good load factor exists, transportation of energy can be secured at the lowest possible cost by means of electric transmission.

We often find ourselves, as engineers, greatly handicapped by the sentiment prevailing amongst bankers and investors to whom we must look for the necessary funds to finance our undertakings. Often the method of presentation will do much to influence, favorably, prejudiced investors capable of furnishing funds for the enterprise.

It was my intention to show that these so-called "long-distance transmissions" constituted a problem in transportation which could be reduced to the same terms as railroad statistics—which are always available—and their commercial efficiency measured by a comparison therewith. It was my belief that such a comparison would put this whole field of work on a more substantial basis in the minds of the public and increase the activity in this line of work, thus greatly increasing our usefulness.

As my own time will not permit, and as there are many men in the Institute who can do this work much better, I am taking the liberty of suggesting this comparison in relation to the consideration of this subject.

I believe the engineer, and the engineering organization, have been undergoing an evolution which will materially change their scope of usefulness. The engineer must now know, with more or less intimacy, other lines of business; and, as an example, he must know more or less of financial methods and possibilities. He must often interject his engineering methods into other lines of work—in fact, it is remarkable how many of our general business matters are capable of solution by engineering methods and engineering laws. He must know how to state his conclusions in such form as to give readily a clear and correct impression to men educated in other lines of work. The engineering society must not only know the needs of its own members, but must know how to influence other classes of people for the benefit of its own class.

The recent conference called by President Roosevelt for the conservation of our natural resources was, to my mind, a great stroke of statesmanship; and, using this occurrence as an illustration of my preceding statement, it seems to me that as the members of the Institute represent an immense sum of knowledge on this problem—which we all recognize is a matter of national importance—it would not be impertinent for the organization to undertake to contribute materially to the solution of this problem and to make all parties in interest know that it is in a position to do so, and that it should be considered as an important factor in this work. It is hard for me to refrain in any discussion from applauding the excellent work done by this Institute in recent years; and I avail myself of every opportunity to urge that it do still more, and still further widen its field.

Regardless of the other contributions made by Mr. Mershon, I want to point out particularly that he has demonstrated that it is possible to secure financial support for practical research work of this character. This demonstration may prove of greater value to the future achievements of the Institute than any specific work heretofore presented to it. I strongly urge that the Institute should place on record, in unmistakable terms, its appreciation of the generosity of these men, and that at this meeting an effort be made to point out to everybody that such contributions cannot help but prove, if properly directed, great factors in our industrial progress and great factors in the commercial prosperity of this country as a whole.

While many may believe that an organization of this sort should attempt to exert influence only over its own members, here is an excellent example of how it may exert influence over men of great wealth, who are willing to contribute liberally to our research work if it promises benefits to the industrial progress of the country. It seems to me not hard to point out the great value of such contributions.

Elihu Thomson: Mr. Mershon some time ago told me of some of his results, and asked me if I could make any suggestions as to the cause; that is, as to the actual thing happening; if I could suggest some physical conception. Before I make any attempt at this—and it can only be an attempt—I should say that there is true gratification felt in finding that funds can be at times available, and may in the future be more frequently available, for engineering research as distinguished from pure scientific research. Yet the results of the paper while quite practical, as applied to transmission lines, have, further, germs of scientific progress in them—progress along the lines of pure science.

I will not take the time to discuss in detail the various matters of the paper. Perhaps it would be well to turn to the last pages and take up the final conclusions, and note, if we can, whether the results appear to be rational or not. The "critical point" is well understood; and I do not see, in reading over Conclusion 1, that there is anything to be said further; there is a certain voltage called the "critical point" at which a very appreciable loss begins, and that point is coincident with the visual manifestations, corona, or decomposition, or static breakdown, of the dielectric surrounding the conductor.

Conclusion 2 reads, "That there may or may not be an appreciable loss existing below this critical point, depending upon the atmospheric conditions." It seems to be a remarkable result, where no visible effect can be found around the conductor, that there is still even with air in a dry condition, a notable loss.

Conclusion 3 reads, "That the presence of floating particles in the atmosphere may produce a loss below the critical point (Ryan)." That, I think, we can readily understand; the particles may be attracted to the conductor, and may be the means of causing a diffusion of energy—they may be pulled, as it were, towards the conductor, breaking down the homogeneity of the dielectric and thus causing a loss of energy.

Then, again, we read in Conclusion 4, "That the presence of moisture in the atmosphere may produce a loss below the critical point." Here we find that Mr. Mershon has given us conditions under which the moisture produces a loss depending apparently on the amount of water present in a given volume, that is, on the absolute vapor pressure of the water and the condition of the water as to its being more or less near condensation, or whether it is further from the condition of condensation or nearer to it. This leads us, I think, to the consideration of the fact that water vapor just above its vaporization point cannot be considered as a true gas. It has to be heated considerably beyond its vaporization point to become a true gas. In a gas, according to the kinetic theory, the molecules are flying about and colliding in all directions; they have a certain velocity, but the velocity is not uniform, some molecules being accelerated beyond the mean velocity and others very definitely retarded below the mean velocity. Now, when a liquid is near condensation, some of these molecules may be robbed almost entirely of velocity, but we know that if this occurs in a vapor, such as water vapor, such molecules may be considered as for a very brief time in the liquid state. Afterwards they are again separated by colliding with other molecules at higher velocities. In other words, in this instance there is an unstable condition in the gaseous substance, and the nearer therefore to condensation, or the greater the humidity—relative humidity—the more this factor will come in; the state of some small fraction of the water vapor being changed from that of a gas to a liquid. A larger number of molecules are then in a state of quiescence, for the time being, or in the state of water for a very short interval, the nearer the approach to condensation. This condition would explain why the vapor product—if our theory is correct—is concerned. According to the result obtained by Mr. Mershon the vapor product must be taken; not only the amount of vapor water present, but the state of that water. The number of molecules of water vapor so condensed, as water—or so losing their velocity and becoming for the time being, only instantaneously, more or less liquid—would be greater with the greater amount of water, as expressed by the greater vapor tension of the water, and greater also the nearer to the point of condensation, or the greater the humidity. We must infer from this, however, that the molecules of water in the quiescent condition have some unusual power of absorbing energy; that they can receive energy, can be accelerated or can be decomposed or driven apart by absorbing energy from the conductor. This action might go on invisibly, it might simply be that the water is, as it were, electrically evaporated or put back into its high-velocity condition by energy expended. This view is merely put forward as a suggestion, depending on the validity of our ideas of the action of molecules of gas; bringing together Conclusions 3 and 4, " That the presence of floating particles in the atmosphere may produce a loss below the critical point " and " that the presence of moisture in the atmosphere may produce a loss below the critical point."

Conclusion 5 is as follows: " That the presence of moisture in the atmosphere affects the loss both above and below the critical point." I think this would naturally follow from the other idea, that the more water the more loss would occur, even above the critical point; that the particles would be in condition to carry away more energy, on the assumption that water particles have an exceptional capacity in that direction.

Conclusion 6 is as follows: " That the presence of moisture or floating particles in the atmosphere affects the position of the critical point." This, again, means that the water particles, or the moisture particles, would act as a sort of intermediary in extending the influence around the conductor, and accentuating the production of the critical point; or in other words, the water would be more readily ionized than dry air, and become the medium for the absorption of larger amounts of energy, causing dielectric breakdown.

Conclusion 7 reads: " That the critical point corresponds to a partial breakdown of the dielectric." That, of course, we can readily assume.

Conclusion 8: " That the critical point coincides with the voltage at which luminosity or hissing (or both) of the conductors begins." Either the sound or the luminosity may be an index of the breakdown—actual sparking into the dielectric, and a reversal of current flowing through a short distance with each alternation, and naturally this would be equivalent to a high-resistance leak around the conductor, causing loss.

Conclusions 9, 10, and 11 are as follows:

9. " That the critical point depends upon the maximum value of the electromotive-force wave and the distance between the conductors.

10. " That the critical point depends upon the local rate of fall of potential or dielectric stress at some point in the atmosphere and, therefore, depends not only upon the maximum value of the electromotive-force wave and the distance between the conductors, but also upon the diameter of the conductors (Ryan).

11. " That there is a loss over insulators which is affected by the moisture conditions of the atmosphere."

I do not think there is much necessity for my commenting upon these conclusions. They seem to be the natural results to be expected and are fairly well known.

Conclusion 12 reads: " That the variation of the atmospheric loss between conductors, the variation of the loss over insulators, and the variation of the critical point due to the moisture conditions of the atmosphere, bear to the vapor product (*i.e.*, the product obtained by multiplying the vapor pressure by the relative humidity) a definite relation, which, so far as it is at present known, is an empirical one." In that conclusion we come to the particularly novel point, namely, that we must have not merely the amount of moisture, not merely the saturation, but we must multiply together the amount of moisture present, or the amount of water vapor, by its state or condition, so to speak, which I have above suggested as that of more or less imperfect gas.

Conclusion 13 reads: " That the loss over insulators in a fog is very much higher than the loss in dry air, and somewhat higher than that in a heavy rain." This can be easily understood. If the whole insulator were covered with water, the conduction over the upper surface of the insulator would undoubtedly be greater than if it were covered with fine particles of water. Its upper surface would be, in other words, of lower resistance and therefore cause less loss than if the fog settled upon the surface and made it a very high-resistance surface.

Conclusion 14 reads: " That the smoother the surface of the conductor, the less the loss and the higher the critical point." We would be naturally ready to admit this, on account of the absence of projections or points; but 15, which reads: "That the stranding of the line conductors reduces the loss and raises the critical point due to the increase of the equivalent diameter of the conductor," seems to be inconsistent therewith. With the stranding of the line conductor we would reach a limit, of course, very soon. The continued finer and finer stranding would not give a continued increase in real diameter, as the spaces between the wires of large or small gauge are relatively the same, so that we seem to be, in Conclusion 15, roughening the conductor and reducing the loss thereby. Conclusion 16 reads: " That the increase of the equivalent diameter of the conductor is greater the greater the number of strands." Here we have roughened still more by making the strands fine and thus have a large number of spiral projections or ridges all over it, and yet the increase of equivalent diameter is greater; whereas under Conclusion 14 it is said that the smoother the surface of the conductor, the less the loss and the higher the critical point. These things seem to need further investigation and some explanation, because they do not seem to be consistent. One can imagine that the stranding could go on until the conductor was covered with extremely fine ridges, and then we should consider it as possessing a rough surface; not an irregular roughness, but ridges nevertheless.

Conclusion 17 reads: " That the weathering of conductors (or at any rate the aluminum conductors and probably copper ones also) does not appreciably increase the loss or lower the critical point." I do not know that we should expect to find any great difference between conductors of aluminum or copper, as referred to in this Conclusion, or that a slight film of oxide would have any particular effect on the losses.

Conclusions 18 and 19 are as follows:

18. " That the loss and critical point are the same for copper and aluminum under the same conditions."

19. " That anything which increases the charging current of the insulator increases the loss over the insulator."

I think this we could admit. If the insulator dielectric were free from all dielectric hysteresis, then, of course, we would not expect Conclusion 19 to hold, but with ordinary materials it would undoubtedly be true that the loss would increase with the increase of the charging current. It would hence follow that if we interpose layers of air between the insulator pin and the wire or conductor, and these layers of air have the total effect of reducing the charging current very considerably, we may save some loss from the insulator. On the other hand, if we introduce only thin or moderately thick layers of air which have to be worked across by blue discharges, or anything of that sort, then of course we increase the loss by dividing the insulation. This comes directly in line with Conclusion 20, which reads, " That the loss over an insulator on a wooden pin is greater than that over an insulator on a metal pin, because the resistance of the wooden pin is in series with the charging current of the insulator," and would point to the fact that if we use a metal pin and want to avoid the losses as much as possible, we should make the metal pin fit the insulator very closely by some sort of conducting filling, and not have any space to be worked over by discharges from the pin to the inside surface of the insulator; but that is probably not of very great consequence.

Conclusion 21 reads: " That the atmospheric loss between conductors and the loss over insulators decreases with the frequency. The law of the decrease is not at present accurately known." I think this might naturally be expected, as the greater frequency would stress the dielectric more times per second, although a single long wave would no doubt give more loss than the shorter wave of equal amplitude.

Conclusion 22 reads: " That neither the critical point nor the loss between cables is affected by variation in the distance of the cables from the ground." I suppose this follows, because in ordinary cases the cables are at such distances from the ground that there is none of the effect of incipient brush discharge and increase of loss, such as we have when the conductors themselves are near together.

I have to say in regard to the paper, that it is certainly a very interesting and valuable one, and points the way to further useful investigation.

Samuel Sheldon: Professor Barus, at Brown University, and members of his department, have carried on very extensive investigations as to the conditions of the atmosphere in the vicinity of Brown University, in Providence, and also on Block Island, which is pretty well out to sea, with the result that he finds ions always present in the atmosphere at both places. Now these ions each have upon them a definite charge of electricity although they are of various shapes and conditions. If one of these ions be placed in an electrostatic field of a given intensity, a force is exerted upon it which is equal to the product of the charge times the strength of the field. Townsend has determined that under a given set of conditions a very definite and measured amount of energy is necessary to produce by impact ionization of un-ionized or non-ionized particles.

Under given atmospheric conditions there is an average free path between the particles that may come into collision; and if the collision represents a sufficient amount of energy, ionization of the particle that is struck will take place. The fact that energy is equal to half the mass times the square of the velocity, and that the velocity squared is twice the acceleration times the distance through which the particle moves, taken in conjunction with the fact that with the same force the acceleration is inversely as the mass, shows that irrespective of the size of the particles, if the latter move over the same distances, under the same force, they will arrive at the end of these distances with the same energy. If, therefore, they come into impact and the field has been sufficiently strong, ionization will take place. There will result more ions than were present before, and these again will set up further ionization, and therefore, ionization will spread like a disease in a field of sufficient intensity, until there are a sufficient number of ions to carry a discharge by convection. This explains many of the results which Mr. Mershon has obtained.

Henry Floy: While the Institute is certainly to be felicitated on being the medium through which the valuable data collected by Mr. Mershon is to be transmitted to the engineering world, and while Mr. Mershon is to be congratulated most heartily at the happy outcome of this long and tedious experimentation, there is another fact brought out by this paper which is particularly interesting to me, one which Mr. Mershon and Mr. Doherty have already touched on, and that is the disinterested generosity of Mr. J. E. Aldred, Mr. Frederic Nicholls, Mr. James Ross, and Mr. George Westinghouse. It seems to me that the engineer has now reached a point, at which I have long hoped he would arrive, where in the minds of commercial business men he is no longer considered only a theorist and scientist. And so we now have a most encouraging example of practical business men entrusting, in an unrestricted manner, their funds to a scientist for carrying on an investigation in his own way. The precedent thereby established seems to me to mark an era in the engineering world. If we can only encourage men of financial ability to subscribe funds to those engineers who are as capable as Mr. Mershon is to originate and carry out a series of tests and experiments, we have gone a long way in advance of previous achievement.

The results and conclusions reached by Mr. Mershon are very complete, surprisingly so to anyone who has attempted to carry on a set of experiments of this sort, particularly when it is realized that any one of an innumerable number of accidents might have vitiated the entire results.

With regard to the deductions made by Mr. Mershon, perhaps the most disquieting in the whole paper is the one shown, for example, by the curves in Fig. 43, showing the relation between the critical point as determined by his experiments and those of Professor Ryan. Mr. Mershon has noted that his critical voltages are about sixty per cent. of Professor Ryan's results. This difference seems to me most extraordinary, and as far as I know Mr. Mershon has not offered any explanation of it. I had hoped we would hear from Professor Ryan with regard to this matter. It is possible that part of the difference may be accounted for by the failure of Professor Ryan to have accurately determined the points of luminosity. As I understand his experiments, his critical points were determined mainly by visual observation, and if he did not fix the points exactly owing to personal equation or to failure to carry on his experiments in a room absolutely devoid of light, his voltages would be higher than Mr. Mershon's, who determines his values by a continuation of the lower curve. This difference in method, however, would not, apparently, wholly account for the great difference in the voltages at the critical points. While I am unable to criticize Mr. Mershon's method, it would seem as if his voltages were somewhat low. Turning to Fig. 43 and assuming the vapor product of 0.4, and the diameter of the conductor as one-quarter inch, we find that the critical point in kilovolts is 75. From what I have heard of the line at Grand Rapids, Michigan, where they are operating at 75,000 volts, and at other very high-tension installations, luminosity does not appear on those lines. It may be that the conductors are more widely separated or of different diameter, or that the vapor product is not as given in this table; but it would seem as if in the course of their operation there would be some fluctuation in the voltage or variation in conditions so that luminosity would be observable. Under the conditions stated by Mr. Mershon, luminosity should appear at 75,000 volts.

It would seem as if further investigation, perhaps by some other or outside party, should be carried on to determine whether Mr. Mershon's results can be absolutely relied upon, or whether the figures previously given by Professor Ryan are nearer correct. It would be unnecessary to repeat the experiment throughout, but simply to check a few points on certain curves. The location of the critical point is certainly a very vital question with engineers who are designing high-tension lines.

The announcement by Mr. Mershon of the less resistance of wooden pins, as compared with iron pins, is certainly astonishing to most of us, but in view of his measurements and the explanation given, we must acknowledge it as a fact.

In regard to the insulators, Mr. Mershon apparently tested no suspension-type insulators. Of course, tests of these insulators would have been especially interesting at this time, as they are coming more and more into use. Although he does not say so, his statement indicates he has learned the ideal form of insulator for a given voltage. Perhaps, in another paper, he will give us an idea as to the form of the insulator, and the number of petticoats which this ideal form of insulator should have, for a given voltage.

Chas. P. Steinmetz: Mr. Mershon has given us a very valuable paper, increasing our knowledge of what takes place in conductors when they reach such high voltages that the air surrounding them ceases to be insulating, begins to break down.

He has proved again the law given some years ago, and illustrated in his paper, that the voltage loss is a curve of two branches—one at lower voltage, of very small loss, and the second branch at high voltage, rising very rapidly. There is one point, however, concerning which I cannot agree with him, and that is the method of determining the critical point as the point where deviation begins from the lower branch, as seen in Fig. 38. I do not think that is the correct critical point. We must expect that the conductor and surrounding conditions are not absolutely uniform, and that the transition from one condition to another one can only be gradual. The first beginning of a brush discharge at some isolated point, some kink in the conductor, cannot be considered as the voltage where the breakdown strength of the system has been reached all over the conductor, and I believe this feature may account for the discrepancy between Professor Ryan's and Mr Mershon's results. I also agree that it would not be proper to take as the critical point the point where the upper or higher curve is perfect. Theoretically if we had the equations of the lower curve in Fig. 38, and of the upper curve, the point of intersection would be the proper critical point. This would be at higher voltage than that assumed by Mr. Mershon. We obviously cannot do this, as we do not have the equations of the two curves; and by estimating it, by continuing such curves, it is very difficult to get anywhere near accurate results; that is, results from which you can derive numerical values of the breakdown strength of air.

I should therefore suggest a slight modification of the method, which may give accurate results or may not. It is to plot the curve in Fig. 38 with the logarithm of the ordinate against the logarithm of the abscissa. Both branches of the curve may theoretically be expected to go through the origin, and such curves going through the origin, can frequently, at least within a limited range, be expressed by some power of the abscissas, that is, by a curve of the equation: $y = A x^n$. If you now plot logarithm y against logarithm x, you get a straight line. If in Fig. 38 you were to plot the logarithm of the ordinate against the logarithm of the abscissa, you may get two straight lines, one extending up to 70,000 volts, and the other extending from 80,000 volts upwards, connected with each other by a curved branch; and where you have straight lines, at least for some distance, it is very easy to extend them and so get the point of intersection very sharply, which gives the critical value with much greater accuracy than it can be derived by extrapolation from the original curve. It all depends on the character of the two curves, whether they will give straight lines in the logarithmic representation or not, but it is quite likely they may do so, or at least the deviation from the straight line will be small, and therefore this method should be tried.

While the results, as derived here, are extremely valuable in showing the existence of the critical value, we really want to have more than that. We would like to get the exact numerical value of that critical voltage, and compare it with the conductor diameter, and from that calculate the potential gradient at the conductor surface, and compare it under different conditions; and also compare it with that derived from the striking distances with needle points, where you get the analogous effect. After determining numerically the breakdown strength of air, or its breakdown gradient, very many of these features could be predetermined theoretically and pre-calculated. For this work, very interesting and valuable data are afforded by this paper. We all agree that the future of power transmission depends on going to higher and higher voltage, and you see that the loss of power from the line increases enormously with the increase of voltage beyond the critical point, and so would limit the permissible transmission voltage. Now, if we know how this loss depends on conditions of conductor surface, conductor diameter, etc., it would be a question of engineering design how to lay out the conductors of the transmission line, so as to be able t) use as high voltage as possible, and still have a negligible static loss into space.

Percy H. Thomas: There are two broad aspects from which this paper may be viewed: first, that of commercial transmission work; secondly, that of the scientist. In the first aspect, what we need to know is whether this loss from conductor to air is going to actually limit the development of power transmission. I think we can say, pretty conclusively, from Mr. Mershon's results, that for the immediate future, at any rate, these losses are commercially practically negligible. The experiments have been so carefully made and their results so carefully stated, that unless some very extreme conditions arise, they will not have to be done over.

In regard to the scientific aspect of the paper, there are a number of comments which I would like to make.

I was very much interested in Dr. Thomson's explanation as to the cause or the physical meaning of the relation between loss and vapor product. Personally, I had not been able to find any consistent hypothesis. His explanation seems to be very reasonable. I ask him if his conception as to the manner in which the molecules or minute particles of water act is as convectors of electricity?

Elihu Thomson: They absorb energy by being separated, again by re-evaporation. The mere addition of energy to a perfect gas would, of course, require more energy to accelerate the molecules, to take up their liquid state; and we know it requires considerable energy to convert this condensed or partially condensed product into a gas.

Percy H. Thomas: As in the case of the pith-ball and charged disc?

Elihu Thomson: It may act as a convector or absorber, and may be ionized more readily also.

Percy H. Thomas: That seems reasonable.

This loss is one of the most difficult scientific measurements the engineer has to make—the measurement of true losses at very low power factor and high tension. The method thus becomes of extreme importance. Mr. Mershon has taken very great pains to get a good method, and to check it in all ways possible. In the absence of further data I do not see any reason to expect any error in it, but I do not feel fully convinced, by his check measurements alone, that he has eliminated all possible sources of error. I do not mean to say I suspect any error, but I do not quite consider these checks themselves wholly conclusive.

I wish to suggest another method of making these measurements quickly, which I have tried, and which I consider much simpler in many ways. This is an old method for high-tension measurements—the use of the quadrant electrometer as a wattmeter. Such an instrument was discussed by Miles Walker in a paper read before the Institute at the annual convention in 1902, under the title of "The Electrostatic Wattmeter in Commercial Measurements." The description of certain measurements made with this wattmeter will be found in a paper presented by me at the annual convention of the Institute in 1905, entitled "The Experimental Study of the Rise of Potential of Commercial Transmission Lines, Due to Static Disturbances Caused by Switching, Grounding, etc."

One other suggestion. Often the best method to eliminate a good many of the difficulties is to use a large-capacity apparatus. It is a great handicap to start off with a small transformer in large work, as it is necessary to correct for the errors introduced by the transformer itself. If it had been possible to use a 1,000-kw. generator, and a 1,000-kw. transformer, a great many of the possible causes of error would not have had to be considered.

There are some minor points I would like to bring forward.

I would like to ask Mr. Mershon if he has any numerical data on the actual loss during a rain storm?

Some of the vapor-product series are straight lines—those which show the loss on insulators—and others are given as curves. The points are such that it is difficult to tell whether they should be drawn as curved or straight lines. Is it possible they should all be drawn as straight lines?

The losses on the insulator with the pin and without the pin are given. I think Dr. Thomson brought out the point there, that the discrepancy may be due not only to the greater distance the charging current has to come without any pin, but to the presence of loss in the dielectric itself. The dielectric loss may be the cause of the decreased loss with the pin removed. When the pin is in, there is a greater concentration of potential on the head of the insulator than in the other case, and that, as is well known, may cause very serious losses in the dielectric when the concentration is somewhere near the puncture point.

There is another difference between Mr. Ryan's method and

Mr. Mershon's, if I remember, Mr. Ryan projected the vertical part of the loss curve down, to get the critical point, while Mr. Mershon estimates the point of tangency. I may be mistaken, but if I am not, there should be quite a difference in the critical point obtained from the same curve by the two methods, and each method should be judged by its own results. When this is considered, the discrepancy appearing between Ryan's and Mershon's curves would be very materially reduced.

I would like also to ask how the points fall on the loss curve above the knee of the curve. The points given do not run up around the knee. It would be interesting to know how well they fall on the curve above the knee.

Ralph D. Mershon: Fig. 22 gives that.

Percy H. Thomas: That is just around the knee?

Ralph D. Mershon: They come very much closer.

Percy H. Thomas: That seems to me remarkable. It would be interesting to have the insulator, *A*, identified, or a more complete drawing of it given. That is the insulator which shows the least loss.

I would like to ask also whether there is any possibility that the losses in the cords from which the wire was held would affect these curves at all? Of course, that loss is pretty small, but if the cords stayed out in the atmosphere even one day, there might be some change.

Ralph D. Mershon: A check was taken on the cords each time.

Percy H. Thomas: It is interesting to link this phenomenon with the usual brush discharge. With needle points, separated somewhere near the sparking distance of the applied voltage, there is a strong brush discharge, and there is an enormous loss. If the distance is not too small—say three feet—the sparking distance will be almost the same with point terminals as with spheres or other large-diameter terminals. This result is different from the results which are given in Mershon's curve, which we still believe to be right, wherein the smaller-diameter conductors show a much larger loss than the larger conductors. These two results must certainly be consistent, and probably indicate a loop curve in the form shown in the figure. If we have transmission lines separated say 29 in., as the closest distance we have in the paper, the loss with different size conductors varies considerably, but with any given conductor will increase as the distance is decreased, until puncture finally occurs. I imagine we can draw the loss curve for a small conductor as the voltage increases, as shown somewhat by the full line. For a larger conductor the loss does not start at so low a voltage, but it does rise in the end as shown in the dotted line, and judging from the fact just stated that needle points and spheres have equal puncture distances, these two curves approach each other. The explanation probably is that as the smaller conductor begins to discharge actively the strain is transferred from the conductor itself to the region of air around it, and is then restrained there. At a little higher voltage, or closer distance, the larger-diameter conductor begins to discharge current, and is then on an equality with the smaller. After this critical starting point is once reached, it is the convective resistance of the air which determines the leakage to the air. If this hypothesis turns out to be true, it will probably be found that the loss curves for the smaller and the larger conductors approach each other as the puncture points come nearer and nearer. Of course, that is beyond any practical limit for commercial work.

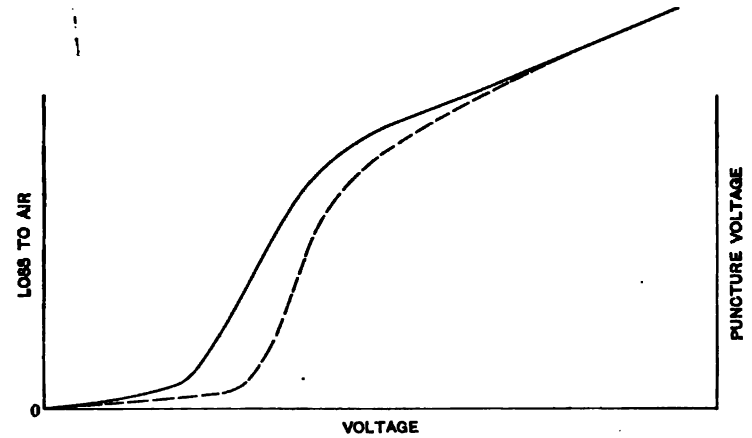


Fig. 1

P. M. Lincoln: A question has occurred to me the answer to which I do not find in the paper—and that is, the question as to the influence of the potential of the ground with respect to the potential between wires. Mr. Mershon's curves are all plotted with reference to the potential between the wires. If he was working with a single phase, as I assume he was, then the ground potential may safely be assumed as half the potential between wires. However, in a three-phase system the potential to ground does not remain one-half of the potential between wires, but approximately 58 per cent. of it. The question occurs to me, whether or not this difference of potential has any effect on the loss which will take place in these wires. Again, in case a three-phase system becomes grounded, what effect does that have on the loss? Will the loss be decreased or increased? The potential between the wires will remain the same, but the potential of the conductors to ground will be very different. That is quite an important question, and has an important practical bearing.

Carl Hering: While at Niagara Falls some ten or a dozen years ago I noticed a phenomenon which may have some bearing on a point raised in the paper, and touched on by Professor Thomson and Dr. Sheldon, namely, a possible loss of energy into the space surrounding the high-tension transmission wires. I was riding on a steam train which was close to the high-tension transmission lines, and the smoke or steam from the locomotive passed across these lines. As it crossed the wires, I noticed that it formed itself into circular rings around the wires as centers. Presumably they were hollow cylinders, but I could see only the ends, which were circles. It was quite a striking phenomenon. I called it to the attention of a number of those near me, and they all perceived it. It was at the time of one of our conventions. It seems to me this shows at least two things, namely, that some phenomenon takes place requiring energy, to arrange the smoke into rings, and that this energy must come from the wires, and therefore represents a loss; it may be trifling, but it is a loss.

Chas. F. Scott: Mr. Mershon's paper deals with certain large and fundamental questions and combines in a very beautiful way the scientific and the engineering. It records a research into unknown fields—not a simple measurement of some facts pretty well outlined beforehand, but an investigation into an unknown field where unexpected results have been obtained. The unexpected very often appears, at first, to be the erratic. If the points do not fall on the curve properly, it seems at first that there are probably some errors of measurement, or instrumental errors. The errors of observation having been eliminated, and the discrepancy still occurring, then it is necessary to look for some new factor. Mr. Mershon intimates some of the troubles he had in discovering these new and erratic elements, and it seems to me that one of the most beautiful things in the paper is the manner in which he set about to discover what was the matter with the vapor. He was pretty sure beforehand that the vapor had not anything to do with the case. His former tests and Professor Ryan's tests had apparently established the fact that the vapor was not a variable factor, but when he finds troubles, he begins to investigate as to the vapor. He does not know what the law is, but he sets his assistants to work to make all sorts of new combinations of vapor constants, and they finally fall upon one which seems to account for and explain the phenomena observed. The law may admit of scientific explanation, and in short be quite obvious now that it has been discovered. The point I want to make is that Mr. Mershon worked in the dark, and worked with no guide, and he finally came across this " obvious " conclusion.

Another thing is brought out in the same connection—a matter which can be worked out in the laboratory, and measured under laboratory facilities and laboratory conditions, may unfortunately, and sometimes in a very discouraging way, prove ineffectual and incomplete when it gets out on a practical transmission line. Lightning arresters may be cited, and observations and measurements like those of Professor Ryan—not because the laboratory may not have gone as far as it could, but because it cannot anticipate all the outside conditions.

This paper, and the things it deals with, are fundamental, because they set certain limits to electric power transmission. Mr. Mershon in another paper, which he worked out with great elaboration some two years ago, sets forth the economic voltage to be applied for transmission to different distances, and the size of conductor which would be used in transmitting power to different distances with the greatest economy. It happens that the diameters of conductors to be used with certain voltages, at certain distances, all fell within the limits of loss. For example, a wire half an inch in diameter was determined by Professor Ryan to have a loss beginning at a certain critical voltage. Mr. Mershon found it was not desirable to use that wire at that voltage, but at a lower voltage. That seems to be a happy circumstance. It would be quite unfortunate if it were necessary, for example, to use a wire an inch in diameter to avoid loss at 25,000 volts.

Now, Mr. Mershon has unfortunately changed one set of these constants, the critical voltages for wires of different diameter, and I am not sure from my recollection of the other paper whether he can now use the diameters of conductors which he found in his paper on transmission economics were desirable. If he can touch on that in his résumé, it will be quite interesting.

This brings into prominence another point, the difference between aluminum and copper. He says in his Conclusion 18 that "The loss and critical point are the same for copper and aluminum under the same conditions." That statement refers presumably to the same conditions of diameter, and not of conductivity, so that in practical work there may be a very marked difference between the use of the two metals, favorable to aluminum.

This paper marks another chapter in an interesting history which has been going on in the Institute TRANSACTIONS for the past ten years. To give a résumé for a moment—a dozen years or more ago, when large transformers of high voltage seemed to be something in the future, a high-voltage transformer giving 40,000 volts was made for the testing of insulators and for a general investigation into unknown regions of high voltage. In these tests, made at Pittsburg, some small wires were strung four or five inches apart, which showed a critical point, a very decided critical point, at which the wires began to give manifestations in three different ways, appealed to three different senses: to the eye by the corona and halo around the wire, manifested in the dark; to the ear by sound, and to the sense of smell by the odor of ozone, or something of that character. These three indications began to manifest themselves at a certain voltage which could be read very closely on the voltmeter, and the manifestations could be repeated in successive tests. Mr. Nunn was in Pittsburg about that time. He was then operating the oldest high-tension single-phase transmission, and was very much interested in these tests, and urged that tests of that kind be continued out at Telluride, offering the facilities of the power plant with the facilities of the apparatus and assistants and observers which were furnished from the east. A little later Mr. Mershon did the work which has been recorded in the Institute TRANSACTIONS, to which he has referred this evening. Professor Ryan became interested in these results. He wanted to get a scientific explanation, a law, and he carried on a very beautiful set of experiments which have been recorded in the papers which he has read and to which Mr. Mershon has referred.

Mr. Mershon has taken up the subject again, and has been able to draw some new conclusions. I think it is hardly worth while to add to the endorsements which have already been given to research work of this kind, which unites so closely the scientific, the engineering, and the commercial. We can hardly realize the difficulties which Mr. Mershon has encountered. He has alluded to them, with that little smile of disgust which he had a while ago, when he referred to the fact that he had to throw away some six months' tests, because something had been overlooked; but all of us who know Mr. Mershon and have known him for years back, realize his persistence. He does not stop at any ordinary obstacles.

A. E. Kennelly: While desiring to add my tribute to the great merit and value of the paper, I would like to ask a question about a point that seems difficult to understand. I refer to the effect of stranding the conductor upon the critical voltage. If I understand Figs. 39 and 42 correctly, the relation between critical voltage and outside diameter of the conductor is substantially the same for the solid wires and for the seven-strand wires. When, however, we come to 19-strand and 37-strand wires, there is a departure from the relation, and the critical voltage increases as though the diameter of the multiple-strand wire had been enlarged. In other words, do I understand the results correctly in deducing that the effect of stranding is only shown in 19-strand and 37-strand wires; and is not shown in seven-strand wires, which behave like solid conductors?

Ralph D. Mershon: Apparently the seven-strand conductor is equivalent to the solid, for the same extreme diameter, but No. 7 is 19-strand and No. 8 is 37-strand.

A. E. Kennelly: That is, the difference begins between seven strands and nineteen strands.

Ralph D. Mershon: Dr. Kennelly's understanding is in part correct and partly in error. The effect of stranding is shown in a seven-strand cable. The effective diameter of a seven-strand cable (that is, the diameter as regards critical point) is apparently that corresponding to the diameter of a solid conductor whose diameter is equal to the outside diameter of the seven-strand cable. The outside diameter of a seven-strand cable is greater than the outside diameter of a solid conductor ***having the same carrying capacity***. The effect of stranding, therefore, in a seven-strand cable is to increase the effective diameter. When however, the cable has more than seven strands, the effective diameter is apparently greater than that of a solid conductor having the same diameter as the extreme outside diameter of the cable. In other words, the seven-strand cable appears to have an effective diameter greater than that of a conductor having the same carrying capacity, and equal to that of a solid conductor having a diameter equal to the extreme diameter of the seven-strand cable; while, with more than seven strands, not only is the effective diameter greater than that of a solid conductor having the same carrying capacity as the cable, but it is even greater than the extreme diameter of the cable. Why this should be the case I am unable to conjecture, unless it be that the smaller the strand used, the smoother and more polished the surface of the strand, because of the greater number of times it has been drawn through the die. Now, we know that the smoother the surface of a solid conductor, the higher is the critical point, and it may be that while the corrugations on the surface of the cable, due to stranding, are too large to amount to roughness, the smoother surface of the individual strands is effective in increasing the critical point.

Before replying to the other speakers I wish to ask Dr. Thomson with reference to Fig. 40, to see whether he has any suggestions to make in regard to that part of the subject.

Elihu Thomson: It is simply a question of the advisability of carrying to extremes the tests that are there shown. Quoting from the paper: " In Fig. 41 corresponding results have been obtained for different sizes of conductors at the same spacing. Here again, the residual differences do not differ widely, except at the high voltages where the loss is very sensitive to any change of condition, such as smoke, etc. On comparison also it will be seen that the residual differences are about the same in the two figures. Apparently, therefore, the increase of loss due to the increase of vapor in the atmosphere is the same for the same vapor product, no matter what the size of conductor or what the spacing between conductors. Why this should be the case, I am unable to conjecture, any more than I am able to arrive at any rational explanation of the relation between loss and vapor product." It seems to me it would be desirable that these tests be carried to extremes, to see whether the condition pointed out will hold through much wider ranges.

W. L. Waters (by letter): I think Mr. Mershon's paper on the losses in high-voltage overhead wires and insulators is the most important paper which has been presented to the Institute this year. Only those who have undertaken similar investigations realize the amount of work which is entailed in such an investigation. Mr. Mershon is greatly to be complimented on the perseverance and skill with which he has continued the investigations which he began twelve years ago.

There is just one point which I wish to discuss in connection with this paper, and that is the general question of insulator loss. Mr. Mershon finds that the insulator loss is considerably greater with wooden pins than with iron pins, and the explanation of this fact given by him does not altogether explain why we get much better results with some insulators with wooden pins than with iron pins. Mr. Mershon in his explanation considers the insulator as a condenser of infinite resistance in series with the resistance of the wooden pin. As a matter of fact, however, the insulator has not an infinite resistance, and under some conditions can have a comparatively low resistance. A more exact way would be to consider the

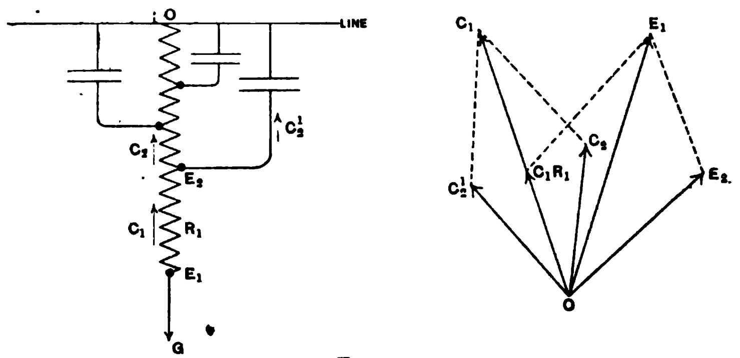


FIG. 1

insulator as a condenser with a considerable amount of leakage, in series with an ohmic resistance. There is leakage between the various parts of the insulator, both through the material of the insulator and over the surface. Assuming this state of affairs, the correct diagram is shown in the accompanying sketch, in which I have taken the system as consisting of a high ohmic resistance shunted at various points by condensers. We see from the vector diagram that the ohmic drop in the wooden pin is not in quadrature with the terminal voltage but makes a smaller angle, say 45 deg. or 60 deg., with it, consequently the voltage on the insulator is appreciably lowered by the use of the wooden pin, as shown in the vector diagram. The voltage from line to ground is $O\,E_1$ while the voltage on the insulator is $O\,E_2$, so we can understand why an insulator which will stand up satisfactorily with a wooden pin may not be able to stand up under the increased voltage to which it would be subjected when used with an iron pin. Of course, it is impossible to say just how much the voltage would be reduced, in any individual case, without making measurements. Probably Mr. Mershon has figures from which he can calculate the reduction in voltage. The ohmic loss in the wooden pin is $C_1^2 R_1$ and the magnitude of this loss will decide whether there will be any trouble due to overheating of the pin; and the variable resistance of the pins on the line will account for the fact that some pins give trouble, due to burning, while others do not.

It will be seen from the above that the capacity and ohmic resistance of the insulator, and the ohmic resistance of the pin, together decide the success of the wooden pin in any particular case, and I would suggest that on account of the experience

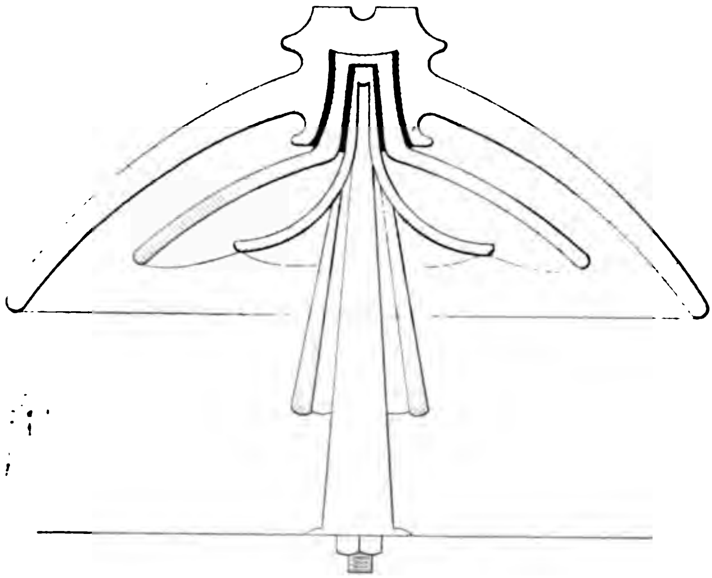


FIG. 1

gained from Mr. Mershon's tests that Mr. Skinner add to his insulator-testing specifications a clause requiring measurement of the average ohmic resistance and electrostatic capacity of the insulators, and the ohmic resistance of the pins when of wood. These values could probably be determined without much trouble, when the insulators are tested in large numbers, and after we have a little more experience in regard to actual figures they would give us some idea as to the probable success of the insulators and pins.

N. M. Snyder (by letter): From observation and personal experience I concur with Mr. Mershon—especially in his Conclusions 13 and 14.

I also believe that the loss over insulators by moisture may be reduced materially by a change in design, and Conclusion 14, for example—the smoother the surface of a conductor the less the loss and the higher the critical point—seems to suggest a remedy. It is quite true, that a point or number of points on a highly charged body will dissipate a charge more quickly than if the body were entirely smooth; and by a series of tests we find that a charge will remain on the exterior rather than in the interior of a conductor.

Now, then, by so designing an insulator as to give the main feature the shape of a cone, as shown, to give a protection to the petticoat and pin from rain, the insulator would have the advantage that the surface would be comparatively free from points to radiate a discharge, and also that the inner petticoats would be protected to a certain extent from moisture, coal-smoke deposits, etc. By this arrangement I believe additional benefits may be derived.

Ralph D. Mershon: Before replying to the technical points raised, I desire to emphasize the appreciation, already expressed in my paper, of the generosity of those who contributed to the expenses of this work; and to express my gratification at the expressions of like appreciation which have come from the various speakers this evening. It is to be hoped that the engineering profession may be benefited by similar contributions in the future.

The interest which Professor Thomson has taken in this matter, and the thoroughness of his discussion are very gratifying, especially as he has thrown a great deal of light upon the relation between loss and "vapor product." In fact, it seems to me most probable that his explanation of this relation is the correct one.

Mr. Floy has brought up the question as to why the critical points obtained by me differ from those obtained by Professor Ryan. I think that his suggestion, especially if extended a little further, may largely account for this difference. Professor Ryan determined his critical points by the criterion of luminosity. There might have been, as suggested by Mr. Floy, an error in this method of determination, due to the personal equation, and to the conditions under which observations were made. But there is another possibility. Luminosity would require a certain definite amount of loss; that is, it would be necessary to be a certain distance above the critical point before luminosity would occur, and it may be that it is necessary to go quite appreciably beyond the critical point before the wires are luminous. There are two other considerations also which might, in part, account for the difference. One of these, touched upon by Dr. Steinmetz, and which I shall refer to again, is that Professor Ryan in his experiments undoubtedly realized the condition of a conductor having a uniformly smooth surface a great deal closer than it would be possible to realize this in a transmission line. The other is that Professor Ryan's experiments were made under such conditions as would give him as nearly uniform density of charge on the surface of the conductor as it is possible to obtain. Now, if instead of making the measurements between a conductor and a cylinder which surrounds it, they are made between two conductors, there is not a uniform distribution of charge on the surfaces of the two conductors. It is true that when the distance between the conductors is great as compared with their diameters, the charge is very nearly uniform, but, nevertheless, it is not exactly uniform, the density being very slightly greater on those portions of the conductors opposed to each other. This difference of charge means an increased rate of fall of potential, which increase, though very slight, would aid somewhat in lowering the critical point, and, therefore, in the discrepancy in results.

Dr. Steinmetz has said that the critical point as determined in my paper is not the true critical point, for the reason that the conductor and surrounding conditions are not absolutely uniform; that the first beginning of a brush discharge will be at some isolated point, some kink in the conductor, and that the voltage at which this occurs cannot be considered as the voltage where the breakdown strength of the system has been reached all over the conductor. The point Dr. Steinmetz raises is a very interesting example of the difference between engineering investigation and scientific investigation. The critical point I have determined in my paper is the engineering critical point of a transmission line, and is the critical point with which transmission engineers are concerned. The critical point of which Dr. Steinmetz speaks is the scientific critical point, and while it is very desirable to have it, after it has been obtained, it will be impossible to make use of it in transmission practice until there has been determined the corrective factor necessary to account for the "kinks" and "surrounding conditions" which will exist in practice. I quite agree with Dr. Steinmetz that it is very interesting and very desirable to have the scientific critical point, but I maintain that for transmission purposes the critical point as I have derived it is the one most immediately desirable and which we must have in any case. It is possible for us to get along without the scientific critical point, but the engineering critical point is absolutely necessary. Later on, I expect to continue this work, and at that time I shall endeavor to obtain the scientific critical point.

Dr. Steinmetz also takes exception to the method I have employed for determining the critical point; that is, by extending the lower limb of the loss curve and finding the point at which the upper limb departs from it. He says that both branches of the curve may theoretically be expected to go through the origin, and that if we had the equations of the lower branch in Fig. 38 and of the upper branch, the point of intersection would be the critical point. I do not see how these statements can be correct. The whole theory of critical voltage, as I understand it, is that the equation of the upper branch does not go through the origin; that the phenomena to which the upper branch is due begin more or less abruptly at the critical voltage. If this be the case, the critical point is the point at which the equation of the upper branch would cut the axis of voltage and is, therefore, in the curve formed by the combination of the upper and lower branches, the point of departure of the upper from the lower branch, and not, as stated by Dr. Steinmetz, the point of intersection of the equations of the upper and lower branches. Resort to the device suggested by Dr. Steinmetz, of plotting the logarithms of the ordinates, does not lead to satisfactory results. The upper portion of the curve obtained is not a straight line, so that if the endeavor is made to determine the critical point from this derived curve, it can be done only by resorting to the same method that I have employed; namely, that of determining the point of departure of the upper from the lower limb. In the derived curve, this method is more difficult of determination than in the original curve, because the rate of departure is less.

As regards the method of measurement used and the remarks made thereon by Mr. Thomas; I would say that while I have not used the particular method he mentions, I have made use of some methods of measurement involving a wattmeter in the high-voltage circuit, and found them extremely unsatisfactory. The chances for error due to electrostatic effects and other effects incident to the use of high voltage directly, are so great, especially when measuring losses as small as those concerned and at the very low power-factor at which they occur, that the use of instruments in the high-voltage circuit does not commend itself to me. On the other hand, having used the present method of measurement in the Telluride investigation, and having made use of it again in the Niagara work with entirely different apparatus, I have come to feel, as the result of such check measurements as could be made, and in view of the compatibility of the results obtained, that this method is absolutely reliable. In addition, it has to commend it the fact that one is working in a low-voltage circuit instead of a high-voltage one. I would say in reply to Mr. Thomas's question, that I have some data for losses which occur under rain conditions, but they are not definite enough to justify their presentation. During a rain storm or a snow storm the wattmeter continually varies with little jerky kicks. Mr. Thomas notes that the "vapor product" curves for the insulators are straight lines, while those for the loss between conductors are curved. As stated in the paper, the loss lines for the insulators would probably have been curves also if the readings had been carried to a higher vapor product. This was not done, because of the great expenditure of time that would be involved in waiting for weather conditions which would make possible measurements under the higher vapor product conditions. Mr. Thomas also refers to Professor Ryan's method of determining the critical point of a loss curve, and thinks that this might in part account for the discrepancy between my critical points and those of Professor Ryan. While there would, undoubtedly, be a discrepancy in the critical points determined in these two ways, this would not, I think, account for the discrepancy which exists, for the reason that the critical points finally given in Professor Ryan's paper were obtained not from loss curves—as I remember it he took no loss curves—but from his observations as to critical point as determined by luminosity.

Mr. Lincoln inquires as to the influence which the potential of the ground relative to the wires might have on loss, with particular reference to the losses on a three-phase circuit, as compared with those on a single-phase circuit. My measurements were all made on a single-phase circuit with the neutral grounded, so that I am unable to reply to Mr. Lincoln's question. I hope, however, to be able to supply this information later, as the three-phase loss is one of the matters which I have in mind for future investigation.

Replying to Mr. Scott's remarks, as to the bearing which these new critical points would have upon the conclusions arrived at in the paper which I wrote for presentation at the International Electrical Congress, I would say that the economic voltages of the International Electrical Congress paper are all well below the critical voltages of the present paper. I agree with Mr. Scott that in the use of higher voltages there may be a very marked difference in favor of aluminum, because of greater diameter of conductor corresponding to a line of given resistance.

Ralph D. Mershon (by letter): There is one point in Professor Thomson's discussion with which I do not agree, and which I feel sure is in error. He assumes, in discussing Conclusion 19, that the only insulator loss which could vary with frequency would be the hysteresis loss in the body of the insulator itself. I do not consider that this would be the case, and my attitude in regard to this matter will be clear from what follows.

The losses which, as I take it, occur in connection with an insulator, are as follows:

a. I^2R loss through the substance of the insulator itself. This is the same sort of loss that would occur with direct current of the same effective voltage.

b. Hysteresis loss in the substance of the insulator itself, due to the variations of dielectric stress which occur with alternating current.

c. I^2R loss in the film on the surface of the insulator, similar to that which would occur with direct current.

d. I^2R loss in the film of the insulator due to the charging or condenser current taken by the insulator.

I think there can be no question that film losses or something equivalent thereto exist. The fact that insulator losses are different for different hygrometric conditions is proof of this, so that there will probably be no dispute as to the existence of the loss (*c*). That there should be a loss of the nature of (*d*) would appear from the following consideration. The several petticoats of the insulator and the different portions of these petticoats are undoubtedly at different potentials. The result is that between the various petticoats and portions of the same and between the petticoats and the insulator pin there is a condenser action; that is to say, the petticoats are analogous to the plates of condensers in series. To make the analogy more nearly exact, we must consider that between the condensers in series there are resistances because the charging current to the various portions of the petticoats must come to these portions through the resistance of the surface film of the insulator. Thus, the charging current to the insulator will cause I^2R loss in the film on the surface of the insulator. Now, the magnitude of this charging current will depend upon the frequency, and the I^2R loss due to this charging current will, therefore, depend upon the frequency, so that the insulator loss would be affected by frequency, even though the hysteretic loss, or loss (*b*), were zero. It was not loss (*b*) that I had in mind in Conclusion 19, but rather loss (*d*). I do not, therefore, agree with Professor Thomson that "if the insulator dielectric were free from all dielectric hysteresis, we should not expect Conclusion 19 to hold." Aside from the above theoretical consideration, that there is a loss of the nature of (*d*) is evidenced, I think, by the fact brought out in the paper that of the various insulators measured, some of the larger ones had the higher losses. We would expect that the larger the insulator, the smaller would be losses (*a*), (*b*) and (*c*), but that loss (*d*), if it exists, would be greater, since the charging current would increase with the size of the insulator, and the increase would be especially marked in insulators whose petticoats were approximately parallel to each other and to the pin. As pointed out in the paper, the measurements seem to bear out these two considerations, and it was because of this fact, and because it seemed to me that the loss in the substance of the insulator would be small, that the difference in the losses shown by the two curves in Fig. 32 was assumed as mainly due to loss (*d*). It has been suggested that this assumption is in error and that a large part of the difference in loss shown in Fig. 32 may be due to (*a*) or (*b*) or both of them.

Since the presentation of this paper, Mr. C. E. Skinner has very kindly had made for me some measurements bearing upon the ohmic resistance of an insulator similar to that to which Fig. 32 applies. He found that with a galvanometer connected in series with the insulator and with 500 volts direct current impressed upon the combination, no indication was obtained, although the galvanometer was such as would, on 500 volts, give an indication with 500,000 megohms in series with it. It would seem, therefore, that this definitely settles the question as to I^2R loss in the substance of the insulator, to the effect that such loss is practically zero. As to the hysteresis loss, I am not in a position at this time to make any measurements bearing upon it, but expect to do so later. Meanwhile I have no reason to believe that any large portion of the difference in loss shown by the two curves in Fig. 32 is due to hysteresis.

I do not see the force of the comment made by Mr. Waters. Even if the insulator were a perfect condenser with no loss, a pin would increase the insulation strength, if it had sufficient resistance, and would also be a source of loss. The fact that the insulator is not a perfect condenser is shown by the measurements upon it. The loss is mainly, if not entirely, over the surface of the insulator. Possibly Mr. Waters has been misled in considering Fig. 37 by having overlooked the fact that, as stated in the paper, the pin, with which the curves of Fig. 37 were obtained, had to be wetted in order to make the measurements.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 30, 1908.

FUNDAMENTAL CONSIDERATIONS GOVERNING THE DESIGN OF TRANSMISSION-LINE STRUCTURES

BY D. R. SCHOLES

Before the work of designing a tower or pole for a given transmission line can proceed, a statement must be made setting forth the loads which the structure should be capable of withstanding. This statement is, in general, based on a forecast of the probable extreme weather conditions which may occur in the vicinity of the line, and also on a prediction as to what accidents will probably occur to the conductors of the line.

There is naturally considerable variation in forecasts of this sort and this variation is due primarily to a lack of accurate data regarding the various factors which enter into the case. The cost of a line is affected very largely by the figures which are selected to represent the probable extreme conditions, and the selection and application of these figures is, therefore, a matter of a good deal of importance. Unfortunately, data on this subject are very meagre, and a rational solution of a problem involving weather conditions and possible accidents is manifestly impracticable. It seems, therefore, that the best guide in selecting figures to represent the probable extreme load conditions is experience with existing transmission line structures and other structures similar to them.

During the last few years many members of the Institute have had occasion to investigate this subject, in preparing specifications for transmission lines. A discussion referring to experience with these lines and bringing out the ideas of each as to what load conditions should be provided for would be very beneficial. It is hoped that there will be such a discussion following this paper.

Figures must, in general, be selected to represent the forces which may come upon a transmission-line structure as a result of one or more of the following influences:

Wind.
Sleet.
Low temperature.
Accidents, as breaking of cables, etc.

It is also necessary to select a factor or factors of safety for use in connection with these figures, and a prediction must be made as to whether or not loads resulting from two or more of these causes are likely to occur at the same time. Considerations of cost often determine to what extent provision shall be made, in a given line, against such combinations of extreme conditions. In a very important line it may properly be considered desirable to provide strength against a combination of conditions likely to occur only once in a hundred years, whereas in a less important line the possibility of such a chance condition might, with equal propriety, be neglected.

Wind pressure on structures. The records of the weather bureau are available as an aid in estimating the maximum wind velocity to be expected in a given locality. The relation between wind velocity, however, as indicated by a government anemometer, and the actual pressure in pounds per square foot produced by a wind of that velocity on a cable or on the members of a tower, is by no means definitely known. In fact this relation is so uncertain that the most one can hope to gain from an examination of the weather reports is a general idea as to whether the winds occurring in a given locality are likely to be high or not. The anemometers of the weather bureau do not take account of sudden gusts of wind. The published velocities are not accurate, but must be corrected according to a correction table which may be obtained from the weather bureau.

The relation between wind velocity and the pressure produced by the wind on a plane surface normal to the direction of the wind is given by the formula,

$M = K V^2$, where
M = pressure in lb. per sq. ft.,
V = wind velocity in miles per hour, and
K = constant.

Experiments in general indicate that the form of this equation is correct, but experimenters differ as to the proper value of K. The values given range from 0.0035 to 0.0048. According to tests by the weather bureau, $K = 0.004$, which is probably the most reliable figure there is for K.

Experiments indicate that, in general, higher pressures are to be expected at the top of a tower than near the ground, but little is known as to how the pressure is distributed. There is considerable doubt as to what should properly be considered the exposed area of a structure; it is certain, however, that both faces are not, in general, subject to the same pressure. It is usually considered that a reduction factor of 0.5 should be used in figuring the wind pressure per square foot of projected area of cylindrical surfaces. The wide use which has been given this factor is its principal recommendation.

The purpose of the foregoing remarks on wind pressure is to point out some of the reasons for uncertainty in wind pressure calculations. In view of these uncertainties, it seems necessary to turn to some empirical method for providing against loads due to wind pressure. In bridge work pressures of from 30 to 50 lb. per square foot are commonly assumed, and these pressures are used in connection with factors of safety of from 4 to 6. Structures built to withstand loads calculated in this way are found to be strong enough. How much too strong they are is a matter of conjecture. The usual transmission line cannot stand the expense of structures built to bridge specifications. Experience with bridges cannot, therefore, be of much help in the present connection.

Steel windmill towers have been in general use for about eighteen years. Such towers are built to withstand wind loads almost exclusively and their use is very widespread. It is known that the provision against wind loads in these structures is not excessive, for there are occasional failures. The windmill tower is, in general, similar to the towers used in transmission lines. The success of a given design of windmill tower depends on what might be called the integrated experience of all the users of towers of such design. Competition has led builders to reduce their weight to a minimum. It is probable; therefore, that a windmill tower of standard design which is widely used has just about enough strength to resist the highest winds, tornadoes excepted, and it would appear that a study of such a windmill tower will probably give the best data available for use in connection with transmission line structures.

An examination of a standard design of windmill tower, of which many thousands are in use, shows that such tower will actually fail under loads calculated on the basis of wind pressures of from 40 to 50 lb. per sq. ft. The tower referred to is of square pyramidal form, and in the calculations it is assumed that the wind is blowing at right angles to one side, and both faces of the tower are considered equally exposed.

It appears, therefore, that it would be good practice in transmission line construction to specify that the poles or towers should, in addition to their other properties, have strength to resist loads on their members due to a wind pressure of 40 lb. per square foot, with a factor of safety of from 1.5 to 2, based on actual test. Such a structure would be suitable for locations where the winds are high; in other locations these figures would be reduced by judgment, aided by a consultation of the weather reports and other such data.

Factor of safety. A few remarks regarding factor of safety may be proper at this point. The factor of safety used in connection with the design of a given piece of engineering apparatus, is, in a sense, a measure of the uncertainty attending the making of calculations of the loads to be sustained or of the strength of the structure under consideration. In designing a complicated structure to sustain a complex system of loads, it would be natural and proper to allow a large factor of safety, particularly if the structure were such that it could not be tested to destruction to check all calculations and methods. On the other hand, a smaller factor of safety would be equally safe in connection with a simple structure to sustain certain definite loads, the actual ultimate strength of the structure having been determined by testing it to destruction.

The structures ordinarily used in transmission lines are simple. They are usually built in large numbers from standard designs. It is proper, therefore, that the design for such a structure should be carefully investigated and that specimen structures should be tested in such way as to remove all doubt as to their ability to withstand the loads for which they are intended. And, notwithstanding the fact that calculations of wind pressure are uncertain, experience with windmill towers removes, to a large extent, the uncertainty which would otherwise surround the figure 40 lb. per square foot which has been suggested.

Wind pressure on cables. The opinion is commonly held that, in providing against wind pressure on a surface such as that of a long span transmission line cable, it is not necessary to allow for as high a pressure as is necessary for a surface extending through smaller linear distances. Data on this subject are, as yet, very indefinite. and there is great need of specific figures for the pressure experienced on the cables of a transmission line. The following experiment is suggested as a means of securing such data.

The experimental apparatus would consist of a typical transmission line span of from 500 to 1000 ft., erected as near as possible to a weather bureau station. The cable would be fixed to the tower at one end and would pass over a pulley at its other end and be secured to a weight, this weight serving to maintain a uniform tension in the cable at all times. The position of the weight would be recorded at all times by means of a pencil and moving drum. Continuous records of temperature and wind velocity are made in the weather bureau stations. An analysis of the three records; namely, those of temperature, wind velocity, and the length of the cable in the span, would give data from which the wind pressure in pounds per square foot of projected area of the cable could be calculated. It would also be desirable to have a continuous record of the direction of wind, and this record could be readily obtained.

Records from such apparatus extending over a period of a year or more would be of much interest. It is to be observed that the readings would furnish a means of checking the coefficient of expansion of the cable. Data obtained in this way would have direct relation with the weather bureau reports, and most questions as to methods of calculation of pressure on conductors would thus be eliminated.

In the absence of specific data relating to wind pressures on the cables of long spans, it seems unsafe to assume a pressure of less than 30 lb. per sq. ft. for localities where the winds are known to be high. The figure 30 lb. per sq. ft. is commonly used in bridge calculations for surfaces extending through horizontal distances of 60 ft. or more. It seems that a factor of safety of 2 should be used in connection with this pressure, so that the conductor will not be stressed beyond its elastic limit, under extreme conditions.

Sleet. Destructive sleet storms occur in the eastern part of the United States at least as far south as Atlanta. During the past winter, a sleet storm occurred in the region of Chicago after which a coating of ice over half an inch thick was observed on conductors of various sizes. In many cases the thickness of ice at the center of the span was much greater than at the insulators, due to the tendency of the water in the sleet to run down to the lowest point while freezing. The sleet formed during this storm was practically solid ice, and it remained on the conductors for several days. In view of observations made after this storm, it is the writer's opinion that, for localities where sleet is known to form, provision should be made against a coating of ice on the cables at least one half inch thick, in combination with a factor of safety of not less than 2 based on the ultimate strength of the conductor.

There is much discussion as to whether the safety of a line demands that provision be made against sleet, low temperature, and high wind all occurring at the same time. If the sleet forms at all, it is certainly possible that it will remain on the wires several days. And if it remains on the wires several days it is certainly entirely within the range of possibility that high wind or low temperature or both will occur before it melts off. Whether or not provision should be made against a combination of these three extreme conditions becomes, therefore, entirely a question of how much the owners of the line are willing to pay for immunity from interruptions of service due to these causes. These are matters to be settled between the engineer and the owner of the line.

Accidents, as breaking of wires, etc. In providing mechanical strength in the line to resist loads due to accidents to the cables, there are two well known plans which may be followed. In the one all structures are given the same strength, each having strength to withstand the loads due to accidents which it is contemplated may occur in any span; in the other plan, two kinds of structures are used—a standard structure intended to support loads transverse to the line only, and a heavy structure having strength against the breaking strength of all the cables. These heavy structures are distributed along the line at intervals of a mile or so. The first mentioned type of construction is best adapted for lines having relatively small conductors, while the second type is favorable where the conductors are heavy.

In designing a line of the first mentioned type it is usual to assume that any two conductors may break in a given span due to the formation of an arc between them, and that the tower or pole should be capable of withstanding the loads so developed without damage to itself. Provision is not, in general, made for the simultaneous occurrence of such breakage and high wind or sleet. It would seem, however, that in such cases the towers might better be designed to withstand wind loads transverse to the line in addition to the loads due to the breakage of any two conductors, since arcing is more likely to occur in a high wind than at other times.

It is believed that the factor of safety used in connection with the loads due to breakage of conductors should be greater in the case of suspension type insulators than with pin insulators. When a conductor supported by suspension insulators breaks, it will suddenly move away from the point of breakage and will be brought to a sudden stop when the insulator comes into line with the cable. The movement will occur, in decreasing amount, all along the line, or until a strain insulator is reached. This sudden application of load and the attending inertia effect

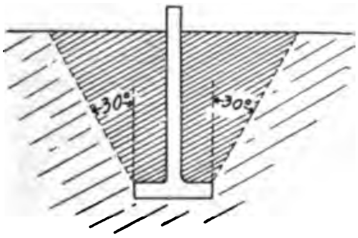


Fig. 1

will subject the cross arm to a greater force than the tension which existed in the cable before it broke. It is suggested that cross arms be tested to loads equal to 1.25 times the elastic limit of the conductor for pin insulators and 1.5 times the elastic limit of the conductor for suspension insulators.

Foundations. It is a usual assumption that the resistance to uplift offered by a foundation is equal to the weight of the foundation plus the weight of earth contained in the frustum indicated in Fig. 1, the angle of inclination of the sides of the frustum being 30°. The results obtained by this method agree quite closely with practice in anchors for windmill towers. In addition to resisting uplift, the foundation must, in general, have strength against horizontal forces at the ground line. The variety of designs of foundations is so great as to make a discussion of them impossible within the limits of this paper. It is suggested that, in developing the design of a foundation for a given line, tests should be made to determine the holding power, density, etc., of the soil of the locality so that the strength of the foundation will be known as accurately as the strengths of the other parts of the line.

DISCUSSION ON "FUNDAMENTAL CONSIDERATIONS GOVERNING THE DESIGN OF TRANSMISSION LINE STRUCTURES."
ATLANTIC CITY, N. J., JUNE 30, 1908

N. J. Neall (by letter): It may properly be said of a transmission line structure that factors of safety are both necessary and expensive. It is naturally desirable, therefore, so to shape the design and so to adjust it to its operating requirements as to secure a maximum of service with a minimum of outlay.

There are two factors of safety that must be considered: 1. That relating to the transmission line conductors directly. 2. That of the tower itself as an independent structure. These will be discussed as follows:

1. Factors of safety relating to transmission line conductors. Once the amount of power to be delivered has been determined, together with the location of definite geographical points of distribution, a fundamental size of transmission wire can be deduced to carry the load economically. This should then enable a decision as to the permissible length of maximum span and an adjustment between conductor size, span, and cost of support (substituting other sizes of conductor if necessary) until the best proportions of cost have been obtained.

The next step is to decide whether the tower is merely to act as a support for the wires without further responsibility, or to hold them up against any ordinary accident; in other words, whether it is to be what is commonly known as a "dead end" structure. If the latter, a much stronger pin support is required and may, because of the generally low natural limit of strength of such a feature, require several or more such supports in tandem and special arrangements for holding the wires to the supports.

2. Inherent structural characteristics affecting factors of safety. Theoretically it is desirable to have a tower or transmission line structure flexible in the direction of the line, and inflexible perpendicularly thereto. It is assumed that any wind blowing parallel to the line does it no harm, while a high wind perpendicular to the line is a serious feature; in other words, the tower must be able to resist being blown over. This consideration immediately affects the shape and size of the tower, to which, in turn, such items as wind pressure per square foot, form of anchorage, nature of soil, etc., add their quota of requirements.

For such a structure pure and simple, the requirements of endurance are obvious and of little concern to the electrical engineer, save that the character of the design will permit of the special loads being applied without any impairment of the structure as such.

It is but a step, then, to combine these two elements; in the usual specification the loads brought on the tower by the transmission wires are the only ones specified. It is here that I think there is much room for improvement over present practice.

When it is borne in mind that both the tower itself and all conductors are simultaneously affected by any of the usual elemental conditions considered in this connection, such as heat, cold, wind, dead weight, ice or sleet, etc., and since moreover it is found in usual requirements that a safe length of span to stand minimum temperature, with ice or sleet, and wind blowing simultaneously at a maximum velocity determined by local observation to be applicable to this case, is taken at the condition of maximum strain on the conductor, it becomes clear that the only true criterion of suitability of a structure is to test it preliminarily for design in such a way as to imitate as closely as possible just those strains that come upon a tower in service. The two extremes usually taken are the one for

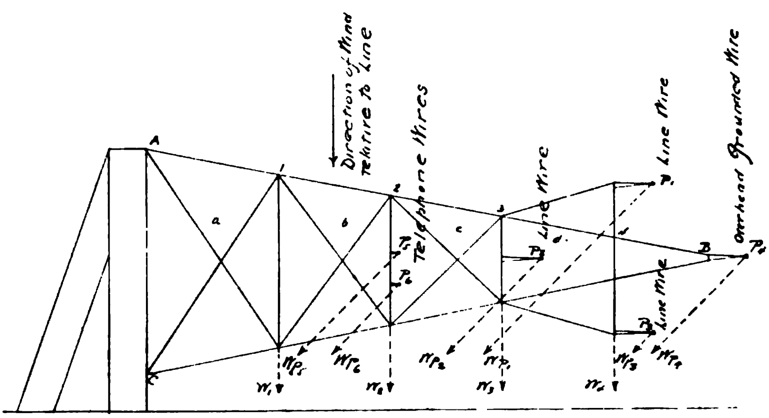


FIG. 1.—Loads on tower for maximum temperature and minimum wind velocity.

minimum temperature, sleet, and relatively low velocity of wind; and the other, maximum temperature and highest wind velocity. With reference to the tower, the former may be considered for the present as less serious than the latter, principally since dead weight is such a large factor in the cold, and sleet condition and structures for transmission lines are thought to carry this more easily than the maximum wind condition.

In order to test the tower, once the line details have been preliminarily worked out, two methods are possible.

First, the tower is mounted on a concrete or other firm foundation and loads applied at various points by means of cables suitably equipped with a dynamometer, or, more conveniently, as follows.

Secondly, the tower fastened horizontally. Let *A B C* represent the structure fastened rigidly to a strong support. Then by calculation the resultant direction and amount of dead weight of wire and wind for the wire when two adjacent spans are in equilibrium can be determined, and the load corresponding thereto can be suitably applied to the tower, as shown by Wp 1, Wp 2, Wp 3, etc. This is the only load brought on the tower by the conductors for this condition of the weather, viz., maximum wind and maximum temperature.

By a proper calculation the total stress on the tower can be determined. For the purposes of this test, these loads are best applied at the girts 1, 2, 3, etc., as shown on diagram, and are indicated by W_1, W_2, etc. For the horizontal position the dead weight of the tower must be taken into account, and W_1, W_2, etc., represent the additional amount to equal the calculated values.

From this it is obvious that any of the conditions applying to tower operation can be properly investigated. For example, one test can be made of "dead end" structures, where the strain in the wire (much in excess of that used as its dead weight and wind exposure pressure per support for towers under the usual service requirements) is appropriately applied at the insulator pins.

Horizontal strength or resistance to twisting from one or more broken wires (known commonly as torsional strength) under any given condition can be easily worked out.

The maximum amount of load which should be applied under these circumstances should depend on the given conditions of the test. I think that an all-round factor of safety of 2 will well meet most commercial requirements, particularly if the test is made for a condition of maximum weather effect.

It is easy to argue that if minimum temperature, sleet, and wind are the worst for the wire and that as a matter of design this strain is made equal to the elastic limit of the conductor, we have roughly a factor of safety of 2 against breakage, coupled with a relatively remote possibility of occurrence. The same basis of selection might apply to the structure as well. The usual specification for a given pull at the insulator pins is only a quasi-wire strain; it is practically a test of the strength of the tower built up to the pin strength as a basis.

Structurally, the insulator pin strength is the weak point of the ensemble; an average pin strength of approximately 1500 lb. to 2000 lb. does not give much leeway in the use of a single pin for dead-end work, nor for a large factor of safety. It is surprising, however, how much can be obtained from a given weight of material if properly disposed.

Some time ago I tested two towers in the manner described, the only difference of importance being the method of holding the diagonal rods *a*, *b*, *c*, and so forth. The builder of the tower employed what I have termed a button and buttonhole method of attachment for the diagonals to the girt. The rod was provided with a button-like end, and the girt was cut button-hole-wise to receive this. In the second tower the diagonals ended in loops and were held by bolts, washers, and nuts to the girts or corner parts respectively, as happened to be. The total weights were not sensibly changed, and yet the latter arrangement showed itself much the stronger.

Strangely enough, the first tower had been successfully tested according to specifications by heavy loads applied to the pins when in an upright position. The second method showed easily how unsatisfactory the other might be as an indication of all-round strength.

The result of all this is a better appreciation of the behavior of the tower under service conditions, and leads directly to the conclusion that in this wise purchasers of towers can obtain a high degree of economy of tower material for a given amount of money.

In these discussions before the Institute cost is more than likely to be treated academically. In no place is it more critical, however, than in the problem of selection of a transmission line support; in fact, despite the many advantages of tower construction over its predecessor—the wood-pole line—the first cost of a metal tower line constantly tends above the wood pole, and of this the tower alone plays a large part, and increasingly so as the factor of safety increases.

With careful selection of design, such a line construction can be kept within reasonable limits; but to this end it is necessary to emphasize the point that not all towers should be dead-end towers; in fact, comparatively few may be so, and the remainder must be something less than this.

Since continuity of service is the final measure of success, in such a case it seems to me that any detail of construction used in this connection should be tested *destructively* for type before acceptance. In my judgment the test herein described is no more burdensome on the tower than standard arcing-over tests, etc., now employed for insulators. Each seeks to determine the ultimate strength of the respective detail under its known possibilities of service.

Ralph Bennett (by letter): The available data on existing transmission lines must necessarily be of a negative character, for a line is built to stand. It does so; therefore it is satisfactory. The margin of security in such a case is difficult, if not impossible, of determination.

A factor of safety of 2 against the greatest combination of known and assumed strains seems inadequate unless the strains themselves contain a large excess allowance. The following uncalculated strains occur to a greater or less extent in every tower: 1. Irregular loading due to settlement of footings and to uneven cable tension. 2. Load due to vertical and horizontal angles too slight to warrant special towers. 3. The results of line and side guying. 4. The racking effect of unequal adjacent spans under temperature changes.

A factor of safety of 2.5 against the extreme load would give a margin between the total calculated loading and the elastic limit of 25% of the load. This should be sufficient to produce a line which would stand in any weather. It is not sufficient to build a line which would just fail under the extreme weather condition which can be expected. A line important enough to warrant steel or concrete towers will warrant sufficient expense to make it secure.

The Kern River Transmission of the Edison Electric Company was designed in 1905 and erected in 1906-7. It has now been in active service slightly over 12 months. Part of the towers have had wire on them for two winters.

The essential features are as follows:

Standard span	700 ft.
Size of cable	4-0 copper, 7 strand
No. of cables	9
Average height above earth	57 ft.
Base at earth	12 by 12
Depth of footing	6 ft. 6 in. extreme.
Diameter of footing plate	24 in.
Type of tower	4 post
Tower bracing	tension rods
Total wind load on wire (22.5 lb.)	6300 lb.
Dead load due to wire	4050 lb.
Extreme longitudinal load (due to to failure of one wire)	3000 lb.

No sleet or snow; no temperature below 20° fahr.; maximum temperature 130° fahr.

At this writing the only difficulty experienced with the line has been due to eagles, which roost on the lower cross-arm between the insulator and the upright. In flapping their wings preparatory to flight they cause a short-circuit from the wire to the structure. In a few cases this has burned the cable off. No tower failures have resulted from this or any other cause. Most of the line has as yet but six cables, and as a consequence there are as yet no data on the effect of side winds. But the absence of three cables renders the effect of cable failures and guying much more serious than it will be in the completed line.

Before issuing the tower and wire specifications an experimental span of 1000 ft. of a single 19 strand 00 copper cable was observed at intervals during about a year. Its performance in respect to temperature change was very closely in accordance with calculated values. The methods of measuring wind proved to be so crude that the results were indeterminate. Other observations were made with rods of various shapes projected from the side of interurban cars. These results were also only approximate. The average of a large number of observations indicated that the pressure on round rods was less than six-tenths of that on thin flat rods of the same projected area for any velocity up to 30 miles per hr.

The method of figuring footings given by Mr. Scholes is based on the angle of repose of loose sand. This can scarcely apply in firm or sticky or flowing soils. Most tower footings are in soils which are at least self-supporting. Such materials as gravel and stiff, dry clay are very slow in recovering their original firmness, even if tamped thoroughly. Meanwhile a footing placed in a straight-sided hole is liable to lift the loose dirt out along the line of disturbance with a much less load than that calculated by any formula based on an assumed cleavage line. If, however, the earth is firm enough to permit of enlarging the hole at the bottom so that the sides overhang the footing plate, the completed footing may hold more than the calculated amount, for the angle of repose of such a soil in a massive body such as that surrounding the foundation may be much greater than that of sand.

In setting guy-plates no lineman would think of so placing them as to bring the strain against the disturbed earth. In ordinary pole-line construction the disturbed earth is subjected to direct compression only, yet the only valid reason for the deep holes commonly used is to bring the point of maximum loading on this earth so low that it will be held from flowing under the pressure by the firm earth surrounding it.

Where economy would demand the minimum cost to such an extreme as to render the skimping of footings necessary, would not the use of guyed towers instead of the self-supporting types result in a more reliable construction? Such towers would be much lighter; they would occupy less room at the base; they could be so constructed that the longitudinal guys would also take the side strains; and all earth plates would be against firm, undisturbed earth. Since the variation in load with length of span affects the wind load as much as the dead load, and as any tower which will stand the stress of erection will stand the dead load, the towers could be adjusted to their loads by changing the number or size of the guy wires. The total weight would be low, as would the erection cost. The parts would be stiff enough to stand erection strains without the elaborate precautions taken on one or more of the lines recently noted in the technical press. It seems almost a truism to say that in construction of this character the parts should be few and rugged so that the money saved on shop cost need not be lost in the field in the handling of fragile members or the locating of a large number of small pieces.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., June 30, 1908.

THE TESTING OF HIGH-VOLTAGE LINE INSULATORS

BY C. E. SKINNER

The specification herein proposed as standard for the testing of high-voltage line insulators was written at the request of the chairman of the High Tension Transmission Committee in order to bring this matter before the Institute for discussion. It is not presented as representing the writer's personal opinion so much as an endeavor to harmonize information which he has obtained from various sources with the view of producing such a specification. Quite a large number of porcelain manufacturers and others interested in the testing and use of high-voltage porcelain line insulators in this country and in various parts of Europe have been consulted, and the information received is embodied as far as possible in the proposed specifications. It is fully appreciated that differences of opinion may exist on any point which is incorporated in such a specification. The fact that such differences do exist, and that tests are so different in different places, seems to the writer to be ample justification for the attempt at a specification which can be used by all as a standard of reference.

At the present time it is almost impossible for one familiar with a certain method of testing to base any judgment whatever as to the bearing of results obtained by some one else who uses a different method. If, after thorough discussion, and any revision which may be found essential, a specification is produced which will allow direct comparisons to be made of the performance of insulators of different types tested in various places and at different times, the object of the specification will have been in part accomplished. If a specification is evolved which can be accepted as a standard performance specification for line insulators in general, the uncertainty regarding comparisons of tests at various places and the widely varying requirements now insisted upon for conditions which are practically the same, will have been eliminated.

The specification naturally divides itself into three general parts. The requirements of routine tests are placed first, as it is considered that any tests of a routine nature which are required on insulators for the acceptance of any lot should naturally form a part of the tests made to determine the limitations of design.

The question of routine inspection for mechanical flaws and other defects is difficult to outline definitely, as a complete description of all points which might constitute cause for rejection would make the specification unduly long and complicated, and it is therefore usually considered better to leave this to the judgment of the inspector.

Some difference of opinion has been expressed as to whether the dielectric test on each individual part of insulators made up of parts should be included in the routine test instead of in the design test. It is the writer's understanding that such tests are invariably made by the porcelain manufacturers for their own information, whether specified as a part of the routine test or not, and it would therefore seem that there should be no very great objection to their being included in the routine test.

It is possible that in some designs the voltage tests specified cannot be met by certain shells which are used next to the pin for the purpose of increasing the dielectric strength when the other parts of the insulator are wet, due to the short surface distance. If the surface distance on the short shell is so small that this test cannot be reached, it is probable that the object of inserting the short shell is in some measure defeated in the particular design. It might be possible to substitute a fixed test for each individual part of a shell independent of the voltage on which the insulator is to be used, as it is difficult if not impossible to get porcelain to stand tests of above 60,000 to 70,000 volts, regardless of the thickness. The better material in the thinner shells gives results equal to the poorer material in the thicker shells, due to shrinkage cracks and other defects which it is ordinarily impossible to eliminate in thick porcelain. The provision to exclude insulators which show excessive local heating is inserted for the reason that those familiar with such tests may at times be perfectly sure that an insulator is unsatisfactory even though it is not punctured. The term "localized discharge" as used does not mean the discharge which occurs uniformly around the insulator at the point of contact, but a discharge at some point on the surface. Such discharge sometimes indicating a spongy material which will eventually give trouble.

Under design tests the amount of pull to be applied to the insulators is not specified for the reason that the strength must necessarily depend on the particular design, and the figures to be inserted of course should be agreed upon in each case between the manufacturer and the user.

The rate of precipitation specified in the rain test is probably greater than will ever be experienced except in very excessive storms, and even then only for a very limited time. The rate specified is less than that used as standard in some parts of Europe. A fairly wide limit of variation in rate is allowed, partly on account of the great difficulty in securing a perfectly definite and uniform rate, and partly from the belief—borne out by rather limited tests—that little difference in the results of tests will be obtained between the limits specified. It is desirable in the discussion of this paper that as much information as possible be brought out relatively to the best possible method of obtaining a satisfactory spray.

The requirement that the insulator shall be tested with the pin at an angle rather than to attempt an angular rainfall is given for the reason that a satisfactory method has not yet come to the writer's attention for the obtaining and maintaining of a satisfactory angular rainfall, under the conditions which usually obtain where insulators are to be tested. It is far easier to incline the insulator and use a vertical precipitation, and it is considered far more probable that results can be repeated than by providing for an inclined precipitation.

The use of a rain gauge for determining the rate of precipitation is specified, as the writer has found it nearly impossible to get the rate of precipitation by measuring the flow of water through the supply pipe, which is frequently done. The diameter is limited to three inches for the reason that a larger gauge would disturb the distribution of the spray to some extent. The type of rain gauge illustrated herewith is suggested as convenient and as having been found satisfactory in tests where it has been used. The diameter of the funnel is so chosen that the precipitation for any elapsed time measured in cubic centimeters and divided by 100, gives the rate of precipitation in inches for that elapsed time. By means of the valve at the bottom the water may be run into a standard chemical burette, and quite accurate measurements made. In use the rain gauge is inserted upside-down into the spray, and then quickly turned to the upright position when the location to be measured is reached. A suitable card or other covering is then quickly placed over the top at the end of the elapsed time. It would,

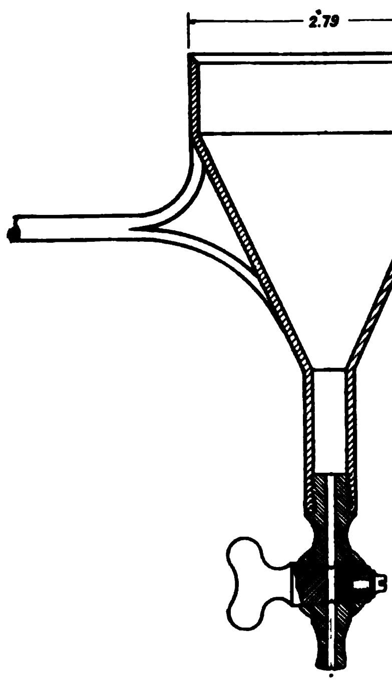


FIG. 1—Special rain gauge

of course, be easy to provide a lid which could be opened and closed, in place of the method specified.

The writer would lay special emphasis on the importance of the dew test as probably determining the most severe condition which the insulator will ever be called upon to meet in practice. Also the fact that such a test is more nearly capable of exact repetition than any precipitation test. Furthermore, in such a test the moisture on the surface of the insulator must be perfectly clean water if the insulator itself is clean, and therefore is more like natural rainwater and eliminates any possibility of variations due to the quality of the water used.

Proposed Standard Specifications for the Testing of High-Voltage Line Insulators

General. This specification is intended to provide a standard method of making tests on porcelain insulators or their equivalent which may be designed for use on transmission systems of 6000 volts or above. The specification is intended to provide a means of determining the performance of any insulator, and is not intended to restrict design in any way whatsoever. The specification is divided into three parts as follows:

1. *Routine tests*, or tests to be made on each individual insulator, to show whether or not workmanship, materials, and dielectric strength are up to the required standard.

2. *Design tests*, or tests to show the limitations of a design under a specific set of test conditions.

3. *Methods of testing.* The methods to be followed in making the various tests specified, are separated from the body of the specification as a matter of convenience for reference.

1. Routine Tests

a. Inspection. Each insulator shall be inspected to see that it is reasonably free from mechanical flaws, defects of glazing and cementing, chipping, etc. Those parts of the insulator which are to be fitted to caps, pins or other fastening devices shall be sufficiently close to designed dimensions to insure first class work in assembling and mounting. The general over-all dimensions shall not vary more than plus or minus 5 per cent. from the designed size.

b. Dielectric tests. When tested dry each shell of insulators of the cemented type and each unit of insulators made up of units, shall withstand for a period of 5 minutes three times its proportion of the line voltage, based on the total number of shells or units of which the insulator is composed. (For example, each shell of a four-part insulator shall withstand $\frac{3}{4}$ times the normal line voltage on which the insulator is to be used, for a period of five minutes.)

When tested dry each completed insulator shall withstand for a period of five minutes 2.5 times the line voltage on which it is to be used.

If any shell or any insulator shows excessive localized discharge without puncture, the test on same may be continued for two additional periods of five minutes each. Excessive local heating or excessive localized discharge shall, if continued, be considered a failure.

Insulators not to exceed 5 per cent. of any lot shall be tested for flash-over by raising the voltage gradually, or by steps of not more than 5 per cent., until flash-over occurs. If there is failure by puncture of more than one-half of those so tested, the flash-over test may be required on all insulators of the lot on order.

2. Design Tests

Tests to determine the limitations of any particular design are to be made on a few insulators, not more than 5 per cent. of any particular lot on order.

a. *Mechanical tests.* Insulators mounted on pins shall withstand a side pull exerted on the tie groove at right angles to the axis of the pin, of ... pounds. Cemented insulators and insulators made up of units shall withstand a direct pull along the axis of the pin, or equivalent, of ... pounds the force being exerted between the crossarm and line fastenings.

b. *Routine tests.* Insulators must successfully withstand tests under the heading of routine tests.

c. *Rain tests.* In addition to the routine dielectric tests, the insulators shall withstand the following dielectric test when subjected to artificial rain. With a vertical precipitation of not less than 0.3 in. per minute and not more than 0.4 in. per minute, the insulator in normal position mounted on a crossarm or its equivalent, with pin with which it is to be used, the complete insulator shall withstand for five minutes 1.5 times the normal voltage of the line on which it is to be used. With the same rate of precipitation and with the crossarm so turned that both the crossarm and the insulator pin are at an angle of 45° to the vertical, the complete insulator shall withstand for five minutes 1.25 times the normal voltage of the line on which it is to be used

d. *Dew test.* With the insulator cooled to 0° cent., or below, and then placed in a moist atmosphere of 30° to 40° cent., it shall withstand 1.25 times the normal line voltage after the insulator has become thoroughly covered over its entire surface by the condensation of moisture from the atmosphere.

3. Methods of Making Tests

a. *Mechanical test.* The strength test of the insulator may be made by any suitable means of obtaining the specified pull. For this test pin insulators should be mounted on the pin with which they are to be used in practice, and a heavy copper wire or cable looped in the tie groove in such a way that there will be no injury exceeding that which would occur from the application of the standing tie. Insulators of other types shall be tested by having the pull exerted between the mounting intended for the crossarm and the line wire.

b. *Dielectric tests. Dry test.* The surface of the insulator shall be clean. The test on pin type insulators shall be made by placing the insulator upsidedown in a pan of water to a depth just sufficient to cover the tie groove or equivalent. This pan with water forming one testing terminal should be as small in diameter as possible and so arranged that the striking distance over the surface of the insulator is not reduced. Water shall be placed in the pin-hole of the insulator, covering that portion of the insulator which would come in contact with the pin or equivalent. When insulators have metal thimbles placed in the pin-hole or are mounted complete with metal pins, the thimbles or pins may be used in place of the water in the pin-hole. Connection to the water in the pin-hole forming the other testing terminal shall be made by means of any suitable metallic conductor, which must be so placed that it is central with the pin-hole and extends far enough above the insulator so that it will not shorten the striking distance from the wire groove to this conductor. In testing the shells of insulators made up of concentric shells, tests shall be made from a pan of water in which the insulator is placed upside-down as one terminal, to water placed inside the shell as the other terminal, the depth of water in the pan and in the shell being so arranged as to cover that part of the insulator which will be in contact with the cementing material. In testing units of insulators made up of units, the testing terminals shall consist of the metal mountings or their equivalent, with which the insulators are to be used in practice.

Rain test. The rain test shall be made by mounting the insulator on a metal pin or equivalent so arranged that it may be placed either vertical or at an angle of 45° to the vertical. Clean water shall be used and the precipitation shall be such that the water falls in a fine spray and in practically a vertical direction and at a rate of from 0.2 in. to 0.3 in. per minute, over the area formed by the vertical projection of the insulator. The rate of precipitation shall be obtained by the use of a suitable rain gauge of not more than 3 in. in diameter.

Frequencies. Dielectric tests shall be made at the standard frequencies of either 25 or 60 cycles per second. Any frequency between 25 and 60 cycles will be considered as meeting the specification. Lower or higher frequencies will be considered as special.

Voltage control. When only a very limited number of insulators are tested in parallel, the test voltage may be taken from a constant potential source and applied directly or it may be raised to the required value gradually. When a considerable number of insulators are tested together, the test voltage shall be raised to the required value smoothly and without sudden large increments and then applied for the prescribed interval. Flash-over tests and other tests requiring variation of voltage shall be made by raising the voltage to the required value smoothly and without sudden large increments.

Discussion on "The Testing of High-Voltage Line Insulators." Atlantic City, N. J., June 30, 1908

Percy H. Thomas: The testing of a high-voltage insulator by application of voltage from a spark-gap or other high-frequency discharge has an extremely important bearing on the type of insulator. A four-part insulator or a three-part insulator, in fact any large insulator, has its resisting power made up of a number of elements all practically in series. The possibility of the concentration of potential on certain parts of these insulators is well recognized. This condition is recognized as greatly reducing the breakdown voltage of an arrester. The feature I wish to bring out is that with very high frequencies this concentration effect is enormously multiplied. I believe that a careful study of this matter will result in some modification of insulator design.

I ask Mr. Skinner if it is worth considering that there will be a difference in local heating, according to the frequency of the test current; for example, a 133-cycle test might show local heating which would not show on a 25-cycle test. Any local heating in a specific insulator is a very important item, as it indicates an inside weakness.

Ralph D. Mershon: Mr. Skinner's paper interests me not so much because it suggests specifications for an insulator but because it recommends certain definite methods of test. There is one recommendation especially with which I heartily concur; that is, make rain tests by using, in all cases, a vertical spray and placing the insulator at a specified angle, instead of endeavoring to imitate rain conditions as regards the angle of precipitation.

Rain conditions are indefinite and variable, so that any assumptions made in regard to them must necessarily cover only one of many conditions. The best that can be done is to assume a maximum rate of uniform precipitation at a definite angle. We know, with a fair degree of accuracy, the maximum rate which will not be exceeded in practice. The angle of precipitation may, however, vary from approximately zero degrees with the horizontal, to 90 degrees. It is usual to assume 45 degrees as a fair compromise. But having made thes assumptions, which can not represent all actual conditions, we are unable accurately and conveniently to produce them. It is extremely difficult to get a water spray to fall at any definite angle, other than 90 degrees with the horizontal, and with uniform precipitation. The result is that flash-over tests made by different observers with different testing equipments, or even at different times by the same observer with the same equipment, are often most discordant.

It seems to me it would be much better to adopt a method of test which, whether it does or does not appear to represent very closely actual conditions, will serve as a criterion and will offer some chance of reproduction, approximately at least, at different times and with different observers. This will be accomplished if the spray be kept vertical and the insulator inclined, for there is a great deal better chance in that case of obtaining a uniform spray at a definite angle with respect to the insulator.

The present method of testing insulators for flash-over conditions under precipitation is very unsatisfactory, so far as repetition is concerned. I have lately made some experiments on a new type of insulator under what was supposed to be a precipitation of one inch in five minutes at an angle of 45°. About a week afterwards another observer made similar tests under as nearly as possible similar conditions, and obtained the same results. About three weeks afterwards the same man endeavored to repeat the results and could not get anywhere near the figures which had been obtained in the first place for flash-over under rain conditions. I have had similar experiences repeatedly and, as the result, have usually, when desiring to compare two insulators as regards flash-over, endeavored to test them together, either simultaneously or by keeping the spray the same and alternately substituting one insulator for the other. The latter is probably the safer method, as more likely resulting in the same precipitation in both cases.

I am disappointed that Mr. Skinner has not gone more fully into the question of the kind of spray nozzle for producing a uniform precipitation. It is difficult to get a nozzle which will give uniform distribution and do so continuously. It would be of great value if Mr. Skinner could tell us where or how to obtain a nozzle which would fulfil the requirements.

The rate of precipitation usually assumed is about one inch in five minutes. The maximum rate of precipitation of which the United States Weather Bureau has any record is less than eight-tenths of an inch in five minutes. Something more than the maximum observed rate is desirable, and it would seem, therefore, that one inch in five minutes would be amply sufficient. I would suggest that this figure, or its equivalent, two-tenths of an inch in one minute, be adopted, instead of the range of precipitation which Mr. Skinner has suggested.

I agree with Mr. Thomas that frequency will, in all probability, have a considerable effect on the flash-over voltage of the insulator; if this is shown to be the case, the insulator should be tested at the frequency for which it is designed. The flash-over voltage will also probably depend a good deal upon the method of voltage control in making the test. The discordant results obtained with different testing equipments are, no doubt, due in part to the fact that the voltage is usually controlled by means of a rheostat in series with the testing transformer. This method of control will, under some conditions, materially modify the wave-form. If the voltage be varied by varying the field of the generator, there is a much better chance for concordant results.

Clayton H. Sharp: In reference to the production of a proper spray or precipitation for use in the testing of insulators, I am having constructed at present an apparatus which has been suggested by an arrangement in use, I believe, in the German Reichsanstalt. This arrangement consists of a large cylindrical vessel, which has a considerable number of little tubes running out from the bottom of it, equally spaced in centers of one and one-half inch squares. The top is closed and has a micrometer valve by which the admission of air can be controlled. On the side there is an ordinary gauge glass to show the level of the water and the rate of precipitation. These tubes will have cotton wicking stuffed into them of proper thickness, and a tail of fine wire, to enable the drops to fall off. With an arrangement of that kind, I hope to get a uniform dropping of the water; in other words, an imitation of a rainstorm, which can be regulated by changing the admission of air by the valve, and in which the rate of precipitation can be measured on the gauge glass at the side. By setting up a fan at one side the effect of a hard wind may be obtained. I do not know that such an apparatus has ever been used, at any rate in this country; if it has I would like to know what the results of experience with a device of that kind have been. It seems to me that such a device has a great advantage, coming nearer to an actual rainstorm than any ordinary spraying arrangement. It gives separate drops of water, and that is what we want.

E. M. Hewlett: Regarding the rain test on insulators, some folks like heavy rain, some like a flood, and some are satisfied with a mist. I do not believe you can satisfy everybody. It looks as though possibly a steam test might give a better average condition; there will not be so many conditions to balance up. You can get quite a film of moisture on the surface with the steam test, and can duplicate it more readily than the rain test, making better average tests. The rain test is of value in determining insulator design.

Chas. P. Steinmetz: There is one test which I do not find mentioned in Mr. Skinner's paper, and that is the test of the insulator after being submerged in water for a considerable period, a week or ten days, or more. A porcelain insulator may well stand all the tests outlined here, and may fail in service after a few weeks or months, by absorption of moisture through hair cracks in the glaze, due to incomplete vitrification or to porosity of the material. Porcelain is by no means an ideal material for an insulator. It is a heterogeneous mass, more or less completely vitrified in its interior, and covered with a glaze. If the vitrification is perfect, it is just as good and uniform as glass, but if the vitrification is not perfect, as long as the glaze keeps the moisture out it will stand, but as soon as by temperature variation of the air the glaze begins to acquire hair cracks, moisture soaks in and the insulation strength goes down. Glass would be preferable except that it is mechanically weak, and is hydroscopic, and so covers itself with a film of moisture. I am not entirely satisfied with such a test as immersion in water, but I bring it to your attention. Perhaps some one else might suggest a better way of testing a small percentage of insulators for absorption of moisture and decrease of disruptive strength resulting therefrom by long-continued exposure to water and atmospheric conditions.

I also raise the question whether it is advisable, as proposed in the paper, to allow any tests of less than twice the rated voltage. Here tests are proposed of one and one-quarter and one and one-half times the rated voltage. Now, you know that in our standardization rules, no voltage test is permitted on any apparatus of less than twice the rated voltage. That is the minimum test permitted. Here we have recommended tests under undesirable, but still actually existing conditions, of one and one-quarter times the rated voltage. The margin is rather small, probably too small. Many of us feel that at the present time the insulator is the weakest link in the long-distance transmission system. The occasional use, for instance, of 60,000-volt insulators on 40,000-volt systems also shows that the engineers feel that the extra high-voltage insulator is really not quite as good, does not have the same margin of safety, as other apparatus. It would be much more consistent if we insisted in testing the insulators to ascertain that they have the same margin of disruptive strength as all other apparatus is supposed to have, instead of so moderating the tests as to allow the insulators to be rated at a voltage to which they are not entitled on the basis of the margin of safety standard for all other parts of the system.

In regard to specifying the number of insulators tested on one trial, etc., my understanding is that the paper proposed additional matter to the rules of our Standardization Committee, so in this connection our standardization rules—which specify how many insulators you can test from one transformer, that is, how large the capacity current can be compared to the tranformer capacity—would apply. All those features that are specified by our standardization rules are implied, and the paper merely brings forward additional matter.

C. E. Skinner: I am a little surprised that nothing has been said in the discussion in regard to the dew test. I do not know whether this test has been used to any great extent or not, but those who have used it claim that it is far more satisfactory than any rain test, and also more severe, and it is one that ought to be relatively easy to repeat.

In regard to the number of insulators which may be tested on a given size transformer, I would say that Dr. Steinmetz has already covered the point. You will note that the paragraphs on voltage control, capacity of testing transformers, etc., are practically copied from the standardization rules. They are partly repeated here, in order to make the specification complete.

In regard to the question of placing the insulator vertically, and having an angular spray, I agree entirely with Mr. Mershon; no method has yet been brought to my attention which could be controlled with sufficient accuracy to allow repetition. This specification is intended to be a performance specification, and to have a performance specification you must be able to repeat your tests. The best we have been able to do in our laboratory in making tests is to get a spray which, measured by the rain gauge, taking successive squares in the spray, showed a variation of very approximately that given in the specification, that is, 0.2 to 0.3 inches per minute. I do not believe it makes very much difference whether it is a little more or a little less. I think that with that amount of water the results will be about the same. Mr. Mershon referred to a rate of one inch in five minutes—which is equivalent to 0.2 inch per minute. In all the communications that I received, I did not hear of the method mentioned by Dr. Sharp, although I was in communication with some of the German porcelain manufacturers. As a matter of fact, their letters did not give me very much information in regard to their method of obtaining the spray. I think that it would be impossible to use a fan and blow the spray the same on two successive days—that it would be out of the question to repeat tests of that kind.

The steam test suggested by Mr. Hewlett I think is virtually covered by what I called the dew test here. It is merely gotten at in a little different way, and in a way which is frequently met in practice; that is, the insulator is cooled down during the night, and in the morning, particularly in the tropics, and in places where the temperature rises suddenly, the insulator becomes completely coated over its entire surface with a layer of moisture.

The immersion test suggested by Dr. Steinmetz I think would be a very good one to add to the performance test, but I think it would be rather burdensome to make on all insulators.

In regard to the over-potential test of one and a quarter and one and a half times, I would refer to the introduction of the paper, in which I disclaim any great responsibility for the data there given; my own idea is that the tests specified are rather too low. I would like to see them higher, and if insulators can be purchased at a reasonable price for use on transmission lines under the more severe conditions, I would like to see the double-potential test substituted for the one given.

Ralph D. Mershon: As regards the dew test, both as suggested by Mr. Skinner and by Mr. Hewlett, it does not seem a definite test. Moist atmosphere—what does that mean? It would make a great deal of difference in the condition of the insulator just how long it is left out, and just what that moist atmosphere means—how can you give any definite condition?

C. E. Skinner: I have not made the dew test myself. It was proposed by two or three other persons who claimed they had made this test and found it perfectly satisfactory. If the air contains a fair amount of moisture, there will be a sufficient collection of moisture on the surface of the insulator to make it flash over, *under the test specified.*

E. M. Hewlett: We have made the steam test, and find it is easier to make the steam test than it is to get up in the morning early so as to find the insulator cool enough to test it with the dew test. We put the steam on the insulator with a hose until the surface is covered with condensed steam, and then make the voltage test.

N. J. Neall (by letter): Routine test. Inspection. I doubt very much the wisdom of allowing so large a variation as ± 5 per cent. in general over-all dimensions from the designed size. This is particularly critical for extra high-potential designs, where an insulator 14 inches in diameter and 14 inches high may have a considerably lower arcing test than one 15 inches in diameter by 15 inches high. I think ± 2.5 per cent. would be safer.

Dielectric tests. This is a good commercial rule for testing parts, but due consideration should be given the potential gradient of the insulator as a whole as well as the individual electrostatic capacities of the parts in series therein. Actually I would expect the top of an insulator to get almost line potential strain, particularly if the pin be grounded. I would call for three times the usual line voltage, 2.5 times is not high enough. There is no designation as to what constitutes a "lot." I should suggest a selective test from each kiln fired.

Design test. This permission to test as many as 5 per cent. is generous indeed. In general, a much smaller percentage than this should suffice.

Rain tests. 1.5 times normal voltage is too low; it should be at least 2 times. Likewise for dew test,—of shorter duration than 5 minutes, say 1 or 2 minutes. In none of these specifications is the important requirement, "with pin grounded," mentioned.

Methods of making tests. Rain test. The proposed angular position of the insulator with respect to a vertical rain does not appeal to me, because the whole combination of static field and reflected rain as found in service is apparently inadequately reproduced in this way. I prefer the rain at an angle instead. This detail requires further working out.

In addition to the preceding details, I wish to add the following: For testing insulators local conductors should be of large diameter to cut down the excessive ionization usually present in such tests. There should be some "table" of the number of insulators of given size that it is permissible to simultaneously test with a testing transformer of given capacity. In testing large insulators in large numbers for extra high voltage, the charging current alone may impair the value of the test. There must be some recognized method of measuring the voltage. Just as soon as the true character of spark-gaps is known it should be applied to this work authoritatively. In the meanwhile I would suggest the use of shielded spark-gaps with spherical tips—needles being more liable to error and burdensome in practice. For general testing, the voltage by ratio is desirable, provided it can be kept free of error; and the specification as to appropriate size of apparatus might include among its requirements a description of the apparatus.

I have already called attention, in my recent paper on lightning before the Institute, to the desirability of equivalent spark-gap measurements of insulators as indications of their non-arcing power. The arcing-over test of the insulator is no more indicative of its efficiency than the same test on a lightning arrester.

The tendency of the proposed specification seems to be a letting down rather than a stiffening of those requirements heretofore generally followed. I think the record of operation of high-tension transmission systems will show the wisdom of making this element thereof stronger than is proposed in these specifications.

Aside from these criticisms of details, I wish to state my belief that such a specification as here proposed by Mr. Skinner is highly desirable.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

VOLTAGE RATIO IN SYNCHRONOUS CONVERTERS WITH SPECIAL REFERENCE TO THE SPLIT-POLE CONVERTER

BY COMFORT A. ADAMS

Introduction. The interesting discussion on the split-pole converter at the February New York meeting of the Institute instigated the following paper, which consists of a purely theoretical analysis, the principal object of which is to show how the field distortion of a split-pole converter does not necessarily involve electromotive force distortion.

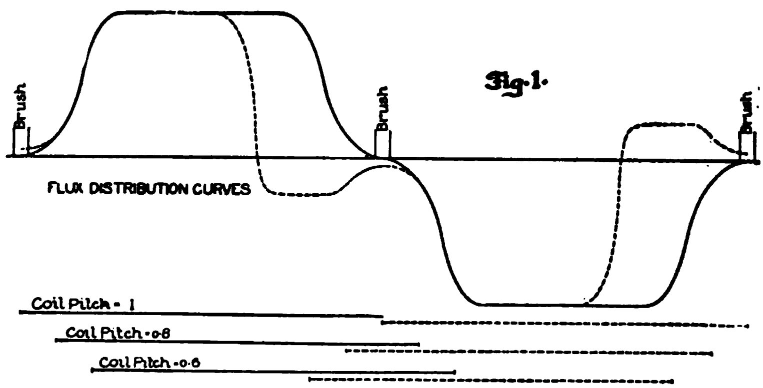
Incidentally there will be developed a method of analysis by which the direct electromotive force or any one of the alternating electromotive forces is determined analytically from the harmonic analysis of the flux distribution curve, thus establishing a simple and direct connection between the shape of the flux distribution curve, and the shapes as well as the magnitudes of the resulting electromotive forces.

Definitions

Brush electromotive force and tap electromotive force. The electromotive force between two adjacent commutator brushes will be called the *brush electromotive force* and that between collector rings the *tap electromotive force.*

Belt. In the ordinary closed-coil, two-layer lap winding, a series path between two adjacent brushes or between two adjacent taps, is made up of a set of coils, half of the sides of which, in the tops of slots, form a *belt* of conductors; the other half, in the bottoms of other slots, form a similar *belt*, but connected in the opposite direction. In the wave winding a series path contains as many such sets of coils or pairs of *belts* as there are pairs of poles. With a full pitch winding the two belts of any pair are 180 electrical degrees apart between centers; but less in a fractional pitch winding.

For a series path between brushes the span of each belt is 180 electrical degrees, and for a path between taps the belt span depends upon the number of phases; 180° for the single-phase or diametral connection, 120° for the three-phase, 90° for the quarter-phase, etc. In Fig. 1 is shown the location of a pair of belts forming a path between brushes, for three different values of the coil pitch. The solid lines indicate the top slot belts and the broken lines bottom slot belts. The brushes are also shown, together with the flux distribution curve. The broken curve shows a split-pole distortion.



Differential-factor. The brush-belts are as a whole stationary, although the individual components thereof are cutting through the flux, each belt remaining under a particular pole so that all of its elements are cutting flux in the same direction; but a tap-belt revolves as a whole and at times lies across parts of two poles, thus generating opposing electromotive forces in the two parts of the belt and reducing the effectiveness of electromotive force generation by introducing what may be called *differential-action.* The average electromotive force induced in this belt is therefore less than that induced in an equal number of active conductors in a brush-belt, and the factor by which it is less will be called the *differential-factor*, since it takes account of the differential cutting of the flux.

A differential action also takes place in the brush-belts, in the case of fractional-pitch windings, or when the brushes are displaced from the neutral points.

Form-factor. The tap voltage differs from the brush voltage induced in the same number of conductors, not only because of the differential action which reduces the average useful electromotive force, but also because the effective or root-mean-square tap voltage differs from the average by a factor called the *form-factor* which depends upon the wave-shape.

If E and E_1 be the brush and the single phase (or diametral) tap electromotive force, respectively, we may write

$$E_1 = k_d\, k_f\, E \tag{1}$$

where k_d and k_f are the differential- and the form-factor respectively. These two constants are closely connected and are commonly combined into a single constant, but they represent two distinct phenomena which are not inter-dependent, since they may be combined in a great variety of proportions. For example, in the split-pole converter it is possible so to proportion the parts as to change the differential-factor, and therefore the voltage ratio over a wide range without producing much change in the form-factor or the wave-shape.

The writer's present purpose is to show how this can be done.

Flux Distribution Curve and Single-Conductor Electromotive Force

Imagine a single conductor to cut through the gap flux at normal speed; the induced electromotive force will be at each instant proportional to the density of flux through which it is cutting, and the resulting electromotive force curve will have the same shape as the *flux distribution curve*. It will be convenient to deal with this elementary electromotive force curve in considering the resultant electromotive force of a whole belt. The elementary electromotive force curve may be expressed algebraically in the form of Fourier's series, thus;

$$\left.\begin{aligned} e' &= a_1 \sin \omega t + a_3 \sin 3\omega t + a_5 \sin 5\omega t + a_7 \sin 7\omega t + \text{ etc.} \\ &+ b_1 \cos \omega t + b_3 \cos 3\omega t + b_5 \cos 5\omega t + b_7 \cos 7\omega t + \text{ etc.} \end{aligned}\right\} \tag{2}$$

where $\omega = 2\,\pi\,n$, and n is the fundamental frequency.

For our present purpose equation (2) may be more conveniently written:

$$e' = a_1 \left| \begin{array}{l} \sin \omega t + q_{a_3} \sin 3\omega t + q_{a_5} \sin 5\omega t + \ldots \ldots \\ \quad + q_{am} \sin m\,\omega t + \text{etc.} \\ q_{b_1} \cos \omega t + q_{b_3} \cos 3\,\omega t + q_{b_5} \cos 5\,\omega t + \ldots \ldots \\ \quad + q_{bm} \cos m\,\omega t + \text{etc.} \end{array} \right| \qquad (3)$$

$$\text{where } q_{am} = \frac{a_m}{a_1} \text{ and } q_{bm} = \frac{b_m}{a_1}$$

Relation of Brush Voltage to the Elementary or Single-Conductor Electromotive Force

It will be assumed in what follows that the harmonic analysis of the single-conductor electromotive force was so carried out that the zero of time in equation (3) corresponds to the instant when the conductor in question is passing under one of the brushes.

With this assumption, the brush voltage, which is proportional to the area or to the average value of the elementary electromotive force wave between brushes, is:

$$E_{dc} = N \frac{2}{\pi} a_1 \left[1 + \frac{1}{3} q_{a_3} + \frac{1}{5} q_{a_5} + \frac{1}{7} q_{a_7} + \frac{1}{9} q_{a_9} + \text{etc.} \right] \qquad (4)$$

or if we designate $\frac{2}{\pi} N a_1$ by E_1

$$E_{dc} = E_1 \left[1 + \frac{1}{3} q_{a_3} + \frac{1}{5} q_{a_5} + \frac{1}{7} q_{a_7} + \ldots \text{etc.} \right] \qquad (4a)$$

where N is the number of conductors in series between brushes.

The cosine terms drop out, since the average value of the cosine over a half cycle beginning with zero, π, or any multiple thereof, is zero. The average value of an odd mth harmonic sine term taken over a half-cycle of the fundamental is $1/m$th of the average value of a half cycle of that term, since the other m-1 half-waves cancel out.

Relation of Tap Voltage to the Elementary or Single-Conductor Electromotive Force

For this purpose write equation (3) as follows:

$$e' = a_1 [q_1 \sin(\omega t + \theta_1) + q_3 \sin(3\,\omega t + \theta_3) \\ + q_5 \sin(5\,\omega t + \theta_5) + \ldots q_m \sin(m\,\omega t + \theta_m) + \text{etc.}] \qquad (5)$$

where $q_m = \sqrt{q^2_{am} + q^2_{bm}}$, and $\theta_m = \tan^{-1} \frac{b_m}{a_m}$ is measured in mth harmonic radians.

A little consideration will show that q_1 $(= \sqrt{1+q_{b_1}^2})$ does not differ much from unity except in cases of considerable distortion, and that it is equal to unity in the ordinary converter where the flux distribution curve is symmetrical about the center of the pole face, or with the three-part split-pole and symmetrical distortion. A displacement of the brushes would have the effect of increasing q_1.

The electromotive force of a single slot will have the same shape as e' and will differ only in magnitude according to the number of conductors per slot. The electromotive force of a given alternating-current belt may be considered as made up of as many components as there are slots per belt, each component being the electromotive force of a single slot and differing in phase from that of an adjacent slot by the electrical angular pitch of the slots.

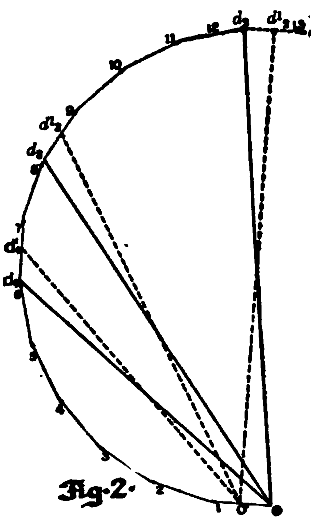


Fig. 2.

Then the electromotive force of the belt will have for its fundamental the vector sum of the fundamentals of the electromotive forces of the several slots of that belt, and for its mth harmonic the vector sum of the mth harmonics of the slots of that belt; but the phase difference between the latter will be (in harmonic degrees) m times as great as that of the fundamentals. For example, Fig. 2 shows the fundamental slot electromotive forces extending over 180 electrical degrees of armature circumference, for a converter with 12 slots per pole, while Fig. 2a shows the corresponding twelve fifth harmonic slot electromotive forces. In both Figs. the electromotive forces are numbered in the order of their respective slots starting at the point o. In Fig. 2, $\overline{o\,d_2}$ is the diametral or 180° fundamental electromotive force, $\overline{o\,d_3}$ the delta or 120° electromotive force and $\overline{o\,d_4}$ the quarter phase or 90° electromotive force. Similarly in Fig. 2a, $\overline{o\,d_2}$ is the vector sum of the twelve fifth-harmonic slot electromotive forces, that is, the 180° fifth harmonic; $\overline{o\,d_3}$ the vector sum of the fifth harmonic electromotive forces of slots numbers one to eight inclusive, that is, the 120° fifth harmonic; and $\overline{o\,d_4}$ the 90° fifth harmonic. A little considera-

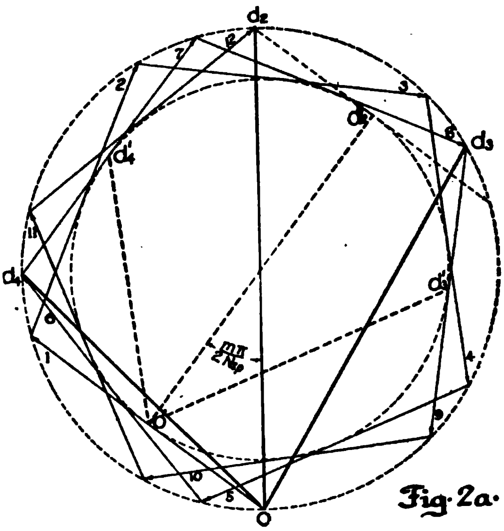


Fig. 2a.

tion will show that the ratio of the resultant to the arithmetical sum of the fifth harmonic slot electromotive forces is small as compared with the ratio of the resultant to the arithmetical sum of the fundamental slot electromotive forces. These ratios are the differential-factors for the fifth harmonic and the fundamental respectively, and will be labelled k_{d_5} and k_{d_1}. The quantity

$$k_{r_5} = \frac{k_{d_5}}{k_{d_1}}$$

will be called the fifth harmonic *reduction-factor*, and is the ratio in which the fifth harmonic of the belt electromotive force is reduced from that of the slot electromotive force (or the flux distribution curve); that is, if the amplitude of the fifth harmonic of the slot electromotive force is q per cent. of the fundamental, the amplitude of the fifth harmonic of the belt electromotive force will be $k_{r_5} \times q$ per cent. of its fundamental.

These relations may be generalized as follows:

Let N_{sp} = slots per pole.

and p' = the number of belt spans per electrical circumference, ($p' = 2$ for diametral connection, and 3 for 3 phase delta or 6 phase double delta).

Then $\frac{\pi}{N_{sp}}$ = the phase difference between the fundamental electromotive forces of two adjacent slots.

$\frac{m\,\pi}{N_{sp}}$ = the phase difference between the mth harmonic electromotive forces of two adjacent slots.

and $\frac{2\,N_{sp}}{p'}$ = slots per belt.

The total phase rotation of the mth harmonic in all the slots of the belt will be

$$\frac{m\,\pi}{N_{sp}} \times \frac{2\,N_{sp}}{p'} = \frac{2\,m\,\pi}{p'}$$

and if the diagram (such as Fig. 2a) be reduced to unit radius the resultant of the $\frac{2\,N_{sp}}{p'}$ m^{th} harmonics will be $2 \sin \frac{m\,\pi}{p'}$.

On the same basis the m^{th} harmonic of the slot electromotive force is $2 \sin \frac{m\,\pi}{2\,N_{sp}}$. and the arithmetical sum of the $\frac{2\,N_{sp}}{p'}$ m^{th} harmonics is $\frac{2\,N_{sp}}{p'} \times 2 \sin \frac{m\,\pi}{2\,N_{sp}}$

Thus the differential factor for the mth harmonic of the belt electromotive force is

$$k_{dm} = \frac{p'}{2\,N_{sp}} \frac{\sin \frac{m\,\pi}{p'}}{\sin \frac{m\,\pi}{2\,N_{sp}}} \tag{6}$$

and that for the fundamental

$$k_{d_1} = \frac{p'}{2\,N_{sp}} \, \frac{\sin \dfrac{\pi}{p'}}{\sin \dfrac{\pi}{2\,N_{sp}}} \tag{7}$$

The reduction-factor for the *m*th harmonic is then

$$k_{r_m} = \frac{k_{d_m}}{k_{d_1}} = \frac{\sin \dfrac{m\,\pi}{p'}}{\sin \dfrac{\pi}{p'}} \cdot \frac{\sin \dfrac{\pi}{2\,N_{sp}}}{\sin \dfrac{m\,\pi}{2\,N_{sp}}} \tag{8}$$

The first factor of the expression for k_{r_m} is independent of the number of slots, and its numerical value for various phases and harmonics is shown in Table I.

Table I giving values of $\sin \dfrac{m\,\pi}{p'} \div \sin \dfrac{\pi}{p'}$

TABLE I.

$m =$	3	5	7	9	11	13	15	17	19
$p' = 2$	1	1	1	1	1	1	1	1	1
$p' = 3$	0	1	1	0	1	1	0	1	1
$p' = 4$	1	1	1	1	1	1	1	1	1
$p' = 6$	2	1	1	2	1	1	2	1	1

The second factor of the formula for k_{r_m} is independent of p', and its numerical values for various values of N_{sp} and m, are shown in Table II.

Table II giving values of $\sin \dfrac{\pi}{2\,N_{sp}} \div \sin \dfrac{m\,\pi}{2\,N_{sp}}$.

TABLE II.

$N_{sp} =$	8	10	12	14	16	18	20	22	24	26	∞
$m = 3$	0.351	0.344	0.340	0.338	0.3368	0.3362	0.3358	0.3355	0.3353	0.3351	$\frac{1}{3}$
$m = 5$	0.235	0.222	0.214	0.2101	0.2081	0.2062	0.2051	0.2042	0.2034	0.2029	$\frac{1}{5}$
$m = 7$	0.199	0.1755	0.1646	0.158	0.154	0.152	0.150	0.149	0.1477	0.1467	$\frac{1}{7}$
$m = 9$	0.198	0.158	0.141	0.132	0.126	0.123	0.1208	0.119	0.1177	0.1166	$\frac{1}{9}$
$m = 11$	0.2345	0.1583	0.1316	0.1185	0.1111	0.106	0.103	0.1009	0.0992	0.0979	$\frac{1}{11}$
$m = 13$	0.351	0.1755	0.1317	0.1124	0.102	0.961	0.0920	0.08916	0.08702	0.08533	$\frac{1}{13}$
$m = 15$	0.995	0.2214	0.1413	0.1126	0.0986	0.0903	0.08492	0.0813	0.07865	0.07665	$\frac{1}{15}$
$m = 17$	1.000	0.3444	0.1644	0.1185	0.0985	0.0875	0.0807	0.0761	0.0729	0.0706	$\frac{1}{17}$
$m = 19$	0.351	1.000	0.214	0.132	0.100	0.0875	0.0788	0.073	0.069	0.0662	$\frac{1}{19}$

Except for the 3 and 6 phase, third, ninth and fifteenth harmonics, the values in Table II are those of k_{rm}, the reduction-factor, or the ratio of the per cent. harmonic in the tap electromotive force to the per cent. harmonic in the flux distribution curve.

An inspection of Tables I and II shows that for the single-phase or diametral connection and for a very large number of slots per pole, the reduction-factor is one-third for the third harmonic, one-fifth for the fifth harmonic, one-seventh for the seventh harmonic, etc; and that for a moderate number of slots per pole, the only change is a slight increase in the reduction-factor, this increase being greater for the higher harmonics.

Table I shows that for the delta or 120° connection ($p' = 3$), the third, ninth and all other harmonics which are multiples of three, drop out. This may be explained in several ways: first, a third or ninth harmonic would mean a circulating electromotive force around the closed-coil winding, which is impossible if the north and south pole fluxes are similarly distributed, since under this last assumption the electromotive forces of the two halves of the winding must be equal and opposite at each instant; secondly, the differential-factor of the third, ninth or fifteenth harmonic is zero for the 120° connection, since its phase rotation which is 3, 9 or 15 times that of the fundamental (120°), is a multiple of 360°. In other respects than in the disappearance of these harmonics the reduction factor is the same for the 120° as for the 180° connection.

For the quarter phase, or 90° connection, the reduction-factor is exactly the same as for the 180° connection; and for the 60° connection the only difference is that the third, ninth and fifteenth harmonic reduction-factors are doubled instead of being eliminated as in the 120° connection; that is, the reduction-factor for the third harmonic is two-thirds for the 60° connection, in place of one-third for the 180° connection and zero for the 120° connection. The explanation of this is, that with a fundamental phase rotation of 60°, the third harmonic phase rotation is 180°, giving a diameter as resultant, which is relatively twice as great as the 60° chord of the fundamental resultant. The ninth, fifteenth, etc., harmonics would add multiples of 360° to the phase rotation, leaving the resultant a diameter as in the case of the third harmonic.

Fractional-pitch or short-chord windings. Thus far the electromotive force of only a single belt has been considered. If the winding has the full pitch of 180°, the electromotive forces of the two belts of any pair will be in exact phase, and their resultant will have the same shape as the electromotive force of a single belt. But if a *fractional-pitch* or *short-chord* winding be employed, the two belt-electromotive forces will differ in phase by an amount depending upon the pitch deficiency, and their resultant may not have the same shape; in general it will not, unless the two component belt electromotive forces are sinusoidal.

Let λ_c designate the coil pitch in terms of full pitch. Then $1-\lambda_c$ is the pitch deficiency, and $(1-\lambda_c)\pi$ is the phase difference between the fundamental electromotive forces of the two belts; see Fig. 3, where $O\,A$ and $A\,B$ are the two equal fundamental electromotive forces and OB their resultant. The differential or pitch reduction-factor for the fundamental electromotive force is

$$\overline{OB} \div (\overline{OA} + \overline{AB}) = \sin \frac{\pi}{2} \lambda_c$$

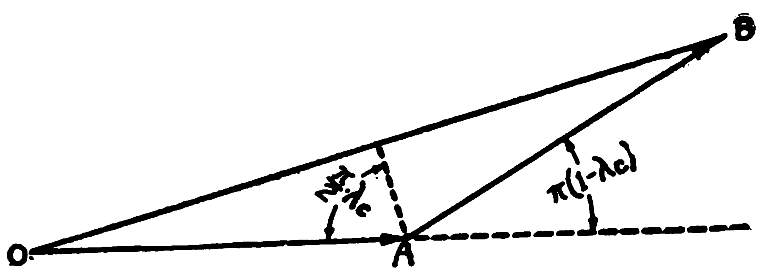


FIG. 3

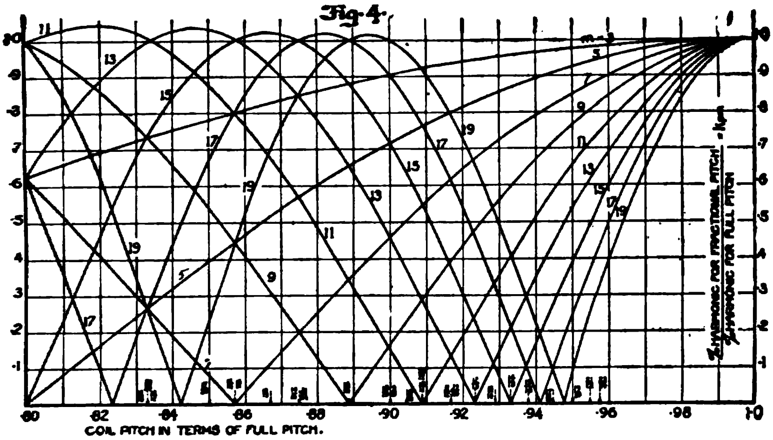
and that for the mth harmonic is $\sin \frac{m\pi}{2} \lambda_c$. The ratio of the latter to the former will be designated the mth harmonic pitch factor, it is

$$k_{pm} = \frac{\sin m \frac{\pi}{2} \lambda_c}{\sin \frac{\pi}{2} \lambda_c} = \frac{\text{per cent. of } m\text{th harmonic in fractional pitch winding}}{\text{per cent. of } m\text{th harmonic in full pitch winding}} \qquad (9)$$

This is plotted in Fig. 4 for the odd harmonics up to the nineteenth and for values of the coil pitch from 0.8 to 1.0. From the curves of Fig. 4, it appears that it would be possible to practically wipe out any one of the higher harmonics by a proper choice of pitch, although the reduction necessary to eliminate, say the fifth harmonic, would probably cause serious commutation difficulties. It is interesting to note in this connection that a pitch of 0.833 (10 slots in 12, 15 in 18, 20 in 24) would reduce the fifth, seventh, seventeenth and nineteenth harmonics to about one-quarter of their full pitch values. This would leave for the delta or double delta connections only the eleventh and thirteenth harmonics, since the third, ninth, fifteenth, and twenty-first harmonics are entirely absent in this case.

Fractional slot connection. Thus far it has been assumed that each phase belt comprised a whole number of slots; but it is obvious that this is not at all necessary, for even when the number of slots per pair of poles is a multiple of the number of phases, the taps may be taken off at junctions in the centers of

Fig. 4.



slots. In this case there would be two half-slot electromotive force vectors at the ends of the whole slot vectors. This is shown in Fig. 2 where $\overline{o' d_2'}$, drawn from the middle of slot electromotive force number one to the middle of slot electromotive force number thirteen, is the 180° fundamental resultant, and in Fig. 2a where $\overline{o' d_2'}$ is in the same way the 180° fifth harmonic resultant. Similarly $\overline{o' d_3'}$ and $\overline{o' d_4'}$ are the corresponding resultants for the 120° and 90° electromotive forces.

An inspection of Fig. 2a will show that this mid-slot connection has reduced the resultant harmonic by the factor, $\cos \dfrac{m \pi}{2 N_{sp}}$ which approaches zero as m and N_{sp} approach equality. Thus if N_{sp} is large, some of the higher harmonics are practically wiped out by this mid-slot connection, and if N_{sp} is small all of the lower harmonics are reduced and some intermediate ones almost eliminated.

For other fractional slot connections the reduction is less. Designate the fractional slot reduction-factor by k_{sm} and it is:

$$k_{sm} = \left\{ \begin{array}{l} \cos \frac{m\pi}{2N_{sp}} \text{ (for half-slot connection).} \\ \\ \cos \frac{m\pi}{2N_{sp}} \div \cos \frac{m\pi}{6N_{sp}} \text{ (for a one-third or a two-thirds slot connection} \\ \\ \cos \frac{m\pi}{2N_{sp}} \div \cos \frac{m\pi}{4N_{sp}} \text{ (for a one-fourth or a three-fourths slot connection.} \end{array} \right\} \quad (10)$$

If the two taps which terminate a given pair of belts, are taken, one from a mid-slot point and one from an interslot point, the slot reduction factor would lie between that for the mid-slot connection and unity.

The mid-slot connection is practically equivalent to a fractional pitch winding, the pitch deficiency of which is larger the smaller the number of slots per pole.

Combining equations (8), (9) and (10) gives as the final ratio of the mth harmonic of the induced electromotive force, to that of the single conductor electromotive force or flux distribution curve,

$$k_m = k_{rm} \times k_{pm} \times k_{sm} = \frac{\sin \frac{m\pi}{2} \lambda_c}{\sin \frac{\pi}{2} \lambda_c} \times \frac{\sin \frac{m\pi}{p'}}{\sin \frac{\pi}{p'}} \times \frac{\sin \frac{\pi}{2N_{sp}}}{\sin \frac{m\pi}{2N_{sp}}} \times k_{sm} \quad (11)$$

Equation (10) includes the effects of all the thus-far-considered factors upon the *magnitudes* of the harmonics.

Consider now the phases of the harmonics.

Phases of harmonics. Referring to Figs. 2 and 2a, the total phase rotation of the fundamental is $\frac{2\pi}{p'}$. Thus the phase of the fundamental of the resultant or belt electromotive force with respect to the fundamental of the electromotive force of slot

No. 1 is, in fundamental radians

$$\alpha_1 = \frac{\pi}{p'} - \frac{\pi}{2\,N_{sp}} = \pi\left(\frac{1}{p'} - \frac{1}{2\,N_{sp}}\right)$$

The total phase rotation of the mth harmonic is $\frac{2\,m\,\pi}{p'}$

Let w = nearest whole number less than $\frac{m}{p'}$ and

w_s = " " " " " $\frac{m}{2\,N_{sp}}$

Then the equivalent phase rotation of the mth harmonic is $2\,\pi\left(\frac{m}{p'} - w\right)$ and the phase of the mth harmonic of the resultant or belt electromotive force with respect to the mth harmonic of the electromotive force of slot No. 1 is, in mth harmonic radians

$$\alpha_m = \pi\left(\frac{m}{p'} - w\right) - \pi\left(\frac{m}{2\,N_{sp}} - w_s\right)$$

$$= \pi\left[m\left(\frac{1}{p'} - \frac{1}{2\,N_{sp}}\right) - (w - w_s)\right]$$

or in fundamental radians

$$\alpha_m{}' = \pi\left[\left(\frac{1}{p'} - \frac{1}{2\,N_{sp}}\right) - \left(\frac{w - w_s}{m}\right)\right]$$

Thus in passing from the slot electromotive force to the resultant or belt electromotive force, the relative phase of the mth harmonic with respect to the fundamental is changed by an amount which is, in fundamental radians

$$\beta_m{}' = \alpha_m{}' - \alpha_1 = -\frac{\pi}{m}\,(w - w_s) \tag{12}$$

or, in mth harmonic radians,

$$\beta_m = m\,(\alpha_m{}' - \alpha_1) = -\pi\,(w - w_s) \tag{13}$$

But w and w_s are whole numbers and β_m is therefore always a multiple of π. Moreover w_s is practically always zero, since when

$$m = 2 N_{sp}$$

the harmonic is so small as to be wholly negligible. With this assumption, β_m is zero when w is even, and π when w is odd. That is, when w is odd the mth harmonic of the belt electromotive force is reversed in phase as compared with the same harmonic in the slot electromotive force.

For example, consider a slot electromotive force which con-

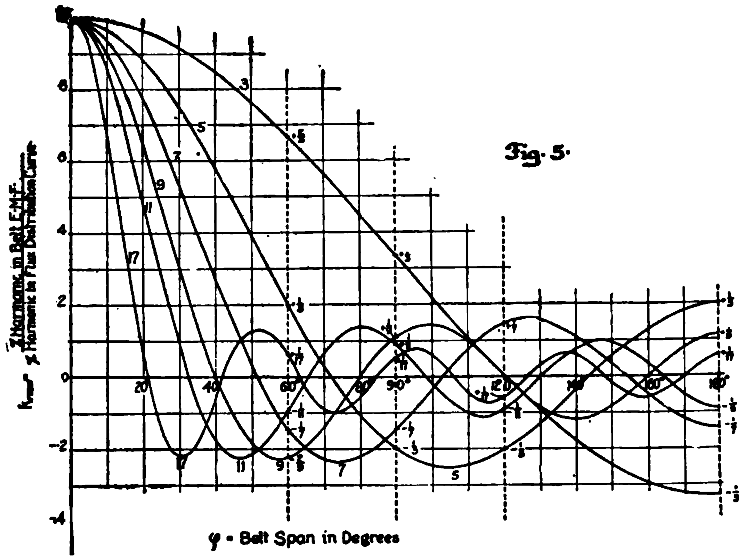


Fig. 5.

sists of a fundamental and a prominent third harmonic; follow this harmonic into the various belt electromotive forces.

For the single phase or diametral connection, $p' = 2$ and $w = 1$. Thus the third harmonic is *inverted* as well as being reduced to one-third of the percentage value which it had in the slot electromotive force. That is, if the slot electromotive force be flat topped, the 180° electromotive force will be peaked and vice versa, but less so than the slot electromotive force.

For the three-phase or delta connection, $p' = 3$ and $w = 1$; but the amplitude of the third harmonic in the 120° electromotive force is zero.

For the quarter-phase or 90° connection, $p' = 4$ and $w = 0$; therefore the third harmonic is not reversed, but is merely reduced to one-third of its slot percentage. That is, if the slot electromotive force be peaked, the 90° electromotive force will be peaked but less so; and if the slot electromotive force be flat topped, the 90° electromotive force will be flat topped but less so.

For the six-phase connection, $p' = 6$ and $w = 0$. The third harmonic is not reversed and is reduced to two-thirds of its slot percentage.

Follow this third harmonic from the six-phase back through the four, three, and two (single) phase. It is relatively large (two-thirds) and positive in the six-phase, reduced to one-third in the quarter-phase, vanishes in the three-phase, and reappears inverted in the single-phase with a relative value of one-third.

Follow the fifth harmonic in the same way. In the 60° belt it is positive and of one-fifth magnitude; in the 72° belt zero; in the 90° belt inverted and of one-fifth magnitude; the same in the 120° belt, having passed through a maximum between these last two points; and one-fifth positive again in the 180° belt. It thus appears that the change of belt span alters the magnitude but not the phases of the harmonics. This is just what was to be expected, since the phase of any given harmonic with respect to the fundamental is the same in each of the elementary e.m.fs. of which the tap e.m.f. is composed, and must therefore be the same in the resultant or tap e.m.f.

In order to see this relation more clearly, start with the flux distribution curve or the single-conductor electromotive force, assume an infinite number of slots and imagine the belt span to be increased gradually. The per cent. magnitude of any harmonic will gradually decrease through zero to a negative maximum, then back again to a positive maximum, etc., without any change of phase other than that involved in reversal. If the angle of belt span be designated by ϕ, the reduction factor for this case of an infinite number of slots will be

$$k_{rm}\,\infty = \frac{1}{m} \sin \frac{m\,\phi}{2} \div \sin \frac{\phi}{2} \tag{14}$$

Values of $k_{rm}\,\infty$ for several harmonics and for all values of belt span up to 180° are shown plotted in Fig. 5. These curves are instructive as showing clearly the general relation between belt span and the relative magnitudes of the several harmonics, although they do not take account of the number of slots per pole.

Negative values of $k_{rm}\,\infty$ indicate that the harmonic is inverted.

Tap Voltage.

Adopting the notation of equation (5) the mth harmonic of any tap electromotive force may be completely expressed thus:

$$k_m\, q_m \sin\,(m\,\omega\, t+\theta_m+\pi\, w),$$

where w is as already defined, and k_m is given by equation (11).

The amplitude of the fundamental of the tap voltage is

$$a_{t_1} = k_{d_1}\,\frac{2\,N}{p'}\,q_1\,a_1$$

where N is the number of conductors in series between brushes, $\frac{2\,N}{p'}$ the number of conductors in series between taps, $q_1\,a_1$ the amplitude of the fundamental of the single conductor electromotive force and k_{d_1} the fundamental differential-factor, which, for all practical purposes, may be taken as equal to $\frac{p'}{\pi}\times\sin\frac{\pi}{p'}$, the limiting value of equation (7) as N_{sp} approaches infinity. The amplitude of the tap voltage fundamental is then

$$a_{t_1} = \frac{2}{\pi}\,N_{a_1}\,q_1\,\sin\frac{\pi}{p'} = q_1\,E_1\,\sin\frac{\pi}{p'} \tag{15}$$

and the complete tap voltage is

$$e_t = q_1\,E_1\,\sin\frac{\pi}{p'}\,[\sin\,\omega\,t+k_3\,q_3\,\sin\,(3\,\omega\,t+\theta_3+\pi\,w)$$
$$+k_5\,q_5\sin\,(5\,\omega\,t+\theta_5+\pi\,w)+\text{etc.}] \tag{16}$$

A consideration of the value of w or of the curves of Fig. 5 will show that in the 180° electromotive force the third, seventh, eleventh, etc., harmonics are reversed; that in the 120° electromotive force, the third, ninth, fifteenth, etc., harmonics vanish and the fifth, eleventh, seventeenth, etc., harmonics are reversed; and that in the 60° electromotive force the seventh, ninth and eleventh, the nineteenth, twenty-first and twenty-third, etc., harmonics are reversed.

To reduce these to a more specific basis assume the case of a 60-cycle converter with twelve slots per pole. Then

$$e_{t-180} = q_1 E_1 [\sin \omega t - 0.34\, q_3 \sin (3\, \omega t+\theta_3) + 0.214\, q_5 \sin (5\, \omega t + \theta_5) - 0.165\, q_7 \sin (7\, \omega\, t+ \vartheta_7) + 0.141\, q_9 \sin (9\, \omega\, t + \theta_9) - 0.1316\, q_{11} \sin (11\, \omega\, t+\theta_{11}) + 0.1317\, q_{13} \sin (13\, w\, t+\theta_{13}) - 0.1413\, q_{15} \sin (15\, \omega\, t+\theta_{15}) + 0.164\, q_{17} \sin (17\, \omega\, t+\theta_{17}) - 0.214\, q_{19} \sin (19\, w\, t+\theta_{19}) \;.\;.\;.\; + \text{etc.}] \tag{17}$$

$$e_{t-120} = 0.866\, q_1 E_1 [\sin \omega\, t - 0.214\, q_5 \sin (5\, \omega\, t+\theta_5) + 0.165\, q_7 \sin (7\, \omega\, t+\theta_7) - 0.1316\, q_{11} \sin (11\, \omega\, t+\theta_{11}) + 0.1317\, q_{13} \sin (13\, \omega\, t+\theta_{13}) - 0.164\, q_{17} \sin (17\, \omega\, t+\theta_{17}) + 0.214\, q_{19} \sin (19\, \omega\, t+\theta_{19}) \;.\;.\;.\; + \text{etc.}] \tag{18}$$

$$e_{t-60} = 0.5\, q_1 E_1 [\sin \omega\, t+0.68\, q_3 \sin (3\, \omega\, t+\theta_3) + 0.214\, q_5 \sin (5\, \omega\, t+\theta_5) - 0.165\, q_7 \sin (7\, \omega\, t+\theta_7) - 0.282\, q_9 \sin (9\, \omega\, t+\theta_9) - 0.1316\, q_{11} \sin (11\, \omega\, t+\theta_{11}) + 0.1317\, q_{13} \sin (13\, \omega\, t+\theta_{13}) + 0.283\, q_{15} \sin (15\, \omega\, t+\theta_{16}) + 0.164\, q_{17} \sin (17\, \omega\, t+\theta_{17}) - 0.214\, q_{19} \sin (19\, \omega\, t+\theta_{19}) + \text{etc.}\;.\;.\;] \tag{19}$$

With a larger number of slots per pole such as would be employed for a 25-cycle converter, the harmonic coefficients would be reduced, especially for the higher harmonics, see Table II. It is interesting to note that with as small a number of slots per pole as here assumed, 12, the reduction factor does not continue to decrease indefinitely as m increases but actually begins to increase from the thirteenth harmonic up. Also its value for the nineteenth harmonic is more than one-fifth, or three times its value when $N_{sp} = 24$.

It is thus possible to determine easily and accurately any tap voltage, from the harmonic analysis of the flux distribution curve. It is obviously much easier to determine the harmonics of the flux distribution curve with a given degree of accuracy, than it is to determine those of the resultant electromotive force wave, since the former are several times larger than the latter.

Root mean square tap voltage. This is obviously,

$$E_t = \frac{q_1 E_1}{\sqrt{2}} \sin \frac{\pi}{p'} \sqrt{1+k_3^2\, q_3^2+k_5^2\, q_5^2+k_7^2\, q_7^2+\text{etc.}} \tag{20}$$

or, for the special case of twelve slots per pole

$$E_{180} = 0.707\, q_1 E_1 \sqrt{1+0.116\, q_3^2+0.0458\, q_5^2+0.027\, q_7^2+0.0199\, q_9^2 + 0.073\, q_{11}^2+0.0173\, q_{13}^2+0.02\, q_{15}^2+0.027\, q_{17}^2+0.0458\, q_{19}^2+\text{ etc.}} \tag{21}$$

$$E_{120} = 0.613\, q_1\, E_1 \sqrt{1 + 0.0458\, q_5^2 + 0.027\, q_7^2 + 0.0173\, q_{11}^2 + 0.0173\, q_{13}^2 + 0.027\, q_{17}^2 + 0.0458\, q_{19}^2 + \ldots \text{etc.}} \tag{22}$$

$$E_{60} = 0.354\, q_1\, E_1 \sqrt{1 + 0.462\, q_3^2 + 0.0458\, q_5^2 + 0.027\, q_7^2 + 0.0795 q_9 + 0.0173\, q_{11}^2 + 0.0173\, q_{13}^2 + .08\, q_{15}^2 + 0.027\, q_{17}^2 + 0.0458\, q_{19}^2 + \text{etc.}} \tag{23}$$

VOLTAGE RATIO

The ratio of brush voltage to tap voltage is obtained from equations (4a) and (20). It is

$$K_{bt} = \frac{E_{dc}}{E_t} = \frac{\sqrt{2}}{q_1 \sin \frac{\pi}{p'}} \times \frac{(1 + \frac{1}{3} q_{a3} + \frac{1}{5} q_{a5} + \frac{1}{7} q_{a7} + \text{etc.} \ldots)}{\sqrt{1 + k_3^2 q_3^2 + k_5^2 q_5^2 + k_7^2 q_7^2 + \text{etc}}} \tag{24}$$

or

$$K_{bt} = \frac{\sqrt{2}}{q_1 \sin \frac{\pi}{p'}} \cdot K_q \tag{25}$$

where

$$K_q = \frac{(1 + \frac{1}{3} q_{a3} + \frac{1}{3} q_{a3} + \frac{1}{5} q_{a5} + \text{etc.})}{\sqrt{1 + k_3^2 q_3^2 + k_5^2 q_5^2 + k_7^2 q_7^2 + \text{etc.}}} \tag{26}$$

THREE-PART POLE.

With a normal flux distribution curve, or with a three-part pole and symmetrical distortion, the q_b's are zero, $q_1 = 1$ and $q_m = q_{am}$. The denominator of K_q will differ little from unity, being always a little greater; for example, with a 30 per cent. third harmonic the denominator is 1.005; with a 15 per cent. third, a 15 per cent. fifth and a 15 per cent. seventh harmonic, the denominator is about 1.002. The numerator, on the other hand, may differ considerably from unity on either side according as the q_a's are positive or negative. With a normal undistorted flux distribution the numerator will be slightly less than unity, and K_{bt} will be slightly less than as given by the usual approximate formula. Any variation in the q_a's, due to a symmetrical distortion, causes a variation in the voltage ratio; but it will also produce a variation in the harmonics of the tap electromotive forces. The relation between these two variations in the case of symmetrical distortion will next be considered.

Assume that the flux distribution curve is completely under control; start with a sinusoidal distribution and add harmonics. First add a third harmonic which will be in phase with or in exact opposition to the fundamental, since for a symmetrical distortion $q_{b_3} = 0$; that is $q_3 = q_{a_3}$. Also assume $N_{sp} = 24$, and a winding of full pitch with interslot connections. Then

$$K_{q_3} = \frac{1 + \frac{1}{3} q_{a_3}}{\sqrt{1 + 0.112\, q_{a_3}^2}}.$$

and the corresponding harmonic in the tap electromotive forces is $k_3\, q_{a_3}$ which is $0.335\, q_{a_3}$ for the 180° electromotive force and zero for the 120° electromotive force.

In curve 3 of Fig. 6, K_{q_3} is shown plotted against $k_3\, q_{a_3}$ for the case in hand; that is, the curve shows the per cent. change in the voltage ratio corresponding to various per cent. magnitudes of the third harmonic in the 180° electromotive force. This harmonic appears with the same per cent. magnitude in the 90° electromotive force and with double this magnitude in the 60° electromotive force, but vanishes in the 120° electromotive force.

The per cent. magnitude of the third harmonic in the flux distribution curve is approximately three times that in the 180° tap voltage.

Curves 7, 11 and 17, of Fig. 6, show for the seventh, eleventh and seventeenth harmonics, the same relation as is shown in curve 3, for the third harmonic.

From these curves it is evident that the several harmonics are almost equally effective in changing the voltage ratio; that is, for a given change in voltage ratio, the resulting harmonic in the tap voltage is of practically the same magnitude whether it be third, fifth, seventh, or higher. It should be noted, however, that to obtain a given change in voltage ratio by means of the higher harmonics, requires a much greater distortion of the flux distribution curve than by means of the lower harmonics, although the resulting electromotive force distortion is approximately the same in the two cases. For example, referring to Fig. 6, to change K_q from 1.0 to 0.9 by the third harmonic, requires a 10 per cent. third harmonic in the 180° electromotive force, which means a 30 per cent. third harmonic in the flux distribution curve; but to produce the same change in K_q by the seventeenth harmonic requires a 12 per cent. seventeenth harmonic in the 180° electromotive force, and $\left(\frac{12}{k_m} =\right)$ 165 per cent. seventeenth harmonic in the flux distribution curve. Such a distortion is out of the question, in fact it is quite obvious that the higher harmonics play a very small part in the voltage ratio or in the distortion of the tap voltage.

Consider now the effect of a combination of harmonics, which

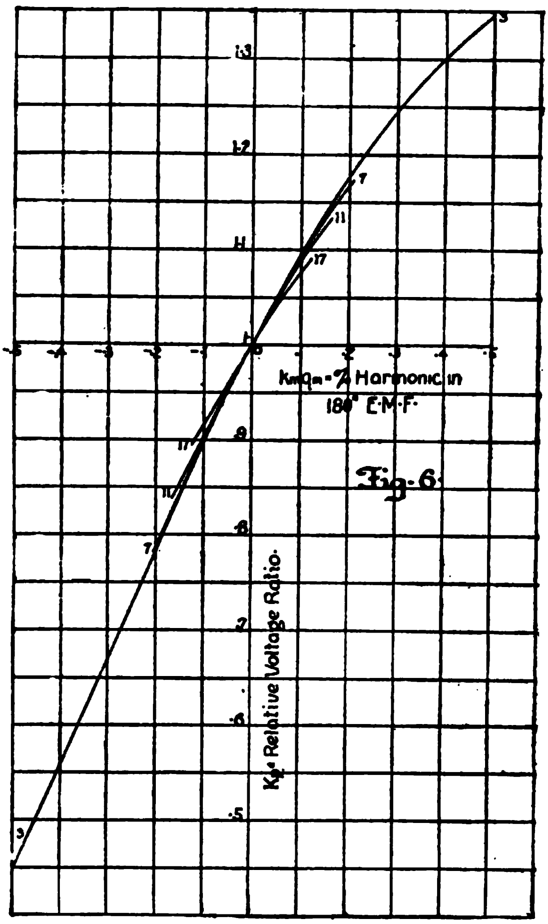


Fig. 6.

corresponds more nearly to the actual conditions. The q_a's must all be of the same sign in order to produce the maximum effect upon K_q for a given electromotive force distortion, and in that case they will have practically the same effect as a single harmonic producing the same electromotive force distortion. For example, consider the following combinations:

(a), $q_{a_3} = 0.60$; and (b), $q_{a_3} = 0.21$, $q_{a_5} = 0.35$, and $q_{a_7} = 0.396$. For (a), $k_3\, q_{a_3} = 0.201$ and $K_q = \frac{1.2}{1.02} = 1.177$; for (b), $k_3\, q_{a_3} = 0.0704$, $k_5\, q_{a_5} = .0711$, $k_7\, q_{a_7} = 0.0585$, $\Sigma k_m\, q_{am} = k_3\, q_{a_3} + k_5\, q_{a_5} + k_7\, q_{a_7} = 0.20$ and $K_q = \frac{1.197}{1.007} = 1.188$. Thus for the same total per cent. (0.20) of harmonics, the per cent. variation of voltage ratio is about the same in the two cases.

It might appear that in the case of several harmonics their maxima would not coincide at any point, and that therefore the "sine deviation" would be less for the same total per cent. of harmonics than in the case of a single harmonic; but if the q_a's are all of the same sign, the harmonics in the 180° electromotive force will be alternately inverted and the central maxima will all coincide, giving the same deviation as for a single harmonic.

If the q_a's are not all of the same sign, the variation in K_q will be appreciably reduced for a given total per cent. of harmonics.

Thus with symmetrical distortion of the flux curve, the q_a's, in order to produce the largest variation in K_q for a given electromotive force distortion, must all be of the same sign; but it makes little difference which harmonics are used.

Starting with a sinusoidal flux curve, the per cent. change in voltage ratio is of about the same magnitude as the total per cent. of harmonics in the 180° electromotive force; but the normal undistorted flux distribution is not sinusoidal. Its harmonics are not large, however, the largest being a negative fifth harmonic of not more than ten or fifteen per cent., which means a fifth harmonic of not more than two or three per cent. in the 180° electromotive force. This would correspond to a point on the curves of Fig. 6, slightly below the origin. Starting thus with negative harmonics in the flux distribution curve, the addition of positive harmonics would, for a given change in K_q, give a more nearly sinusoidal 180° electromotive force than would the addition of more negative harmonics; that is, for a given range in K_q, the least electromotive force distortion occurs when the mean value of K_q is unity or slightly less.

For example, for a total range of 20 per cent. variation in K_q, the least possible harmonics will be about 10 per cent., provided a full pitch winding and interslot connections be employed. With a fractional pitch winding, mid-slot connections, or both, this may be reduced according to the winding pitch and the harmonics employed, see Fig. 4. If it were possible to employ only fifth and seventh harmonics, and if commutation requirements did not prevent the use of a coil pitch as low as 0.83, the two harmonics in question would be reduced to about one-fourth of their full pitch values or to about 2.5 per cent. of the fundamental in the 180° electromotive force. As a matter of fact the fifth harmonic is naturally the most prominent one in a three-part pole symmetrical distortion, and it is not difficult to avoid a large third harmonic, but the use of the fifth harmonic involves much more flux distortion for the same change in voltage ratio, as has already been pointed out.

With the 120° connection the third harmonic disappears, but the fifth harmonic has the same value as in the 180° electromotive force. It is thus less advantageous to use the 120° connection with a three-part pole than with a type of distortion in which the third harmonic predominates.

It is possible to make the third harmonic the most prominent by shortening up the interpolar arc, and thus to take advantage of the fact that the third harmonic disappears in the 120° connection; but this makes the commutating field very steep and the commutation sensitive.

Two-Part Pole.

In this case the distortion results in a lateral shifting of the center of gravity of the flux wave and an introduction of cosine terms into the equation for the single conductor electromotive force (equation 3); that is, the q_b's are no longer zero and at least some of the q's are larger than the corresponding q_a's. Unless the q_b's are very large, they will not affect the denominator of K_q (equation 26) appreciably, and K_q will be only a very little smaller than when the q_b's are zero, the same q_a's assumed. But the increase in the q's will increase the per cent. harmonics in the tap voltage, corresponding to a given K_q.

A little consideration will show that one of the principal cosine terms introduced by the lateral distortion here considered is that of the fundamental, and that, therefore, q_1 $(= \sqrt{1 + q_{b_1}^2})$ is no longer unity and constant, but increases with the distortion. Thus the first factor in the expression for the voltage ratio, K_{bt} (equation 24), decreases with increasing distortion owing to the increase of q_1. If at the same time the q_a's have been increasing positively, the two effects will tend to neutralize, but if the q_a's are decreasing, the two effects are in the same direction and it is obviously possible to obtain a greater change in voltage ratio for a given total per cent. of harmonics in the 180° tap electromotive force than with the symmetrical distortion. Moreover, it will be remembered that q_m is the amplitude of the mth harmonic of the flux distribution curve in terms of a_1, the amplitude of the fundamental sine term; but the real measure of distortion is the amplitude of the harmonic in terms of the whole fundamental; it is, therefore,

$$q_m \div q_1 = \sqrt{q_{am}^2 + q_{bm}^2} \div \sqrt{1 + q_{b_1}^2}$$

which may be considerably smaller than q_m, and $\frac{k_m q_m}{q_1}$ the real per cent. harmonic may be considerably smaller than $k_m q_m$. For example it is quite possible to shift the center of gravity of the flux distribution curve by 40°, which means that q_1 is about 1.3 and that the mth harmonic in the 180° electromotive force is $0.77\ k_m q_m$.

It may be instructive at this point to compare the two-part pole lateral distortion with a shifting of the brushes and a fixed flux distribution. Assume the latter to be sinusoidal, imagine the brushes to be shifted by an angle β and disregard commutation difficulties. Imagine the flux distribution curve to be subdivided into two components, one in space phase with the new brush position and in magnitude proportional to cos β, and the other in quadrature with the brush position and proportional to sin β. The brush voltage and therefore the voltage ratio will be reduced by the factor cos β, but there will be no harmonics in the flux distribution curve or in any of the tap voltage waves. This may be described as a method of altering the differential factor of the brush voltage without changing or distorting the tap voltage.

Were it not for commutation difficulties, this would be an ideal method of varying the voltage ratio of converters. The case of extreme two-part pole distortion such as shown in Fig. 7 looks very much like a shift of brushes with notches cut in the flux curves for commutation purposes.

It is thus obvious that the two-part pole distortion affects the voltage ratio in quite a different way from that of the symmetrical distortion, and that this way does not necessarily involve such serious harmonics in the resulting electromotive force wave. This difference may be most easily specified by means of equation 25: the symmetrical distortion affects the voltage ratio only through K_q, and there is a fairly definite minimum electromotive force distortion involved in a given change in K_q; but the lateral distortion affects the voltage ratio mostly through q_1, and were it not for the necessity of the brush notch in the flux curve, K_q might be kept nearly constant and the harmonics largely eliminated. Unfortunately the necessity for the notch or weak field at the brush position does introduce some harmonics but the worst of these is usually the third, and it can be eliminated by employing the double delta connection, or a diametral connection with the transformer primaries connected in star.

There appears to be in the case of the two-part pole, no simple method of determining the minimum electromotive force distortion corresponding to a given change in voltage ratio, but it is evident from the above analysis that it may be considerably less than with the symmetrical distortion of a three-part pole.

As an example consider Fig. 7, which shows a flux distribution curve for extreme distortion calculated from the dimensions of an actual machine. A rough harmonic analysis of this curve gives the following constants.

Table III.

$m =$	1	3	5	7	9	11	13	15
$q_{am} =$	1.	−0.417	+0.0438	+0.0353	−0.1446	+0.01434	+0.0522	−0.026
$q_{bm} =$	+0.822	−0.268	−0.359	+ .0093	−0.0084	−0.064	+0.069	−0.043
$q_m =$	1.295	0.495	0.362	0.0364	0.145	0.0657	0.0827	0.0503
$180° k_m =$	1.	0.3353	0.2034	0.1477	0.1177	0.0992	0.087	0.0786
$180° \frac{k_m q_m}{q_1}$	1.	0.128	0.0568	0.00415	0.0132	0.005	0.0058	0.00305
$120° \frac{k_m q_m}{q_1}$	1.	0	0.0568	0.00415	0	0.005	0.0058	0

k_m is taken for full pitch winding, interslot connections 24 slots per pole, and 180° taps. $k_m\ q_m \div q_1$ is the per cent. harmonic in the tap electromotive force.

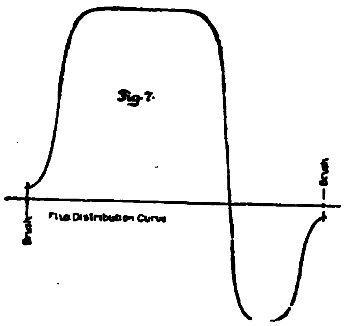
There are doubtless higher harmonics of appreciable magnitude, but they do not influence the quantities with which we are at present interested. The voltage ratio is (from equation 24)

$$\frac{E_{dc}}{E_{t\text{-}180°}} = \frac{\sqrt{2}}{q_1} \frac{(1+\frac{1}{3}q_{a_3}+\frac{1}{5}q_{a_5}+\text{etc.})}{\sqrt{1+k_3^2\,q_3^2+k_5^2\,q_5^2+\text{ etc.}}}$$

$$= \frac{\sqrt{2}}{1.295}\ \frac{0.862}{1.016} = \sqrt{2}\times 0.655$$

This ratio is less than two-thirds of the ratio for a sinusoidal flux distribution central between brushes, and just about two-thirds of the ratio with a normal undistorted field. Taking this as the no-load ratio, the increase when the auxiliary pole is fully excited in the positive direction, will be 50 per cent, or the decrease in this ratio for the reverse change will be about 33 per cent.

The total of harmonics in the 180° electromotive force is 21.6 per cent. and in the 120° electromotive force only 7.2 per cent, both of which are considerably less than is possible for the three-part pole and an equal variation in voltage ratio. But it is quite possible that a more careful design of pole faces might result in a yet lower per cent. of harmonics. For example, an increase in the width of the auxiliary pole would decrease the fifth harmonic and thus the per cent. harmonics in the 120° electromotive force.



The above figures are for full pitch winding and interslot connection. With mid-slot connections and a coil pitch of twenty-three slots the per cents. are reduced to 19 and 6 respectively, and sufficient reduction of pitch would practically wipe out all the harmonics from the 120° electromotive force and all those of the 180° electromotive force except the third. But the conditions cited in this example are extreme and it is quite possible, with a reasonable range of voltage ratio, to reduce the harmonics in the 120° electromotive force to very low figures without employing a coil pitch so low as to endanger commutation. Moreover, these harmonics are of the induced electromotive force and will not appear in full magnitude at the transformer primaries since they will be partly consumed by the small harmonic currents, flowing through the reactance of armature and transformers.

Damping. Mr. Lincoln stated in his discussion at the February New York meeting that the result of the harmonics in the induced electromotive force would be harmonic currents which would react upon the field in such a way as partly to prevent the otherwise flux distortion, and thus require greater unbalancing of excitation. This is true, but the use of low resistance squirrel-cage dampers would preserve a constant flux distribution and would relieve the circuit of any considerable damping currents.

Commutation. It might seem at first sight that commutation in a field such as shown at the brushes in Fig. 7, would not be satisfactory, but if the direction of rotation of the armature be taken away from the auxiliary poles the armature magnetomotive force will at least partly neutralize the field magnetomotive force in the commutation zone, and the commutation may be even better at some loads than under ordinary conditions. It should be remembered, however, that at full load when the armature magnetomotive force is the greatest the polarity of the auxiliary pole is normal and there is no flux to neutralize, and that at light or no load when the auxiliary pole is reversed as in Fig. 7, the armature current is very small and the flux to be reversed is a maximum. But with a sufficiently low reactance voltage a little flux (more or less) in the commutating zone is comparatively harmless.

In this connection it is interesting to note that a machine with a short air-gap and a weak field would probably have better commutation under conditions of extreme distortion, since the armature magnetomotive force would then even at light load be able to materially reduce the flux in the commutation zone. A saturation of pole tips would also be an aid to this end as well as to the end of eliminating some of the higher harmonics from the flux distribution curve.

The extent to which the winding pitch can be reduced without endangering commutation depends upon many details of design such as reactance voltage, width of brush, length of air-gap, etc., and can only be determined satisfactorily by experiment; the writer therefore hesitates to make a bold guess, although if forced he would place the lower limit at 90 per cent. under reasonably favorable conditions.

Summary. With the 180° connection and a large number of slots, an mth harmonic in the flux distribution appears in the tap electromotive force, but reduced to about $1/m$th of its per cent. value. Thus the higher harmonics rarely appear in the tap electromotive force unless they are very large in the flux distribution curve. Moreover, the higher harmonics are of little value in changing the voltage ratio, (see equation 24), so that they need have little consideration.

With the 120° connection the third, ninth, fifteenth, etc., harmonics do not appear in the tap electromotive force, and there will be no corresponding harmonic currents set up. This assumes that the flux distribution curves under north and south poles are exactly alike, otherwise there may be a third harmonic even in the 120° electromotive force.

With the diametral connection and the transformer primaries star connected, these harmonics do not appear in the line electromotive force nor their currents in the line.

With three-part poles and symmetrical distortion, there is a definite minimum per cent. of electromotive force harmonics for a given range of voltage ratio and as in this case the fifth harmonic is usually prominent (especially with equal part poles), less advantage can be taken of the fact that the third harmonic vanishes in the 120° electromotive force.

With two-part poles, the total per cent. of electromotive force harmonics for a given range of voltage ratio, may be made considerably less than for the symmetrical distortion, and as the third harmonic is naturally predominant in this case, the employment of the 120° connection, or of the 180° connection with star-connected transformer primaries, may result in the practical elimination of electromotive force or current harmonics.

The conditions in this respect may be still further improved by the use of mid-slot connections and a fractional pitch winding, as far as the latter is consistent with good commutation.

The first part of this paper applies as well to alternators as to synchronous converters; the writer expects shortly to put it into such shape that it will be more useful in the study of alternator wave shape and in the design of alternators.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

APPLICATION OF STORAGE BATTERIES TO REGULATION OF ALTERNATING-CURRENT SYSTEMS

BY J. L. WOODBRIDGE

The general function of a storage battery in connection with an alternating-current system is the same as in a direct-current system, namely, to relieve the power plant and in some cases the transmission lines of the fluctuations of load, permitting the generating machinery and conductors to be utilized to the greatest advantage and at maximum economy by subjecting them to a steady load equal to the average, instead of a load whose fluctuations in some instances, as in heavy interurban railway work and in many industrial plants, are exceedingly rapid and severe. In many cases where alternating currents are developed, the advantages of a regulating storage battery are even more pronounced than in direct-current service, for the following reasons:

1. Alternating-current generation is particularly applicable to long distance interurban railway work where steam railroad conditions prevail, involving heavy units operating at comparatively infrequent intervals, producing fluctuations of load abnormally high as compared with the average. These conditions are comparable, except on a largely magnified scale, with those obtaining on the four or five car electric roads so common in the early history of electric railways.

2. In many cases alternating current is developed at water-power plants and transmitted for considerable distances, and it is well known that a storage battery affords special advantages in connection with the operation of a water-power supply to which are added the advantages of a practically constant current on the transmission lines.

3. In many such cases of water-power development with alternating-current transmission, power is sold on the maximum demand basis, and the saving in power bills effected by removing the peaks and fluctuations with a storage battery makes the battery installation a very attractive investment.

4. The auxiliary apparatus required in connection with a storage battery adapted for regulating an alternating-current system may also be used for changing the frequency or controlling the power factor.

5. The increasing use of gas engines in steel plants for utilizing the furnace gas offers a large field for the application of storage batteries, and in most of these plants the heavy loads and the large area covered call for alternating-current generation and distribution:

The conditions to be met in the various alternating-current systems include a number of different combinations which may be classified broadly as follows:

A. GENERATION ENTIRELY ALTERNATING-CURRENT

I. Load wholly direct-current. Distribution by alternating-current transmission lines to direct-current sub-stations.

II. Load wholly alternating-current, polyphase.

III. Load wholly alternating-current, single-phase.

B. GENERATION AND LOAD PARTLY ALTERNATING-CURRENT AND PARTLY DIRECT-CURRENT

IV. Direct-current fluctuations preponderating.

V. Alternating-current fluctuations preponderating.

Below is given an outline of the methods which have been adopted or suggested for handling each of these cases. In every case it is of course necessary to have converting apparatus for interchangeably transforming alternating and direct currents as well as automatic apparatus for controlling the transfer of energy between the alternating- and the direct-current circuits. Such apparatus is taken up in detail in the latter part of this paper.

I. Under this heading are included systems in which alternating current is generated and transmitted to direct-current sub-stations for supplying a direct-current fluctuating load such as that of the ordinary interurban direct-current trolley system. The early applications of storage batteries to systems of this kind involved the installation of a battery at each of the several sub-stations to regulate the fluctuations of direct-current load at that point. This required nothing more than the ordinary direct-current controlling apparatus. In some recent installations, however, the conditions have been such that it has been found advantageous to concentrate the battery capacity at one point, a single battery being installed arranged to control the total combined fluctuations of load on the entire system. Such an arrangement obviously reduces the first cost for two reasons. First, the total battery capacity required will usually be considerably smaller where the battery is concentrated in this way, inasmuch as the maximum fluctuations of the combined load will ordinarily be less than the sum of the maximum fluctuations occurring at the several sub-stations, since the latter do not occur simultaneously. Second, the cost per unit of capacity of a single large battery will obviously be less than that of a number of smaller installations. This is particularly true of the auxiliary apparatus.

In order to permit of this concentration of the battery capacity it is necessary to select some point for installing the battery, from which the conductors carrying the total output of the generating plant are readily accessible. In many cases a converter station located at the power house itself affords the most suitable location. In other cases where the transmission is wholly in one direction from the power house the nearest sub-station may be selected. In such cases the battery with its regulating booster is connected to the direct-current sub-station bus-bars in the usual manner, but the booster is controlled by means responsive to the total alternating-current output of the generating plant. Its control is accomplished by means of current transformers located in the main alternating-current bus-bars or transmission lines, between the converter station and the generators, whose secondaries will furnish alternating-current to the battery controlling apparatus proportional to the total output of the power station. In some cases current transformers are located in the individual generator circuits, their secondaries being connected in parallel to give a combined output proportional to the total generator load.

By this arrangement the same converters which are required for supplying the local direct-current load on the sub-station are also used for transforming the battery current, and in many cases no greater converter capacity is called for on account of this latter function. For example, the total load on the system may vary from 500 to 3000 kw. with an average of 1500 kw. The direct-current demand on the sub-station where the battery is located may vary from zero to 1500 kw. Converting apparatus must therefore be installed at this sub-station of sufficient capacity to handle a momentary load of 1500 kw. It is evident that with the battery regulating on the total load this converting apparatus will never have to deliver more than 1500 kw. to the sub-station, since this is the total output from the generators and this amount of energy will be supplied to this sub-station only at rare intervals when the total load on the rest of the system is zero. Nor will this converting apparatus be called upon to transmit energy in the reverse direction to a greater amount than 1500 kw. since the total maximum load on the entire system is 3000 kw. of which the main generator is always supplying 1500. At times of maximum total load there will

CASE I

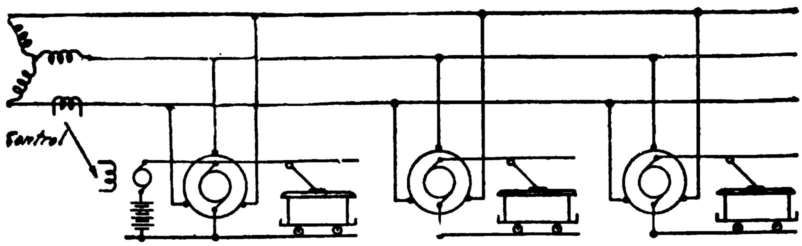


FIG. 1

usually be a direct-current load of considerable magnitude on the regulating sub-station which must first be taken up by the battery before the latter can invert the converters and feed back into the alternating-current mains, thus reducing to that extent the amount of energy inverted. The general operation of the battery in this scheme is to relieve the sub-station of more or less of its direct-current load at times of increased load on the rest of the system, thus preserving a constant load on the generating units. It being necessary on account of the direct-current load to maintain constant voltage on the direct-current bus-bars, a battery booster will be required to cause the battery to charge and discharge, this booster being controlled from the alternating-current circuit. This case is illustrated diagrammatically in Fig. 1.

II. In this case the current is generated and utilized as polyphase alternating current, there being no direct-current circuit. Under these conditions the battery would be located either at the main generating station or in some cases at the center of load, and inasmuch as the entire output of the battery must be converted into alternating current, converting apparatus, either motor-generator sets or synchronous converters, must be provided of sufficient capacity to handle the maximum battery discharge. If a motor-generator set is used for control, the charge and discharge of the battery may be accomplished by varying the voltage of the direct-current machine in response to fluctuations of the alternating-current load. Synchronous converters are undoubtedly preferable for this purpose, particularly if the fluctuations of load are extremely rapid. If a motor-generator is called upon to transfer energy from the battery to the alternating-current circuit it will deliver energy to the latter circuit only after its armature has been driven somewhat ahead of its normal phase position, and owing to the inertia of the armature some appreciable time must elapse after the battery has begun to discharge into the direct-current machine before the alternating-current machine is delivering the equivalent energy to the alternating-current circuit and a time lag is thus introduced. With a synchronous converter this effect is practically eliminated.

With the latter type of machine, however, special means must be provided for varying the direct-current voltage at the battery terminals to compel the battery to charge and discharge. If a standard converter is employed a battery booster will be required to produce this voltage variation. This booster will be provided with field control responsive to the fluctuations of alternating-current load, such controlling apparatus being described below.

On account of the absence of direct-current load, however, a constant direct-current voltage is not required, and in order to dispense with the battery booster a synchronous converter providing for a variable ratio between the alternating- and direct-current voltages has been developed. This machine has been designated the " split-pole " converter owing to the fact that the control of the transformation ratio is brought about by dividing each pole into two or more sections whose excitations are separately controlled. By this means the direct-current voltage of the converter is varied to cause the battery to charge and discharge while the alternating-current voltage is maintained practically constant. This type of machine offers very considerable advantages in the way of simplification and increased efficiency of operation over apparatus involving boosters or motor-generator sets. It is found to be particularly adapted for handling extremely rapid fluctuations of alternating-current load. It must be borne in mind that in dispensing with the booster the means for maintaining a constant-potential direct-current bus-bar is eliminated and any circuit connected directly to the battery terminals will be subject to more or less variation of voltage due to the charge and discharge of the battery. This case is illustrated in Fig. 2.

III. In this case power is generated as alternating current and is supplied to a single-phase circuit. One of the most noted examples of this class of installation is found at the frequency-changing station of the Spokane & Inland Ry. Co. at Spokane,

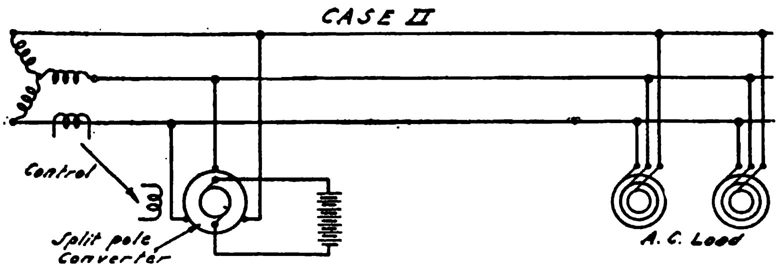


FIG. 2

Washington. The equipment of this station includes four frequency- and phase-changing motor-generator sets, each set consisting of three direct connected machines, viz., a 60-cycle, 3-phase induction motor of 1000 h.p. capacity, receiving power from the incoming transmission line; a 1000-kw., single-phase, 25-cycle, 2200-volt generator, delivering current to the railroad; and a 750-h.p., 500-volt, direct-current machine, which is connected to the battery and acts as a generator or a motor when the battery is respectively charging or discharging. The battery consists of 275 cells having a capacity of 1920 amperes at the one-hour rate, and is provided with two motor driven boosters which may be operated in parallel. The boosters are controlled by apparatus which responds to the current in the incoming 60-cycle transmission line in such a way that small variations of this current vary the booster voltage, thus causing the battery to charge from or discharge into the direct-current machine to absorb the fluctuations of load on the single-phase circuit. This is one of the cases in which the financial results from a battery installation may be very clearly demonstrated, inasmuch as the power for the operation of the railway system is purchased on the maximum demand basis, and it is estimated that the reduction in the power bills effected by the battery in

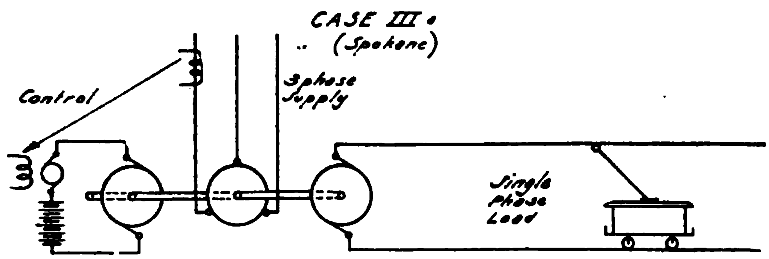


FIG. 3

relieving the transmission line of load fluctuations will be sufficient to repay the initial investment in about three years. This application is illustrated in Fig. 3.

Instead of the scheme above described it will be quite practicable to utilize the split-pole converter operating as a single-phase machine connected to the single-phase circuit with suit-

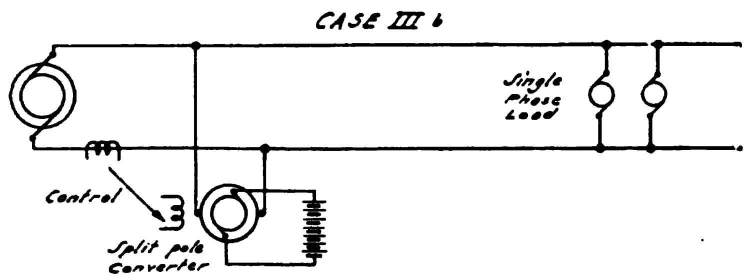


FIG. 4

able transformers. The output of the battery will then be delivered to the railway circuit without the intervention of mechanical transmission, and the efficiency on discharge and therefore the net alternating-current output for a given battery discharge will be greater. This is illustrated in Fig. 4.

IV. This and the following case include systems in which both alternating-current and direct-current generators are operated to supply a load which is partly alternating- and partly direct-current, the two circuits being interconnected by suitable converting apparatus. In case IV, the fluctuations of the direct-current load preponderate, and inasmuch as the direct-current bus-bar must ordinarily be maintained at constant voltage a battery booster is required, sufficient at least for the direct-current fluctuations. Since these latter constitute the greater part of the total load fluctuations, the booster should be made of sufficient capacity for the total battery output. The booster control would not differ in any way from the standard direct-current controlling apparatus. In order to transmit the fluctuations originating on the alternating-current circuit to the direct-current bus-bar special automatic control of the converting apparatus must be provided. The split-pole converter would undoubtedly be a most satisfactory method of providing this control, and inasmuch as the direct-current bus-bar voltage is practically constant only a very small change in ratio of conversion would be necessary, just sufficient to overcome the internal drop in the converter and static transformers.

Another method of providing for this automatic control of the converting apparatus consists in mounting on the shaft of the converter an alternating-current booster whose windings are connected in series between the alternating-current terminals of the converter armature and the collector rings. The field of this booster is controlled automatically in response to fluctuations of load on the alternating-current circuit, thus adding to or subtracting from the alternating-current voltage of the converter and thereby causing it to transmit energy in either direction between the alternating- and direct-current circuits as may be required to maintain constant load on the alternating-current generators. This latter arrangement is illustrated in Fig. 5.

V. The conditions included in this case are the same as in case IV except that the alternating-current load fluctuations preponderate. Inasmuch as the direct-current bus-bar voltage must be maintained constant a battery booster is required of sufficient capacity to handle the fluctuations of direct-current load. These fluctuations being comparatively small this booster capacity may be reduced to a minimum provided the fluctuations of alternating-current load are transmitted directly to the battery terminals instead of to the direct-current bus-bars. This latter may be accomplished by means of the split-pole converter which in this case is connected between the alternating-current circuit and the battery terminals, and is so controlled as to transmit directly to the battery the fluctuations of load originating on the alternating-current circuit. This is the scheme which has been adopted for controlling the load fluctuations at the electric power plant of the Indiana Steel Company, at Gary, Indiana.

For this plant there is being installed a battery consisting of two series of cells arranged to operate in parallel, each series consisting of 125 cells having a one-hour capacity of 4320 amperes. Provision has been made for the future extension of this plant by the addition of two more series of cells of the

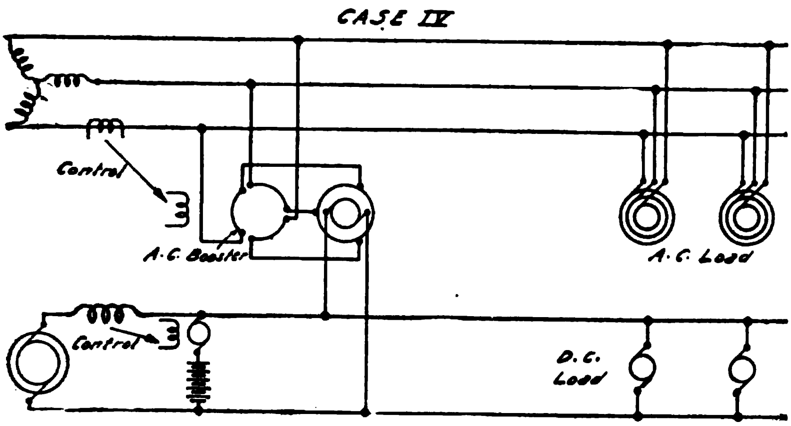


FIG. 5

same capacity. Two direct-current boosters are provided, having sufficient capacity to handle the fluctuations of direct-current load, which will be only a small part of the total load fluctuations. For handling the fluctuations of alternating-current load a split-pole converter of 2000 kw. capacity is being installed, designed to provide a range of direct-current voltage from a minimum of 200 to a maximum of 300 volts with constant alternating-current voltage. The regulating field of this converter will be controlled by a special alternating-direct-current exciter, designed as described hereinbelow. Provision has been made for a second converter of the same design and capacity. This case is illustrated in Fig. 6.

Auxiliary apparatus. Under this heading are described three pieces of auxiliary apparatus which have been developed for controlling the operation of a battery in connection with an alternating-current circuit.

The first of these devices, the carbon regulator, calls for only passing notice, as its general design and operation are well known in connection with direct-current regulating plants. It is shown diagrammatically in Fig. 7, in which x and y are two piles of carbon discs subjected to variable pressure by means of the lever, L, which is pivoted between the two piles in such a way that any motion of this lever will increase the pressure on one pile and decrease that on the other. At one end of the lever is an adjustable spring, T, while from the other end is suspended the iron core of a solenoid, S, which carries the current

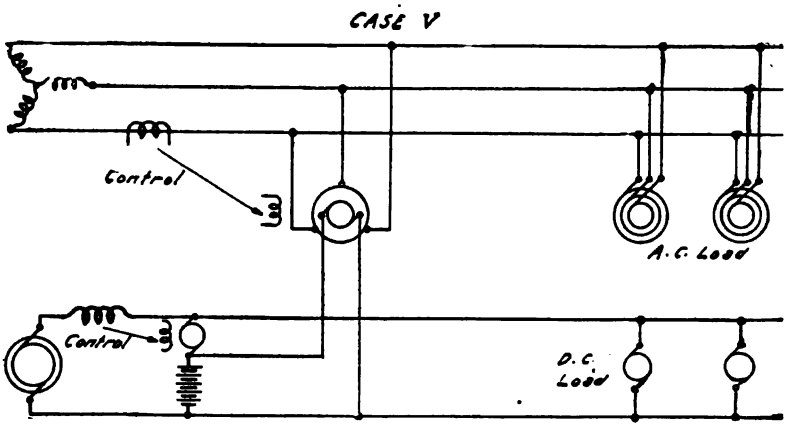


FIG. 6

to be regulated, or a current proportional thereto. The two carbon piles are electrically connected in series across the battery, B, while the field coil, F, which is to be controlled, is connected between a point in the circuit between the two carbon piles and the middle point of the battery. The variation of resistance in the two piles due to change of pressure causes a flow of current in one direction or the other through the field coil, F, which is therefore made responsive to the current in the solenoid, S. The application of the regulator to alternating-current control involves the substitution of an alternating-current solenoid for the usual direct-current coil. In those installations where the power factor is normally maintained approximately at unity, such as the usual alternating-current generating plant supplying current to converter sub-stations for direct-current railway service, a single coil supplied with alternating current proportional to the current output of the alternators will give entirely satisfactory results, this current being practically proportional to the true watt output of the machines. If, however, the power factor is normally considerably less than unity or if it is subject to wide variations, it may be necessary to control the carbon regulator by means responsive to the energy component of the current and two coils may be used, one of which will produce a constant magnetic flux, while the other carries current which is in phase with the flux at unity power-factor, or whose energy component is in phase with the flux at power-factors other than unity.

The second piece of regulating apparatus which has been developed for this class of service is the split-pole converter.

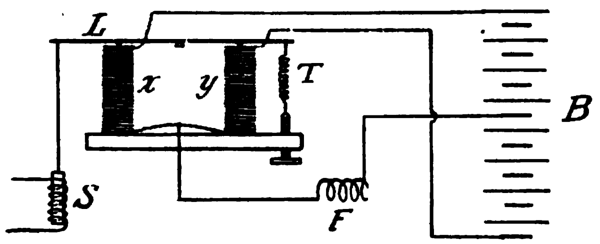


FIG. 7

This machine has already been mentioned in a recent paper before this Institute, but I wish to take up somewhat more in detail certain points in reference to the theory of its operation and the detail of its design.

As originally suggested this type of machine was constructed with each pole sub-divided into three parallel sections with suitable windings for varying the relative excitation of the three sections, the object being to secure a variation of direct-current voltage while maintaining a constant alternating-current voltage. The theory upon which this design was based may be illustrated by reference to Fig. 8, in which is shown an assumed distribution of field flux around the periphery of the armature. The line, A B, represents the developed armature periphery, the direct current brushes being located at A and D. The lines a, b, and c, represent the field strength of the three sections of one pole and the lines d, e, and f that of the other pole under normal conditions of excitation. The result of this excitation will be a certain ratio between alternating- and direct-current voltage. If, now, the excitation of the outer pole sections *a*, *c*, *d*, and *f*, be increased, and that of the middle sections *b*, and *e*, be decreased, the field distribution may then be represented as the resultant of the auxiliary field *g*, *h*, *i*, *j*, *k*, and *l*, superposed upon the normal field. If, now, we consider the electromotive force developed in a section of the armature between the points *P*, and *Q*, which may be taken to represent two of the three phase taps located 120° apart, it will be seen that the auxiliary field will have absolutely no effect on the voltage in this section at any point in the revolution; for the sum of the shaded areas *x*, and *z*, is always equal and opposite to the shaded area *y*. The auxiliary field therefore interposes into

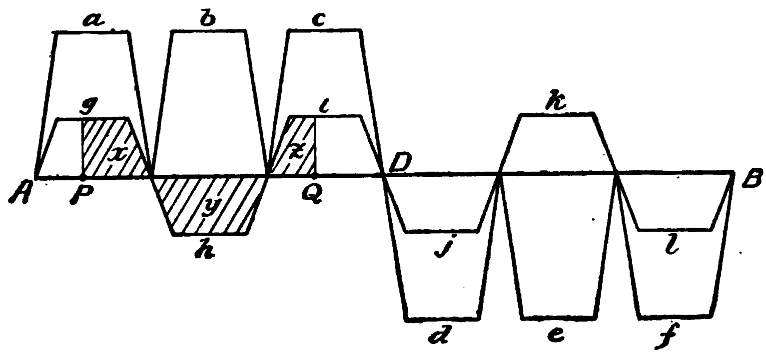


FIG. 8

the armature section between the points *P*, and *Q*, equal and opposite amounts of field flux, which exactly neutralize each other in the generation of electromotive force between *P*, and *Q*. The effect however of the auxiliary field on the direct-current voltage between the points of brush contact *A*, and *D*, is to produce an increase in that voltage for the reason that between these two points there are two areas of positive flux and only one area of negative flux due to the auxiliary field. Theoretically, therefore, this auxiliary field would produce an increase or decrease in the direct-current voltage without any effect whatever on the magnitude or wave shape of the alternating voltage, between the points *P*, and *Q*, provided these points are spaced 120° apart.

Practically, however, it is necessary to allow a neutral area of zero field strength at the points A, and D, for commutation. This modification produces some disturbance of the wave shape of the alternating counter electromotive force, which, however, is not sufficient to interfere with the practical operation of the machine. In Figs. 9 to 14 inclusive, are shown oscillograph

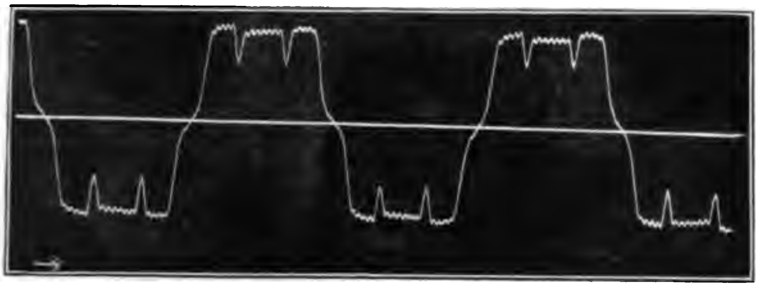


FIG. 9

records giving the distribution of field flux obtained by an exploring coil and the corresponding wave shape of alternating electromotive force across two of the three phase collector rings, taken from a 500 kw. split-pole converter of this type. Fig. 9 shows the normal field distribution with all three sections of

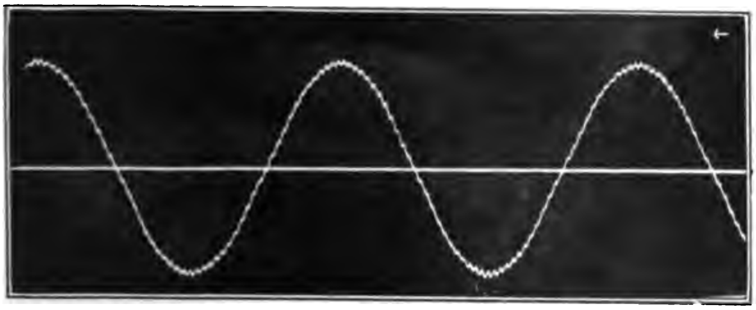


FIG. 10

the pole equally excited and Fig. 10 is the corresponding wave shape of electromotive force; direct-current voltage 250. Fig. 11 shows the field distribution with the outer sections increased and the middle section reduced in excitation, while Fig. 12 shows the corresponding wave shape of electromotive force; direct-current voltage 280. Fig. 13 shows the field distribution with the outer sections weakened and the middle section strengthened, and Fig. 14 shows the corresponding wave shape of electromotive force; direct-current voltage 232. It will be noted that the difference in the wave shapes for these three different conditions of field excitation is scarcely perceptible. The alternating electromotive force was held constant throughout.

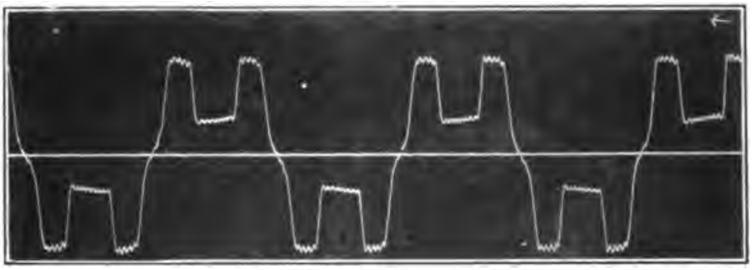


FIG. 11

In a later development of the split-pole converter suggested by Mr. J. L. Burnham, only two pole sections are employed. One of these has a pole arc covering a considerable portion of the total pole pitch, and constitutes the principal pole, while the other covers a smaller arc and is used as an auxiliary or regulating pole. A simple explanation of the theory of this machine

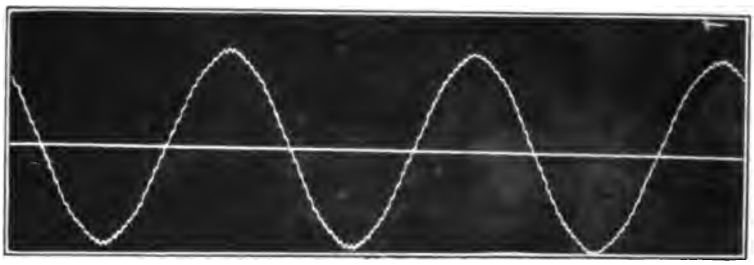


FIG. 12

may be had by reference to Figs. 15 and 16. In Fig. 15 the line, OP, represents the alternating voltage developed by the principal pole section. The line, OF, may be taken to represent in amount and phase position the electromotive force developed by the auxiliary pole, these two electromotive forces being displaced by the angle, ρ, which represents the angle between the axes of the two fields. The line, $O\,R$, will then represent the resultant alternating electromotive force. If the excitation of the auxiliary field is zero, the line, $O\,P$, will represent the alternating electromotive force. It will be noted that the line, $O\,R$, is but very little longer than the line $O\,P$. The direct-current

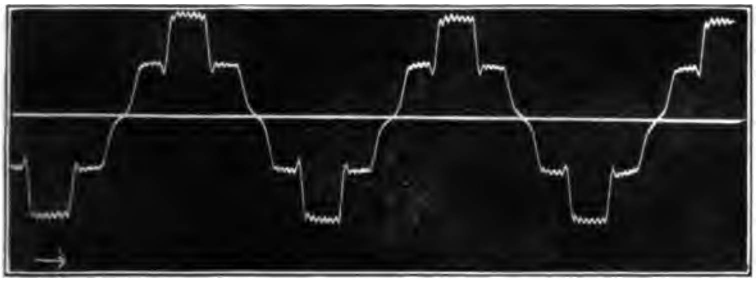


Fig. 13

electromotive force will be represented by the algebraic sum of $O\,P$, and $O\,F$. With zero excitation of the auxiliary pole this direct-current electromotive force will be represented by $O\,P$, while with an excitation corresponding with the line $O\,F$, the direct-current electromotive force will be represented by $O\,D$. If the main field excitation is reduced to $O\,P'$, when

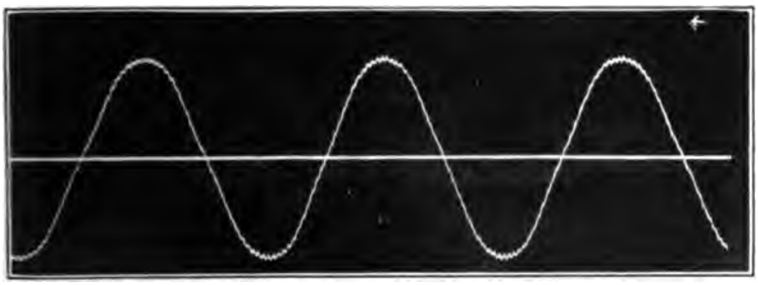


Fig. 14

the auxiliary field has an excitation corresponding with $O\,F$, the resultant alternating electromotive force will be represented by $O\,R'$, equal to $O\,P$, while the direct-current electromotive force will be represented by $O\,D'$. It will thus be seen that the direct-current electromotive force has been increased from $O\,P$ to $O\,D'$ without any change in the magnitude of the alternating current electromotive force. If the auxiliary pole be given an initial negative excitation corresponding with the line $O F_0$, such that the resultant $O R_0$ is at right angles to $O F_0$, then any variation of excitation of the auxiliary pole on either side of this point will produce a minimum change in the alternating electromotive force, provided the main field excitation is maintained constant, or will require the least variation in the excitation of the main field to maintain constant alternating electromotive force. Thus if the negative excitation of the auxiliary field be decreased to F_1, the resultant alternating electromotive force will be represented by $O R_1$, while if the negative excitation of the auxiliary pole be increased to F_2, the alternating electromotive force will be represented by $O R_2$. The corresponding direct-current electromotive forces will be

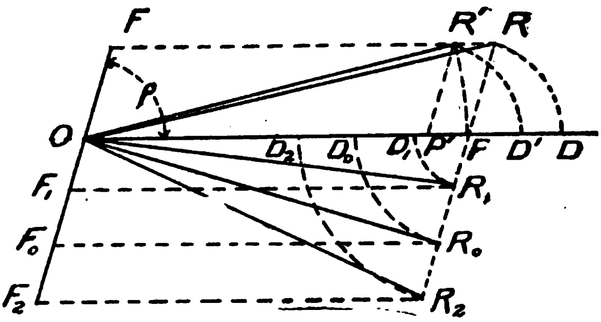


FIG. 15

represented by $O D_0$, $O D_1$, and $O D_2$, respectively. For hand control it is of course permissible to vary the excitation of both the main and auxiliary fields, in order to vary the direct-current voltage without disturbing the power-factor. With automatic control it is, however, very desirable to limit the control to a single field coil; and to accomplish this with minimum disturbance to the power factor, it is necessary to work on either side of the point F_0 which represents the mean excitation of the auxiliary pole. In Fig. 16 are illustrated the results obtained by designing the machine with a pitch of 90° between the main and auxiliary pole sections. In this case the mean excitation of the auxiliary pole for minimum change in the alternating-current electromotive force is zero, the excitation of this section being varied on either side of zero to produce a change of direct-current electromotive force above and below the mean. It can be readily shown that if the lines $O F_1$, and $O F_2$, are each equal to 20 per cent. of the line $O P$, corresponding with a variation in direct-current electromotive force of 20 per cent. above and 20 per cent. below the mean, the lines $O R_1$, and $O R_2$, will be about 2 per cent. greater than the line $O P$, resulting in a total variation of 1 per cent. in the alternating electromotive force on either side of the normal.

In addition to the change in direct-current voltage it will be noted that a change in the phase position of the resultant alternating electromotive force is brought about by varying the excitation of the auxiliary pole section. If this pole section is made the trailing section, this shifting of the phase position will coöperate with the change in direct-current voltage in bringing about a quick transfer of energy between the battery and the alternating-current circuit, by advancing the phase

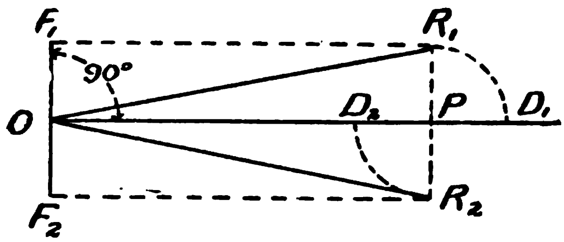


FIG. 16

of the alternating-current voltage, thus causing the converter to deliver energy to the alternating-current circuit at the same time that the direct-current voltage is reduced and energy is thus taken from the battery.

The effect of the two-section split-pole construction on the wave shape of the alternating electromotive force may be determined in the following manner. Referring to Fig. 17, the line $A\ D\ B$ represents as before the developed periphery of the armature, the ordinate f representing the field density at any point, this ordinate having the values b, and c, at the face of the main and auxiliary pole sections respectively. A neutral space of 30° is allowed for commutation and the pole section pitch is taken at 90° which requires a space of 30° between the main and auxiliary pole sections. This leaves 120° for the combined arc of the two pole sections. Assuming that a variation of direct-current voltage from 20 per cent. above to 20 per cent.

below the mean is desired, and that the maximum density in the two pole sections is to be approximately the same, the arc of the auxiliary pole section should be about 20 per cent. of that of the main section. I have therefore shown the main section covering 100° of arc and the auxiliary section 20°.

Let θ represent the angular distance of any point on the circumference of the pole faces from the neutral point A which is taken as the origin.

Let f represent the ordinate corresponding with the field density at that point.

Let $X\ Y$ represent any section of the armature, having an angular span β.

Let ϕ equal the angular distance of the center point of the armature section $X\ Y$ from the origin A at any instant in the revolution of the armature.

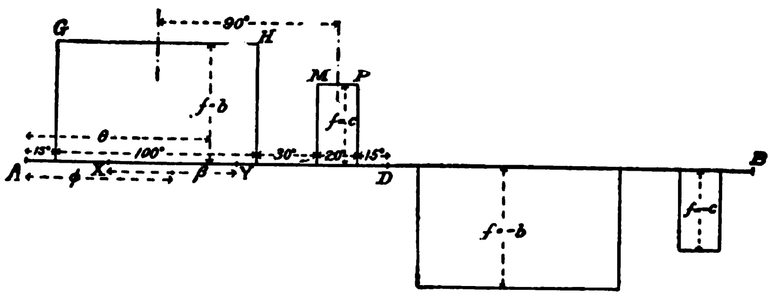


FIG. 17

Let e represent the momentary electromotive force developed in the armature section between the points X and Y at any instant.

Then by Fourier's theorem the value of f may be represented by

$$f = a_1 \sin(\theta - \alpha_1) + a_3 \sin 3(\theta - \alpha_3) + a_5 \sin 5(\theta - \alpha_5) + \dots + a_n \sin n(\theta - \alpha_n) \qquad (1)$$

Also we have

$$e = k \int_{\phi - \frac{\beta}{2}}^{\phi + \frac{\beta}{2}} f\, d\theta \qquad (2)$$

from which, by substituting the value of f in (1) and integrating we have

$$e = k \left\{ a_1 \left[\cos \left(\phi - \frac{\beta}{2} - \alpha_1 \right) - \cos \left(\phi + \frac{\beta}{2} - \alpha_1 \right) \right] + \frac{a_3}{3} \left[\cos 3 \left(\phi - \frac{\beta}{2} - \alpha_3 \right) - \cos 3 \left(\phi + \frac{\beta}{2} - \alpha_3 \right) \right] + \frac{a_5}{5} \left[\cos 5 \left(\phi - \frac{\beta}{2} - \alpha_5 \right) - \cos 5 \left(\phi + \frac{\beta}{2} - \alpha_5 \right) \right] \ldots + \frac{a_n}{n} \left[\cos n \left(\phi - \frac{\beta}{2} - \alpha_n \right) - \cos n \left(\phi + \frac{\beta}{2} - \alpha_n \right) \right] \right\}$$

in which k is a constant depending upon the design of the machine, the speed of rotation, and the units employed. This expression may be reduced to

$$\epsilon = 2\,k \left\{ a_1 \sin \frac{\beta}{2} \sin (\phi - \alpha_1) + \frac{a_3}{3} \sin \frac{3\beta}{2} \sin 3 (\phi - \alpha_3) + \frac{a_5}{5} \sin \frac{5\beta}{2} \sin 5 (\phi - \alpha_5) \ldots + \frac{a_n}{n} \sin \frac{n\beta}{2} \sin n (\phi - \alpha_n) \right\} \qquad (3)$$

It will be noted that the expression within the brackets consists of the fundamental sine wave $a_1 \sin \frac{\beta}{2} \sin (\phi - \alpha_1)$ upon which is superposed a series of harmonics of the general form $\frac{a_n}{n} \sin \frac{n\beta}{2} \sin n (\phi - \alpha_n)$ The coefficient of each of these harmonic terms is made up of two factors, namely $\frac{a_n}{n}$ and $\sin \frac{n\beta}{2}$ whose product will determine the magnitude of the harmonic. Let us first consider the factor $\frac{a_n}{n}$. In order to determine the value of a_n, multiply both members of equation (1) by $\sin n\theta\, d(n\theta)$ and integrate between the limits $\theta = o$ and $\theta = \pi$. The second member will then consist of a series of terms of the form $\int_0^{\pi} a_m \sin m (\theta - \alpha_m) \sin n\,\theta\, d\,(n\,\theta)$.

It can be shown that $\int_0^\pi a_m \sin m (\theta - \alpha_m) \sin n \theta \, d (n \theta) = o$ when m and n are unequal odd integers. Every term in the second member of the equation therefore reduces to zero except the one term in which m equals n and the equation reduces to

$$\int_0^\pi f \sin n \theta \, d (n \theta) = \int_0^\pi a_n \sin n (\theta - \alpha_n) \sin n \theta \, d (n \theta) =$$

$$\frac{n \pi a_n \cos n \alpha_n}{2} \qquad (4)$$

To eliminate α_n assume the zero point, or origin from which the angular measurements are made, moved forward along the periphery of the pole face by an angle $\frac{\pi}{2n}$ and let θ' represent the angular distance of any point on the pole faces from this new origin. α_n will then become $\alpha_n - \frac{\pi}{2n}$ and equation (4) becomes

$$\int_0^\pi f \sin n \theta' \, d (n \theta') = \frac{n \pi a_n \cos \left(n \alpha_n - \frac{\pi}{2} \right)}{2} = \frac{n \pi a_n \sin n \alpha_n}{2} \qquad (5)$$

Combining (4) and (5) by squaring and adding we have

$$a_n = \frac{2}{n\pi} \sqrt{ \left\{ \int_0^\pi f \sin n \theta \, d (n \theta) \right\}^2 + \left\{ \int_0^\pi f \sin n \theta' \, d (n \theta') \right\}^2 } \qquad (6)$$

The value of the expression $\int_0^\pi f \sin n \theta \, d (n \theta)$ can readily be obtained from the flux distribution. Where f is a succession of constant values as in Fig. 17, the integration gives

$$- f \cos n \theta \Big|_{\text{limit}}^{\text{limit}}$$

For the fundamental wave $n = 1$, and the value of the first integral under the radical sign may be derived as follows from the data given in Fig. 17;

Limits, θ	f	Integral	Value
0° to 15°	o		
15° to 115°	b	$-b$ (cos 115° - cos 15°)	$+1.3885\,b$
115° to 145°	o		
145° to 165°	c	$-c$ (cos 165° - cos 145°)	$+\ .147\,c$
165° to 180°	o		
		Total	$(1.3885\,b + .147\,c)$

For the second integral assume the origin moved 90° ahead and proceed as above.

Limits, θ'	f	Integral	Value
0° to 25°	b	$-b$ (cos 25° - cos 0°)	$+.0937\,b$
25° to 55°	o		
55° to 75°	c	$-c$ (cos 75° - cos 55°)	$+.315\,c$
75° to 105°	o		
105° to 180°	$-b$	$+b$ (cos 180° - cos 105°)	$-.7412\,b$
		Total	$-.6475\,b + .315\,c$

Substituting these two values in (6) we have

$$a_1 = \frac{2}{\pi}\sqrt{(1.3885\,b + .147\,c)^2 + (-.6475\,b + .315\,c)^2},$$

or

$$a_1 = \frac{2}{\pi}\sqrt{2.347\,b^2 + .121\,c^2} \tag{7}$$

If $c = o$ $\qquad a_1 = \ .975\,b$
If $c = +b$ or $-b$, $a_1 = 1.000\,b$

Thus we have a variation of $2\frac{1}{2}$ per cent. in the magnitude of the fundamental sine wave of electromotive force with a 40 per cent. change in the direct-current voltage.

To determine the value of a_5 we have $n = 5$. Proceeding as above:

Limits, θ	f	Integral	Value
0° to 15°	o		
15° to 115°	b	$-b$ (cos 575° - cos 75°)	$1.078\,b$
115° to 145°	o		
145° to 165°	c	$-c$ (cos 825° - cos 725°)	$+1.255\,c$
165° to 180°	o		
		Total	$1.078\,b + 1.255\,c$

Moving origin 18° ($= \pi/2n$) ahead,

Limits, θ'	f	Integral	Value
0° to 97°	b	$-b$ (cos 485° - cos 0°)	$1.574\,b$
97° to 127°	o		
127° to 147°	c	$-c$ (cos 735° - cos 635°)	$-.879\,c$
147° to 177°	o		
177° to 180°	$-b$	$+b$ (cos 900° - cos 885°)	$-.034\,b$
		Total	$1.540\,b - .879\,c$

From which

$$a_5 = \frac{2}{5\pi}\sqrt{3.532\, b^2 + 2.347\, c^2} \tag{8}$$

$$\begin{aligned} &\text{If } c = o & a_5 &= .239\, b \\ &\text{If } c = +b \text{ or } -b, & a_5 &= .308\, b \end{aligned}$$

By the above method the value of a_n for any harmonic term for any type of synchronous machine may be determined when the field distribution is known. (For a more general solution of the two-sectional pole type, see Appendix.)

Turn now to the other factor in the coefficient, namely $\sin \frac{n\beta}{2}$. The angle β represents the angular span included between two alternating-current taps from the armature to the collector rings. When this angle is known this factor in the coefficient is determined. For the ordinary diametric connection we have $\beta = \pi$ and $\sin \frac{n\beta}{2}$ is always $\pm$ 1, n being an odd integer, and it can be shown that when $\beta = \pi$, each harmonic in the series is passing through a maximum value with respect to the fundamental. By making $\beta = \frac{2\pi}{3}$, $\sin \frac{3\beta}{2} = 0$, and the third harmonic disappears. Likewise all the multiples of the third harmonic are reduced to zero. This corresponds with the ordinary three-phase delta connection. With this value of β we have

$$\sin \frac{n\beta}{2} = \pm \tfrac{1}{2}\sqrt{3}$$

for all values of n except multiples of three. The ratio to the fundamental, of all harmonics except the third and its multiples, will therefore be the same as with the diametric connection.

The third harmonic and its multiples may however be similarly suppressed by star-connecting the primaries of the statics. By doing this the value of β may be chosen with reference to some other harmonic than the third. By making $\beta = .8\,\pi$ we have $\sin \frac{5\beta}{2} = 0$ and the 5th harmonic disappears. By making $\beta = \frac{6\pi}{7}$, $\sin \frac{7\beta}{2} = 0$ and the 7th harmonic disappears. By assuming a value for β intermediate between $.8\,\pi$ and $\frac{6\pi}{7}$, say $0.82\,\pi$, we have $\sin\frac{5\beta}{2} = 0.1564$ and $\sin\frac{7\beta}{2} = 0.397$, and both the 5th and 7th harmonics are reduced to small percentages of their maximum values.

In Fig. 18 is illustrated a combination designed to suppress the third harmonic and reduce the 5th and 7th to a minimum as above described. The armature A, of the converter is provided with alternating-current taps dividing it into three phase-sections, each spanning an angle of $0.82\,\pi = 147°\,36'$. The three phase-sections are however displaced by angles of 120°, and can therefore be independently connected across the sec-

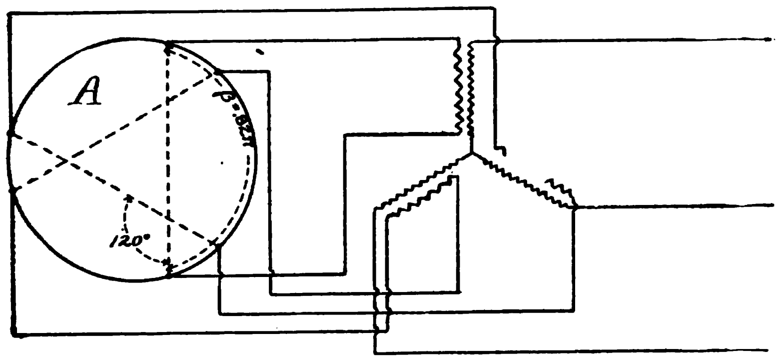


FIG. 18

ondaries of the statics whose primaries are star-connected to the three-phase circuit as shown.

From equation (3) it will be seen that the ratio of the amplitude of any harmonic to that of the fundamental is

$$\frac{a_n \sin\frac{n\beta}{2}}{a_1 \sin\frac{\beta}{2}}$$

Substituting in this the values of a_1 and a_5 as given in equations (7) and (8) and similarly determining the value of this ratio for the 7th harmonic, the values of these two harmonics in proportion to the fundamental will be given in terms of the two field densities b and c. The direct-current voltage will be proportional to the total field flux, *i.e.*, to $b + .2\,c$ (the smaller pole section arc being 20 per cent. of the greater). From these data the curves shown in Fig. 19 have been plotted, showing the ratio of the 5th and 7th harmonics to the fundamental for various direct-current voltages covering a range of 20 per cent. on each side of the mean. In the curves to the left the diametric connection has been assumed; the same ratios would hold for three-phase delta connection. In the curves to the right the relative values of the same harmonics are given, using the scheme of connection as shown in Fig. 18. In the first case the maximum values of the 5th and 7th harmonics are seen to be 6.2 per cent. and 2.5 per cent. of the fundamental, respec-

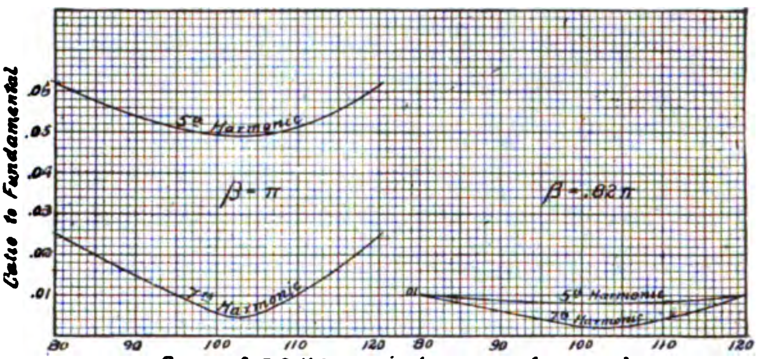


FIG. 19

tively, while in the latter case they are both reduced to a maximum of 1 per cent. of the fundamental.

In Fig. 20, is shown the resultant wave shape obtained by combining the fundamental with the 5th and 7th harmonics, the maximum values of the latter being taken, corresponding with minimum direct-current voltage and diametric connection (see Fig. 19). In Fig. 21, the wave shape is given for the same conditions except that the connections are as shown in Fig. 18 ($\beta = 0.82\,\pi$). In these curves, only the 5th and 7th harmonics have been considered. The 3d and 9th are eliminated by star-connecting the static primaries, and those of higher frequency may be ignored.

The scheme shown in Fig. 18, produces some increase in the heating developed in the armature winding of the converter as compared with the diametric connection. In Fig. 22, curves have been plotted showing this comparison. By reference to these curves it will be seen that the mean $I^2 R$ loss has been increased about 12 per cent., while the maximum loss in any single bar is about 50 per cent. greater than with the diametric connection. The maximum heating per slot is however increased only about 10 per cent. since the two bars in which the heating is maximum are not located in the same slot. These curves are based on field distribution corresponding to the minimum ratio of direct- to alternating-current electromotive force, the value of c in equation (7) being taken as $-b$. This voltage ratio

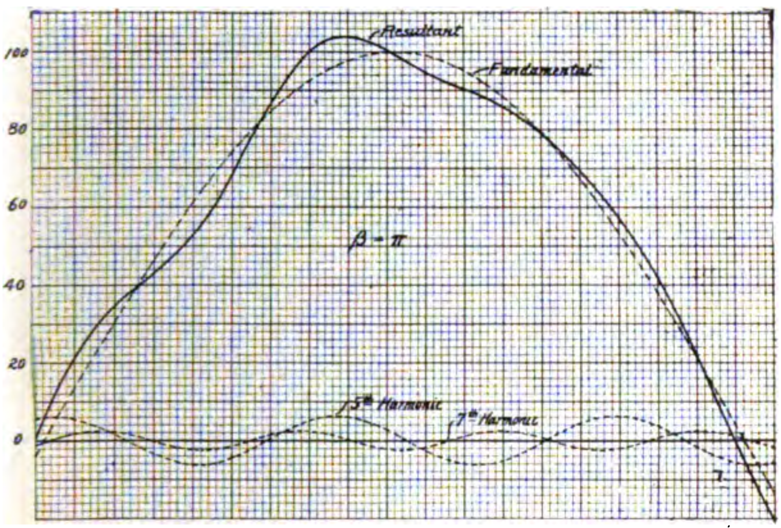


FIG. 20

has of course a material effect on the armature heating since it fixes the ratio between the superposed alternating and direct currents in the armature winding. The curve showing the distribution of heating with diametric phase connection in Fig. 22 is therefore somewhat different from that corresponding to a converter of normal ratio. For intermittent service, such as would be involved in storage battery regulating work, the average heating of the armature winding will usually be of minor importance, as the design of the machine would ordinarily be determined by other considerations, such as commutation at maximum momentary output and flux distribution. Thus the scheme shown in Fig. 18 would appear to offer no appreciable disadvantages. Furthermore, this increase in heating will be at least partially offset by the reduced losses due to the elimination of high frequency currents superposed on the fundamental.

The third piece of auxiliary apparatus which I will describe is an exciter designed to control the regulating field of a booster or split-pole converter in response to variations of load on an alternating-current circuit. This machine is illustrated in Fig. 23, and as there shown consists of an armature, B, provided with a bi-polar winding revolving in a four-pole field frame. This armature is connected by suitable collector rings (not

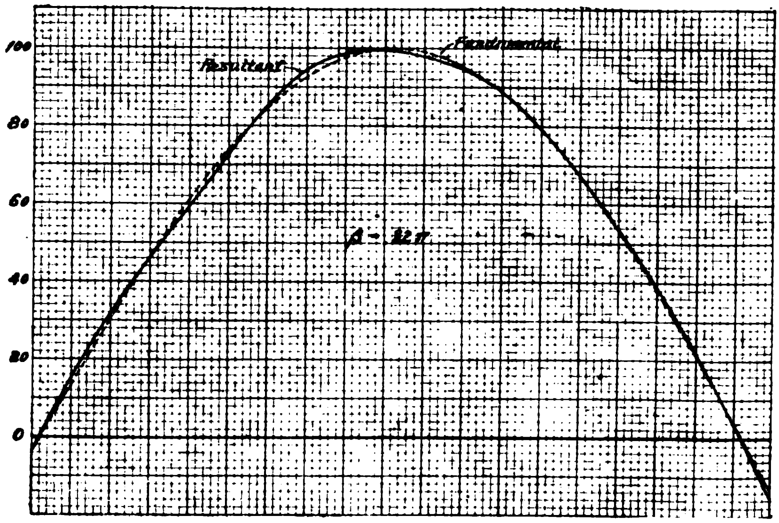


FIG. 21

shown in the drawing) to the secondary windings of three current transformers T_1, T_2, T_3, whose primaries are connected in series with the alternating-current circuit 1, 2, 3, supplied by a source, A, whose load is to be controlled. The alternating current thus transmitted through the armature winding would, if the latter were stationary, produce a revolving field proportional to the output from the source, A. The armature is, however, rotated by means of a synchronous motor in the opposite direction to that of the field rotation so as to hold this field stationary in space. There will result a stationary field whose axis is in the line of the arrow K_1. A commutator, C,

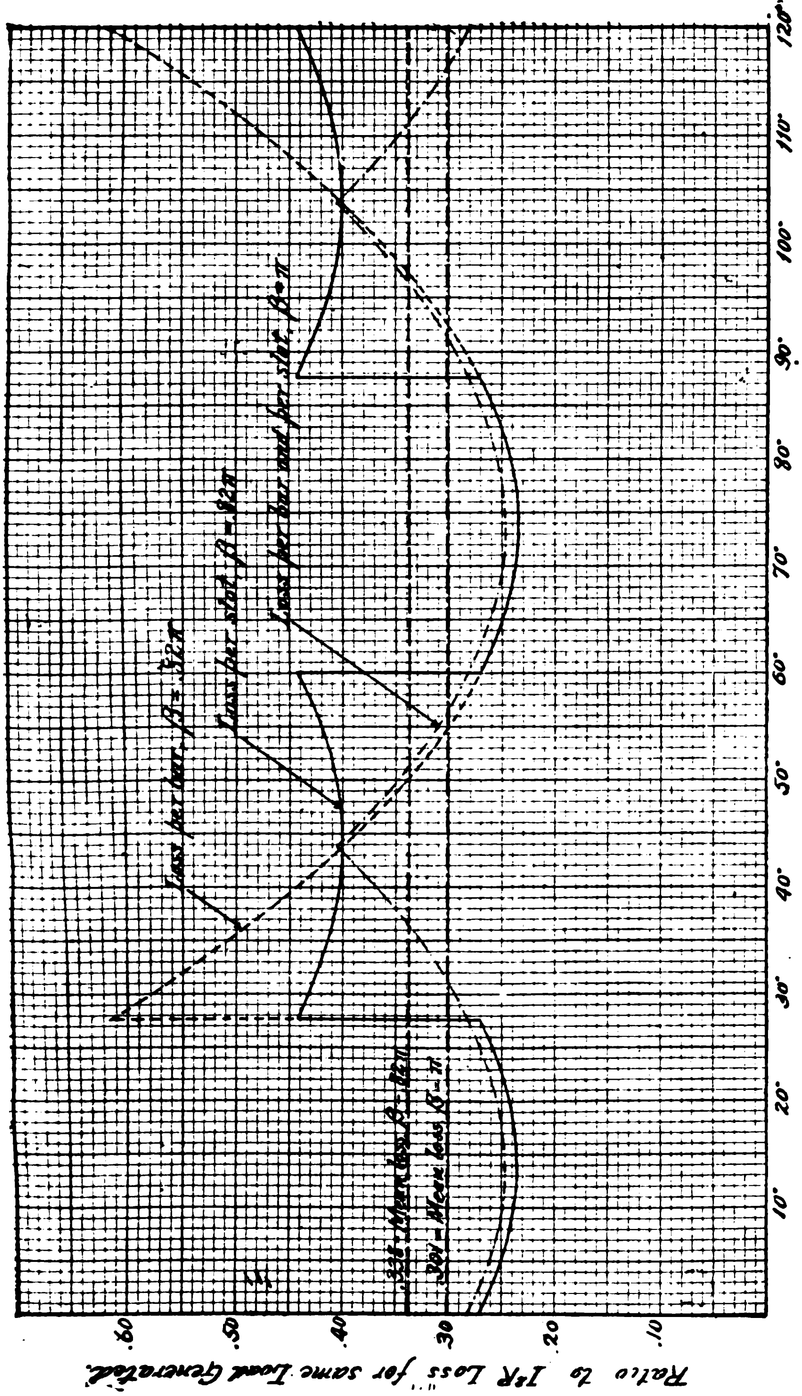

Loss per bar β = .52π
Loss per slot β = .82π
Loss per bar and per slot β = π
.338 - Mean loss β = .82π
.301 - Mean loss β = π
10° 20° 30° 40° 50° 60° 70° 80° 90° 100° 110° 120°
.60 .50 .40 .30 .20 .10
Ratio to I²R Loss for same Load Generated.

FIG. 22

is connected to the armature winding upon which bear two pairs of brushes, 4-5 and 6-7. The effect of the field K_1 would be to produce a direct-current electromotive force between the brushes 4 and 5. A shunt winding, 8, on the four poles of this machine, controlled by the rheostat, R, may be adjusted to neutralize the field K_1 at any predetermined output from the source, A, thus reducing the electromotive force between the brushes 4 and 5 to zero. These latter brushes are then short circuited. If now, a slight increase of load on the source A should occur the field K_1 would no longer be neutralized by the shunt field winding and an electromotive force would be

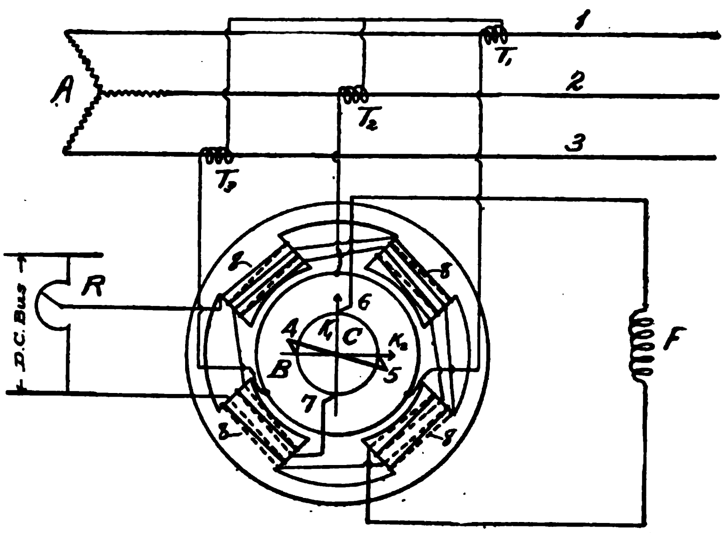


FIG. 23

produced between the brushes 4-5 causing a considerable flow between these brushes. This flow produces a secondary field K_2 at right angles to the first and of appreciable magnitude; and this second field produces an electromotive force across the other pair of brushes 6-7, which are connected to the regulating field, F. In series between the brush 7 and the corresponding terminal of the field, F, is connected a series winding on the poles of the exciter, designed to neutralize the magnetomotive force of the current through the armature winding between the brushes 6 and 7, so that the output from these brushes will have no disturbing effect on the exciter field.

This type of exciter possesses a number of important features in connection with alternating-current regulation, which may be briefly enumerated as follows:

1. It will be seen that it acts as a multiplying device by reason of the magnifying effect of the current between the short circuited brushes, and can therefore be made exceedingly sensitive to small changes in the alternating-current load.

2. It can be made to respond to any desired phase component of the alternating current by a suitable angular relation between the exciter armature and the armature of the synchronous motor which drives it. Thus for controlling the charge and discharge of a storage battery this exciter would be adjusted to respond to the energy component of the alternating current.

3. For controlling the power factor on the alternating-current circuit the two pairs of brushes 4-5 and 6-7 may be interchanged so that the latter are short-circuited while the former are connected to a field winding on any synchronous apparatus connected to the alternating-current circuit. The exciter will then be responsive to the wattless component of the alternating current and can be made to control the field of the synchronous apparatus in such a way that this apparatus will supply practically all or any desired proportion of this wattless current. For controlling both the energy fluctuations and the power-factor two such exciters may be operated, one of which is responsive to the energy component and the other to the wattless component of current.

Another advantageous feature of this exciter lies in the fact that the secondaries of the current transformers may be short-circuited at any time with the result of merely killing the regulating function. It is not necessary to disconnect the connections to the collector rings before doing this, even with considerable excitation in the shunt field winding of the exciter, since the armature reaction is so great that a short-circuit will produce only a nominal flow of current sufficient to demagnetize the fields. Similarly, in putting the apparatus into service the rheostat, R, if properly calibrated may be set for any desired constant load on the source, A, and after all the main connections are closed so that the battery is merely floating on the system, the short-circuiting switch across the secondaries of the current transformers may be opened and the load on the source, A, will immediately be held at the constant predetermined value for which the rheostat, R, has been set.

APPENDIX

A more general solution for the wave shape of alternating electromotive force in the split-pole converter of the two section type is given below, using the nomenclature shown in Fig. 24 as follows:

γ_b = the arc covered by the principal pole section.
γ_c = the arc covered by the auxiliary pole section.
$2\,\delta$ = the neutral arc between pole tips.
ρ = the pole section pitch.
θ = the angular distance of any point on the pole faces from the origin A.

$\theta' = \theta - \dfrac{\pi}{2n}$ = the angular distance of the same point from the secondary origin O.

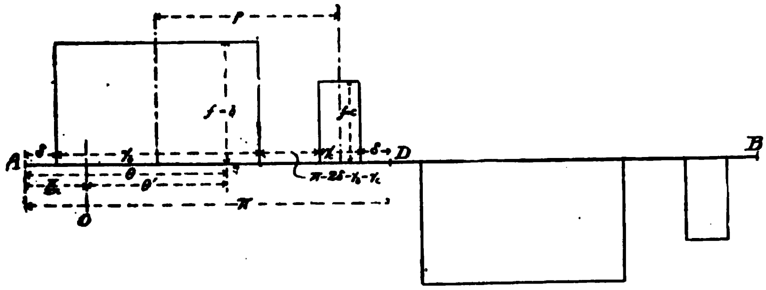


FIG. 24

b = the flux density under the principal pole section.
c = the flux density under the auxiliary pole section.

Repeating equation (6) we have

$$a_n = \frac{2}{n\pi}\sqrt{\left\{\int_0^{\pi} f \sin n\theta \, d(n\theta)\right\}^2 + \left\{\int_0^{\pi} f \sin n\theta' \, d(n\theta')\right\}^2} \qquad (6)$$

But the expression $\int_0^{\pi} f \sin n\theta \; d(n\theta)$, after integrating and substituting the values in Fig. 24 becomes

$$b\,[\cos n\delta - \cos n(\delta+\gamma_b)] + c\,[\cos n(\pi - \delta - \gamma_c) - \cos n(\pi - \delta)]$$
$$= b\,[\cos n\delta - \cos n(\delta+\gamma_b)] + c\,[\cos n\delta - \cos n(\delta+\gamma_c)]$$

likewise $\int_0^{\pi} f \sin n\,\theta'\, d\,(n\,\theta')$ becomes

$$b\left\{1-\cos n\left(\delta+\gamma_b-\frac{\pi}{2n}\right)\right\}+c\left\{\cos n\left(\pi-\delta-\gamma_c-\frac{\pi}{2n}\right)-\right.$$

$$\left.\cos n\left(\pi-\delta-\frac{\pi}{2n}\right)\right\}-b\left\{\cos n\left(\pi+\delta-\frac{\pi}{2n}\right)+1\right\}$$

$$=b\,[\sin n\delta-\sin n(\delta+\gamma_b)]-c\,[\sin n\delta-\sin n(\delta+\gamma_c)]$$

Substituting these values in equation (6) and making the necessary trigonometrical transformations, remembering that

$$\rho=\pi-2\,\delta-\frac{\gamma_b+\gamma_c}{2}$$

we have

$$a_n=\frac{4}{n\pi}\sqrt{b^2\sin^2\frac{n\gamma_b}{2}+c^2\sin^2\frac{n\gamma_c}{2}+2\,b\,c\sin\frac{n\gamma_b}{2}\sin\frac{n\gamma_c}{2}\cos n\rho} \tag{9}$$

this gives for the amplitude of any harmonic in equation (3)

$$\frac{8k}{\pi n^2}\sin\frac{n\beta}{2}\sqrt{b^2\sin^2\frac{n\gamma_b}{2}+c^2\sin^2\frac{n\gamma_c}{2}+2bc\sin\frac{n\gamma_b}{2}\sin\frac{n\gamma_c}{2}\cos n\rho} \tag{10}$$

and representing the ratio of any harmonic to the fundamental by h we have

$$h=\frac{1}{n^2}\frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}}\sqrt{\frac{b^2\sin^2\frac{n\gamma_b}{2}+c^2\sin^2\frac{n\gamma_c}{2}+2\,b\,c\sin\frac{n\gamma_b}{2}\sin\frac{n\gamma_c}{2}\cos n\,\rho}{b^2\sin^2\frac{\gamma_b}{2}+c^2\sin^2\frac{\gamma_c}{2}+2\,b\,c\sin\frac{\gamma_b}{2}\sin\frac{\gamma_c}{2}\cos\rho}} \tag{11}$$

This expression gives the relative value of any harmonic in terms of the two pole section arcs γ_b and γ_c, the corresponding field strengths b and c, the pole section pitch ρ and the angular span of the armature section β. Equation (11) may also be written

$$h = \frac{1}{n^2} \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \sqrt{\frac{\sin^2\frac{n\gamma_b}{2} + \frac{c^2}{b^2}\sin^2\frac{n\gamma_c}{2} + \frac{2c}{b}\sin\frac{n\gamma_b}{2}\sin\frac{n\gamma_c}{2}\cos n\rho}{\sin^2\frac{\gamma_b}{2} + \frac{c^2}{b^2}\sin^2\frac{\gamma_c}{2} + \frac{2c}{b}\sin\frac{\gamma_b}{2}\sin\frac{\gamma_c}{2}\cos\rho}} \tag{12}$$

in which the ratio of the harmonic to the fundamental is expressed in terms of the ratio between the field strength of the auxiliary section and that of the principal section.

Let $b = b_0$ when $c = 0$, also let the amplitude of the fundamental electromotive force be kept constant by slightly varying the value of b as c is varied. Then from equation (9) we have

$$b^2\sin^2\frac{\gamma_b}{2} + c^2\sin^2\frac{\gamma_c}{2} + 2\,b\,c\sin\frac{\gamma_b}{2}\sin\frac{\gamma_c}{2}\cos\rho = b_0^2\sin^2\frac{\gamma_b}{2} \tag{13}$$

Substituting this in equation (11) we have

$$h = \frac{1}{n^2} \cdot \frac{\sin\frac{n\beta}{2}}{\sin\frac{\beta}{2}} \cdot \frac{\sqrt{\frac{b^2}{b_0^2}\sin^2\frac{n\gamma_b}{2} + \frac{c^2}{b_0^2}\sin^2\frac{n\gamma_c}{2} + \frac{2bc}{b_0^2}\sin\frac{n\gamma_b}{2}\sin\frac{n\gamma_c}{2}\cos n\rho}}{\sin\frac{\gamma_b}{2}} \tag{14}$$

For any assumed value of c/b_0 the value of b/b_0 may be determined from equation (13) and these two in (14) will give the corresponding value of h.

The direct current voltage V will be (see equation 2)

$$V = k\int_0^\pi f\,d\,\theta$$

and using the nomenclature in Fig. 24, this becomes

$$V = k\left\{b\,\gamma_b + c\,\gamma_c\right\} \tag{15}$$

From (10) the mean effective value of the fundamental is

$$E = \frac{4\,k\sqrt{2}}{\pi}\sin\frac{\beta}{2}\sqrt{b^2\sin^2\frac{\gamma_b}{2} + c^2\sin^2\frac{\gamma_c}{2} + 2\,b\,c\sin\frac{\gamma_b}{2}\sin\frac{\gamma_c}{2}\cos\rho} \tag{16}$$

and the ratio of alternating- to direct-current voltage

$$\frac{E}{V} = \frac{4\sqrt{2}}{\pi\,(b\,\gamma_b + c\,\gamma_c)} \sin\frac{\beta}{2} \sqrt{b^2 \sin^2\frac{\gamma_b}{2} + c^2 \sin^2\frac{\gamma_c}{2} + 2\,b\,c \sin\frac{\gamma_b}{2} \sin\frac{\gamma_c}{2} \cos\epsilon} \tag{17}$$

or

$$\frac{E}{V} = \frac{4\sqrt{2}}{\pi\left(\gamma_b + \frac{c}{b}\gamma_c\right)} \sin\frac{\beta}{2} \sqrt{\sin^2\frac{\gamma_b}{2} + \frac{c^2}{b^2}\sin^2\frac{\gamma_c}{2} + 2\frac{c}{b}\sin\frac{\gamma_b}{2}\sin\frac{\gamma_c}{2}\cos\epsilon} \tag{18}$$

which gives this ratio in terms of the ratio c/b

If $c = o$, (17) becomes

$$\frac{E_0}{V_0} = \frac{4\sqrt{2}}{\pi\,\gamma_b} \sin\frac{\beta}{2} \sin\frac{\gamma_b}{2} \tag{19}$$

If the amplitude of the fundamental is held constant, we have $E = E_0$ and by substituting (13) and (19) in equation (17)

$$V = V_0 \left\{ \frac{b}{b_0} + \frac{c}{b_0} \cdot \frac{\gamma_c}{\gamma_b} \right\} \tag{20}$$

From equations (13) and (20) the variation of the direct-current electromotive force may be derived for any change in the auxiliary field strength, on the assumption that the alternating-current electromotive force is held constant by varying the strength of the principal pole section. If the converter is connected to a source of constant alternating electromotive force, this variation in the field strength of the principal pole section, if not obtained by manual adjustment, will be brought about by the magnetizing effect of the wattless currents in the armature winding.

From equations (13), (14), and (20), the curves shown in Fig. 19 were obtained.

In Fig. 25 curves have been plotted giving the variation in field strength of the main and auxiliary poles for various direct-current voltages, the amplitude of the fundamental wave of alternating electromotive force being held constant, these curves being plotted from equations (13) and (20), the following values (corresponding to the design of a machine actually built) being assumed for the pole face arcs:

$$\gamma_b = 104^\circ$$
$$\gamma_c = 44^\circ$$
$$\rho = 74^\circ$$

It will be noted that a certain maximum direct-current

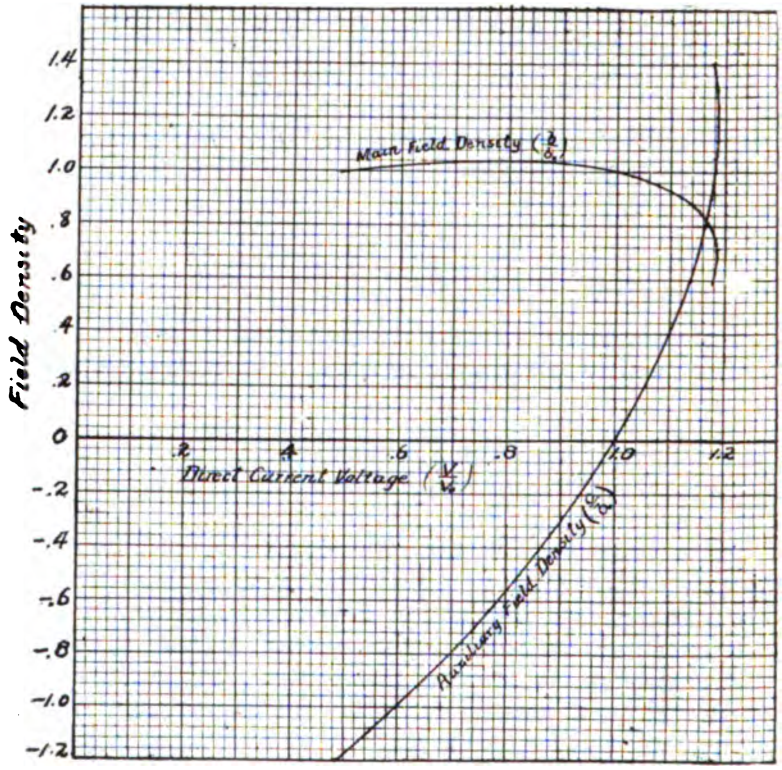


FIG. 25

voltage is reached which cannot be exceeded under the condition of constant alternating electromotive force, since the reduction in the main field strength necessary to maintain this latter condition more than offsets the further increase in the auxiliary field strength. It will also be noted that over a wide range of variation of the direct-current voltage the main field strength remains nearly constant, passing through a maximum point from which it recedes as the direct-current voltage is increased or diminished.

By reference to equation (12) it will be seen that certain of the harmonics may be eliminated by giving suitable values to γ_b and γ_c. For example, if $\gamma_b = \frac{4\pi}{7}$ and $\gamma_c = \frac{2\pi}{7}$, the 7th harmonic will disappear. These values for the pole section arcs leave but very little neutral area for commutation, even if the pole sections are contiguous, and call for an auxiliary pole section arc one-half that of the principal pole section, while the angle ρ is $\frac{3\pi}{7}$, necessitating control of both pole sections in order to maintain unity power factor (unless the mean value of c is negative, which will reduce the kw. output). The 5th harmonic cannot be entirely eliminated in this way, and the higher harmonics are not of sufficient importance to warrant a design which is otherwise disadvantageous.

Some of the higher harmonics may also be suppressed by using a fractional pitch winding. This method however cannot be carried far enough to have an appreciable effect on the value of harmonics below the 9th.

DISCUSSION ON "VOLTAGE RATIO IN SYNCHRONOUS CONVERTERS, WITH SPECIAL REFERENCE TO THE SPLIT-POLE CONVERTER" AND ON "APPLICATION OF STORAGE-BATTERIES TO REGULATION OF ALTERNATING-CURRENT SYSTEMS." ATLANTIC CITY, N. J., JULY 1, 1908.

P. M. Lincoln: These papers give a much clearer insight into the theory and expected performance of the so-called "split-pole" synchronous converter than anything which has been produced up to date. With these papers before him, the engineer who contemplates using this type of machine is in much better position to judge of its merits than ever before.

In both papers, almost the only question discussed, so far as the split-pole synchronous converter is concerned, is that of wave form. Although of prime importance, this question of wave form is by no means the only one which is to determine whether this type of construction will stand the test of time, or whether, on the other hand, it will find its way into the oblivion of the monocyclic system, and other disappointed hopes. This split-pole method of obtaining a variable voltage must be compared with other methods of obtaining the same results, not only with respect to wave form, but also with respect to relative capacity, cost, efficiency, commutation, power-factor, weight, floor space, heating, reliability, noise, possibility of resonance, rapidity of voltage change, liability of hunting, etc. It is only the test of time that can prove out this machine. Some light, however, can be thrown on the matter by further discussion of some of the questions mentioned above.

Wave form. In my discussion of this machine before the Institute in February, the following statement occurs:

> With a field form other than a sine, the wave form across the electrical diameter deviates from sine form less than that across any chord.

While this statement is perfectly true for the general case, I wish to take this opportunity to acknowledge that it does not apply to the case of the synchronous converter as ordinarily constructed. As Professor Adams has pointed out, harmonics of any one order can be eliminated from the electromotive-force wave of a synchronous converter by suitably choosing the angle between adjacent taps. For instance, in a converter in which the adjacent taps are spaced at an angle of $\frac{2\pi}{3}$, or 120 deg., third harmonics will be eliminated. With adjacent taps spaced at an angle of $\frac{2\pi}{5}$, or 72 deg., fifth harmonics will be eliminated. With taps $\frac{2\pi}{7}$, or 51.4 deg. apart, seventh harmonics will be eliminated, etc.

In the above not only are the third, fifth and seventh harmonics eliminated from the electromotive-force wave, but also in each case multiples of these harmonics are eliminated. Likewise, with taps spaced apart by an angle of $\frac{2\pi}{2}$, or 180 deg., all second harmonics are eliminated and multiples thereof; in other words, all even harmonics. With taps $\frac{2\pi}{4}$, or 90 deg. apart all fourth harmonics and multiples thereof are eliminated, etc. Professor Adams has pointed out that in general, harmonics enter into a wave form in values in inverse proportion to their order; that is, ordinarily the third harmonic will be of greater value than the fifth, the fifth of greater value than the seventh, etc.

In the most general case, that is, where even harmonics enter as well as odd, the same relation will hold. In general, the second harmonic will be of greater value than the third, the third greater than the fourth, etc. It follows, therefore, that in this general case the 180-deg. taps give rise to a wave form which deviates from a sine to the minimum extent, since it eliminates all even harmonics.

It must be borne in mind, however, that so long as the negative and positive fluxes in a synchronous converter, or other electrical machine, are of the same shape, even harmonics are eliminated owing to this fact. We can, therefore, entirely neglect the matter of even harmonics in selecting angular distance between taps in synchronous converters, and can make the selection such that the maximum of odd harmonics will disappear. From the above it is evident that the connection which will eliminate third harmonics and multiples thereof will give rise to the wave form which deviates the least from the sine wave. It is evident, therefore, that in a synchronous converter or other electrical machine in which the positive and negative field forms are of the same shape, the 120-degree taps, not the 180-degree, will give a wave form which deviates a minimum amount from a sine.

In a split-pole synchronous converter, therefore, the secret of obtaining a variable voltage by variation of the field form, and at the same time a wave form fairly free from harmonics, is: first, to use a three-phase converter; second to design the field form so that it is composed as far as possible of nothing but a fundamental form upon which is superposed a variable third-harmonic form. It is evident that a constancy of wave form can be realized in a three-phase synchronous converter only as we approach the condition that the superposed variable-field form contains nothing but third harmonics or multiples thereof. Further consideration will show that in the three-part pole converter it is not difficult to obtain a close approximation to this condition.

This fact is quite readily shown by an inspection of Figs 9, 11 and 13 of Mr. Woodbridge's paper. An inspection of these figures shows that by far the larger part of the field distortion takes the form of a superposed third harmonic. In Fig. 9 this superposed third is about 33 per cent. of the maximum field value, in Fig. 11 about 54 per cent., and in Fig. 13 the third is practically absent. During this field variation the total voltage variation is about 19 per cent., thus proving out in a beautiful manner the theoretical curve shown by Professor Adams in Fig. 6. Since these third harmonic variations in the field form do not affect the 120-degree electromotive-force wave, comparatively slight changes in the resulting wave form may be expected, as shown by Figs. 10, 12 and 14.

With a two-part pole the problem of eliminating the higher harmonics is not so simple. Refer, for instance, to Fig. 7, of Professor Adams's paper. The typical two-part field form shown therein gives a rather large third, but inspection also shows that with such a two-part field form it is impossible to confine the variations to thirds exclusively, as is the case in the three-part pole machine. In order to have practically nothing but the third harmonic enter, it would be necessary to have a downward projecting lobe on the left-hand side of this field form as well as on the right-hand side as shown. Since it is impossible to confine the field-form variations to thirds, it follows that the wave-form variations will be much greater in the two-part pole synchronous converter than in the three-part.

Mr. Woodbridge suggests an ingenious scheme in Fig. 18, for further reducing the effects of the fifth and seventh harmonics in the case of a two-part pole synchronous converter. He, however, suggests a rather serious objection to the scheme, in noting that the loss in a single bar of a converter so connected is 50 per cent. higher than the average loss. That the average per slot is not so high will assist matters inside the slot, but it is well known that heavily loaded windings suffer from heating at points outside, not inside the slots. This objection becomes still more important when we consider that the single-bar loss becomes still more than 50 per cent. above the average with power-factors other than 100 per cent.

I might note, in passing, that in order to arrive at the true electromotive-force curves both Figs. 20 and 21 of Mr. Woodbridge's paper should have a large third harmonic added, the values of which should be nearly 20 per cent. in Fig. 20 and nearly 10 per cent. in Fig. 21. While these third harmonics do not appear in the external three-phase circuit, they do cause transformers placed across the diameter (Fig. 20) and the 82 per cent. chord (Fig. 21) to pass through a magnetizing cycle which will increase their iron losses in the approximate ratio of the entering third; that is, 20 per cent. increase for Fig. 20 and 10 per cent. increase for Fig. 21.

A class of service from which this type of synchronous converter is effectually barred is the Edison three-wire circuits in which the neutral is brought out from the transformers and connected to the neutral of the lighting system; as any attempt to use the machine in this service would superpose the third harmonic upon the direct-current circuits, and also cause circulating currents in the transformer windings, if more than one transformer were tapped into the neutral.

In general, so far as wave form is concerned, it seems possible so to design the three-part synchronous converter pole that no appreciable circulating current will be caused in the outside circuits on account of wave-form variation, provided three-phase converters are used. In the case of the two-part pole, the circulating currents caused by variable wave form will probably be appreciable, but not disastrous, unless very large alternating and direct-current voltage variations be attempted.

Capacities and costs. In order to obtain the variable voltage feature of the split-pole converter, it is necessary to make a considerable sacrifice in the output that can be obtained from a given amount of material. This can best be shown by reference to Figs. 11 and 13 of Mr. Woodbridge's paper. Fig. 11, for instance, shows the resulting field form of one of these split-pole converters when it is delivering its maximum voltage. The deep notch out of the middle of the field form in Fig. 11 as measured by a planimeter amounts to about 33 per cent. of the area under the curve. In other words, if the notch were absent and the converter armature winding suitably changed, an increase in capacity could be obtained from the same converter of some 25 per cent. without increasing the cost of the rotary over one designed with a field form like Fig. 11. This means that a sacrifice of 33 per cent. in capacity must be made in order to obtain the variable voltage feature. Coupled with its other disadvantages, this is probably enough to eliminate the three-part-pole converter from any active competition with other modes of obtaining variable voltage.

Referring to Fig. 13, the same criticism applies, although in this case it is not of so much importance, as this Fig. shows the minimum voltage condition and not the maximum voltage as indicated in Fig. 11. The converter with a field form as in Fig. 13 suffers a loss in capacity of about 25 per cent. as compared with a converter of normal field form, the material of the teeth therein being worked to the same magnetic density in both cases.

Referring to the two-part pole scheme, the same criticism applies, but the capacities do not suffer quite so large a decrease. For instance, the arrangement shown by Mr. Woodbridge in Fig. 17 shows a notch between the main and auxiliary poles of 20 per cent of the available pole width. This would have the effect of decreasing the capacity of such a machine in approximately the same ratio. Another criticism of the arrangement shown in Fig. 17 is that the notch in which the brushes rest is given a width of only 30 degrees or 16⅔ per cent. of the pitch. Experience indicates that this is too narrow for good commutating conditions. A width of 20 to 25 per cent should be allowed for the inter-polar space even in a converter of normal design, and this split-pole type should have a still wider neutral. Taking this into consideration, a still further decrease in capacity will result unless the designer is willing to take considerable risk with the commutation.

While it is probable that Mr. Woodbridge's suggested pole arrangement of Fig. 17 can be considerably bettered so far as capacity is concerned, the output that can be obtained from a given amount of material will always be much less in a split-pole converter than in one of normal design. The cost of obtaining a given voltage variation by this means is at best probably as great or greater than that of obtaining the same effect in other ways.

Efficiency and heating. In addition to the split-pole converter requiring considerably more active iron in its armature, this iron is carried through a flux cycle which introduces much greater losses than in the case of a converter of normal design. For instance, in Fig. 11, of Mr. Woodbridge's paper, the introduction of the notch in the field form not only increases the amount of iron in the teeth by about 33 per cent. but also increases the losses that take place in this iron by about 15 per cent. The total iron losses including the body of the armature as well as the teeth at the maximum voltage are increased by about 35 per cent. to 50 per cent. over those of a normal design.

With a two-part pole design as shown in Fig. 17, of Mr. Woodbridge's paper, the iron losses will be increased about 15 per cent. at maximum voltage and about 35 per cent. at minimum voltage. This coupled with the fact that this design requires about 20 per cent. additional material increases the total iron losses in such a machine from 40 per cent. to 60 per cent. The field copper losses will be increased on account of the doubled or trebled number of fields to be magnetized. Armature copper loss will be increased, first, because the machine will be larger and, second, because of the circulation of the higher harmonic currents.

It is evident that the best efficiencies that can be obtained from such a machine will be as low, and probably lower than the alternative method of obtaining voltage variation—by means of a separate synchronous booster mounted on the same shaft as a normally designed converter.

Commutation. So far as commutation is concerned, the design of the three-part pole machine should not introduce any particular difficulties or problems, provided the notch between the poles is not made too small. The field strengths adjacent to the brushes, although variable, are always equal and opposite, so that the point at which commutation takes place is not subjected to a variable field. This cannot be said of the two-part pole machine. Refer to Fig. 17, for instance, of Mr. Woodbridge's paper, and note that the brushes are set at points *A* and *D*. The small field M. P. is assumed to vary from a magnitude equal to field G. H. in the positive direction to the same magnitude in the negative direction. As this variation occurs it is very evident that the field strength at the points *A* and *D* must vary, since at the one extreme the adjacent fields are equal and opposite, and at the other, equal and the same. If, therefore, proper commutating conditions are secured at one extreme, it will be necessary to sacrifice commutating conditions at the other extreme. It is true that the field at the point of commutation can be made to act in a direction to assist commutation. But it is impossible to take advantage of this feature, since in this case we would have a condition very similar to that of a commutating-pole generator the strength of whose commutating pole varies from full value to zero, depending upon conditions totally independent of the current to be commutated. It is evident that under these conditions the field at the commutating point is a hindrance, not a help, to commutation.

Resonance. In long-distance transmission systems, or systems wherein considerable amounts of high-tension cable are used, there is of course, always a considerable amount of capacity. This capacity, coupled with the reactance of transformers presents the possibility of resonance being set up therein. Although the three-phase connection of a split-pole converter eliminates the third harmonic and all multiples thereof, Professor Adams has shown that all other harmonics appear in the system to exactly the same extent in the 120-degree electromotive-force wave as in the 180-degree wave.

It is evident, therefore, that such a machine, even connected three-phase, will impress harmonics upon the outside circuits, which will vary both in order and in magnitude. That at some point in the voltage range a condition of resonance will occur is certainly not without the bounds of possibility. This is particularly true with the two-part-pole converter, since, as pointed out above, it is impossible with this type to eliminate the higher harmonics.

Noise. Not all of the causes for noisy electrical machines are known. Some of them, however, have been determined and one of the most important is the sudden transition of armature teeth from one magnetic density to another. The fact that a relatively small number of armature teeth are at a given magnetic density at once is another contributing cause. Evidently both of these causes enter to make the designing of a noiseless split-pole converter difficult, whether it be of the three-part- or the two-part-pole type. In order to obtain a design to give noiseless operation, it may often be necessary to make a considerable additional sacrifice in capacity.

Rapidity of voltage change. In the application of storage batteries to what Mr. Woodbridge calls the "regulation" of alternating-current systems; or, in other words, to assuming load fluctuations that would otherwise fall on the generating plant, ability to change the voltage of a rotary rapidly is highly essential, since rapidity in voltage change means rapidity on the part of the storage battery in assuming the fluctuations of load.

In order to change the voltage of either a split-pole converter or a normal converter equipped with a separate synchronous booster (similar to Fig. 5, Mr. Woodbridge's paper) it is necessary to cause a certain amount of change in the field flux, in the first case of the main converter poles, and in the second case of the booster poles. Professor Adams has pointed out that the amount of field flux necessary to produce a given voltage change in the split-pole machine is approximately in proportion to the harmonic introduced into the converter in order to produce that voltage change. Thus the total flux change necessary to produce a given voltage change is three times greater when that voltage change is caused by superposing a third harmonic than if caused by changing the fundamental, five times greater if caused by a fifth harmonic, etc. The creation, or the changing, of location of magnetic fluxes requires energy in proportion to the volume of flux created or changed. With a given force acting to produce flux changes it is evident, therefore, that the rapidity of voltage change is at least three times greater when acting upon the fundamental frequency (as is the case in a converter with synchronous booster) than when acting through the agency of a third or higher harmonic (as is the case with the split-pole converter). In this particular, therefore, the split-pole converter is at considerable disadvantage when compared with the alternative of a normal converter equipped with a synchronous booster.

Another important advantage accruing to the separate booster arrangement in point of rapidity of voltage change, is that the flux upon which the regulator must operate in order to change the voltage of the booster field may be left unhampered with dampers. In order to insure proper operation, the main rotary fields, no matter whether split pole or otherwise, should be equipped with relatively heavy dampers.

In the case of the split-pole machine the regulator, of whatever kind, must cause magnetic fluxes to change their positions and magnitudes in spite of the enveloping damper. In the separate booster machine the omission of dampers from the booster fields, while it does not interfere with the running characteristics, gives the regulator an opportunity of operating upon a magnetic flux not enveloped by a damper, and this gives a much higher rapidity of action. Just how much this point will affect the relative speed of voltage changes is problematical, but it is probable that this effect will be still greater than the one mentioned in the preceding paragraph, thus placing the separate booster machine in an entirely different class so far as the rapidity of voltage change is concerned.

It is on this score of rapidity of action that Mr. Woodbridge bases an argument for the converter as against the motor-generator. He points out that in the motor-generator set there is a lag between the time when the battery begins to discharge the direct-current machine and the time when this energy begins to appear in the alternating-current circuit, owing to the fact that the "lost motion", so to speak, in the motor-generator set must be taken up. While this time lag is theoretically present, it hardly deserves the term "appreciable" applied to it by Mr. Woodbridge. I have considered a number of typical motor-generator sets and find that the time lag mentioned by Mr. Woodbridge is of the order of 0.001 seconds. Compared to the time required by mechanical movement on the part of a regulator and the time required for field flux changes, the above time lag is zero. I agree with Mr. Woodbridge's conclusion that a converter is preferable to a motor-generator for the purpose described, but my conclusion is not arrived at by the same process of reasoning.

In conclusion, it is my opinion, for reasons set forth above, that the split-pole converter has disadvantages that far outweigh its advantages, when compared with the alternate, a normally developed converter with a synchronous booster of the same number of poles, mounted on the same shaft.

A. S. Hubbard: The paper by Mr. Woodbridge is interesting as outlining some of the methods for compelling and controlling storage-battery charge and discharge to steady the load on alternating-current generators. These methods must necessarily be mainly suggested and prospective, because so far as I am informed, there have been installed in this country but five storage-battery plants for the regulation of alternating-current loads. One of these five plants, however, is not now in operation as an alternating-current regulating plant, but was installed for the purpose of controlling the output from two generators at different voltages, tied together and to the transmission lines by transformers. Two current coils, each taking current from one of the generators, actuated a single carbon regulator which controlled a direct-current booster in the battery circuit. The regulation of this plant was very erratic, being only within about 33 per cent.; that is, the load on the generators varies from 33 per cent. above to 33 per cent. below the average.

The single-phase plant mentioned by Mr. Woodbridge, although included in the five, cannot be taken as a fair example of alternating-current load regulation, except for isolated, special cases, such as the plant of the Spokane and Inland Railway, where frequency or phase-changing is necessary. The regulation of this plant is effected by a single coil regulator actuated by current variations. As this coil is installed in the circuit leading to the induction motor, its current will be proportional to the current supplied to the motor, which current is affected by the line voltage, load, and frequency. Though I believe this plant to be successful, owing to peculiar local conditions, I think Mr. Woodbridge will agree that in general, definite, close regulation cannot be obtained with current regulation, except where there are no power-factor changes, or where the power-factor changes with load in a definite way; that is to say, that successful current regulation has been and will be the exception rather than the rule. That certainly is the conclusion I came to after considerable experimenting, as a result of which I devised and put in operation regulation by current times power-factor in a plant where regulation by current alone had been a failure.

The third plant includes a two coil regulator, of the dynamometer type. With this type of regulator the battery may be compelled to charge and discharge in response to variations of the true energy; that is, account can be taken of current, voltage, and power-factor variations. I have no information as to the completion of this plant, but the regulation obtained some months ago was but 25 per cent; that is, the watt load varied from 25 per cent below to 25 per cent above the average, with considerably smaller charges and discharges than the size of battery would seem to warrant. As a carbon regulator with the usual retarding spring is used at this plant the lower the average load the worse the regulation.

The fourth plant has been in operation continuously for something over two years, except for slight delays due to the grounding of the series transformers on two occasions by lightning. Water power is here used to generate 13,200-volt, three-phase, 25-cycle alternating current. This is all transmitted approximately five miles to a sub-station, where a portion of the power is transformed and converted and used for direct-current railway work and alternating-current lighting. The rest of the power is used at other sub-stations, one being located 12 miles away. The common sub-station contains two converters, 150 kw. each, a frequency changer, and a 280-cell, 400-ampere, one-hour battery, with its booster and regulating apparatus. The regulation at this plant is in accordance with variations of amperes times power-factor and is accomplished as follows:

In the incoming high-tension lines, three current transformers are installed, with their secondaries connected to a second set of three current transformers located back of the switchboard and provided with sectional secondaries connected to a compensator head. From the compensator head, the current is led to the collector rings of the rectifier. The rectifier consists of a number of discs of armature steel perforated near their edges and mounted on a shaft. A bipolar winding is threaded through the holes and connected at one end at points 120 degrees apart to the collector rings and at the other end to a commutator. The rectifier is connected to and driven by a bipolar synchronous motor built with special reference to minimum hunting.

The alternating currents from the series transformers produce a rotary field in the rectifier, which latter is mechanically rotated at the same speed, but in the opposite direction, by the synchronous motor. As the synchronous motor armature turns in synchronism with the voltage of the system, and the rotary field in synchronism with the current, there is produced in space a field whose position is stationary so long as the power-factor remains unchanged. A change in current without a change in power-factor merely increases the magnitude of the field, but does not change its position.

A single pair of brushes, *a*, arranged diametrically opposite and bearing on the commutator at points of maximum voltage, when the power generated is at 100 per cent power-factor, will give a direct current which will vary in accordance with variations of the alternating current transmitted, independent of change of power-factor within very wide limits, because the current transformers feeding the rectifier tend to preserve their current transformation ratio irrespective of changes in resistance of their secondary circuits. And while the position of the field changes, the voltage on the commutator of the rectifier also changes and rises until of sufficient value to deliver a direct current over the circuit connecting the *a* brushes, equal to the transformation ratio value of the current transformers. When, however, as is the case at the plant in question, a second set of brushes, *b*, is used on the commutator at 90 electrical degrees from the others and connected to a circuit having resistance and inductance characteristics corresponding to the characteristics of the circuit connected to the other brushes, a by-pass is provided and two direct currents are obtained, one of which, *a'*, varies as the energy current component and the other, *b'* as the wattless current component of the current in the alternating current line. *a'* is used to regulate the battery charge and discharge, and *b'* can be used to correct the power-factor by changing the excitation of a synchronous motor or a synchronous converter. The direction of *b'* is in one direction for lead and in the other direction for lag. The watt regulation at this plant is 3.5 per cent; that is, the kilowatt load on the generators varies only 3.5 per cent from any average.

The load regulation is so good that the voltage is constant, and the Tirrell regulator formerly used at the power house has been taken out and used on the lighting circuit supplied from the induction-motor-driven frequency-changer. The 3.5 per cent regulation is obtained with battery charges up to the one hour rate and with discharges of two and one half times the one-hour rate and with booster voltages up to 140 volts.

The sectional series transformers and the compensator head are provided to enable the energy load to be changed at will, during operation, from about 50 kw. to 1000 kw. The regulation is 3.5 per cent. of any value between these limits.

Safety devices are provided so that if the synchronous motor goes out of step, or a short-circuit comes on the lines, the series transformers are automatically short-circuited and the regulation "killed."

It will be noted that variations of output from this rectifier do not affect the load on the synchronous motor driving it, nor has the rectifier any motor action, both very important points.

The fifth plant is a 60-cycle, 2300-volt, three-phase system, a turbine being used ordinarily for the generation of power. The regulating equipment of this plant is of the same type as that previously described, except that the apparatus is designed for 60 cycles. The rectifier and its synchronous motor have six poles, and but one series transformer is used in one phase. The regulation obtained here is 4 per cent of any average, the usual average being 800 kw. The variation is but 32 kw. In this plant also, the load is kept so steady that it is not necessary to operate the Tirrell regulator for voltage regulation, even though a portion of the output is used for lighting. This plant has been in operation for about six months.

While the multiplying features mentioned by Mr. Woodbridge are present in the rectifiers of the fourth and fifth plants here described, as well as in that described by him, this multiplication is not due so much to any shift of field or current in the bypass circuit as it is to an inherent characteristic of the rectifier fed by series transformers, and can be made of any desired value within wide limits by proper design. The direct-current output from these rectifiers is about 40 watts (10 volts and 4 amperes) the direct current from the *a* brushes varying from about 2.75 amperes to about 5.5 amperes for an energy change on the line of only 3.5 per cent plus or minus. By changing the relation between the characteristics of the *a* and *b* circuits, various effects and any desired combination of currents and of energy regulation can be obtained. But one of these rectifiers is required for both load regulation and power-factor correction, the *a* circuit controlling the load regulation, and the *b* circuit the power-factor.

The dynamometer control is an electromechanical device that has no pronounced inertia; in my opinion it is not so good as the rectifier control of plants, with water wheels, steam turbines, or gas engines as prime movers.

The question as to what type of apparatus to use for the connecting link for the transfer of energy between the alternating-current and the direct-current circuits, is one in which enters the efficiency of the apparatus and the magnitude and direction of the loads to be transferred. If an alternating-current booster is used, in most cases this belongs to the high-tension side.

The split-pole converter is undoubtedly a distinct contribution to alternating-current distribution, although the changing shape of the voltage wave of this machine as its alternating-current direct-current ratio is changed, may seriously affect the operation of the small synchronous motor proposed for driving the rectifier at the Gary plant. My experience is, that to insure stability and absence of hunting in a synchronous motor, the wave shape of this motor must correspond to the wave shape of the nearest preponderating synchronous machine.

W. L. Waters: Messrs. Adams and Woodbridge have worked out in an interesting way the mathematical theory of the split-pole converter. In working out any such problem mathematically it is necessary to make a number of assumptions in order to simplify the problem. Both Messrs. Adams and Woodbridge show that it is mathematically possible by means of a split-pole converter to obtain a considerable voltage variation without introducing serious distortion of wave form, or any great number of higher harmonics into the wave form of the converter. It is then simply a question as to whether these mathematical deductions are borne out in practice.

The statement that any particular harmonic can be entirely eliminated by suitably choosing the angle between the various taps on the armature and the pitch of the winding may be mathematically correct, but I doubt very much whether this will be found true in practice. The fact that the poles are not all exactly equal, and that consequently there are local circulating currents in any multiple-circuit converter armature is neglected; as is also the fact that there is a certain amount of electrostatic capacity between different parts of the armature winding and the frame, which causes local higher harmonics circulating currents and a tendency for the higher harmonics to reappear in the voltage wave form. There are a number of similar assumptions that are necessarily made in any mathematical treatment, but the extent to which they will modify the wave form obtained under actual operating conditions and under the special conditions which at times occur in an operating power circuit, is a matter of actual experience.

Mr. Lincoln has cited a number of features in which this split-pole is inferior to the standard converter: its increased size; increased core loss; increased field copper loss; increased armature heating, due to the fact that the alternating and direct currents do not properly neutralize each other with the varying voltage ratio; and also that on such a converter provided with dampers the voltage control will be extremely slow-acting compared with the standard alternating- or direct-current booster. I think, however, that Mr. Lincoln has not sufficiently emphasized the comparatively poor commutation of these converters under general operating conditions of the power circuit.

If there is any disturbance on the system causing hunting and sudden change in voltage or speed of the converter, there will be local equalizing currents flowing through the armature conductors, and the rotating field of the armature will fluctuate rapidly both in magnitude and in position. Similar fluctuating fields will be induced by currents in the dampers in the pole-faces, with the result that the distribution of magnetism on the pole-face will fluctuate and in consequence the voltage ratio and the terminal voltage will fluctuate. This will make the loads on the converter and its voltage and current fluctuate, and will cos τ with a sine wave. Thus, by shifting the brushes from the position A, B, at right angles with the magnetic flux, to the position E, F, in line with the magnetic flux, any direct voltage between $2e$ and O can be produced with the same wave of alternating voltage a.

As seen, this variation of direct voltage between its maximum value and zero, at constant impressed alternating voltage, is independent of the wave shape, and can, therefore, be produced whether the alternating voltage is a sine wave or any other wave.

Instead of shifting the brushes on the commutator, with fixed position of the commutator brushes, the magnetic field poles may be shifted in the opposite direction, by the same angle, as shown in Fig. 4, A, B, C.

Instead of mechanically shifting the field poles, they can be shifted electrically, by having each field pole consist of a number

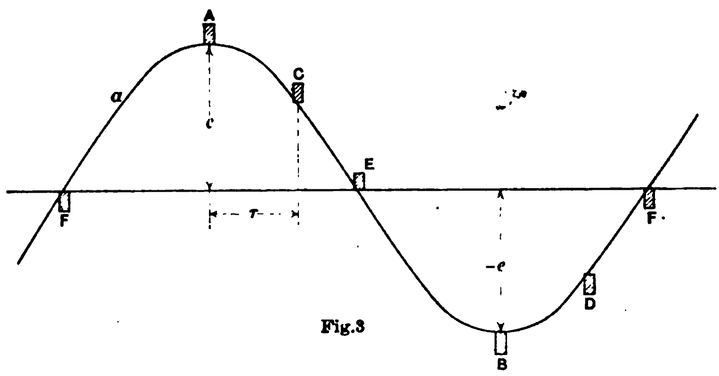


Fig. 3

of sections, and successively reversing the polarity of these sections, as shown in Fig. 5, A, B, C, D.

Instead of a large number of field-pole sections, two sections are obviously sufficient, and the same gradual change can be brought about, by not merely reversing the sections, but by reducing the excitation to zero and bringing it up again in the opposite direction as shown in Fig. 6, A, B, C, D, E. In this case, when reducing one section the other section must be increased by approximately the same amount to maintain the same alternating voltage.

In changing the direct voltage by mechanically shifting the brushes past the field poles, as soon as the brushes come under the field-pole faces self-inductive sparking on the commutator results, if the iron of the field poles is not kept away from the brush position by having a slot in the field poles, as indicated in the dotted line in Fig. 2, and Fig. 4B. With the arrangement in Figs. 2 and 4, this is not feasible mechanically, and these arrangements are therefore unsuitable. It is feasible, however, in the arrangements shown in Figs. 5 and 6; that is, when shifting the resultant magnetic flux electrically, to leave a commutating space between the polar projections of the field at the brushes, as shown in Fig. 6, and thus obtain as good commutation as in any other commutating machine.

Such a variable-ratio converter, then, comprises an armature with the brushes in fixed position, and field poles separated by interpolar spaces of such width as required for commutation. Each field pole consists of two parts, usually of different size, separated by a narrow space, and provided with independent windings. By varying the relative excitation of the two polar sections, an effective shift of the resultant field flux, and a corresponding change of the direct voltage, is produced.

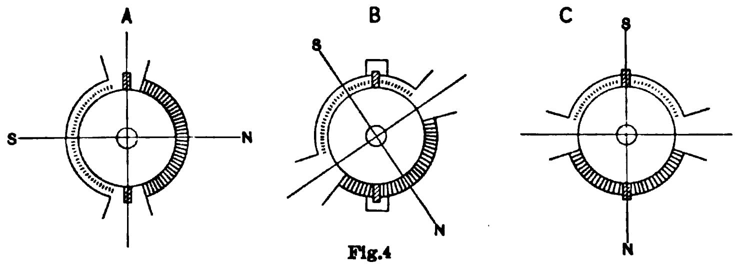


Fig.4

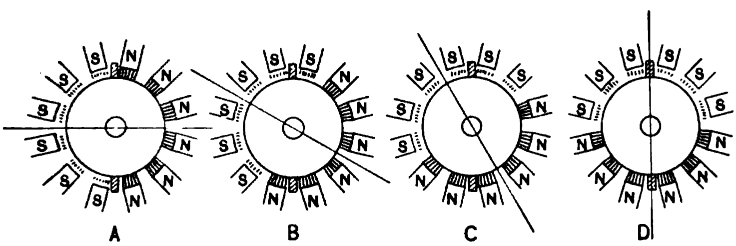


Fig.5

As this method of voltage variation does not depend upon the wave shape, by proper designing of the field pole faces and the pitch of the armature winding, the alternating voltage wave can be made as near a sine wave as desired. Experience shows, however, that the usual distributed winding of the commutating machine produces a sufficiently close approach to sine shape.

B. Variable ratio by change of wave shape of the star voltage. If in the converter shown diagrammatically in Fig. 7 the magnetic flux disposition and the pitch of the armature winding is such that the potential difference between the point a of the armature and the neutral O, or the star voltage, is a sine wave, Fig. 8A, then the voltage ratio is normal. Let us assume, however, that the voltage curve differs from sine shape by the superposition of some higher harmonics—the third harmonic in Fig. 8, *B* and *C*, or the fifth harmonic in Fig. 8, *D* and *E*. If, then, these higher harmonics are in phase with the fundamental, as in Fig. 8, *B* and *D*, they increase the maximum of the alternating voltage, and thereby the direct voltage; if these harmonics are in opposition with the fundamental, as in Fig. 8, *C* and *E*, they decrease the maximum alternating voltage and thereby the direct voltage, without appreciably changing the effective value of the alternating voltage. For instance, a higher harmonic of 30 per cent of the fundamental, increases or decreases the direct voltage by 30 per cent, but varies the effective alternating voltage only by $\sqrt{1 - 0.3^2} = 1.044$, or 4.4 per cent.

The superposition of higher harmonics thus offers a means of

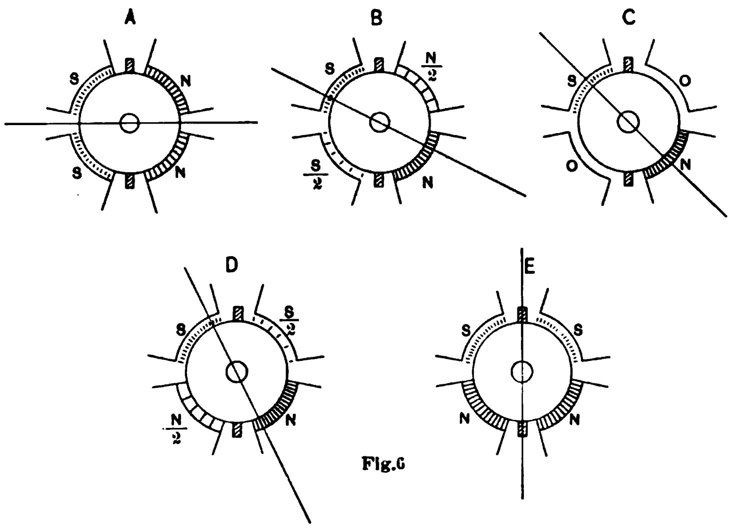


Fig. 6

increasing, as well as decreasing the direct voltage, at constant alternating voltage and without shift of the angle between brush position and resultant magnetic flux.

As, however, the terminal voltage of the converter depends not only on the induced electromotive force of the converter, but also on that of the generator, and is a resultant of the two electromotive forces in approximately inverse proportion of the impedances from the converter terminals to the two respective induced electromotive forces; only such higher harmonics can be used for varying the converter ratio, which may exist in the star voltage, without appearing in the converter terminal voltage.

In general, in an *n* phase system an *n*th harmonic existing in the star voltage does not appear in the ring or delta voltage, as the ring voltage is the combination of two star voltages displaced in phase by $\frac{180}{n}$ degrees for the fundamental, and thus by 180 degrees, or in opposition, for the nth harmonic. Thus in a three-phase system, the third harmonic may be introduced into the star voltage of the converter, as in Fig. 8, *B* and *C*, without affecting, or appearing in the delta voltage, and can, therefore, be used for varying the direct voltage. The fifth harmonic can not be used in this way, but would reappear and cause a short-circuit current in the supply voltage, hence should be made sufficiently small to be harmless.

The third harmonic thus can be used for varying the direct voltage in the three-phase converter diagrammatically shown in Fig. 9*A*, and also in the six-phase converter with double-delta connection, as shown in Fig. 9*B*, or double star connec-

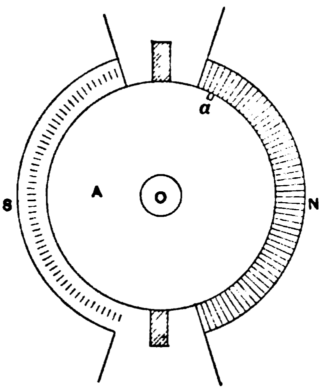


FIG. 7

tion as shown in Fig. 9*C*, since this consists of two separate three-phase triangles of voltage supply, neither of them containing the third harmonic. In such a six-phase converter with double star connection, Fig. 9*C*, the two neutrals must not be connected together, as the third harmonic voltage exists between the neutrals. In the six-phase converter with diametrical connections, the third harmonic of the star voltage appears in the terminal voltage, as the diametrical voltage is twice the star voltage. In such a converter, if the primaries of the supply transformers are connected in delta as in Fig. 9*D*, the third harmonic is short-circuited in the primary voltage triangle, producing excessive currents which causes heating and interferes with the voltage regulation. This arrangement is, therefore, not permissible.

If, however, the primaries are connected in star, as in Fig. 9*E*, and either three separate single-phase transformers, or a three-phase transformer with three independent magnetic circuits, is used, as in Fig. 10, the triple-frequency voltages in the primary are in phase with each other between the line and the neutral, and thus with isolated neutral cannot produce any current. With a three-phase transformer as shown in Fig. 11; that is, in which the magnetic circuit of the third harmonic is open, triple-

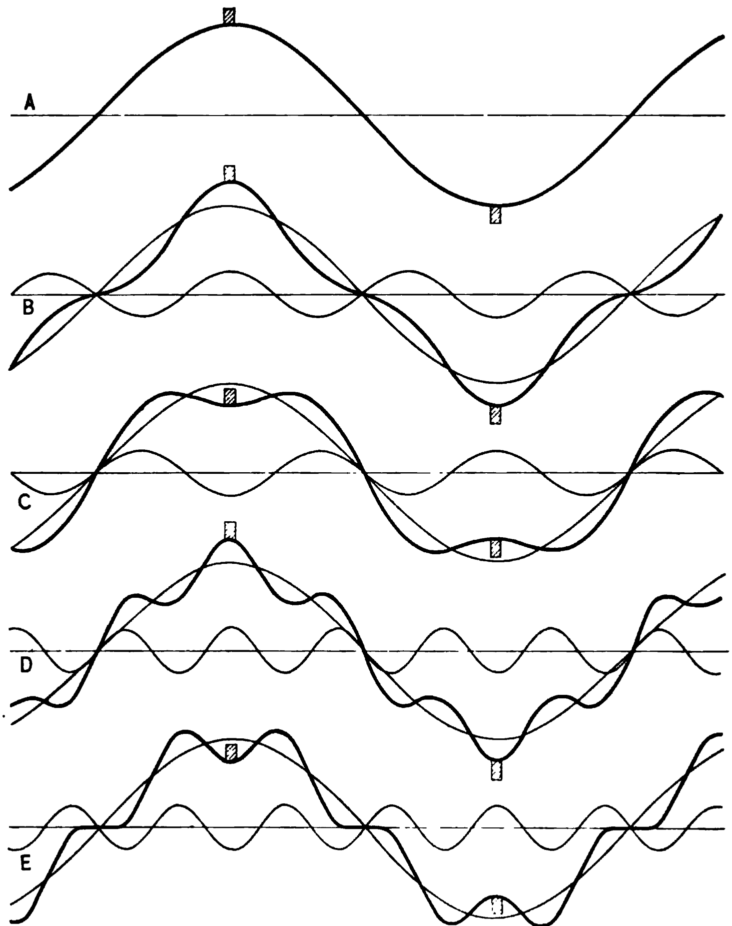


Fig.8

frequency currents can flow in the secondary. This arrangement is therefore not satisfactory.

In two-phase converters higher harmonics can be used for regulation only if the transformers are connected in such a manner that the regulating harmonic, which appears in the converter terminal voltage, does not appear in the transformer terminals; that is, by a connection similar to that in Fig. 9*E* and 10.

As the direct voltage regulation of a three-phase or six-phase converter of this type is produced by the third harmonic, the problem is to design the magnetic circuit of the converter so as to produce the maximum third harmonic, the minimum fifth and seventh harmonic.

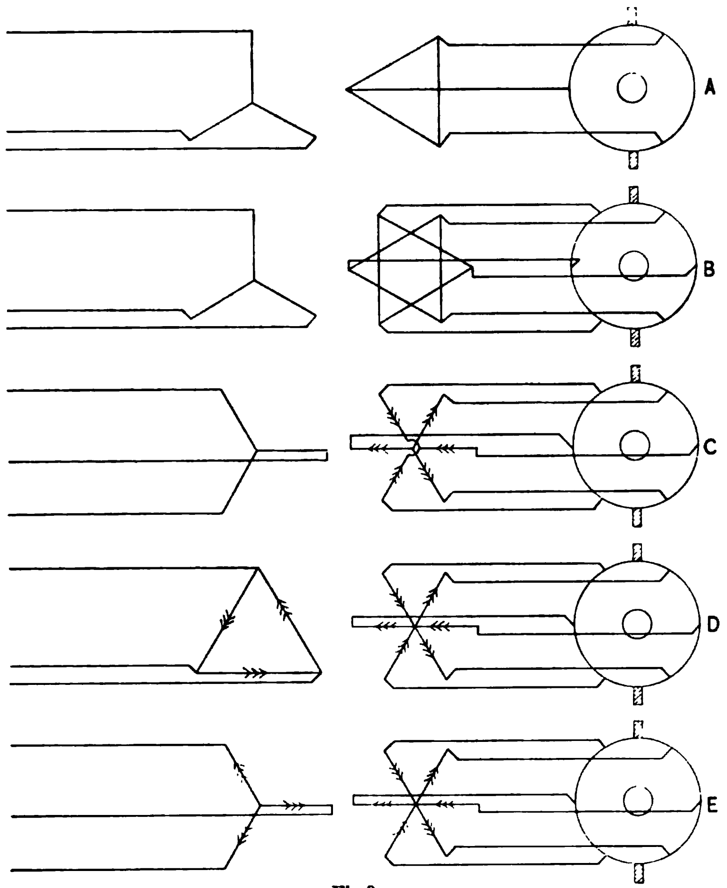


Fig. 9

There are thus two methods of producing a variation of direct voltage at constant alternating supply voltage. We can use either the one, or the other, or a combination of both. Here the question of commutation comes in. In the converter, at normal voltage ratio, the alternating current and the direct current have a fixed ratio, derived from the voltage ratio by the law of conservation of energy. At this ratio, we find that the resultant ampere-turns of the direct-current output equals the resultant ampere-turns of the alternating-current input, neglecting losses and phase displacement, and is opposite thereto. The converter, at normal voltage ratio, has no armature reaction, except the small armature reaction due to the energy current driving the armature round, and the magnetizing or demagnetizing reaction of the lagging or leading current.

Assume, however, that the ratio is changed by any one of the above described methods and the direct voltage is lowered below the value normally corresponding to the alternating voltage; the direct current is raised beyond that value corresponding to the alternating current, and there is a resultant armature reaction; that is, part of the direct-current armature reaction is not neutralized by the alternating current armature reaction, and so a small direct-current armature reaction remains.

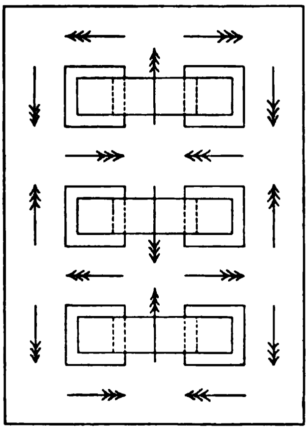


Fig. 10

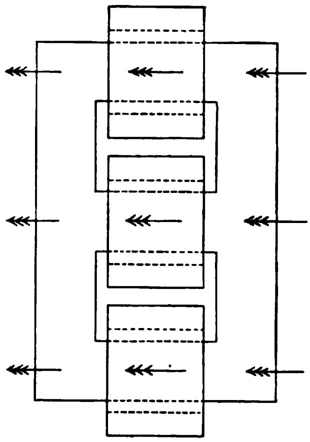


Fig. 11

By raising the direct voltage or lowering the alternating voltage, the direct-current armature reaction is less than that of the alternating current, and thus a part of the alternating current-synchronous motor armature reaction remains uncompensated. A variable-ratio converter, deviating from the normal voltage ratio, gives a small armature reaction, depending on the deviation from the normal voltage ratio, which at raised direct voltage is a synchronous motor reaction; that is, produces a commutating field and so takes care of the commutation; but at lower direct-current voltage it is a direct-current generator armature reaction; that is, tends to impair the commutation by opposing the current reversal. The armature reaction must therefore be taken care of.

In this case of raising the direct voltage, the synchronous motor armature reaction commutates, and if it is not sufficient for the desired commutation, we may shift the brushes to the edge of the next pole. If it should be too high, we can shift the brushes back to the preceding pole. But if we lower the voltage, we get a generator reaction, and therefore need a commutating field. In that case the outer side of the field poles is weakened and the center strengthened, and just where we need the commutating field in the three-part converter—because we have to take care of a generator armature reaction—the field corners weaken. Therefore this type of converter is liable to have poor commutation, when lowering the direct voltage, and thus it is preferably not used for this purpose.

Consider now the type where we may shift the magnetic field electrically. It can be shifted one way or the other. If shifted one way, it means that there is impressed on the brushes a part of the magnetic field of the main pole. If shifted the other way there is impressed a reverse field. In the latter case, there is interference with commutation; in the former, a commutating field is produced. The shift of the field for voltage regulation thus must be designed in the right direction. This commutating field may be too large. The type regulating by wave shape distortion does not give the required commutating field; the type regulating by shifting the flux gives too high a commutating field. The problem, then, is to combine both types; that is, to use wave-shape distortion, and in addition just as much shifting of the resultant flux as is necessary to produce the commutating field required for perfect commutation. Hence, this variable-ratio converter by its design permits perfect control of the commutation.

If there are three sections of the pole, the outside ones are weakened and the middle one is strengthened, or inversely, for regulation by wave shape. Weakening but one outside section, leaving the other outside section the same as the middle one, gives a combination of regulation by shift of flux and by change of wave shape in any proportion, depending on the ratio of the pole arcs of the sections. In this case, however, two of the sections can be combined into one, and there results a converter with two unequal pole sections, as discussed in to-day's papers. This is why in these converters there is used a two-part split-pole of arcs in the proportion roughly of one to two.

This disposes of the question of commutation. The next question is that of output. There is no doubt that if such a machine of 1000 kw. is designed to give any voltage from 200 to 300, it will be larger than an ordinary 1000-kw. machine of constant voltage, as it must be built to produce 300 volts, and to carry current giving the output at 200 volts. This is an inherent feature of all apparatus designed for a range of voltage, whether this range is produced as described above, or by phase control, or by a synchronous generator on the converter shaft, or in any other way.

From what I have said regarding armature reaction, the heating can be understood. At normal voltage ratio, the direct current and alternating current neutralize each other; when raising or lowering the voltage, this slightly modifies the resultant current, and also the heating. Without any further discussion thereof, it may, however, easily be seen that in any case the armature reaction and the excess of direct current over alternating current can never be the same as in the direct-current generator, and the heating thus must remain essentially less than in the direct-current generator, and nearer to that of the standard converter.

Charles P. Steinmetz (by letter): *Armature reaction and commutation of variable ratio converter.* With the brushes in quadrature position to the resultant magnetic flux, and at normal voltage ratio, the direct-current generator armature reaction of the converter equals the synchronous-motor armature reaction of the alternating energy current; thus at unity power-factor the converter has no resultant armature reaction, while with a lagging or leading current it has the magnetizing or demagnetizing reaction of the wattless component of current.

A. In a variable ratio converter of type *A*, if by a shift of the resultant flux from quadrature position with the brushes, by angle τ, the direct voltage is reduced by factor $\cos\tau$, the direct current and therewith the direct-current armature reaction are increased, by factor $\frac{1}{\cos\tau}$, as by the law of conservation of energy the direct-current output must equal the alternating-current input (neglecting losses). The direct-current armature reaction F therefore ceases to be equal to the armature reaction of the alternating energy current, F_0, but is greater by factor $\frac{1}{\cos\tau}$

$$F = \frac{F_0}{\cos\tau}$$

The alternating-current armature reaction, F_0, at no phase displacement is in quadrature position with the magnetic flux. The direct-current armature reaction, F, however, appears in the position of the brushes, or shifted against the quadrature position by angle τ; that is, the direct-current armature reaction is not in opposition to the alternating-current armature reaction, but differs therefrom by angle τ, and can therefore be resolved into two components:

A component in opposition to the alternating-current armature reaction, F_0; that is, in quadrature position with the resultant magnetic flux:

$$F'' = F\cos\tau = -F_0;$$

that is, equal and opposite to the alternating-current armature reaction and neutralizing it and:

A component in quadrature position with the alternating-current armature reaction, F_0, or in phase with the resultant magnetic flux; that is, magnetizing or demagnetizing:

$$F' = F \sin \tau = -F_0 \tan \tau$$

That is, in the variable-ratio converter, the alternating-current armature reaction at unity power-factor is neutralized by a component of the direct-current armature reaction, but a resultant armature reaction, F', remains, in the direction of the resultant magnetic field: that is, shifted by angle $(90 - \tau)$ against the position of brushes. This armature reaction is magnetizing or demagnetizing, depending on the direction of the shift of the field τ.

It can be resolved into two components, one, at right angles with the brushes, and equal:

$$F_1' = F' \cos \tau = F_0 \sin \tau$$

and one, in line with the brushes, and equal:

$$F_2' = F' \sin \tau = F \sin^2 \tau = F_0 \sin \tau \tan \tau$$

as shown diagrammatically in Figs. 12 and 13.

There thus exists a resultant armature reaction in the direction of the brushes that is harmful for commutation, just as in the direct-current generator, except that this armature reaction in the direction of the brushes is only $F_2' = F \sin^2 \tau$; that is, $\sin^2 \tau$ of the value of that of a direct-current generator.

The value of F_2' can also be derived directly, as the difference between the direct-current armature reaction F and the component of the alternating-current armature reaction, in the direction of the brushes: $F_0 \cos \tau$, that is:

$$F_2' = F - F_0 \cos \tau = F\ (1 - \cos^2 \tau) = F \sin^2 \tau = F_0 \sin \tau \tan \tau.$$

The shift of the resultant magnetic flux, by angle τ, gives a component of the magnetomotive force of field excitation $P_2 = P \sin \tau$, where P = magnetomotive of field excitation in the direction of the commutator brushes, either in the direction of armature reaction, thus interfering with commutation, or in opposition to the armature reaction, thus improving commutation.

If the magnetic flux is shifted in the direction of armature rotation; that is, if that section of the field pole towards which the armature moves, as in Fig. 12, is weakened, the component P_2 of field excitation at the brushes in the same direction as the armature reaction, F', thus adds itself thereto and impairs the commutation. Such a converter is hardly operative. In this case the component of armature reaction in the direction of the field flux, F', is magnetizing.

If the magnetic flux is shifted in opposite direction to the armature reaction; that is, that section of the field pole which the armature conductor leaves, as in Fig. 13, is weakened, the component P_2 of field excitation at the brushes is in opposite direction to the armature reaction F'—reversing it if sufficiently large—and gives a commutating or reversing flux R; hence improves commutation, so that this arrangement is used in such converters. In this case, however, the component of armature reaction in the direction of the field flux F' is demagnetizing,

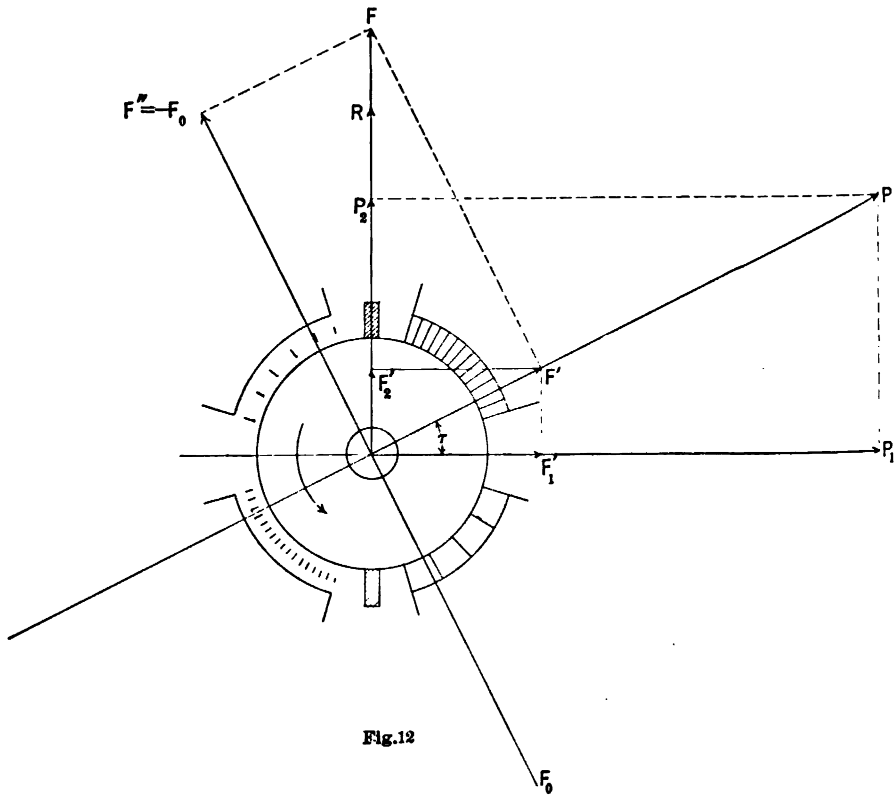


Fig.12

and with increasing load the field excitation has to be increased by F', to maintain constant flux. Such a converter requires compounding, as by a series field, to take care of the demagnetizing armature reaction.

If the alternating current is not in phase with the field, but lags or leads, the armature reaction of the lagging or leading component of current superimposes upon the resultant armature reaction, F', and increases it, with lagging current in Fig. 12, leading current in Fig. 13; or decreases it, with lagging current in Fig. 13, leading current in Fig. 12. With lag of the alternating current, by phase angle τ, under the conditions of Fig. 13, the total resultant armature reaction vanishes; that is, the lagging component of synchronous motor armature reaction compensates for the component of the direct-current reaction, which is not compensated by the armature reaction of the alternating energy current. It is interesting to note that in this case, as regards heating, output based thereon etc., the converter equals that of normal voltage ratio.

B. In a variable-ratio converter of type *B*; that is, operating

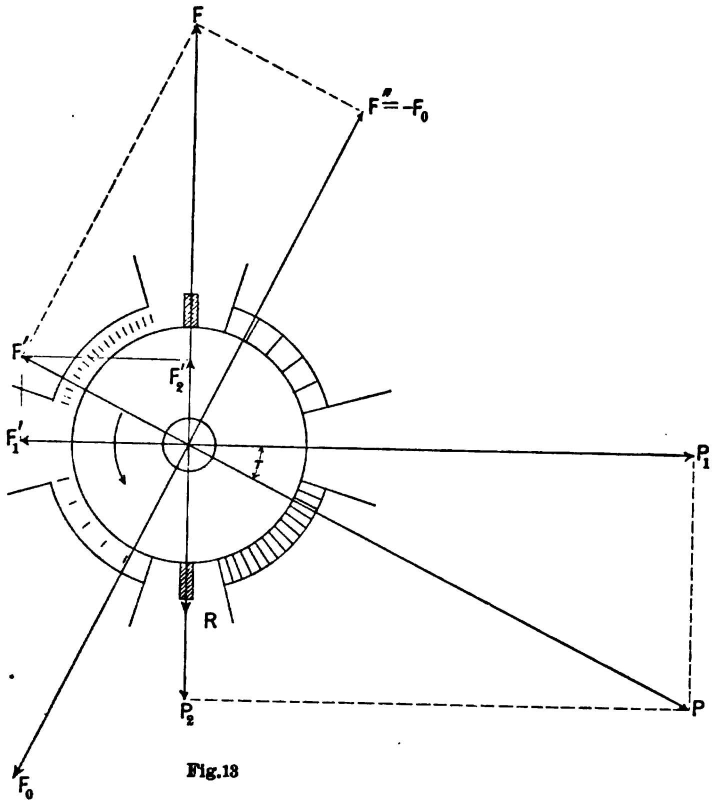


Fig. 13

by wave-shape distortion, let p equal the ratio of direct voltage to that voltage which it would have with the same alternating impressed voltage at normal voltage ratio, where $p > 1$ represents a super-normal voltage, $p < 1$ a subnormal direct voltage. The direct current, and thereby the direct-current armature reaction, then is changed from the value which it would have at normal voltage ratio by the factor $1/p$, since the product of direct volts and amperes must be the same as at normal voltage ratio, being equal to the alternating power input, minus losses.

With unity power-factor, the direct-current armature reaction F in a converter of normal voltage ratio is equal and opposite, and is neutralized by the alternating-current armature reaction F_0, and at a change of voltage ratio from normal, by factor p, thus changing the direct current by factor $1/p$; the direct-current armature reaction is then:

$$F = \frac{F_0}{p}$$

hence leaves an uncompensated resultant.

As the alternating armature reaction at unity power-factor is in quadrature with the magnetic flux, and the direct-current armature reaction in line with the brushes, and as with this type of converter the brushes stand at the magnetic neutral, that is, at right angles to the magnetic flux, the two armature reactions are in the same direction in opposition with each other, and so leave the resultant, in the direction of the commutator brushes:

$$F' = F - F_0$$
$$= F\ (1 - p)$$

The converter thus has an armature reaction, proportional to the deviation of the voltage ratio from normal.

If $p > 1$, or super-normal direct voltage, the armature reaction is negative, or motor reaction, and the magnetic flux produced by it at the commutator brushes is commutating flux. If $p < 1$, or subnormal direct voltage, the armature reaction is positive; that is, the same as in a direct-current generator, but less in intensity, and the magnetic flux of armature reaction tends to impair commutation. In a direct-current generator, by shifting the brushes to the edge of the field poles, the field flux is used as reversing flux to give commutation. In this converter, however, decrease of direct voltage is produced by lowering the outside sections of the field poles. The edge of the field may not have a sufficient flux density to give commutation with a considerable decrease of voltage below normal. Therefore, a separate commutating pole is required, or this type of converter is used only for raising the voltage. For lowering the voltage the other type is used, which operates by a shift of the resultant flux, and gives a component of the main field flux as commutating flux.

With a polar construction consisting of three sections this can be done by having the middle section at low excitation, and the outside sections at high excitation for maximum voltage. To decrease the voltage, raise the excitation of the center section, but instead of lowering both outside sections, leave the section in the direction of the armature reaction unchanged, while lowering the other outside section twice as much, and so produce in addition to the change of wave shape, a shift of the flux, and thereby produce at the brushes a magnetic commutating field of any desired strength.

The distribution of current in the armature conductors of the variable-ratio converter, and the wave form of the actual or differential current flowing in the conductors, and the effect of the wattless current thereon, are determined in the same manner as in the standard converter, and from them is calculated the local heating in the individual armature turns, and the minimum armature heating.

The results in general are the same, showing a superiority of the six-phase over the two-phase, and of the two-phase over the three-phase arrangement. As regards the rating of the machine based on armature heating, the variable-ratio converter is intermediate between the standard converter and the direct-current generator, approaching the former, the less the voltage ratio differs from the normal. The rating, of such a converter of a constant direct voltage differing from that corresponding to the alternating supply voltage, is far higher, and the size therefore smaller, than that of a direct-current generator. It approaches that of the standard converter the nearer the voltage ratio is to the normal.

Where, however, the converter is intended for operation over a considerable voltage range, as when connecting a storage-battery to an alternating-current system, the size of the converter is greater than that of a converter operating at constant voltage, in the same way and by the same amount as any apparatus intended for operation over a considerable voltage range; that is, the magnetic circuit of the machine must be ample to carry the magnetic flux of maximum voltage, while the conductor section and commutator size must be sufficient for maximum current. This increase of size is a necessary result of operation over a voltage range, irrespective of the method by which the change of voltage is brought about, whether by a change of the alternating impressed voltage, or by a change of the converter ratio, and so has nothing to do with the method of voltage regulation.

P. M. Lincoln (by letter): The description by Mr. Steinmetz of the actions taking place in a split-pole converter is beautifully simple and lucid. I fail to see, however, wherein Mr. Steinmetz has substantiated the claims made in opening his discussion; namely, that he would clear up the difficulties in regard to commutation, decrease in capacity, lower efficiency, variable wave form, etc., mentioned in my discussion.

As to the three-part-pole machine, the reduction in capacity necessary to obtain a reasonable voltage variation seems to put this type of machine out of the running entirely. As to the two-part-pole machine, Mr. Steinmetz has failed to reconcile the conflicting requirements of wave form and commutation. Wave form demands that the commutating notch in the field be of zero width. Commutation, on the other hand, demands a notch in the field form of at least 20 per cent to 25 per cent of the space between brushes. This has been demonstrated by years of experience. Just how much will be required under the peculiar conditions of the two-part-pole construction is problematical, but it is impossible for me to conceive how it can be less than in a machine of normal design; and in view of the variable field strength at the commutating point, it seems that this notch should be considerably wider than in the normal design. In fact the necessarily wide variations of field strength at the commutating point raise serious questions as to the possibility of obtaining satisfactory commutation even with the most favorable notch widths.

I am deeply interested in Mr. Steinmetz's description of the variable-voltage converter, and I give mental assent to every one of his deductions, but I am disappointed that he does not explain away the above questions as well as the others raised by my discussion.

In closing his discussion, Mr. J. L. Woodbridge takes issue with my statement that rapidity of voltage change can be largely increased in the separate alternating-current booster converter over anything possible in the split-pole machine, owing to the possibility of omitting dampers from the separate booster field, coupled with the impossibility of omitting them from the split-pole machine. He suggests that I did not take into consideration that in the case of a separate alternating-current booster the strength of the main converter fields, (which must be damped) as well as the booster fields, must change before the voltage ratio of the combination can change. I wish to assure Mr. Woodbridge that I fully considered the matter in this light before making my statement. It is perfectly true that so long as its speed remains constant, neither the alternating nor the direct voltage of a converter can change without changing its field strength. Immediately upon a change in the voltage applied to the alternating current end of a converter, a current begins to circulate through its armature of such a nature as to cause the field value to change rapidly to that necessary to accommodate the changed alternating current value. The whole power of the system to which the converter is attached is available for bringing about this change in field flux. If the shunt field of the converter is excited from its own commutator, the shunt ampere-turns will assume a new value such as will, with proper design, virtually eliminate the armature current which causes the immediate adjustment of the field flux. It is the duty which falls upon the regulator that determines its speed of action, and in the case of the separate booster converter, the changing of the flux in the main converter fields is not a part of this duty. This condition was all gone over mentally before making the statement appearing in my discussion. I considered it so obvious as not to require statement.

Mr. Woodbridge further states that he has designed a damper to apply to the split-pole machine of such a nature that it does not interfere materially with the speed of voltage changes. To cause voltage changes in a converter of Mr. Woodbridge's design, a shifting of magnetic flux from one part of the pole-face to another is required. To prevent hunting, a damper must be so designed as to prevent the shifting of magnetic flux from one part of the pole-face to another. Mr. Woodbridge's requirement is, therefore, diametrically opposed to the essential object of a damper. I must, therefore, remain skeptical as to the efficacy of the new damper until further details are disclosed.

J. L. Burnham (by letter): Since the papers and discussions so far have been theoretical, it may be of interest to have some results from machines that have been built. The results given are based on observations of the operation of one 200-kw. three-phase; one 500-kw. six-phase; and four 1000-kw. six-phase converters all having a range in voltage of 4 to 5; a 1000 kw. converter having a 6 to 7 voltage range; and a 2000-kw. six-phase converter having a range in voltage of 2 to 3. None of these machines had unusual armature winding pitch or tap connections to eliminate harmonics, as suggested in the papers by Messrs. Adams and Woodbridge, since all experimental data obtained showed that it was not necessary.

These machines were operated from a 13200-volt, 2000-kw. alternator over a one-half mile line through transformers connected delta primary, double-delta secondary; delta primary, diametral secondary; star primary, diametral secondary; star primary, double-delta secondary; and two-phase primary, six-phase, double-T secondary. When running at no load, the alternating-current minimum input for various excitations throughout the voltage range did not in any case exceed 8 per cent, except for the delta primary, diametral secondary connection of transformers. This 8 per cent alternating-current input includes all of the odd harmonics, except the third and its multiples, which are eliminated by transformer connections, in addition to no-load losses of the machine, which would amount to about 3 per cent. This result seems to confirm the calculated negligible value of harmonics given in the papers by Messrs. Adams and Woodbridge. At full load these harmonics are so small that they do not make an appreciable difference between kilovolt ampere and kilowatt readings.

The 200-kw. and 500-kw. machines mentioned above have passed the probation period of two years mentioned by Mr. Lincoln and Mr. Waters and have shown themselves to be the equal of machines with usual pole construction in general operating characteristics, such as commutation, stability, alternating-current and direct-current starting, quiet running, etc. The efficiencies are, of course, dependent upon the voltage for each load. For voltages obtained by operating the two sections of the pole with excitation in the same direction, there is virtually no difference between sectional-pole machines and the machine of standard pole construction. When the auxiliary section is excited to reverse polarity of the main section, the core-loss and loss in armature conductors, due to load current, will be greater than for a machine of standard construction, amounting at extreme lower voltage to between 1 and 2 per cent. It will, therefore, be seen in the table given later that the split-pole machine will be more efficient at the higher voltages of its range than the converter with a pole-piece of usual construction and its induction or booster regulator, and approximately the same efficiency at the lower voltages of the range.

In regard to three-wire operation, it may be stated that this has been successfully accomplished with transformers connected star primary, diametral secondary, deriving the neutral from the transformers. Throughout the 3 to 4 range of voltage, with an unbalancing between the two sides of 70 per cent of the full load current of the machine, the machine was perfectly stable, commutating sparklessly and running quietly. The harmonic currents were negligible and the direct-current voltage steady. This is a guaranteed condition of operation.

Mr. Waters's statement that a slight displacement of poles of the split-pole machine from their proper position would cause excessive equalizing current to flow in a multiple-wound armature has no weight, since the same result would be obtained from the displacement of poles of the usual construction or with interpole construction.

No mention has been made of the performance of a split-pole converter as a compound-wound machine. From tests made on a machine having a two-section pole with shunt winding on the main pole and a series winding on the auxiliary pole, an overcompounding of approximately 15 per cent. was obtained without taking a wattless component of current in excess of that given at approximately 98 per cent. power-factor full load the alternating voltage at collector rings being held constant. With the usual arrangement of compound-wound converters with 15 per cent reactance coils in the collector lines, a variation in wattless component of over 100 per cent would be required to give the same overcompounding. Assuming that the shunt field is excited to give 50 per cent of the full-load current lagging at no load, the leading wattless component at full load would be somewhat in excess of 50 per cent full-load current, which would give a power-factor in the converter of less than 90 per cent. In addition to the advantage of running at practically unity power-factor with a compound-wound split-pole converter, the commutation will be fully as good when the machine is operating with rotation such that the auxiliary pole will be on the leading side of the main pole, as with the reverse, or usual rotation. With proper proportions of auxiliary field, the commutation may be improved by depending upon the auxiliary pole for commutating flux, which will be proportional to the load, the same as in a commutating pole machine. The auxiliary section will then serve as a commutating, as well as a compounding pole.

In the following table approximate costs, floor space, and efficiencies, are based on 1000-kw. units with a voltage range 4 to 5, all to meet the same guarantees. The machine of usual pole construction with induction regulator is taken as the basis for comparison.

CONVERTER WITH USUAL POLE CONSTRUCTION

	With induction regulator per cent.	With revolving armature a.c-booster per cent.	With revolving field a.c-booster per cent.	Split pole converter per cent.
Cost	100	112	106	100
Efficiency mid-voltage full-load	94.5	94.25	94.5	95
Weight	100	106	106	97
Floor space	100	103	107	96

J. L. Woodbridge: Professor Adams has presented a very interesting discussion of the theory of the split-pole converter, differing in many respects from mine. A comparison of the two methods of analysis will show that while starting from the same point, namely, the harmonic analysis of the curve of field distribution, they both arrive at practically the same conclusions by two different routes and they therefore fit in very well together. In each analysis certain points are covered which do not appear in the other.

There are one or two points brought up by Mr. Lincoln to which I would like to refer. He has called attention to the limited width of the neutral space for commutation shown in Fig. 17 of my paper. It must be understood that Fig. 17 shows the field distribution at the armature periphery including the field fringe, and the distance between the pole tips would therefore be greater than the 30° space shown in this Fig. In fact this Fig. was taken from a machine actually designed, built, and operated. This question of commutation has, however, been so thoroughly covered by Dr. Steinmetz and by Mr. Burnham, from both theoretical and practical standpoints, that further comment appears unnecessary.

In comparing the split-pole converter with a series alternating-current booster, Mr. Lincoln has taken the stand that the latter would act more quickly, owing to the fact that there is a smaller amount of field flux to change. Mr. Lincoln does not seem to have taken into account the fact that with a converter having an alternating-current booster mounted on the same shaft, if it is desired to vary the direct voltage without altering the alternating voltage or the power-factor, it will be necessary to vary the fields of both machines. It is impossible to vary the direct voltage of the converter without changing its field strength. While this may be brought about without changing the excitation of the field windings, by simply changing the voltage applied to the collector rings and permitting the wattless current thus produced in the armature to magnetize or demagnetize the field, nevertheless this change of field strength must be brought about. The voltage at the brushes cannot be reduced without actually reducing the flux through which the armature winding is passing. Some reduction in excitation will result from the reduced voltage applied to the shunt-field winding of a self-excited converter, but unless a prohibitive amount of iron is used in the magnetic circuit, so as to work below the knee of the saturation curve, this change in excitation will produce but little effect on the field strength, and the remainder must be produced by wattless currents in the armature. Such a combination cannot, therefore, operate at unity power-factor without providing for simultaneous control of both fields.

In regard to damping devices, I would say that a special design of damper has been worked out for the three-part pole, which will not interfere with the quickness of action. It permits the field flux to vary symmetrically with respect to the axis of the field, while retarding any tendency to shift the entire field one way or the other. For the two-part field the main section can be provided with an ordinary copper-ring damper, since the field strength in this section is not varied. Where the auxiliary field section is controlled automatically, no damping device will be required, as the automatic controller will counteract any tendency to a variation of flux in this section, other than that which is required for operation.

Many of Mr. Lincoln's objections to the split-pole converter appear to be based on the assumption that it is impossible to eliminate the higher harmonics from the wave form. In this connection it may be well to emphasize what has already been brought out in this discussion; namely, that the variation of voltage ratio in the split-pole converter is not dependent upon these higher harmonics, but that these are merely incidental, and their elimination by a proper design of the machine does not limit the change of ratio obtainable. In the three-part pole the third harmonic determines the voltage ratio, but this does not appear in the wave form with three-phase connection. In the two-part pole it is the shifting of the entire field that produces the change of voltage ratio, and this result may be obtained with a pure sine wave of fundamental frequency, as has been so clearly shown by Dr. Steinmetz. Mr. Burnham has also shown that in the machines of this type actually built, these harmonics are quite negligible, even without the introduction of special features in the design adapted to minimize them.

G. E. Brown (by letter): Referring to Fig. 25, a test made on a 1000-kw. converter of the two-part-pole type recently put into commercial service, gave data very similar to that indicated in these curves. The converter was operated at no load with constant alternating voltage, and the direct voltage varied from 185 to 310. The normal direct voltage is 270, which requires no current in the auxiliary field circuit.

Referring to the statement that the minimum disturbance of power-factor occurs when working on either side of *Fo* as shown in Fig. 15, it is also evident from Fig. 25 that less change in the main field current is necessary to hold unity power-factor when the converter is operating below normal voltage than when it is operating above normal voltage. In this test, with the converter operating at no load from 185 to 270 volts, it was found that a variation in the main field current of about one per cent was necessary in order to maintain unity power-factor.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

A NEW LARGE GENERATOR FOR NIAGARA FALLS

BY B. A. BEHREND

A new generating plant of considerable magnitude has been completed recently at Niagara Falls. This plant is the new plant of the Niagara Falls Hydraulic Power and Manufacturing Company. A number of large direct-current generators to be used for the manufacture of aluminum has been installed in this plant, each generating unit consisting of two direct-current generators connected to 11,000-h.p. turbines. A large alternating-current generator, one of an aggregate of three wound for 12,000 volts, has also been installed in this station. The power house is located at the foot of the falls, on the American side, below the old power house of the Niagara Falls Hydraulic Power and Manufacturing Company.

The generator which is described in this paper offers a number of interesting features and is remarkable among the generators at Niagara Falls on account of its speed of 300 rev. per min., which is greater than the speed of any of the other large generators installed in the power houses at the falls. The generator is wound for 12,000 volts, three-phase, 25-cycles. The waterwheel is mounted on a horizontal shaft and has a capacity of 11,000 h.p. The generator is rated at 6500 kw. with a capacity for continuous operation of 7320 kw., or approximately 7500 kw. The runaway speed of the waterwheel is given as 506 revolutions; the generator, therefore, had to be designed to be safe at this speed.

The experience which has been gained from the design of generators for direct connection to steam turbines has greatly minimized the difficulties of a problem like the one under consideration. Nevertheless, 11,000-h.p. generating units are neither so plentiful nor so readily designed and built as not to afford considerable interest, especially if the results obtained with such machines are remarkably satisfactory.

The object of this paper is to illustrate, by a number of photographs and drawings, the design and construction of this interesting unit, and to show by the tests the electrical characteristics.

Fig. 1 shows the generator assembled on the test floor.

Fig. 2 shows the stator of the generator with the rotor removed.

Fig. 3 shows the rotor by itself.

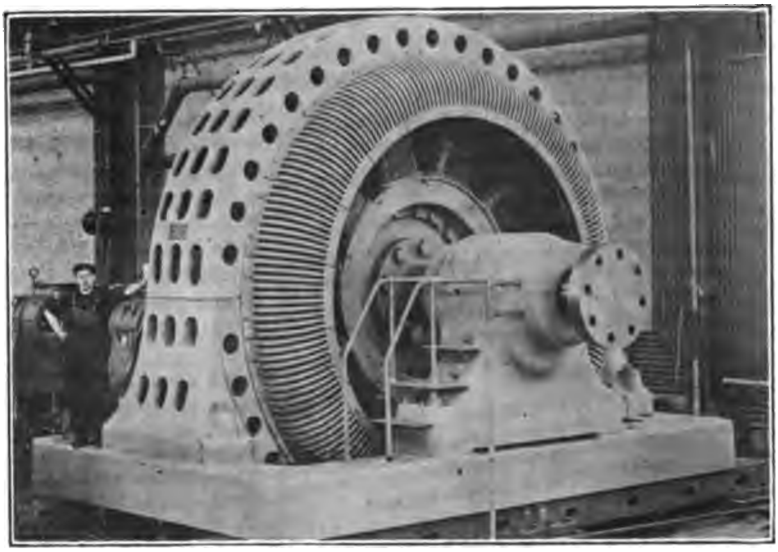


FIG. 1—10,000 h.p. generator

Fig. 4 shows one of the pedestals and the two stands for supporting the brush studs for the field excitation. The field excitation is derived from 220 volts.

Fig. 5 is an assembly drawing showing the most important details.

The construction of the rotor is worthy of careful study. A disc of nickel-steel, without a hole in the centre, forms the middle part of the revolving element. Two nickel-steel rings are mounted on each side of this web and are bolted and keyed to it. The nickel-steel used for this construction must have great mechanical strength and high magnetic permeability. A nickel-steel containing 3.5 per cent. nickel has been used for this purpose. Its elastic properties are.

Elastic limit	50,000 lb. per sq. in.
Ultimate strength	80,000 lb. per sq. in.
Elongation	20 per cent. in 2. in.
Reduction of area	40 per cent.

The magnetic qualities of this steel are given in the curve

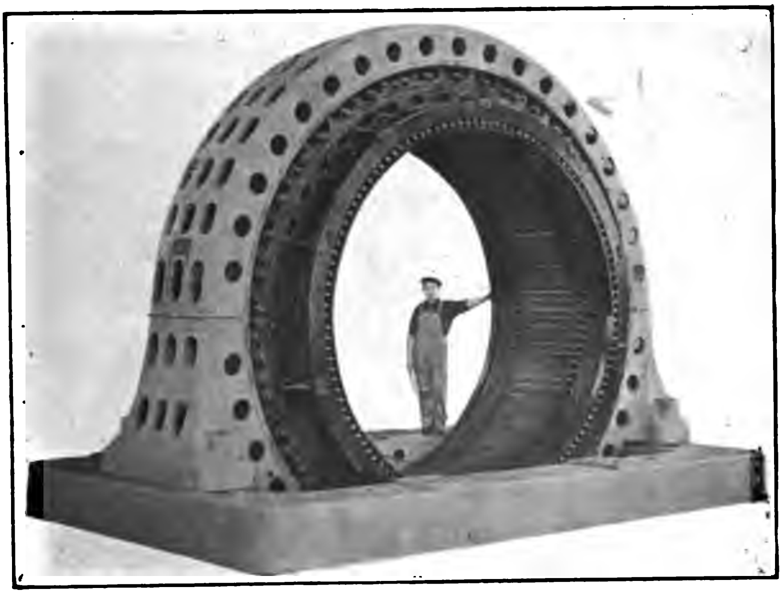


FIG. 2—Stator of 10,000 h.p. generator

illustrated by Fig. 6. The nickel-steel forgings as used in this construction enable the designer to produce the strongest and lightest construction, as the mechanical strength of the material is great and the magnetic permeability high. The weight of the complete rotor is only 92,900 lb. and, the bearings being 16 in. by 50 in., the specific pressure is only 58 lb. per sq. in. The weight of the stator is 116,700 lb. and the weight of the entire machine is 275,000 lb.

It may be of interest to discourse briefly upon the theory of elastic stresses in rotating discs and rings. The theory of

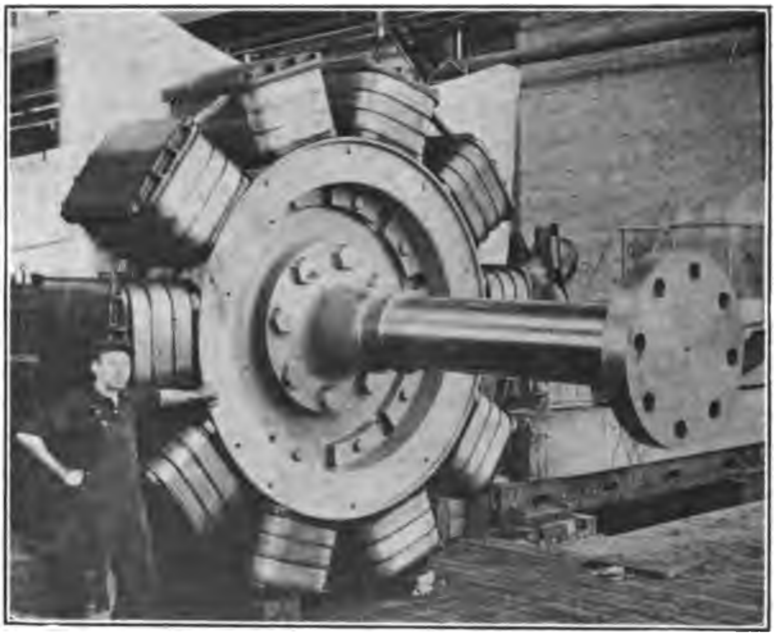
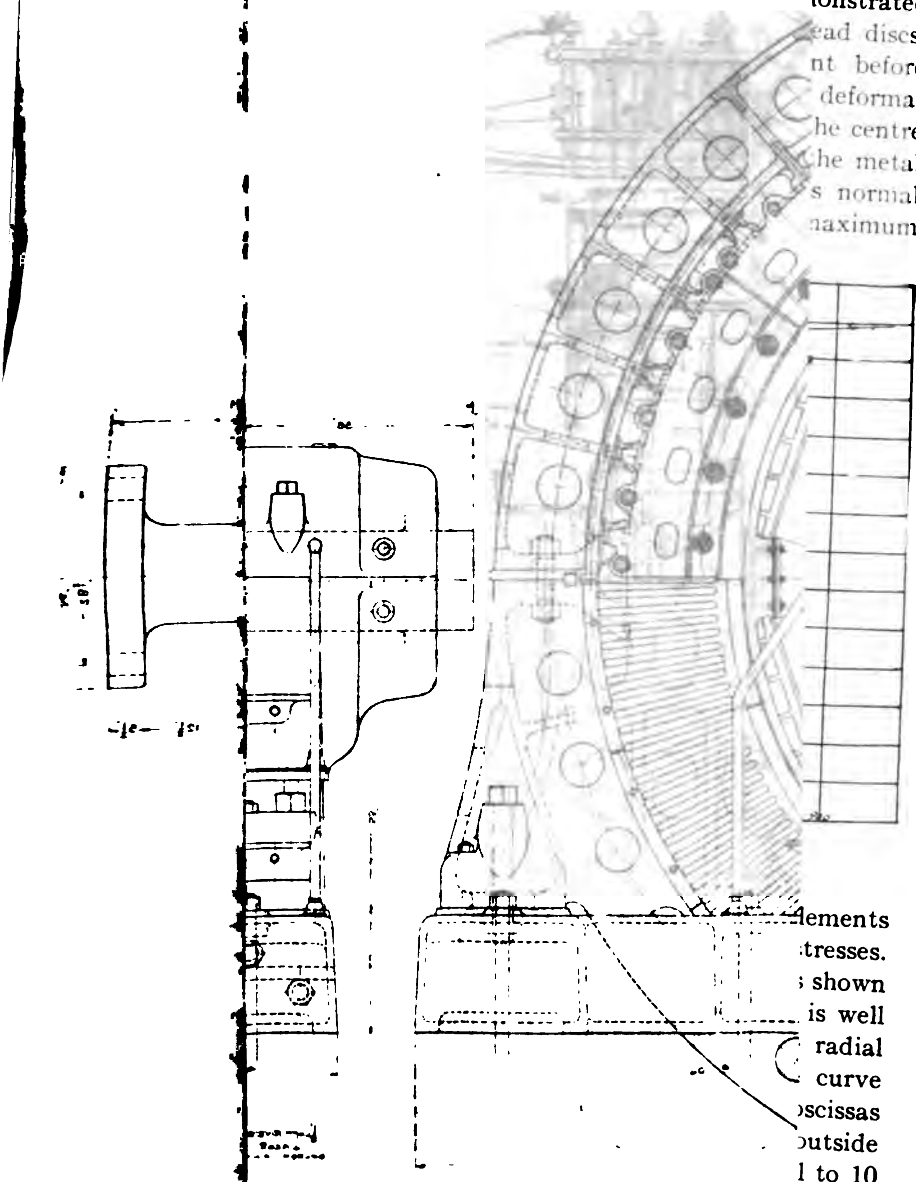


FIG. 3—Revolving field of 10,000 h.p. generator

FIG. 4—Pedestal and brush stands of 10,000 h.p. generator

n rotating
: centre of
ıonstrated
ead discs,
nt before
deforma-
he centre
he metal
s normal
naximum



lements
tresses.
shown
is well
radial
curve
oscissas
outside
l to 10
n zero
e then

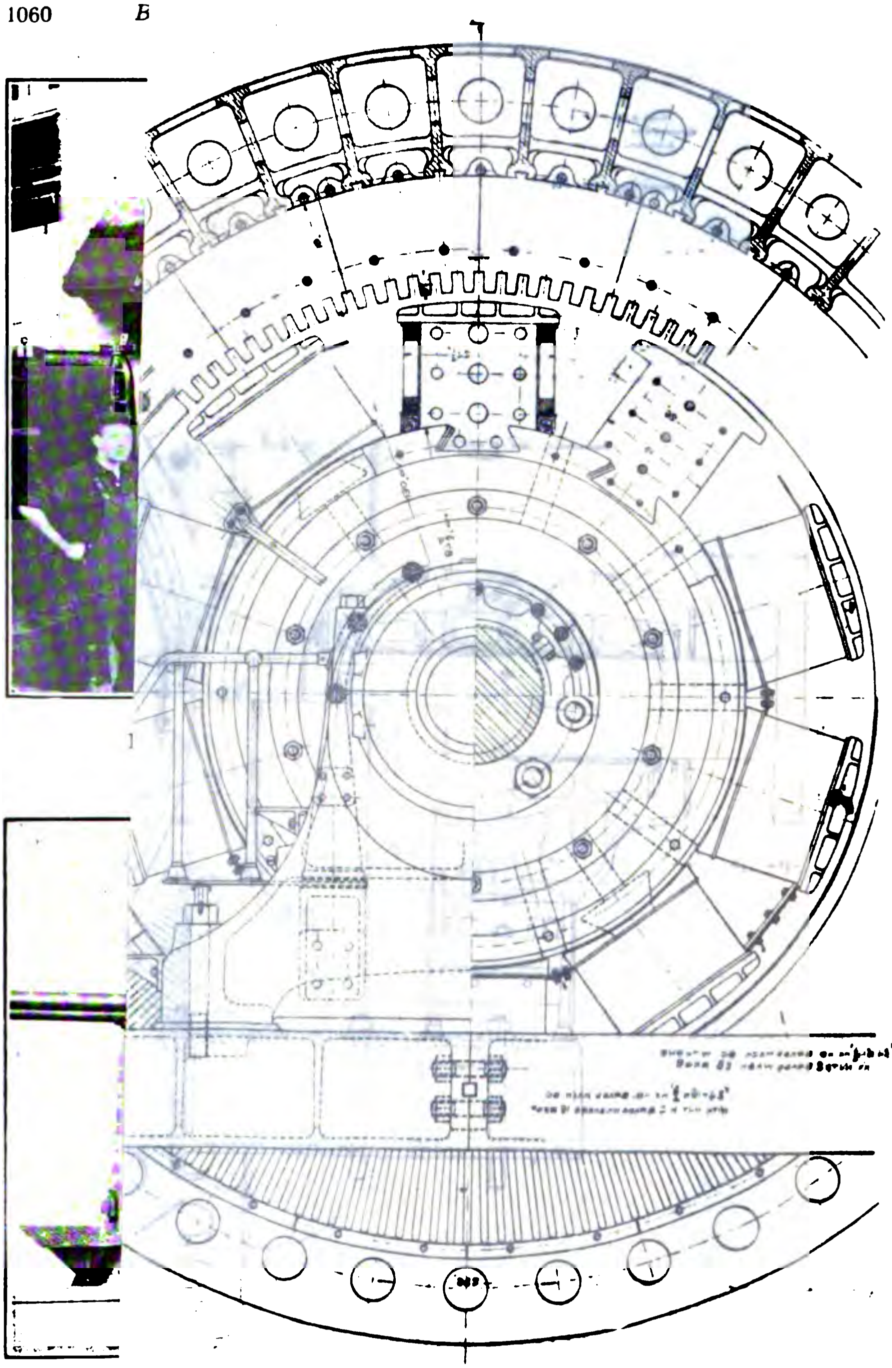


FIG.

elasticity applied to the radial and tangential stresses in rotating discs shows that the point of maximum stress is at the centre of a disc and at the inside surface of a ring. We have demonstrated the correctness of this theory by experiments with lead discs, the deformation of which, as obtained by measurement before and after the test, readily indicates that the maximum deformation, as shown by the lateral contraction, appears at the centre of the discs or at the inside of the ring from where the metal flows toward the outside portions. The radial stress normal to a free surface must be zero, and, therefore, the maximum

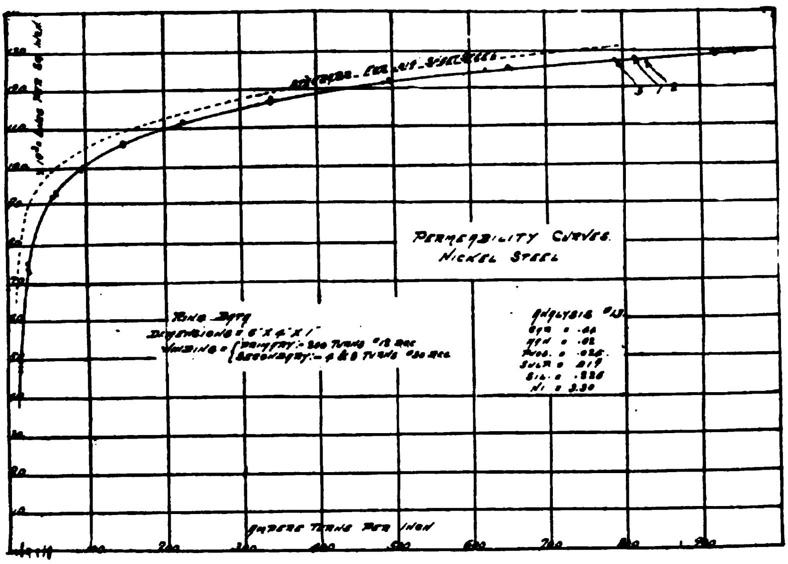
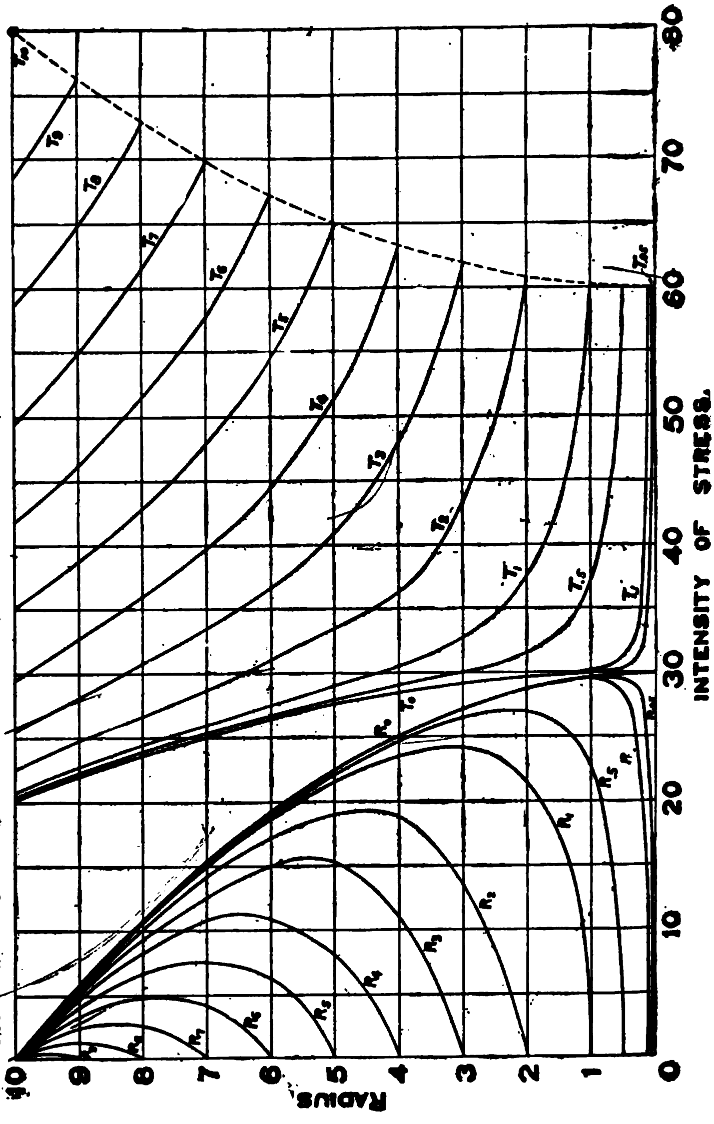


FIG. 6

stress appears as a tangential stress. In a solid disc the elements at the centre are subject to both radial and tangential stresses. A hole in the centre eliminates the radial stress, and, as is shown by the theory, this doubles the tangential stress. This is well illustrated in the curves shown in Fig. 7, in which the radial stresses are shown by the abscissas on the left of the curve sheet, and the tangential stresses are shown by the abscissas on the right of the curve sheet. It is assumed that the outside radius of the disc, or the ring, is kept constant and equal to 10 units of length, while the inside radius is increased from zero to 10 units of length. The radial and tangential stresses are then represented by the abscissas of the two sets of curves shown in the figure. Great mechanical strength and lightness are obtained in this rotor construction, and although it would have



CURVES OF STRESSES IN A ROTATING DISC OR ANNULUS.
OUTER RADIUS=10. R=RADIAL STRESS. T=TANGENTIAL STRESS. SUBSCRIPT INDICATES INNER RADIUS.

FIG. 7

been possible to use a hub mounted on a shaft, according to the practice with slow-speed machinery, as the stresses on the inside of the hub would not have been prohibitive, the design adopted

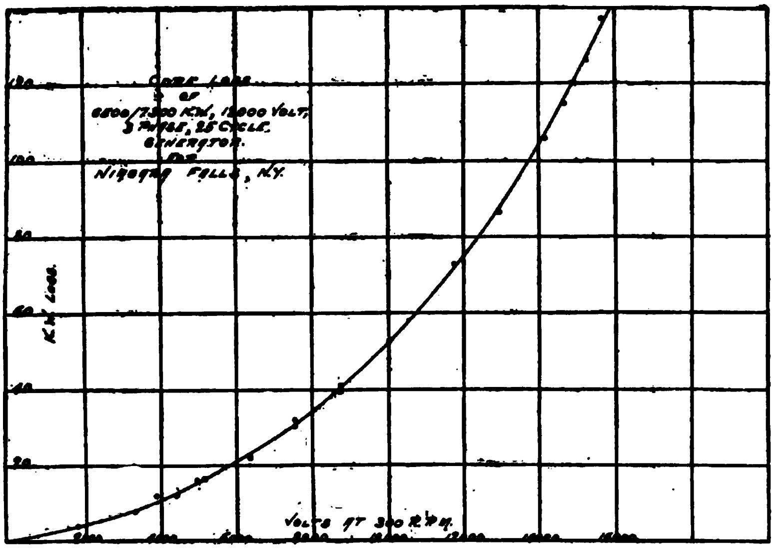


Fig. 8—Core loss of 10,000 h.p. generator

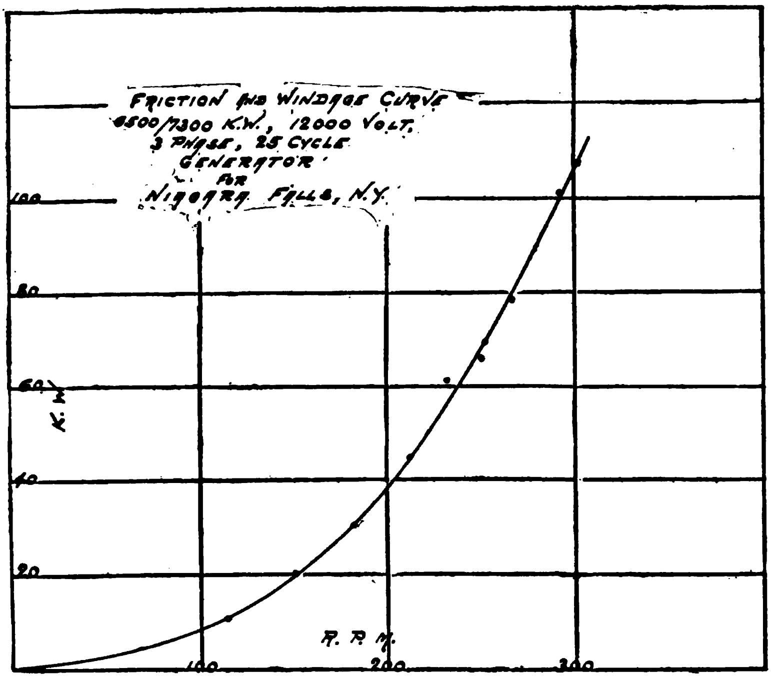


Fig. 9—Friction and windage curves of 10,000 h.p. generator

for this large generator is elegant and mechanically superior. The electrical characteristics of this unit are well shown in the curves, Fig. 8 to 12, accompanying this paper.

Fig. 8 represents the core-loss which is equal to 75 kw. at 12,000 volts; this is approximately one per cent. of the output.

Fig. 9 represents the power necessary to overcome the friction and windage at 300 rev. per min. which is equal to 106 kw.

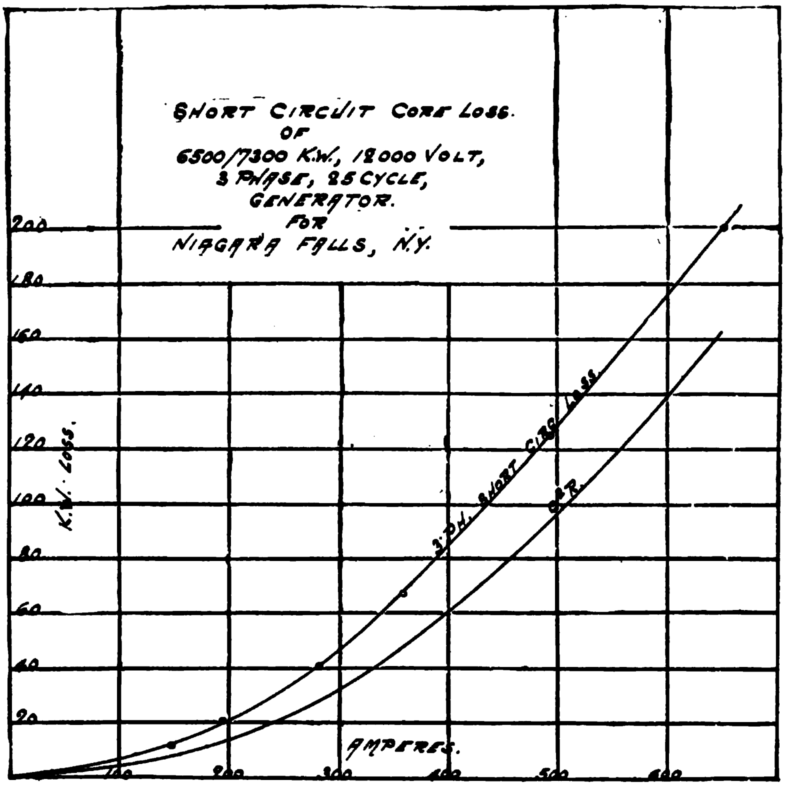


FIG. 10—Short-circuit core loss of 10,000 h.p. generator

Fig. 10 represents the short-circuit core-loss of the generator, demonstrating that the ratio of the short-circuit loss to the $I^2 R$ loss, at 7500 kw., is equal to 1.45, which is a very excellent result.

Fig. 11 represents the saturation and regulation curves of the generator with a regulation of 8.4 per cent. at full load and 100 per cent. power-factor at 12,000 volts. The generator is

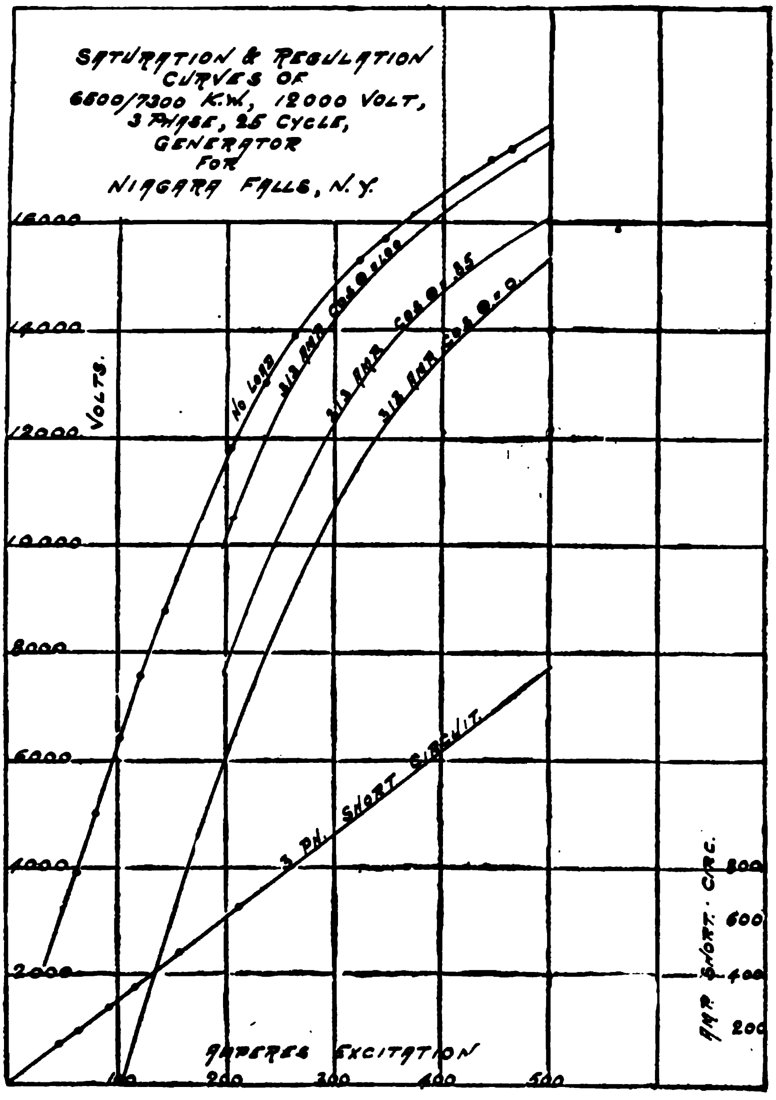
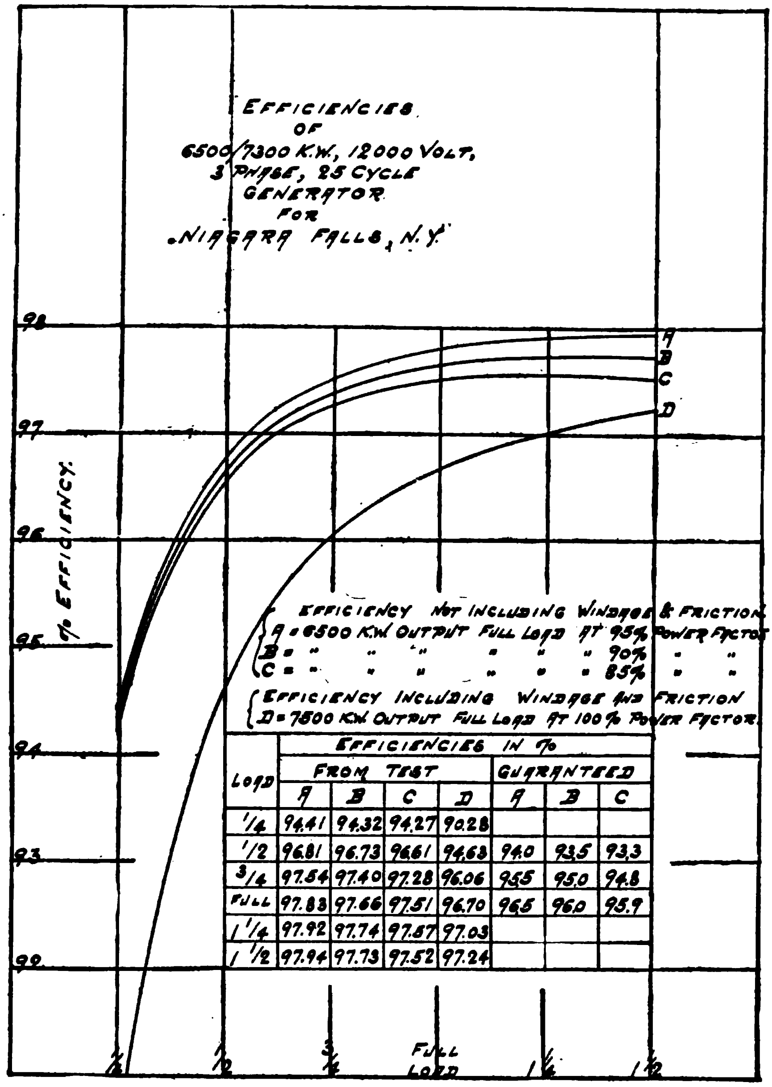


FIG. 11—Saturation and regulation curves of 10,000 h.p. generator



LOAD	EFFICIENCIES IN %						
	FROM TEST				GUARANTEED		
	A	B	C	D	A	B	C
1/4	94.41	94.32	94.27	90.28			
1/2	96.81	96.73	96.61	94.63	94.0	93.5	93.3
3/4	97.54	97.40	97.28	96.06	95.5	95.0	94.8
FULL	97.83	97.66	97.51	96.70	96.5	96.0	95.9
1 1/4	97.92	97.74	97.57	97.03			
1 1/2	97.94	97.73	97.52	97.24			

Fig. 12—Efficiency curves of 10,000 h.p. generator

capable of giving 13,000 volts under any condition of load that is likely to occur, should this be required.

Fig. 12 shows several efficiency curves and a tabulation of the guaranteed efficiencies compared with the actual efficiencies.

Fig. 13—Revolving field on its way down the cliff

Fig. 13 shows the revolving field suspended from a crane from the cliff, 240 ft. above the Niagara River, as it was being lowered into the gorge.

From the tests made regarding the heating of this unit, the temperature will remain within 35 degrees at normal load and 40 degrees at 7500 kw. The power of this generator is going to be used chiefly for the operation of induction motors in Niagara Falls and its vicinity.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N.J., July 1, 1908.

MODERN DEVELOPMENT IN SINGLE-PHASE GENERATORS

BY W. L. WATERS

The single-phase alternator has been in commercial use now for twenty years and it may seem surprising that there should be new developments at this late date. However, single-phase alternators have been used in the past almost exclusively for lighting work, and in units of comparatively small output and low speed. Recently, on account of the adoption of single-phase current for traction work, an important demand has arisen for large high-speed, low-frequency, single-phase generators. It is in the design and manufacture of such units that the engineer has had to overcome new difficulties. In large, high-speed, single-phase generators for 15 and 25 cycles the difficulties met with are due almost entirely to the large pole-pitch and high armature reaction which it is necessary to adopt. A 500-kw., 60-cycle, 72-pole, single-phase generator would have a pole-pitch of about 7 in., while a 6000-kw., 15-cycle, 2-pole machine would have one machine of about 120 in. It is easily seen that the design of these will be radically different.

These difficulties in single-phase generators of large pole-pitch are the result of:

1. Pulsation of the armature reaction.
2. Mechanical stresses on the end-connections of the armature coils.

The pulsation of the armature reaction causes hysteresis and eddy-current losses throughout the machine, often resulting in dangerous heating and low efficiency. The mechanical stresses due to the current in the ends of the armature coils result in vibration and distortion of the windings, and often in damage to the insulation or complete destruction of the coils, these stresses being particularly serious in single-phase railway generators, on account of the sudden variations in load and numerous short-circuits to which these machines are subjected. As the effect of the mechanical stresses on the armature coils, and the losses due to the pulsation of armature reaction, practically increase proportionally to the square of the pole-pitch in generators of standard design, it is easily seen why these effects which were negligible in the old single-phase alternators of small pole-pitch have become quite serious in the modern turbine-driven generator. The seriousness of these difficulties when they were first met with was so great that even within the last two years responsible engineers have stated it was impossible to build satisfactory low-frequency, high-speed, single-phase generators of large capacity, and it is only by careful study and experimenting that the modern machine of this type has been developed.

Losses due to pulsation of armature reaction. In a polyphase generator the armature current produces a magnetic flux which rotates synchronously with the field magnet. This magnetic flux being of practically constant magnitude, causes very little loss in the iron of the magnetic circuit. On the other hand, the armature current in a single-phase generator produces a pulsating magnetic flux which is stationary in space. It is easily seen that this pulsating flux will cause hysteresis and eddy-current losses throughout the whole magnetic circuit. The exact effect of the armature reaction flux on the rotating magnets depends, of course, on the relative phase of the armature current and electromotive force; that is, on the power-factor of the load on the generator. When the power-factor is unity and the armature current is in phase with the electromotive force, the armature reaction flux is a cross-magnetization; when the power-factor is zero and the armature current is 90° out of phase with the electromotive force, the armature reaction flux is a demagnetization. In the special case in which the rotating field magnet is cylindrical, without projecting poles, the effect of the armature reaction flux on the magnets is, of course, more nearly independent of the power-factor of the armature current. But in any case this flux is a pulsating one, and there are important losses in the field magnets, due to their rotation through this pulsating cross-flux or demagnetization flux.

An estimate of the combined losses in the armature and field magnets due to the pulsating armature reaction can be obtained in a number of ways. We can measure the increase of the power required to rotate the field magnets due to normal root-mean square current in the armature coils, with:

1. Direct current in the armature.
2. Alternating current of synchronous frequency in the armature.
3. Armature short-circuited and field excited.

Or with the magnets stationary we can:

4. Send normal frequency alternating current through the armature and measure the losses by a wattmeter.

These methods must all be regarded as convenient tests which are found by experience to give a good indication of the magnitude of the losses. Method (4) has the additional advantage that we can vary the relative position of the armature reaction flux and the pole faces, and thus investigate the variation of the losses in a single-phase generator with the power-factor of the load.

The only exact methods of measuring the losses are:

1. As unknown losses in a motor-generator-method efficiency test, or
2. From a comparison of the temperature rise obtained on full load with that obtained with known losses.

Unfortunately, both of these tests are difficult to make accurately, especially on a large machine, and probably in practice they do not give results which are any more accurate than the other methods. So at the present time we have to acknowledge that though we know a great deal about the relative values of the losses under various conditions, our knowledge of their absolute values are only approximate.

Pole-face dampers. Losses caused by a pulsating flux in the magnetic circuits are due to:

1. Hysteresis.
2. Eddy currents.

And the relative magnitudes of the two depend on the amount of solid metal in the path of the flux. If the whole magnetic circuit is laminated, then the losses are practically all due to hysteresis. On the other hand, if we have solid cast-steel poles there will be eddy currents in these poles which will partly choke back the pulsation of the flux and the hysteresis loss will be reduced. But in this case there will be eddy-current losses in addition to the hysteresis, and the way in which the total loss is changed will depend on the proportions and design of the magnetic circuit. If we place a heavy copper damper in the path of the pulsating flux, this will provide a low-resistance path for the eddy currents, and the pulsating flux, and consequent hysteresis loss, will be reduced practically to zero, while on account of the low resistance of the damper circuit the eddy loss will not be appreciable. The way in which the losses and the pulsating flux vary according to the presence of eddy currents can be determined for any particular design by changing the thickness of the laminations, or changing to solid poles or

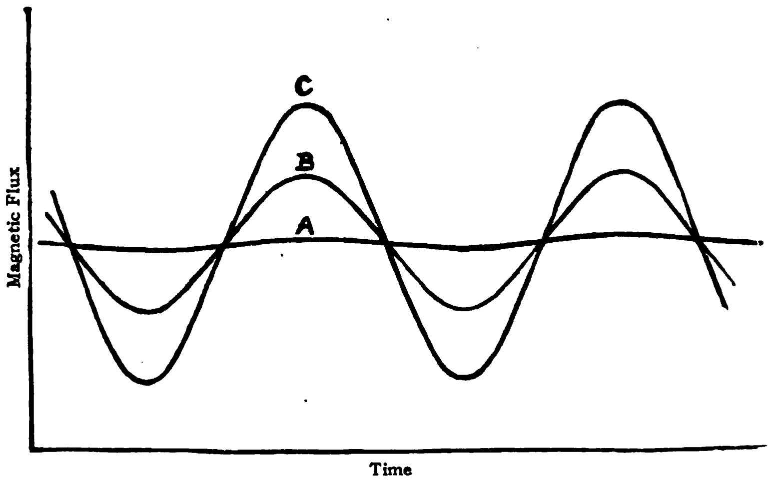


FIG. 1

dampers. It is usually found that the losses are greatest with heavy laminations or solid poles; that they are less for thin laminations, and practically zero when heavy low-resistance dampers are used either with solid or laminated poles.

Fig. 1 shows the pulsation of the armature reaction flux in a 500-kw., single-phase, 20-pole generator, as determined by means of search-coils wound on the pole-faces. "*C*" shows the pulsation for laminated poles, No. 29 gauge: "*B*" shows the same machine with solid poles: "*A*" shows the same solid pole faces covered with a $\frac{3}{8}$-in. copper plate. The magnitude of the pulsations in the three cases is about in the ratio of from 30 to 15 to 1; thus the copper plate has practically damped out all pulsations, the armature reaction flux becoming constant. In practice, a copper damper usually takes a form similar to the squirrel-cage secondary of an induction motor. Heavy copper bars are dovetailed into the pole faces, and short-circuited at the ends by copper rings or discs. Fig. 2 shows such a cage damper on the field magnet of a 6000-kw., 2-pole generator.

The question of losses due to the pulsating armature reaction in a single-phase generator may be considered in another and possibly a simpler way. The single-phase pulsating field is equivalent to, and produces the same effect as, two rotating fields each of half its maximum value, one rotating at the same speed and in the same direction as the generator field magnet, and the other rotating at the same speed but in the opposite direction. The flux rotating synchronously with the field magnet, being constant in magnitude, causes very little loss.

Fig. 2

The flux rotating in the reverse direction causes losses throughout the whole magnetic circuit due to hysteresis and eddies. If we have a squirrel-cage damper enclosing the field magnets, this damper system acts in regard to this reverse rotating flux in the same way as the short-circuited secondary of an induction motor or transformer, a current is induced in the damper which produces a field that neutralizes the rotating flux. The eddy and hysteresis loss in the iron of the magnetic circuit which would be caused by this rotating flux is thus eliminated, and the only loss is that due to the current circulating in the damper. If we make the conductors forming the squirrel cage of sufficiently low resistance, this damper loss becomes negligible, with the result that the entire loss due to the pulsating armature reaction of the single-phase generator is practically eliminated.

To show how serious this matter of losses becomes in two-pole single-phase machines without dampers, the following table is given, showing the losses and full-load temperature rises on three turbo-generators, both with and without dampers:

TWO-POLE, 25-CYCLE, SINGLE-PHASE GENERATORS. SAME CURRENT PER ARMATURE CONDUCTOR ONE AND THREE-PHASE UNDER ALL CONDITIONS AND ALL LOSSES IN PER CENT. OF SINGLE-PHASE RATING

Size	Type field	Three phase		Single phase	
		Without dampers		Without dampers	With dampers
		Loss	Temperature	Loss temp.	Loss temp.
750 kw.	Solid	0.53	27°c.	3.75 95°c.	0.8 34°c.
1000 kw.	Solid	0.3	31°c.	3.0 122°c.	0.5 37°c.
1000 kw	Laminated	0.2	19°c.	3.8 150°c.	0.3 18°c.

It will be seen that in these three machines, operating single-phase, there is due to the pulsating flux an average loss of 3.5% and an average temperature rise of 125° cent., without dampers; with dampers the average loss is 0.5% and the temperature rise 30° cent. Figures are given only on comparatively small machines on account of the difficulty of measuring losses on large machines. But tests on larger generators up to 6000-kw. capacity show that the improvement due to heavy copper dampers is even more striking in large machines than it is in small. So far as experience goes at the present time, it may be said that the use of such dampers is the complete solution of the difficulties due to pulsating armature reaction met with in large, low-frequency, two-pole, single-phase generators.

Mechanical stresses on armature coils. That it was necessary mechanically to brace the end-connections of the armature coils on a direct-current machine subjected to sudden loads and short-circuits has been known for many years. But until quite recently additional supports for alternator armature coils were seldom provided. The reason for this was that as the continuous short-circuit current of an alternator is only about two or three times normal, it was not considered that the mechanical stresses on the ends of the small pole-pitch coils generally in use were sufficiently great to cause any trouble. Only during the last few years has it been demonstrated by experience that coil supports on large pole-pitch alternators are not only advisable but necessary, and that on account of the numerous short-circuits, they are particularly necessary on single-phase machines operating on traction circuits.

When an alternator is suddenly short-circuited, the first rush of current is limited only by the self-induction in circuit. In the case of an alternator of very low self-induction, this first rush of current on sudden short-circuit will be 15 or 20 times normal full-load current. As the mechanical stress on the end-connections varies as the square of the current, this means that the stress on the armature coils is 200 to 400 times normal. A 6000-kw., 2-pole, 25-cycle, single-phase generator will have a pole pitch of about 100 in., and the length of the end-connection at one end of one armature coil will be about 180 in. We find

FIG. 3

that the mechanical stresses on the end-connections at one end of one armature coil of this machine on a sudden short-circuit is something like five tons; and we usually find that on low-frequency high-speed generators of large capacity the mechanical stresses on the end-connections at one end of one armature coil on sudden short-circuits are from 2 to 10 tons. When we consider that this comes as a sudden mechanical shock on the winding, we can realize the kind of coil supports that are required, and can understand the disastrous results sometimes obtained on short-circuits, when such supports are omitted.

We can see from these stresses that coil supports must be of metal of heavy cross-section. The objections to metal are, of course, those of insulation, but though coil supports of wood, porcelain, and similar insulating materials have been tried, it is easily understood that they have proved unsatisfactory on machines of large pole pitch. Fig. 3 shows a form of coil support which has been developed and proved satisfactory. It is of bronze and of heavy girder, section and insulated for the full generator voltage. The coil support and its method of application are evident from the illustration; it is placed in position after the machine is wound and is removable in a few minutes at any time. It is not suggested that this is the only satisfactory type of coil support that can be used; it is given as a type which has proved successful in actual operation on machines up to 10,000-kw. capacity, and it has apparently solved the difficulties due to mechanical stresses on the end-connections of large pole-pitch generators which are liable to sudden variations in load and frequent short-circuits.

The two main difficulties met with in large low-frequency, high-speed, single-phase generators, which have been described above, can at the present time be regarded as having been successfully overcome. The use of heavy copper dampers on the pole faces and heavy bronze coil supports applied to the ends of the armature coils in such a way as to take directly the mechanical stresses which develop on short circuits, have now made such machines a practical success. Like every other new type of electrical machinery, the large turbine-driven, single-phase generator has had to go through a period of development, but at the present time it may be said that such generators for 15 and 25 cycle, and in units of 5000 to 10,000 kw. capacity can be, and are, built with the same success as that obtained on standard slow-speed polyphase generators.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

APPLICATION OF FRACTIONAL PITCH WINDINGS TO ALTERNATING-CURRENT GENERATORS

BY JENS BACHE-WIIG

Fractional pitch windings have been treated by various authors in the past, especially with regard to the influence they have upon the self-induction of the armature winding. The object of this paper is to deal briefly with the points leading to the use of a chorded winding for alternating-current generators from the standpoint of manufacture and design, and to indicate the influence this winding has on the performance of a machine.

Reasons for chording the winding. In general, the chorded winding has been adopted to facilitate the manufacture of armature windings. As generators are manufactured, there are certain standard frames used for a number of ratings at different speeds and voltages. Group windings, with the number of slots per pole per phase equal to an integer, are generally preferred, and the number of conductors is fixed within a limited range for a given voltage. This often necessitates the use of a chorded winding in order to get the proper effective number of conductors. This is especially the case for low-voltage machines of large size. Further, a winding often works out in such a way that a better arrangement of conductors in the slot can be obtained through chording. A two- or a four-pole high-speed generator is another example where the chorded winding facilitates manufacture. Such a machine will naturally have a comparatively small bore, and if open slots and form-wound coils are used, it is impossible with a coil-throw equal to the pole pitch to bring the coil through the bore of the armature without bending it entirely out of shape. The only solution, therefore, is to chord the winding a sufficient amount to allow it to pass through the bore. In this case, therefore, the chorded winding is of great advantage, as it permits the use of form-wound coils. This may not hold in cases where the coils are made in halves.

With regard to the construction of the end-connections of form-wound armature coils for large alternating-current generators, especially for a generator with a small number of poles, the space occupied by the end-connections is considerably smaller for a chorded winding than it is for pitch, even taking into account the increased number of armature conductors made necessary by the chord. The effect of chording upon the number of armature conductors is to increase the number in the ratio of the sine of half the electrical angle between the two sides of the coil, whereas the length of the end-connection and the distance the coils build out decreases directly in proportion to the electrical angle. The result is a saving in space. This is of particular benefit to all two-pole machines, but applies also to four- and six-pole machines above 500 kw. It may also be of advantage to chord the winding for these reasons for smaller machines when wound for high voltage.

Take, for example, a 300-kw. 11,000-volt three-phase 500-rev. per min. 25-cycle generator. This machine has six poles, and with six slots per pole per phase will have 108 slots total. Having form-wound coils with the coil-ends extending in a parallel plane to the shaft, one coil per slot, and a pitch winding (1 and 18) the distance between the armature iron and the extreme end of the coils is approximately 16.5 in. Between adjacent coils 0.25 in. air space is provided. Chording this winding down to 1 and 14, or 130 electrical degrees, reduces the above distance to approximately 14.9 in. or decreases the width of the machine 3.2 in. The same winding arranged for two coils per slot, builds out approximately 20.5 in. for pitch winding and 16.75 in. for throw 1 and 14. This means a 7.5 in. reduction in all-over width of the machine. The width of the armature iron being 8 in., it can readily be seen that this insures relatively large saving in space. As indicated by the above dimensions, the winding with two coils per slot extends farther beyond the armature core than does the winding with one coil per slot, and, accordingly, chording the winding is more advantageous for a two-coil-per-slot winding.

As stated above, space can be saved in the way the coils build out even for a six-pole, 300-kw. machine. As space is saved, the mean length of one armature turn is reduced. On the other hand, the number of conductors on the armature is increased, and thus a certain throw will give the most efficient winding in regard to copper loss and amount of copper. Taking the above mentioned 300-kw. generator and working it out on the basis of equal iron losses, with one coil per slot there is practically no difference in the weight of copper with different throws—the decrease in length of mean turn is balanced by the increase in the number of turns. With two coils per slot there is a saving of approximately 25% in the weight of copper and of copper loss in favor of the chorded winding having coils lying in slots 1 and 14 over the pitch winding having coils lying in slots 1 and 19. The gain by reducing the throw, therefore, amounts to gaining space for the one-coil-per-slot winding only, and means a saving in space and a reduction in copper loss and weight of copper as well for the two-coil-per-slot winding.

As mentioned above, the winding with two coils per slot builds out farther than does the winding with one coil per slot, and the comparison between the pitch winding and the chorded winding will, therefore, show up more in favor of the winding with two coils per slot.

As the six-pole 300 kw. generator here referred to is a comparatively small machine, it is evident that for large generators, or generators with a smaller number of poles, the above figures will show up still more favorably for the chorded winding; the example indicates that even down to this size of machine it is advantageous to chord the winding.

One may say that chording the winding is to take the copper out of the end-connections and put it in the slots, which is another point in favor of the chorded winding, for it is easier to get rid of the heat in that part of the coil which is imbedded in iron than it is to cool the end-connections of the winding. The ventilating conditions, therefore, are often improved by chording. As above stated, this applies particularly to large machines and windings having a comparatively large throw.

It is stated above that the length of the end-connections decreases in proportion to the chord and that, even if the weight of the copper is not decreased, the chording has the effect of decreasing the amount that the coil-ends build out. As the question of bracing the coil-ends of a generator with a small number of poles and a large throw is of great importance, it will readily be seen that shortening the end-connections is a great benefit in this respect; it makes them stiffer, so that even if coil supports are used, both in the case of pitch winding and chording, the chorded winding is superior in mechanical strength.

It may be argued in connection with the above that a chorded winding requires heavier insulation of the end-connections than does a pitch winding as the phases are intermixed and points of higher potential are brought together. As a rule it is true that a chorded winding, will have more points of higher potential between adjacent coils than will a pitch winding, but considering that the coil-ends for larger generators always should be arranged with an air-space between adjacent coils, and further that they are braced and prevented from touching one another, in only extreme cases will it be necessary to provide for any extra heavy insulation. In an ordinary pitch winding, points of high potential are brought together at the beginning of the phases. As a rule no extra heavy insulation is provided for at such points. Considering the part of the coil lying in the slot, even if the full-line voltage exists between two coils in one slot, it should not cause any trouble as each coil is already insulated against ground.

The effect of chorded winding upon armature reaction. As, for a given voltage, the effective number of turns must be the same whether the winding is pitch or chorded, the effective number of armature ampere-turns is also the same in both cases; consequently the demagnetizing effect will remain practically unchanged. The chord, however, has an influence upon the self-induction of the machine. This effect is very much the same as in the case of an induction motor, a subject considered at length in a paper read before the Institute.* The reducing influence of the chording upon the slot leakage, the tooth-tip leakage, and the coil-end leakage, as pointed out in that paper, could be applied to similarly wound alternating-current generators. In many cases, however, the self-induction does not have much influence upon the short-circuit ratio, as it is small compared with the demagnetizing ampere-turns, and, consequently, a change in self-induction will not change the short-circuit ratio to any extent, so that in ordinary cases the self-induction can be figured without considering the chord. There are cases, however, where the decrease of the self-induction due to the

* Fractional Pitch Windings for Induction Motors, by C. A. Adams, W. K. Cabot and G. A. Irving, Jr., Niagara Falls, June 28, 1907.

fluence of the chording upon the slot leakage, the tooth-tip

chording of the winding has to be carefully considered. One such case is when a delta connection is used for a three-phase generator with chorded winding with two coils per slot. It is a well known fact that when the wave-form is not a sine, the third harmonic present in such a winding causes current to flow round the delta. In most cases these currents are small and the loss due to them is negligible. This, however, does not hold in the case referred to; for as the same current flows in the

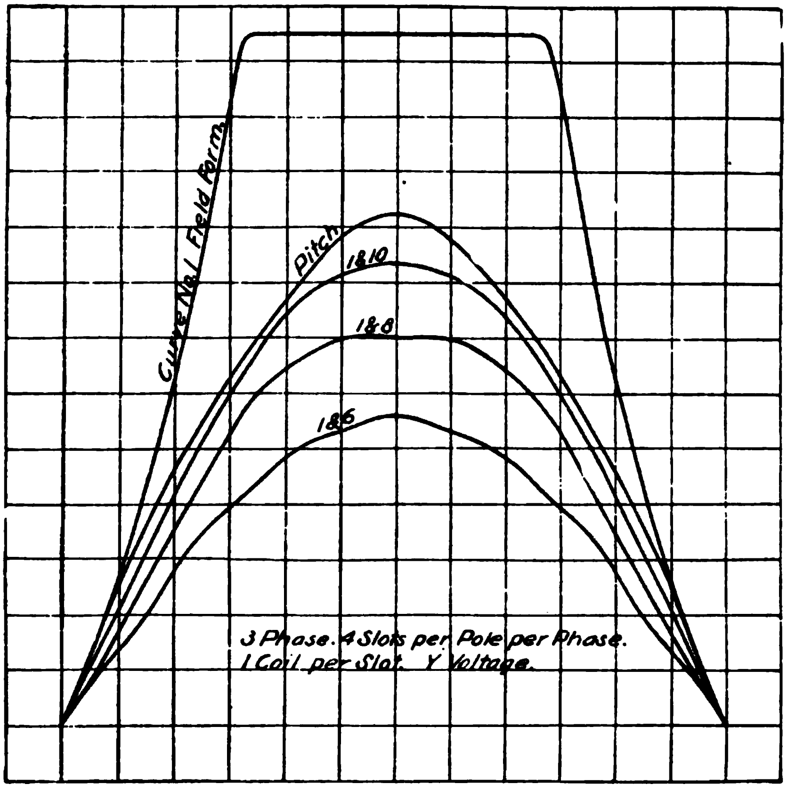


FIG. 1

three legs of the delta through all the coils in succession the slot and the tooth-tip leakage is eliminated for those slots in which the current flows in opposite directions, thus reducing the self-induction of the circuit. The self-induction, however, is the main thing that opposes the flow of this current, and thus it can easily be seen that when the chording of a delta-connected winding is carried out in such a way that the third harmonic is present in the winding, it will set up an internal circulating current. A one-coil-per-slot winding with the same chording will have far less influence upon the local currents flowing, as in this case the sides of the coils opposing each other do not lie in the same slots.

It may be added that the chording of a two-coil-per-slot winding has also some influence upon the eddy-current losses set up in the armature conductors. As is well known, eddy-current losses are set up in the conductors, and, in cases of large

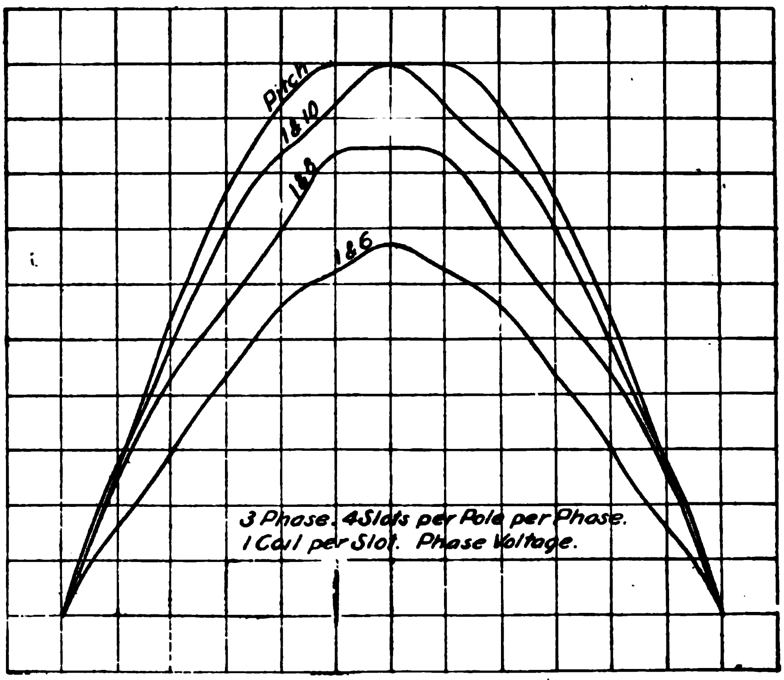


Fig. 2

copper section are considerable. The chording has the effect of bringing conductors belonging to different phases into the same slots, and thus the currents flowing in these conductors are not in phase, which again tends to reduce the eddy-current losses.

The effect of the chorded winding upon the wave-form. If the field form of the generator follows a sine wave, the chording of the armature winding will not change the form of the electromotive force wave; the wave-form will remain a sine wave. The more the field form departs from the true sine wave, the more will the influence of the chord show up in the shape of the wave-form. In the case of high-speed generators with a small number of poles, it is often, for mechanical reasons, not possible to bevel the poles at all, or else it is not possible to bevel the poles in such a way as to give a smooth field form approximating a sine curve. At the same time a generator having a small number of poles will usually have a large number of slots per pole per phase, so that this will tend to smooth out the wave-form and make it approximately a sine wave.

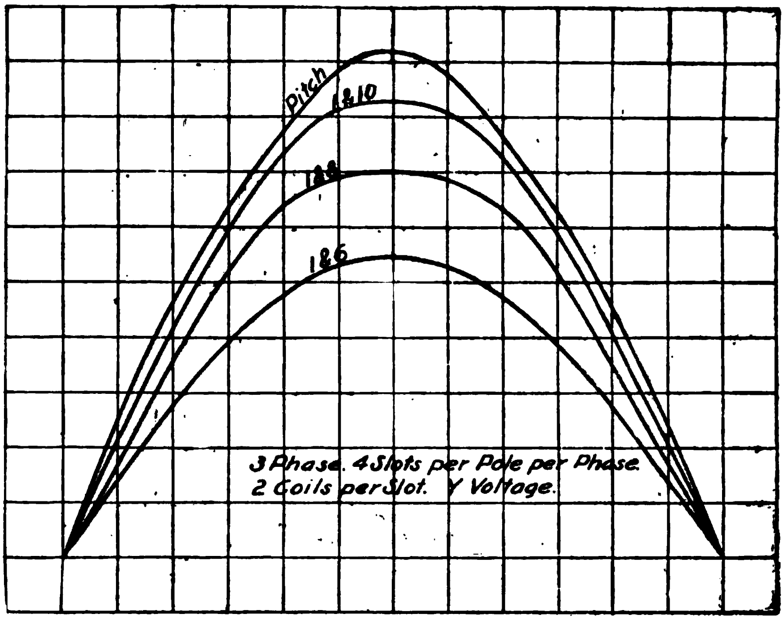


FIG. 3

Considering a generator having a cylindrical field construction, as in the case of a two-pole turbo-generator, the field form will be approximately as shown in Fig. 1, curve 1. For this filed form the electromotive force wave-forms are plotted in Fig. 1 for a three-phase Y-connected winding having four slots per pole per phase; the wave forms are plotted for pitch winding and for chord winding: 1 and 10, 1 and 8, and 1 and 6. It will be seen that the chord 1 and 10 improves the wave-form, but that 1 and 8, and 1 and 6, are distorted.

For the same winding and the same throws, the wave-forms of each leg are plotted in Fig. 2; these wave-forms would also be obtained when connecting the winding in delta. As is to be expected, these wave-forms are in all cases considerably more distorted than those shown in Fig. 1 and indicate the influence of the higher harmonics.

Figs. 3 and 4 show the wave-forms for the same field forms and same number of slots per pole per phase when the winding is connected in Y and delta, but with two coils per slot. As the phases by these combinations are more distributed over the pole face,

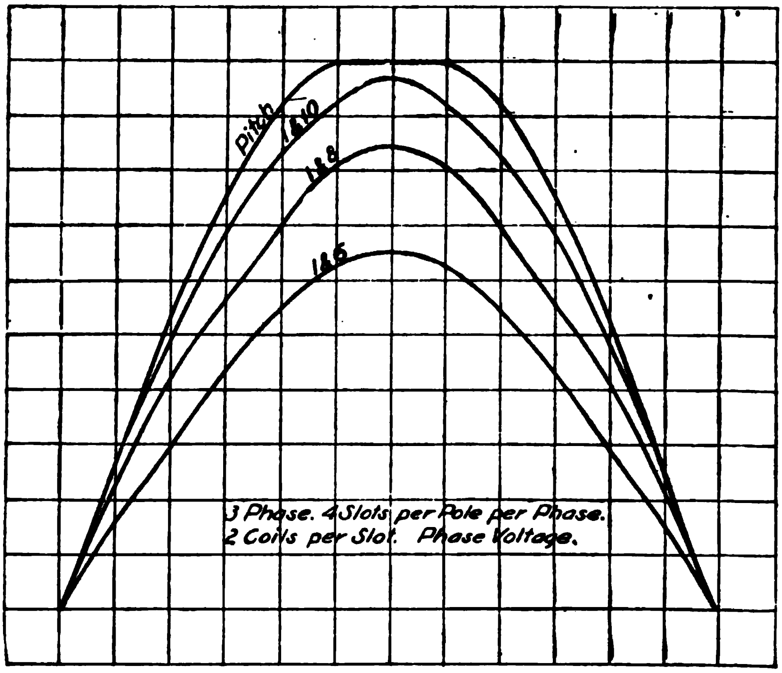


FIG. 4

the wave-forms are less distorted than for the one-coil-per-slot winding; in the case of the Y connection the shape of the wave approximates as nearly a sine for the chord 1 and 6 as it does for the pitch winding. The wave-forms in Fig. 4, however, show evidence of higher harmonics of similar nature, but less pronounced than the one-coil-per-slot winding, Fig. 2.

As noted above, these wave-forms are plotted for a nearly rectangular field-form. It is therefore evident that the chord has a more pronounced influence upon the wave-form than would be the case if the field form would more nearly approximate a sine curve. However, it is evident that the chord does have an influence upon the wave-form, and when chorded windings are used the wave-form should, therefore, be considered.

Conclusion. In determining the most efficient amount of chording, so many points are to be considered in each case that no general rules can be formulated. In cases where the chord is not made necessary solely for mechanical reasons, the most efficient chord will depend upon the number of poles, the ratio of pole-pitch to pole-length, the voltage, and the size of machine. These must be worked out in each individual case. The reasons then for chording the winding are:

1. To obtain the proper number of effective turns.
2. To enable form-wound coils to be used in generators of small bore and few poles.
3. To reduce the space occupied by the end-connections.
4. To improve ventilation.
5. To save copper and insulation.

Discussion on "A New Large Generator for Niagara Falls", "Application of Fractional-Pitch Windings to Alternating-Current Generators", and "Modern Developments in Single-Phase Generators." Atlantic City, N. J., July 1, 1908

Wm. J. Foster: There are two or three points in connection with the generator described by Mr. Behrend to which I would like to call attention. I think that in many respects it is a most remarkable machine. The efficiency as given by Mr. Behrend in the tabulation is something unusual, when we consider that it is a generator of 12,000 volts. I would like to point out how, in my opinion, the high efficiency has been attained.

The writer of this paper has given us a complete assembly drawing. It seems to me that it will be of great value to the professors and representatives of our educational institutions to have such complete data giving every essential dimension. From the curves showing the characteristics, taken in connection with the assembly drawings, it is apparently possible to reproduce the windings exactly. The first thing I notice is the remarkably small amount of magnetic material in the armature, which results in a rotor of unusually small weight, considering the output of the machine. Mr. Behrend has taken a step in advance over all other designers in this respect. The reduction of magnetic material is something that is to be striven for and that can be accomplished, provided we can take care of the heating, which is what has troubled all of us heretofore. The amperage in the slots of this armature is very high, considering the potential, but Mr. Behrend says the temperatures are very low, hence it would appear that the problem has been solved.

A word further with reference to the efficiency. What used to be called the electrical efficiency is evidently given, but it does not include a percentage of the load losses. It simply includes the core losses, which, according to the curve, are 75 kw., or only about one per cent.

The efficiency is unusually high, and has been attained, in my judgment, by using an extremely high armature reaction for a 12,000-volt machine. If you increase the armature reaction twenty-five or thirty per cent, as I consider has been done in this case, you attain a very high efficiency, much higher than has been attained heretofore. But this requires a different design of the rotor, involving a heavier pole. It is evident on the face of it that the pole must be of greater length, to take care of the increased ampere turns. It must be lengthened about as the square of the ampere turns, hence I consider the pole forty or fifty per cent longer than the usual type. This lengthening can be accomplished only by an unusually strong construction in the rotor, which Mr. Behrend dwells upon in his curves pertaining to the peculiar disc construction and in his discussion of the characteristics of the nickel steel. I am especially glad to see that nickel steel can also be obtained of such high permeability. This is another thing which appeals to the designer as a step forward, namely, to be able to obtain nickel steel with high characteristics mechanically, and at the same time very high in its magnetic properties.

In the matter of windage, I am somewhat disappointed in this machine. The curve showing the variation of the friction and windage with the speed is a peculiar one. You will notice how rapidly these factors go up with increased speed. At 300 rev. per min. it is nearly five times what it is at half that speed. If it continues to go up at that rate it will take 300 or 400 kw. to drive the generator at the runaway speed. It seems to me that eddies exist and that the most of the windage is really lost, as far as cooling effect is concerned. What takes place as the speed increases is that a new set or system of eddies is established, requiring more and more energy to enable the speed to be obtained. When we take into account the friction and windage losses in connection with the electrical losses and the load losses, it seems to me there is not very much gain in commercial efficiency on this machine.

The reduction in magnetic material does result in assisting the problem of the bearings, but the bearings must be very conservative with the pressure per square inch as mentioned, and the rubbing speed so conservative. This same diameter of shaft is in use on many generators running at 400 revolutions with which I am acquainted. Hence, the bearings problem in this Niagara generator is not especially difficult.

B. A. Behrend: I have made the efficiency on the basis of short-circuit and core losses, but not including friction and windage, one-third of the short-circuit losses.

Wm. J. Foster: In working up efficiencies it has been customary to take one-third of the short-circuit losses as the "load losses." I think the Institute should appoint a committee to consider whether that is the correct value to take for the load losses.

L. Schuler (by letter): When the first large turbo-alternators for single-phase current were designed about two years ago by the Felten & Guilleaume-Lahmeyerwerke, the question of avoiding losses by the pulsating armature reaction was carefully considered. It was clearly understood that a minimum of losses could be attained either by perfectly laminating the field or by providing heavy dampers, but we were at that time not quite sure which way would be best. We therefore made the following experiment: An ordinary three-phase induction motor with slip-rings was coupled to a direct-current motor and used as a single-phase generator by exciting one stator-phase with direct current. The other two stator phases could be connected to a variable resistance. Two slip-rings were short-circuited so that the whole arrangement represented a

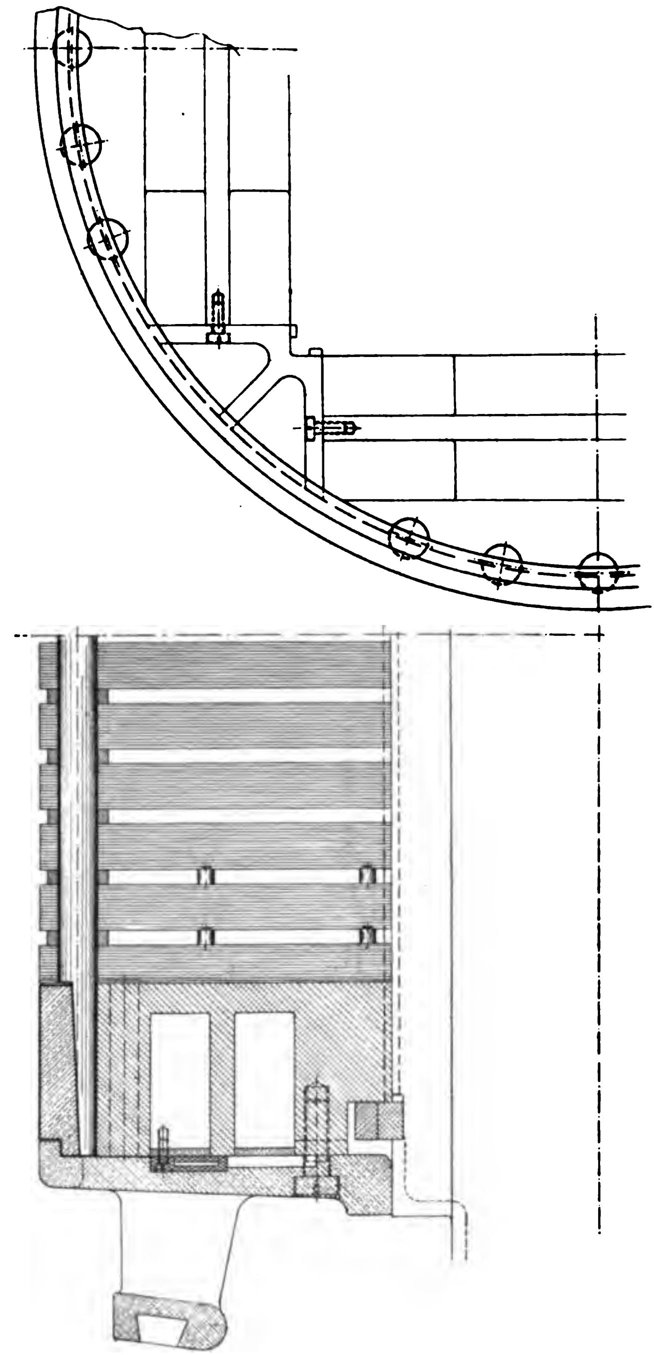


FIG. 1

single-phase generator with stationary laminated field, working under short-circuit. The two stator phases not used for excitation acted as "dampers" when short-circuited. By opening these phases or closing them through the variable resistance it was possible to work the generator with perfectly laminated field or with dampers of variable resistance. The losses were determined by the energy consumed by the direct-current motor. The curve, Fig. 1, was taken with constant armature current. It shows clearly that with single-phase load the hysteresis loss in a perfectly laminated field is much larger than the total hysteresis and eddy-current loss with low-resistance dampers, We therefore used heavy dampers in all single-phase generators,

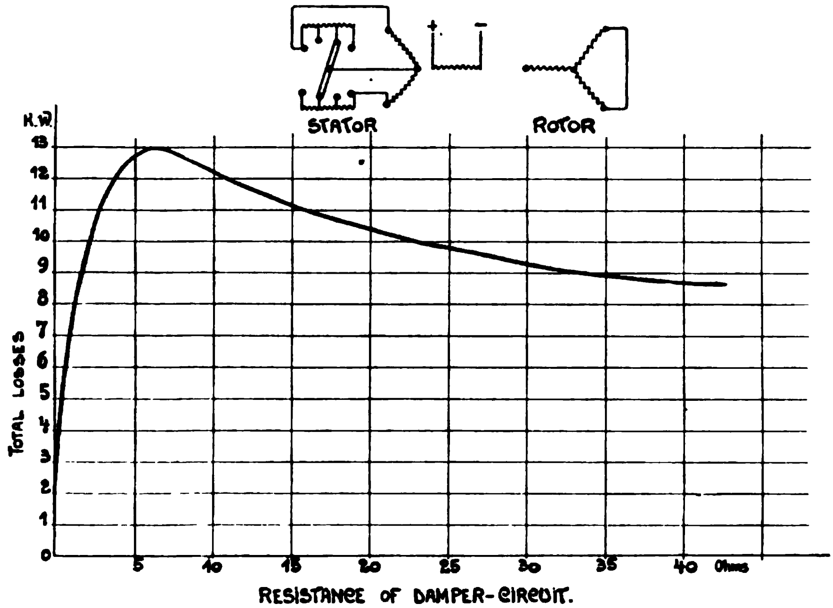


FIG. 2

for which we adopted the design shown in the sketch, Fig. 2, which has given full satisfaction.

F. H. Clough (by letter): I have read Mr. Waters's paper with considerable interest, as probably most of us who have attempted the design of single-phase generators for coupling to steam turbines have experienced the difficulties described and have endeavored to eliminate the deleterious effects of single-phase armature reaction. As the author suggests, the simplest way of looking at the matter is to regard the armature magnetization as consisting of two components, each of half the maximum value and revolving in opposite directions.

If the rotor is provided with a complete squirrel-cage winding, the machine can then be regarded as a single-phase squirrel-cage induction motor. The currents in the rotor bars under these conditions are such that taken in conjunction with the armature reaction of the stator a uniformly rotating field is produced. If the machine be assumed to be running at synchronous speed, the rotor will be stationary with regard to the field, and, therefore, an additional exciting winding supplied with continuous current can be placed on the rotor. By this means we come to the single-phase alternator as described in the paper.

The company with which I am associated recently built a 1,000-kw. generator of this type, and its operation has been entirely satisfactory. The squirrel-cage winding consists of copper bars carried in slots round the periphery of the rotor and short-circuited at their ends by the brass rings used for retaining the exciting winding. By this means a very compact structure is obtained.

Before building this machine, some experiments were made

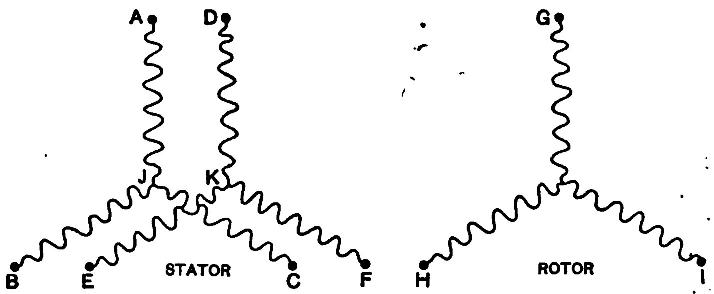


Fig.1

with a slip-ring type of induction motor, a description of which may be of interest. The stator of this motor had been wound with two conductors in parallel in the same slot, and for the purposes of this test these two windings on the stator were kept entirely separate. Fig. 1 shows the arrangement of the windings. Exciting current was supplied through the winding *A J B* and single-phase current taken from the two slip-rings of the rotor *G* and *H*. The other winding *D E F* of the stator was short-circuited through ammeters. The short-circuit rotor current produced for a given excitation with and without the winding *D E F* closed is shown in Fig. 2. In Fig. 3 is shown the single-phase short-circuit core loss with and without this winding short-circuited. The short-circuit current in winding *D E F* agreed very closely with the value expected, and consequently allowed the size of the squirrel-cage winding on the main alternator to be accurately determined.

I agree with the author that this squirrel-cage winding forms a complete solution to the troubles experienced with single-phase turbo-generators. In addition to the reduction in the core loss, it will be noticed from Fig. 2 that the reactance has been reduced to a very great extent, so that a machine with this type of rotor—apart from the $I^2 R$ losses in the squirrel-cage winding, which are small—can be wound to give the same output three-phase or single-phase with approximately the same efficiency and regulation.

With regard to the second portion of the paper, the method of securing the stator coils, although apparently mechanically

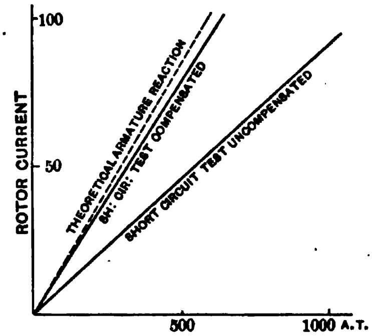


Fig.2

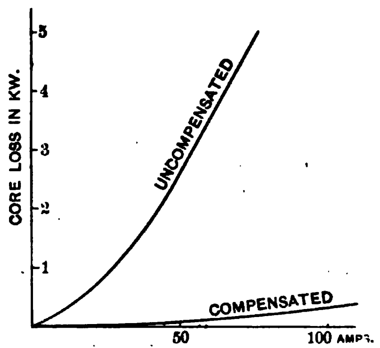


Fig.3

sound, appears to be open to two objections: first, the difficulty of ventilation, although perhaps this may not be serious; second, that the coils are set back a long way from the rotor. This location of the coils would tend to increase the self-induction of the latter considerably, as, although the medium surrounding the coils is only air, still the area of the magnetic path for a machine such as the one described by the author is very big, and it will probably be found that such a winding causes a considerably higher self-induction than a winding which is brought out in the form of a barrel closely surrounding the end connections of the rotor.

Chas. P. Steinmetz: Since the question of single-phase alternators has come up, I desire to call attention to another way of accomplishing the same result of operating a single-phase load on a big alternator, without the objectionable features of the single-phase machine. This way is by using a polyphase alternator, that is, making the machine polyphase and storing the energy of the quadrature-phase.

The arrangement discussed by Mr. Waters is essentially a squirrel-cage winding on the field poles, or an equivalent thereto, in which currents are induced by the pulsation of the armature reaction, thereby giving the quadrature magnetomotive force, that is, the polyphase armature reaction. Theoretically, to effect this result completely would require a copper cross-section in the squirrel-cage winding, or in the dampers, infinitely large compared with the armature section, as otherwise you get additional losses. If you give the dampers the same total section as the armature winding, to produce quadrature polarization, it means that you get the same (or, at any rate, not greatly differing) losses in the dampers as in the armature; in other words, you about double the $I^2 R$ loss.

This method, however, although it eliminates the pulsation of the field, and so also the losses in the field due to pulsation, does not eliminate pulsating strains on motor shaft and revolving machine, because the power is still pulsating; and where you come to very low frequencies, of fifteen to ten cycles, in very large machines, such as have been proposed, the enormous pulsation between zero and maximum power may become objectionable.

One way of eliminating this pulsation I will outline very briefly. Suppose you have a single-phase load of, say, thirty degrees lag, corresponding to average conditions of a single-phase railroad. Suppose you take a three-phase machine and put the load on one phase. Now, connect across another phase a reactive coil, to store the energy in the reactance. In the load the current lags thirty degrees behind the voltage, and the curve of power then is pulsating with double frequency, with a maximum 15 degrees behind that of the voltage. The electromotive force in the reactive coil is lagging behind the electromotive force on the load by 60 degrees, being the other phase. The current in the reactive coil lags 90 degrees behind its electromotive force, and the power of the reactive coil being pulsating (or rather alternating) with double frequency, has a maximum 45 degrees behind the voltage—a negative maximum, corresponding to power return, 45 degrees ahead of the voltage. Now, you see what happens. As the voltage on the reactive coil lags 60 degrees behind the load, the negative maximum of power in the reactive coil, 45 degrees ahead of its voltage, coincides with the power maximum of the load, which is 15 degrees behind its voltage. Superimposing then the alternating power of the reactive coil on the pulsating power of the load, you get constant power; that is, on the alternator by such a combination you can get constant uniform polyphase load with a single-phase external load, by a reactive coil displaced in phase by 60 degrees, storing energy, and returning it in the single-phase system.

If you look at the distribution of current, you will find that one generator coil carries only the load current, lagging 60 degrees behind the voltage of this coil, or Y voltage; one generator coil carries only the reactance current, also 60 degrees lagging behind the voltage of the coil; and the third coil carries the resultant of the load current, in phase with the coil, and the reactive current, lagging 120 degrees, so combining to a resultant current of 60 degrees lag. In other words, all three coils carry equal currents of equal phase displacements. In short, the machine becomes a perfect polyphase alternator, in armature reaction, in stresses, and in everything, but is storing power during a part of the cycle, to return it into the single-phase system during the other part.

This arrangement may be used occasionally, and may become necessary, if we have to come to the use of very low frequencies.

W. L. Waters: Referring to Mr. Steinmetz's comments on my paper on single-phase generators: The system of balancing the load on a single-phase generator advocated by Mr. Steinmetz was, as a matter of fact, the first method which was suggested for overcoming troubles due to heating and loss of efficiency mentioned in my paper. Mr. Steinmetz's scheme is shown in the sketch. Phase A of a three-phase generator carries the single-phase load, this load being usually that of a single-phase traction system. Phase B carries a choke-coil, which at the full voltage of the machine has a charging current equal to the single-phase rated load of the generator and a power factor of 87 per cent. As Mr. Steinmetz has shown, if the current in the choke-coil B is equal to the current on the loaded phase A, and if the current in phase A has a lag of 30 deg. behind its electromotive force, then the currents in the three phases of the generator will be equal in magnitude and in phase, that is, the load on the generator will be a perfectly balanced three-phase load. That this method of balancing the load on a single-phase generator is only of theoretical interest is at once evident when we consider that the load on such a single-phase generator varies continually, both in magnitude and in power factor. If the choke-coil B is adjusted so as to balance the generator when phase A carries full load and power factor of 87 per cent, then as soon as the load changes from this value and power factor, the machine becomes unbalanced. If the load falls off to a very low value, then the generator will be as badly unbalanced and will heat just as much as it would if running on full single-phase load without the choke-coil. Thus we can see that even if we use such a choke-coil, we would also require dampers on the pole faces when operating such a machine under the practical conditions of varying load. Such a choke-coil can only be adjusted for one particular value of the current, and can only be adjusted for one power factor—otherwise two choke-coils must be used. Thus we can see that the choke-coil would have little value in practice. It also has numerous disadvantages. As it loads all the phases of the generator, it thus decreases the latter's efficiency and raises its temperature without increasing the output in any way. The cost of such a choke-coil would be substantially equal to the cost of the generator, while the addition of dampers only increases the cost about 10 per cent. We have greater complication, more chance of breakdown, lower efficiency, and an arrangement which is only good for one particular value of the load and power factor. Thus I think it is not difficult to see why this choke-coil method was considered as being of only theoretical interest and as having no practical value compared with the simple, efficient, reliable, and universal method of applying heavy dampers to the field magnets, as described in my paper.

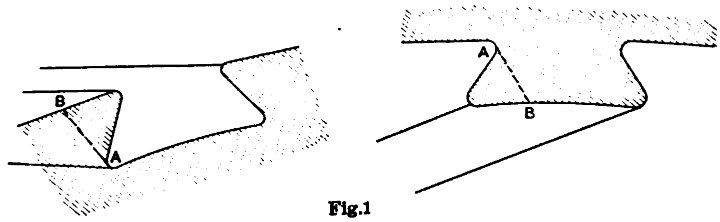


Fig.1

Mr. Behrend, begins by discussing the stresses in rotating disks and rings. The general statements made by him have been well known for the past twenty or thirty years, although there has not been any great necessity to apply them to electrical machinery, as high-speed units are only of comparatively recent date. A large number of approximate formulas have been worked out by various engineers for calculating the stresses in rotating disks and rings, most of them making various assumptions to simplify the problem. The most accurate of these formulas is the one given by Professor Ewing in his treatise on " Elasticity ". Mr. Behrend, however, evidently figured his curves given in Fig. 7 from some formula which is only roughly approximate, as they differ 10 or 15 per cent from the stresses given by the most accurate formulas.

The generator described by Mr. Behrend is a 6500-kw., 300-rev. per min., 12,000-volt, 25-cycle, water-wheel driven alternator. The speed of this generator is comparatively high for a large water-wheel driven alternator, but is of very moderate speed compared to steam-turbine driven units, as there are a number of generators of this size operating to-day at 1500 rev. per min., that is, five times the speed of the generator described by Mr. Behrend. We can therefore see that the question of designing such a generator would not mean dealing with any new problems, but would simply mean choosing the design most suitable for this particular unit from a number of existing designs which have been thoroughly tried out. Mr. Behrend omits the electrical design entirely, merely giving us the performance of the machine; but he goes into the mechanical design in detail, and therefore I think we can assume that the latter is considered exceptionally good and that criticism is invited.

The construction adopted is a center disk of forged nickel steel without a hole in the center and with two forged nickel-steel rings, one on either side, and laminated poles dovetailed into the circumference. Before an opinion can be formed as

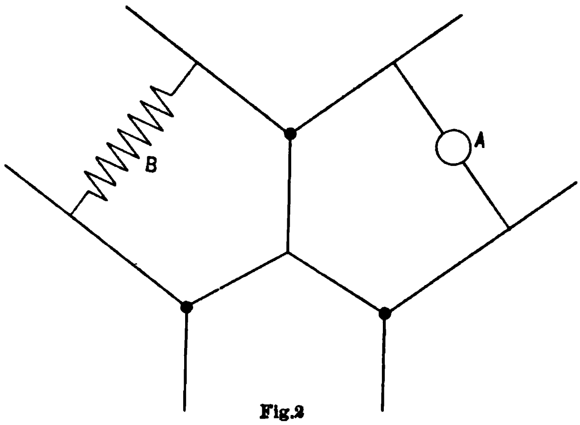


Fig.2

to the suitability of this construction, an idea must be obtained as to the distribution of stresses throughout the field structure. As the stresses figured out considerably too high for conservative practice, the precaution was taken of having them checked before any opinion was formed. Considering first the tangential stresses in the disk, Mr. Behrend's curves give approximately the stresses in a disk of uniform thickness; but taking the disk used in this generator, as shown in Fig. 5, it will be seen that its thickness is by no means uniform. The side rings are registered into, and bolted to, the center disk, evidently in order that the center disk might strengthen the side rings as much as possible, and the result of this design is a disk virtually $32\frac{1}{2}$ in. wide at the circumference and thinned down to 4 in. at the center. We see from Mr. Behrend's curves that even in a disk of uniform thickness the maximum stress in the disk is at the center, and in this machine we have 4 inches in the center of the disk carrying the maximum stress due to the 32½-in. rim, and the poles. The stress at the center of the disk at 500 rev. per min. figures out about 35,000 lb. per square inch, which is more than double the stress in the rim. From the drawing we see that the shaft is registered into the disk at the center, and that in order to accomplish this registering the thickness of the disk has been reduced to about one-third, this thinning down being done by approximately right-angle variations in the thickness of the disk. The result of this design is that the stresses as estimated above are by no means uniformly distributed, and will probably run up considerably higher than the estimated 35,000 lb. per square inch around the sharp corners in the disk. It is moderately certain that the center of the disk is stressed in parts above the elastic limit, and that the metal stretches locally to allow other parts of the disk to assist in carrying the stress. The tangential stress in the side rings figures approximately 20,000 lb. per square inch.

The above estimated stresses are all figured at 500 rev. per min. and assume that there are no original shrinkage stresses in the disk and rings. Assuming that the disk and rings are good forgings, these shrinkage stresses should not be very great, but it is extremely improbable that there are not shrinkage strains of some sort in such large forgings. Test pieces show, of course, nothing in regard to shrinkage stresses in the material and merely show what the metal will do under the best conditions. The writer has known of castings in which shrinkage stresses of 25,000 lb. per square inch have remained even after annealing, and it is quite probable that the shrinkage stresses in these nickel-steel forgings are large. Assuming that there are such shrinkage stresses, it would easily be possible for the total hoop stress at some part of the disk to be greater than the ultimate strength of the metal. Thus it is decidedly doubtful whether the disk and rings are strong enough to stand the tangential stresses due to rotation at 500 rev. per min.

Considering next the stresses in the dovetails, the bending stresses tending to break the dovetail across the section $A\ B$ can only be figured approximately. Making the most favorable possible assumptions, the stresses at 500 rev. per min. figure out about 45,000 lb. per square inch at the point A. But with a laminated pole and taper wedges, we cannot assume that the stress due to the centrifugal force of the pole is uniformly distributed along the length and width of the dovetail. Also, the overhanging end winding on the pole puts a greater stress on the corner A than it does on the center of the dovetail; and the fact that the corner A of the dovetail slot in the spider has a comparatively small radius again increases the stress, and may increase it to very high values. If these unfavorable features are neglected, the stress on the dovetail figures out as given above, but taking them into account we find that the stresses may run up to 150,000 lb. or 200,000 lb. per square inch at the same speed. Of course, the exact extent to which these unfavorable features increase the stresses cannot be decided, but it is very probable that the actual stress on the dovetails at 500 rev. per min. is above the ultimate strength of the material. All of this analysis of the stresses assumes, as in the case of the disk and rings, that there are no shrinkage stresses or local flaws in the forging. If there were such bad places, the complete wheel would not be safe even at a lower speed.

It would be difficult to bring the stresses in the dovetails down to a safe working limit; and their intensity in this machine merely confirms the general impression of engineers that the magnet construction, in which complete individual poles are dovetailed into a spider, cannot be used successfully in such large high-speed, machines. The difficulty of accurately fitting the dovetails and the difficulty in estimating the exact stresses in them make it necessary to abandon them in favor of some construction which does not possess this disadvantage.

Considering everything, I think it is extremely probable that we would find that the disk and rings had been strained locally above the elastic limit even at the normal speed of 300 rev. per min., and if we were to run the machine at this speed and then shut down and gauge the disc and rings all over we would find that there were evidences of local permanent distortion. Inasmuch as Mr. Behrend has so carefully pointed out that the maximum stresses in the disk are at the center, and since the thickness there could be increased five or six times without any change in overall dimensions, it hardly seems logical to thin down the disk to such a small section.

The design of the 6500-kw. alternator described in Mr. Behrend's paper can hardly be considered as a conservative one or one which it is desirable to duplicate, and I think Mr. Behrend is hardly justified in his statement that the design is " elegant and mechanically superior. "

Chairman Armstrong: Mr. Behrend, do you wish to reply to Mr. Waters's remarks?

B. A. Behrend: I have nothing to say.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

DOUBLE-DECK STEAM TURBINE POWER PLANTS

BY J. R. BIBBINS

In the rapid development attending the introduction of the steam turbine, there has been but comparatively little tendency to introduce radical departures from accepted engine practice. The most important changes have taken place in the boiler room, while in the rest of the plant very few changes have been inaugurated other than a general increase of compactness in the turbo-generating room. Outside of the tendency to structural foundations for generating units, there is little opportunity for originality in design without involving a radical change in the ensemble.

There are now in service three important examples of a new type —the double-deck station. This type of station embodies striking originality in its departure from long-established practice, and in a most fortunate manner reserves for the turbine the benefits of certain advantages inherent to this high-speed prime mover. And it is the object of this paper to present in a general way some of the engineering features of this new type of station which, the author believes, deserve careful consideration and ultimate acceptance as standard, at least for power sites of restricted area, as in cities, harbor frontages, hillside locations, etc. No attempt can be made at this time to make more than a cursory study of the important technical points involved, to each of which might properly be devoted an extended investigation.

The West Point station. At the outset it will be well briefly to review the most characteristic features of this station before proceeding with a discussion of the double-decked type in general; the features are:

1. Generating machinery above boilers on second floor of station.

2. Weight of generating apparatus supported entirely by steel building skeleton. All footings carried down to the same stratum—rock. 12 in. box columns 33 ft. high.

3. Floor structure continuous, tied in from wall to wall, thus reinforcing over entire structure.

4. Water-cooled floor column passing through middle of boiler setting to reduce weight and cost of beams spanning battery; all other columns straddle boiler setting.

5. Barometric condenser hung from floor girders directly under turbine exhaust nozzle. Designed for moderate vacuum; no air pumps; adjustable cones; centrifugal type circulating pump.

6. Steam piping simple and direct with few bends, draining back from turbine throttle; no superheat; separators in each supply line to turbine.

7. All important steam header and supply lines controlled by pedestal extension valves from turbine floor; non-return valves in delivery from each boiler to prevent back-flow.

8. Header serves largely as equalizer, hence small in size (8 in.); sectioned by valves; operated practically on the unit system.

9. Designed for 200 lb. pressure, high steam velocities; largest steam supply pipe in plant, 8 in., all hung from girders.

10. All auxiliaries steam-driven for feed heating; open type heater; feed taken from condenser hot well.

11. Circulating pumps interconnected for relay working, primed by steam syphons or service pump.

12. Pump suctions submerged 8 ft. in intake bay to avoid surface air; no foot-valves.

13. Two-story side addition accommodating transformers, control and protective apparatus above; heaters and auxiliaries below; also serves as transmission tower.

14. Operating switchboard located in division wall, leaving a clear rectangular operating room, and separate switch room.

15. Generators air-cooled by positive blast piped from outside to reach cooler air and to avoid the possibility of steam from boiler room reaching insulation.

16. Turbine bedplate leveled on plate girders throughout its length with cast lead pad 1 in. thick.

17. Reserve exciter units, steam-driven, exhaust to heater.

18. Transformers mounted on trucks to facilitate replacement by reserve unit provided. See Fig. 4.

19. Ash hoist to be installed from level of ash tunnel to level of siding track.

20. Crane reaches basement through removable section of flooring.

21. Self-supporting, brick lined, steel stack.

22. Flue connections direct to stack; no bends.

23. Elevated tank in turbine room to provide head for gland water and building service.

FIG. 1.—Front view, West Point station, switch room extension in rear of building.

The West Point station was designed to serve interurban railway lines through alternating-current sub-stations, and its location is reasonably near the ultimate center of the system. It is but a short distance from some coal mines owned by the company, and the site chosen, at the foot of a hill, permits the use of the impounded waters of two passing streams for condensing purposes. With this site, coal-handling machinery was avoided and barometric condensers used, these several factors combining in the form of the double-deck arrangement.

Compactness. As this is perhaps the controlling feature, it is interesting to note that the problem now lies not with the generating, but with the boiler plant. Familiar Corliss engine practice of a decade ago, involving parallel engine and boiler rooms of about equal size, has been superseded by a ratio of one to three, respectively, in turbine plants, unless the boilers be double-decked, as in the large New York plants. It might, then, seem unwise to return to a station arrangement of equal area, as in the two-story building; but it will be found that other advantages accruing in the direction of low cost and simplicity render this arrangement most effective, especially as the extra space on the turbine floor is usually occupied by direct-current converting apparatus serving nearby sections of line.

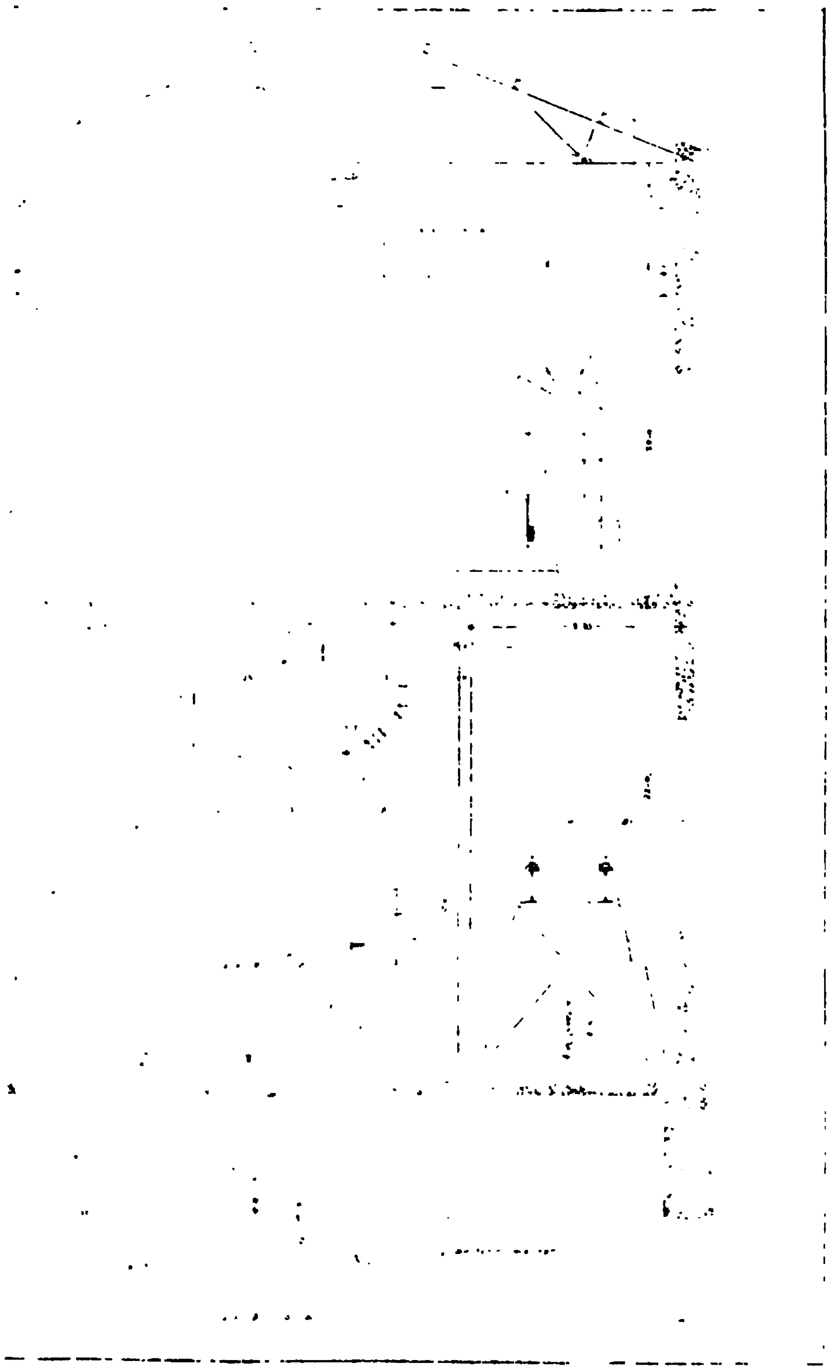
For several years there has been a continual reduction in the bulk and cost of the turbine unit. To what extent, can best be appreciated by comparison with Corliss practice.

Type of prime mover.	Over-all floor space, square feet per electric horse power.
Horizontal Corliss, 500 to 1500 e.h.p.	0.7 to 1
Vertical Corliss, 1000 to 3500	0.35 to 0.4
Horizontal vertical compound Corliss, 7000	0.46
Vertical 3-cyl. Corliss, 5000	0.2
Single-flow turbine, 1000 to 5000	0.17 to 0.75
Double-flow turbine, 15,000	0.05

Thus, in large sizes, the turbine has reduced floor areas to about 20 per cent. of that required by the modern vertical Corliss engine, and to about 10 per cent. of the horizontal-vertical type. A detailed comparison between single-flow and double-flow types follows:

Unit size. kw.	Type.	Floor space required per sq. ft. per kw.	per e.h.p.
1000	Single-flow	0.200	0.149
1500	"	0.165	0.123
2000	"	0.141	0.105
3000	"	0.101	0.075
5000	Double-flow	0.092	0.068
10000	"	0.063	0.047

Similarly in weight per kilowatt capacity. Data are not at hand for comparison with complete reciprocating units, but without generators, large vertical Corliss engines, including flywheel, weigh from 320 to 500 lb. per kilowatt, the weight increasing with the size, whereas the large turbine unit complete



weighs but a fraction, 15 to 20% of the above, and, moreover, the weight decreases with the size. Considering in addition that the horizontal turbine permits the installation of auxiliaries beneath, this extreme compactness practically eliminates it early in the study of reduced floor area.

The West Point station covers a floor area of two square feet per kilowatt. Considering the small size, this represents a high degree of compactness, relatively better than in some large turbine stations. The Ft. Wayne double-deck station covers 1.42 square feet per kilowatt, ground floor plan, the New York

FIG. 3.—General view of operating room, West Point.

Edison Waterside, No. 2, 0.818, and the Boston Edison 2.64, these latter, twenty to twenty-five times the capacity, and one with double-decked boilers.

Building and foundation construction. In spite of the relatively high speed, the construction of turbine machinery has been brought to such a state of perfection that structural foundations are permissible in certain types, which at once opens up an extensive field for originality in the arrangement of units and auxiliaries. For example, 1000-kw. units in a power house at Piedmont, W. Va., are supported entirely upon 8-inch cast iron pipe columns with units spaced 10-ft. centers, and all auxiliaries beneath. In a power house in Philadelphia, reinforced concrete posts tied into the concrete floor structure carry the entire weight of the generating plant. As the height of these columns is 20 ft. or more, it is but a step further to employ standard column sections and elevate the machines to a height sufficient to place the boilers beneath. This has been done in the three stations already referred to. At Fort Wayne the turbine floor stands 28.5 ft., and at West Point, 33 ft., above the boiler room on ground floor, by no means an excessive height.

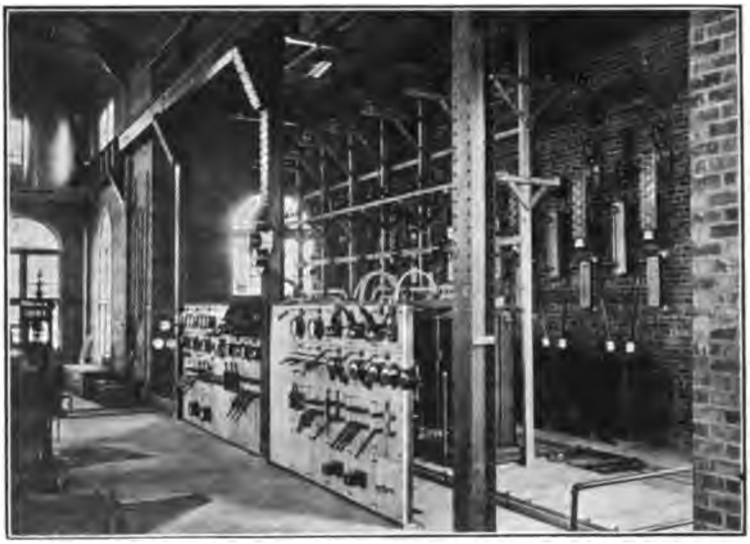


FIG. 4.—Switch room, West Point, showing extension to main operating room.

Here seems to be one of the most advantageous features of the horizontal type turbine—foundation loadings sufficiently distributed to permit of a light foundation structure involving, preferably, but two principal points of support. It might, of course, be possible to provide a supporting column under each of the three bearings; but with the possibility of unequal settling at the footings and temperature changes resulting from the proximity to boiler settings, it is evidently a much simpler solution of the problem to employ two points of support. At West Point only 12-in. box columns are required.

It is, of course, impossible entirely to concentrate the weight of the unit on these columns. A certain proportion must be carried by the building walls. It is therefore desirable to employ a steel skeleton building, carrying the machine crane and roof truss loadings independently of the brick wall. This evidently simplifies to a large degree the problem of maintaining proper turbine alignment. Although strictly necessary only for the turbine supporting structure, the steel work may be extended upward to relieve the wall loading, with little, if any, increase in cost over that of heavier building walls otherwise necessary.

At West Point, the longitudinal members between the turbines are tied together into a *continuous* floor structure from wall to wall, with the sole object of reinforcement, resulting in an exceedingly rigid structure integral with the building skeleton, which has proved to be entirely devoid of any vibration. This applies also to the steel supporting columns which show no evidences of vibration.

Girder span. In the double-deck station the problem of relative arrangement of boiler and turbine capacity resolves itself into three important factors: first, number of units required; secondly, boiler grate surface for the required rate of combustion; and thirdly, cost of supporting steel work for different spans. Assuming the first fixed by the necessity of meeting a given station load curve, and the second by standard boiler practice, there remains the problem of restricting girder span over the boiler batteries. With the floor structure designed for uniform loading between supports, the weight and cost of girders will increase approximately with the span, as shown in Fig. 5, plotted for three typical girder sizes.

A rather unusual precedent has been set at West Point, with the object of reducing the dimensions of these cross-members. Instead of a single girder, spanning the entire width of the battery, a row of columns has been located midway. The forward column falls in front of the boiler setting, the rear pierces the setting at the division wall, thus reducing the span to 14 ft. 7 in., and the steel work to 18 in. I-beams. This column must, of course, be artificially cooled. For this purpose, air spaces equal in depth to the column flanges have been provided on both sides, next the furnaces, to permit air circulation from below, and, in addition, cold water is circulated from top to bottom from the building service, with the result that these columns are the coldest in the plant. It is evident that any expansion or contraction would have no effect whatever upon the alignment of the turbine, this occurring midway between the turbine piers. By this unusual expedient, longitudinal girders have been entirely obviated, and the weight of these members reduced theoretically 50 per cent, an appreciable percentage of the whole building structure. In the Ft. Wayne station, 36-in. girders span the boiler batteries with 15-in. I-beams interlacing.

Boiler duty. The steam turbine is, of course, admirably adapted to meet the rapidly fluctuating and frequently ex-

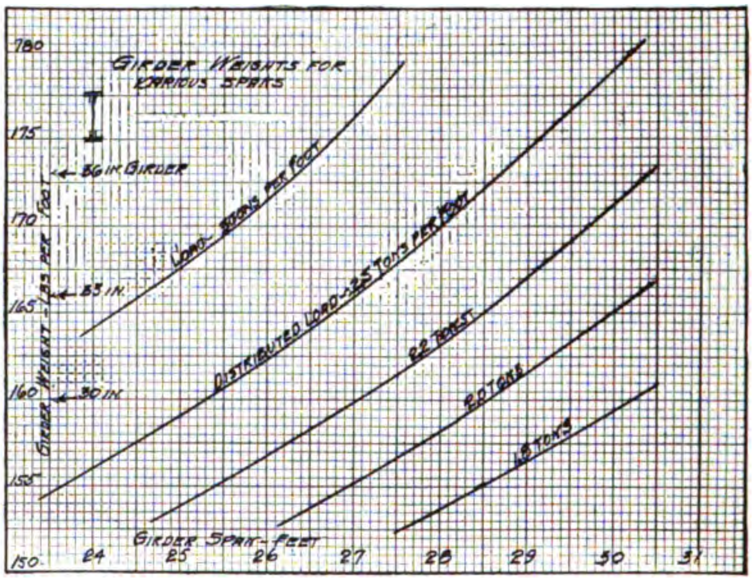


FIG. 5.—Weight of floor girders for different spans and loadings.

cessive demands of a high-speed railway system due to a comparatively small number of cars and the universal practice of arranging interurban schedules so that a number of cars start at the same time (the even or the half hour) from important points along the line. This overload capacity of 50 per cent. or more, is readily accomplished by means of secondary admission of steam to the turbine, and fortunately without decreasing its economy to any considerable degree. To meet these fluctuations, a corresponding forcing capacity is required in the boiler plant. Recent investigation and tests have revealed an important fact;

viz., that the steam boiler is capable of absorbing heat with very nearly the same efficiency, over a very great range of load. In other words, the boiler itself; that is, the type of heating surface, is a matter of little concern, except in detail proportions and arrangements of baffling, etc. This brings us to the furnace, which involves so many complexities, and upon which such a difference of opinion exists that it would scarcely be prudent to venture an opinion in this brief review of the subject. One thing is certain, however, that in order to avail ourselves of

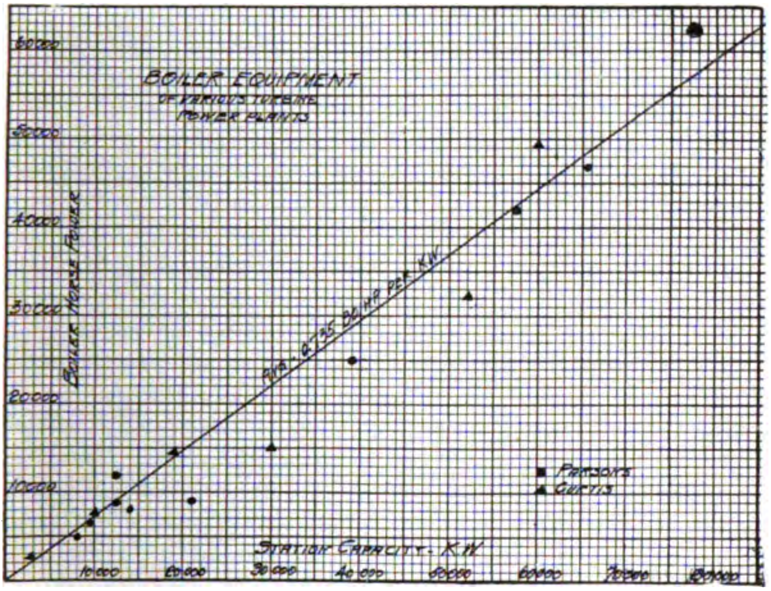


Fig. 6.—Plot representing modern practice in boiler plant equipment.

the forcing capacity of steam boilers, a type of furnace must be adopted in which a rate of combustion suited to the work in hand, may be efficiently maintained. For this reason, the mechanical stoker has proved an exceptionally fortunate addition to power plant equipment, insuring a range of generating capacity* far beyond the possibilities of hand-firing.

The high rate of combustion desired has been made possible

* See Table No. 1.—Tests conducted upon a standard type water-tube boiler with mechanical stoker fitted with some recent improvements in the form of sectional grate bars designed to promote high rates of combustion. Note that a rate of 46 lb. per sq. ft. per hour (125% overload) was maintained for six hours with less than 0.5 in. furnace draft.

chiefly by improvements in air distribution, both under the coking arch and in the grate itself. Present indications point to a combustion rate of from 20 to 25 lb. per sq. ft. per hour as a possible normal rating, whereas this formerly stood some 5 lb. lower. This performance involves one fundamental, namely, that the *effective* draft area of the furnace grate should increase with the rate of combustion—a principle quite opposed to increasing the draft pressure *alone* with the rate of combustion in order to obtain the desired quantity of air. The latter is, of course, a fundamental weakness of the hand-fired flat grate furnace, in fact any furnace in which the effective draft area is fixed. Mechanical stoking has its limitations, especially in small plants with cheap labor; but in large plants subject to severe peaks it is often indispensable.

A study of modern practice in the equipment of turbine stations shows a comparatively uniform proportion between boiler and generating capacity. Eighteen modern turbine stations for railway and lighting service average (see Fig. 6) close to one-half boiler horse power per electric horse power generating capacity, including, of course, a considerable boiler reserve.* In this connection, it is well to remember that the greater the range of boiler capacity per unit, the less the reserve necessary, the lower the investment in boiler plant, and the less space occupied. The handicap of high fixed charges to a power plant has already been called to the attention of the Institute by Messrs. Stott and Finlay. Suffice it to say that idle plant should be minimized in the double-deck station design. Several stations are below 0.75 h.p. per kilowatt. Ft. Wayne is 0.57, Yonkers and Port Morris 0.50; Carville at Newcastle, England, is only 0.44, due to liberal economizers.

Boiler construction. With one or two exceptions, boiler design has followed along well-established lines. And in the endeavor to keep pace with the great increase in capacity of prime movers, the boiler plant has increased largely in *number*, instead of in *size* of units. But to-day boiler units of 1000 to 2000 h.p. are in active demand. The limitations of the hand-fired furnace have long since been exceeded, and even those of the mechanical stoker in single units. The problem is essentially one of grate surface rather than heating surface; for this reason, the double-fired boiler has come in use. One

* Assuming one boiler out to every nine in operation, this is equivalent to an average evaporation of 22 lb. per kilowatt-hour, including steam for auxiliaries.

of this type has already been described by Mr. Finlay. Another is illustrated in Fig. 8, a moderate size unit, employed extensively in a Philadelphia traction station. Quite recently an 1100 h.p. boiler unit of the semi-vertical type has been designed with double stokers and a central combustion chamber of large proportions—a very desirable feature. In the Bow Street station, Charing Cross, London, 1100 h.p. units of an English vertical type and 800 h.p. units of the horizontal type are in operation, the latter with three banks of tubes, superheater between second and third pass. These boilers are fired on three

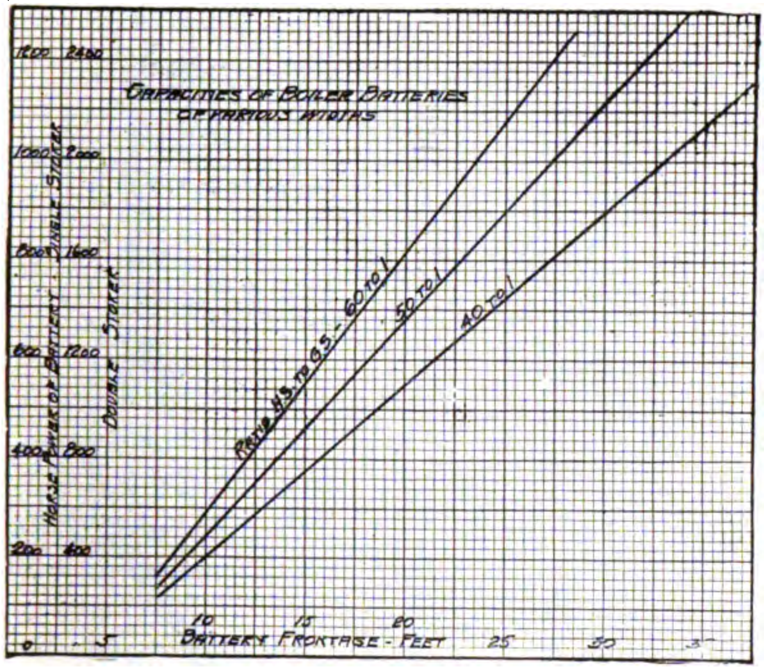


FIG. 7.—Maximum battery capacity for various widths of frontage.

sides with flat grates at a ratio of 65 to 1. Coal is delivered from an overhead bunker to both front and side furnaces and from a point somewhat below the level of the drums, and without exceeding the critical angle, 45°. In the Interborough station, coal reaches both front and rear stokers from a central bin, without incurring an excessive angle to chutes or bends beyond that encountered in any double-deck boiler plant where chutes for the lower boilers must be carried round obstructions.

In Table II the general dimensions of a number of large boiler settings are given, together with those at West Point and Ft. Wayne, for comparison. At Ft. Wayne, the stokers are served by an exterior coal bunker running along the face of the building. It is evident that any attempt to apply the double-fired boiler arrangement, would finally reduce to a question of *critical angle* and head room. With the height between floors

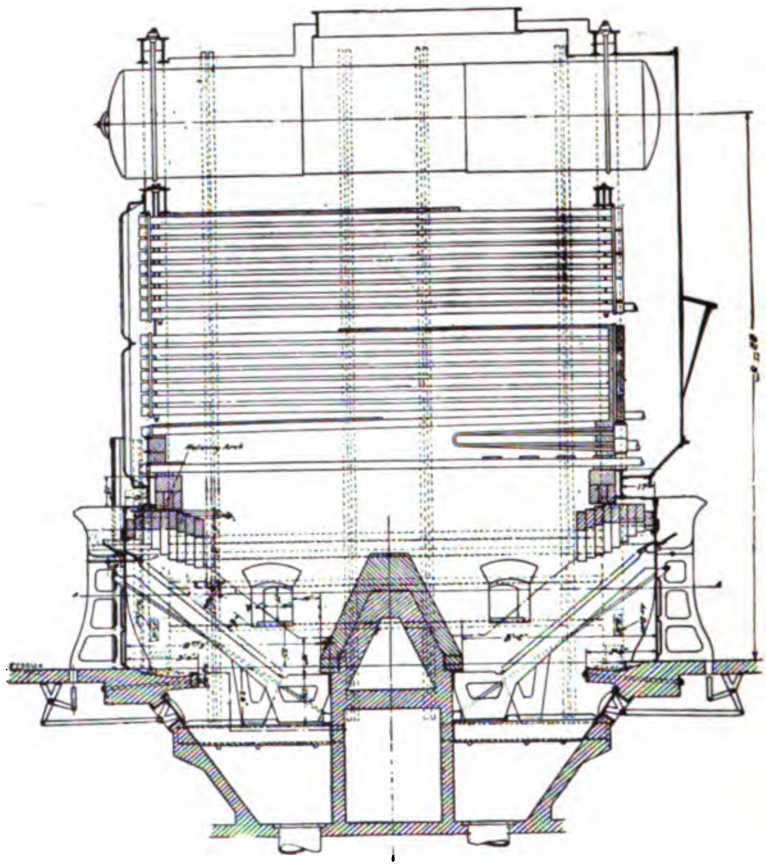


Fig. 8.—Typical setting for large double-fired boiler, 800 h.p., Philadelphia Rapid Transit Co.

available in the larger settings, it would seem possible to reach both ends of the boilers from a single side bunker, especially with a boiler setting of moderate depth, such as is shown in Fig. 8. At West Point, the boilers are set farther back than at Ft. Wayne, in order to accommodate a coal trestle forming the storage. But with an external bunker, the settings

TABLE I.—RESUME OF CAPACITY TESTS

Water-tube boilers; inclined rocking grate stoker; Pittsburg slack coal

No. of test	1	3	4	5	6	8	10	11	12
Duration, hours	7	6	8	6	7	8	9	9	6
Per cent. rating developed	117.2	158.0	179.1	224.7	150.3	130.0	153.0	191.0	134.6
Rate of combustion, lb. per sq. ft. per hr.	23.2	28.3	38.0	46.2	27.5	22.4	27.2	37.9	23.6
Furnace draft, in. water	0.14	0.27	0.295	0.424	0.24	0.21	0.31	0.30	0.22
Evap. lb. per sq. ft. heating temp. per hr.	4.05	5.45	6.18	7.74	5.18	4.48	5.28	6.59	4.65
Equiv. evap. (dry coal) lb.	8.77	9.80	8.23	8.50	9.41	10.20	9.70	8.70	9.86
Effcy. (dry coal) per cent.	66.8	74.0	61.8	63.4	70.4	75.0	73.0	64.0	72.2

Equipment: Inclined header water tube boiler, 3750 sq. ft.=375 h.p. Ratio heating to grate surface, 49 to 1, natural draft.

TABLE II.—DATA ON LARGE BOILER SETTINGS

Name of Boiler and stoker	B. & W. Roney	Parker Roney	Stirling Roney	Hornsby Hand	Hornsby Hand	Stirling Hand	B. & W. Roney
Location	Interboro, 59th St., New York	P. R. T. Phila.	Penna. R.R. Harrison, N.J.	Charing Cross London	Charing Cross London	West Point, Ohio	Ft. Wayne, Indiana
General type	Horiz. No Sup.	Horiz. Supht.	Vert. Supht.	Vert. Supht.	Horiz. Supht.	Vert. No Supht.	Horiz. Supht.
Where fired	Both ends	Both ends	Both ends	Sides and front	Sides and front	Front	Front
Boiler h.p. per battery (10 sq. ft.)	1200	1600	2200	2170	1620	800	800
Ratio grate to heating surface	30	49	53.5	64.5	65	50	48.3
Area required (sq. ft. per b.h.p.)	0.56	0.363	0.352	0.464	0.445	0.59	0.69
Frontage of setting at floor	31 ft. 0 in.	25 ft. 4 in.	31 ft. 0 in.	37 ft. 6 in.	25 ft. 8 in.	28 ft. 0 in.	25 ft. 0 in.
Depth of setting at floor	22 ft. 1 in.	22 ft. 10 in.	25 ft. 0 in.	27 ft. 0 in.	28 ft. 0 in.	16 ft. 9 in.	22 ft. 1 in.
Height to center-line of drums	19 ft. 0 in.	22 ft. 9 in.	31 ft. 3 in.	28 ft. 0 in.	27 ft. 5 in.	20 ft. 0 in.	15 ft. 3 in.
Height to top setting	22 ft. 3 in.	26 ft. 9 in.	34 ft. 0 in.	—	—	22 ft. 6 in.	17 ft. 3 in.
Height between floors	—	—	47 ft. 2 in.	—	—	33 ft. 0 in.	28 ft. 5 in.

could be located well forward with arches in the building wall communicating with the room under the bunkers. Furthermore, it would seem possible to make use of the space above the coal bunker for a switch room, as it would be approximately at the proper level.* This rearrangement permits of sufficient space in the rear of the boilers for flues overhead and pumps beneath.

In order to indicate in a general way the possibilities of the large boiler units, the author has prepared Fig. 7 showing the maximum stoker grate area that may be installed in batteries of two boilers each, with varying spans between turbine piers, presupposing, of course, that the proper heating surface may be obtained by extending the boiler upward, whether in the vertical type or with an extra bank of horizontal tubes. Thus with a ratio of 50 to 1, a frontage of 30 ft. will accommodate 2240 boiler h.p maximum, sufficient to serve a 3000 kw. turbine. This is approximated in the 2200 h.p. battery noted in Table II, which is 31 ft. over all.

Condensers. The West Point station is an excellent example of the possibilities of a compact condensing plant. The barometric condensers are *attached directly to the turbine* exhaust. Here the condensers are arranged in line with the boiler fronts, but with a passage of over 6 ft. between boiler settings, it is quite evident that the condenser may be placed wherever most convenient, irrespective of the position of the batteries, and with a discharge tunnel conveniently located.

It seems almost superfluous to emphasize the advantages of a short and direct connection to the condenser which obviates both the friction of bends and the bulk of large exhaust piping. The suspension at West Point permits of an atmospheric relief connection between the supporting girders, resulting in a distance of only about 2.5 ft. from the floor level to the condenser head. Here the barometric condenser and horizontal turbine are evidently at their best advantage, both as regards compactness and efficiency—an arrangement which cannot be duplicated by any other type of apparatus.

* In fact boiler fronts might be advanced to the building line, omitting the brick wall entirely from floor to roof, excepting the side wall of the bunker, leaving only the steel columns with a 6-ft. gallery overhanging the turbine room. Switchboards might then be located as at West Point, in line with the steel work, with all control and protective apparatus behind.

The degree of vacuum desirable appears to be a matter of opinion. With a turbine that is not unduly sensitive to vacuum, it would seem unwise to incur the additional complexity of extremely high vacuum apparatus. Indeed, for the majority of inland plants, the high temperature of circulating water available during the summer months would render of no avail the condenser plant designed for higher than 28 in. vacuum. The general preference seems to indicate a moderate vacuum of 27

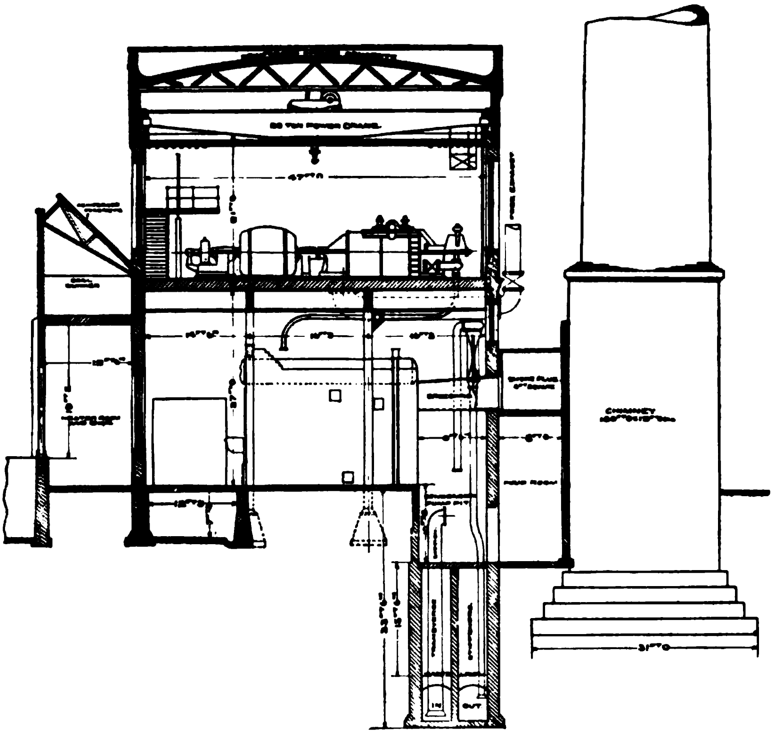


FIG. 9.—Sectional elevation, Ft. Wayne station.

to 28 in., and several condenser manufacturers are now manufacturing barometric condensers to maintain this vacuum without the use of auxiliary dry air pumps. Taking into account the three or four per cent. gain per inch of vacuum, it is indeed questionable whether it would pay to incur the additional cost and complexity of operation with comparatively cheap coal found in the Middle States. But this is always an individual engineering problem. To give some idea of the limitations of cooling water temperature, it is sufficient to note that with a ratio of 60 of water to 1 of steam, and a condenser working to within 15° of the exhaust temperature, the *maximum possible* vacuum will be 28 in. with 70° and only 27.4 in. with 80° water, not an excessive temperature for midsummer.

Auxiliary plant. The Ft. Wayne station employs direct-driven exciters at the end of the turbine shaft, each of sufficient capacity to operate two machines. These direct-connected exciters add 2.6 per cent. to the cost of the main unit, but as compared with the cost of an equivalent independent exciting plant, including reserve, they show some 20 per cent. in favor of direct drive. This subdivision evidently assures uninterrupted service to a degree almost equivalent to the standby storage battery, for with half the exciters inoperative there would still be ample capacity remaining.

On the other hand, the same degree of voltage regulation cannot be obtained except with automatic voltage regulators and the exhaust steam of exciters such as are used at West Point, is unavailable for feed heating. In spite of this, a feed temperature of 160° to 180° is obtained at Ft. Wayne and even 200° at peak loads.

The high-speed centrifugal pump for condenser injection works well into the scheme at West Point, and with syphon or other means for priming is admirably adapted to the moderate head to be pumped against. Moreover it is considerably more compact and delivers a constant injection, which cannot be said of the long stroke, slow-speed reciprocating pump unless an air chamber of ample proportions be provided to prevent "dip" in vacuum to tide over dead center. As entrained air is the greatest deterrent to high vacuum, it is important to submerge the intake well below surface water, as at West Point, to avoid the aeration existing at the surface of the intake crib.

The question of auxiliary drive is important. Recently it developed during a test that a 1000 kw., well equipped turbine station was using on the average nearly 24 per cent. of the total steam in auxiliaries. A common figure is 10 to 12 per cent. for plants of 1000 to 2000 kw. Hence the necessity of moderately economical auxiliaries. At one turbine station in Philadelphia the condenser discharge pump is driven by a belt from the turbine shaft. This, however, would not be applicable in the double-deck station. At Newcastle, England, all auxiliaries are motor-driven, economizers being used to heat the feed water. It is evidently a nice engineering problem to find a happy medium between excessively wasteful auxiliaries, and low feed temperatures.

Ventilation. In the double-deck station it is important to draw ventilating air from the outside to avoid not only the heat above the boilers, but also the possibility of escaping steam reaching the generator insulation. Some little difficulty here arises in avoiding structural work, but hardly to any greater degree than in the ordinary station. At West Point, one vertical and one horizontal offset to the ventilating ducts suffice to clear both columns and girders. At Ft. Wayne, a rectangular steel duct extends from one end of the building to the other in front of the boilers, drawing air from either end, according to the prevailing winds. Each generator then draws from this air header through comparatively short and direct connections. An auxiliary steam-driven blower in the adjoining area way has also been installed to assist in delivering more air to the generators during very hot weather and peak loads. With this provision against excessive temperature rises in the generators, even under the worst conditions of atmosphere and load, there follows the maximum security against interruptions and the possibility of tiding over emergencies.

Piping. The avoidance of long runs and bends in steam piping is, of course, fully as important, if not more so, than in exhaust piping, and with boilers beneath turbines a most simple and desirable arrangement is possible. In the Ft. Wayne station the steam has but 30 to 35 ft. to travel between any turbine throttle and the boilers supplying this particular machine, and with only two long-sweep bends intervening. Moreover, piping may be most readily drained—a point of value, even with superheat. At Ft. Wayne, there is a continuous slope to the boilers. At West Point, where no superheaters are used, the header is drained by steam traps, separators being provided in each turbine line as an additional precaution.

Under normal working conditions, therefore, the plant operates almost entirely on the *unit system*, with the interconnected header properly sectioned to isolate trouble in emergencies. In both stations each turbine line connects to the header close to the feeders from the boilers supplying it, so that the header operates simply as an equalizer between the various points of demand. Owing to the directness of the piping, comparatively high steam velocities and small pipe sizes may be employed, thus reducing radiation without serious drop in pressure. The great simplicity of this piping arrangement is well attested by the fact that at Ft. Wayne the cost, complete, from boiler nozzle to turbine throttle, was only $2.50 per kilowatt.

Switch room. There is an excellent building arrangement at West Point to provide for switch room and auxiliaries. An addition to the rear of the main building accommodates the electrical apparatus above, and the feed pumps, circulating pumps, and heater below. This addition leaves a rectangular engine and boiler room of minimum proportions, with centralized auxiliary and electrical control plants, permitting the in-

FIG. 10.—Ft. Wayne station, showing coil handling and storage arrangements.

stallation of the switchboard in a most convenient position between the building columns, with all the high-tension apparatus in the rear, including high-tension bus-bars, circuit-breakers, and protective apparatus with transmission line outlets near the roof. In the 33 ft. between floors, there is space for a mezzanine floor or gallery for some of the small auxiliaries, but this has not been made use of at West Point. To be sure an elevated switchboard might have been used as at Ft. Wayne, but the extra room required for heaters and auxiliaries, would necessarily have been provided in the main building at the cost of extra ground floor space.

Summary. We have touched briefly upon certain features of the double-deck type power station, which constitutes the most important factors in working out a successful design. Inasmuch as many of the refinements of modern power stations are absent, the highest operating efficiency could scarcely be looked for, and in the three stations already built *simplicity* has perhaps been the controlling factor, leading to minimum labor cost. On the other hand, non-indulgence in refinement has greatly reduced the first cost of the complete station. Incidentally, lower fixed charges are incurred against operation, also a somewhat lower rate of depreciation. The general result is a tendency to make the total cost of operation lower than in a more efficient plant with correspondingly high investment.

The Ft. Wayne station is an excellent example of this principle. Based upon the present cost, the power station when completed—8500 kw. in capacity—will have cost $66.25 per kw. exclusive of all sub-station apparatus, property, and siding. For the present uncompleted plant, the cost per kilowatt is, of course, higher—around $75 per kilowatt, including sub-station

TABLE III.

COST OF COMPLETED POWER STATION, 8500 KW., NO SUB-STATION APPARATUS.

	$	$ per kw.
Building: Including general concrete and steel work, coal bunker, smoke flue, condenser pit, coal storage pit, etc.	93,217	10.97
Generating plant: Including turbines, generators, exciters, cables, switchboards, transformers, and ventilating ducts	259,711	30.55
Boiler plant: Including boilers, superheaters, stokers, piping, pumps, heaters, settings, breechings, and tank	118,313	13.92
Condenser plant: Including condensers, pumps, piping, free exhausts, water tunnels, and intake screen	33,790	3.98
Coal handling plant: Including gantry crane, crusher motors and track	7,990	0.94
Erection, superintendence and engineering	50,500	5.94
Total, excluding property and siding	563,520	66.25

equipment. To show the distributed cost, Table III is quoted from a former paper. To many, these costs will appear exceptionally low, but they are correct as far as it is possible to segregate the various items from the numerous vouchers. This at least will serve to show the tendency toward low station costs, resulting from simplicity of design. In smaller stations, like that at West Point, such low cost would not, of course, be expected; in larger ones it should be even lower.

The price of coal and labor is in all cases a controling factor. With cheap coal, high vacuum and superheat, high-efficiency auxiliaries and economizers are evidently non-essential. But with coal above $3.00, and with city labor, conditions might at least justify the use of superheaters, and amplified condensing plant, with rotative dry-air pumps. The internal type superheater is now a simple and durable apparatus for moderate temperatures when protected by sufficient steaming surface; that is, installed between first and second pass of tubes. There is at present a tendency in boiler practice toward the addition of so-called economizer surface to the boilers themselves, rather than in the form of external economizers.

For the average power plant, the more moderate conditions of operation appear to be in favor—150 to 175 lb. pressure, 75° to 125° superheat, and 27 to 28 in. vacuum. The examples we have studied show that an extremely simple plant design embodying these features, is possible. That future development will result in still further improvement, goes without saying. Much credit is due the designers of these present stations for their courage and initiative in testing out the possibilities of the double-deck type.

DISCUSSION ON "DOUBLE-DECK STEAM TURBINE POWER PLANTS." ATLANTIC CITY, N. J., JULY 1, 1908

C. W. Ricker: The additional data which I have to present should properly have been included in Mr. Bibbins' paper, but as an analysis of all the vouchers had to be made to determine these data, which could not be done in time, they will be offered as discussion as briefly as possible. The data are as follows:

YOUNGSTOWN & OHIO R. R.R.—WEST POINT POWER STATION
Analysis of Cost of Completed Plant, 3000 kw.

Building and fixtures,	*Cost*	*Per kw.*
Foundation, general excavation, and concrete work including condenser wells and overflows and ash tunnel; steel frame and building superstructure, ash-handling apparatus, coal trestle, chimney, smoke flue, and crane	$64,204	$21.40
Boiler plant,		
Six 400 h.p., No. 200 water-tube boilers, settings, furnaces, pumps, heater, piping and covering	42,726	14.24
Condenser plant,		
Three barometric ejector-type condensers with separate centrifugal pumps, water intake and dam including deepening channels	19,224	6.41
Generating plant,		
Three 1000-kw., three-phase, 25-cycle, 400-volt turbo-generators. Six 375-kw., 22,000-volt step-up transformers, duplicate exciters, switching, and protective apparatus	112,764	37.59
General expense,		
This includes items not susceptible of distribution and part of the expense of supervision	7,252	2.42
Complete	$246,170	$82.06
Sub-Station equipment in power station.		
Two 300-kw. synchronous converters with five-panel switchboard	12,600	4.20
Total	$258,770	$86.26

This, of course, is a comparatively small station and the items of general character, mainly in the building and fixtures, are necessarily larger in proportion than they would be in a station of greater capacity. These costs were also increased materially by the position of the station, which is some seven miles from the nearest steam railroad station at which freight could be received. The entire building frame and steel work, and all of the cement and sand used in the foundation and water and ash tunnels, and the machinery and tools used in preparing and erecting these items, had to be hauled over seven miles of bad country roads with steep grades at an expense of from $2 to $4 per ton, which amounts to a considerable item.

The station building has a complete steel frame, which was erected on the foundations, before the brick work was begun. The accompanying photograph shows the completed frame.

To show the steadiness of the station, a photograph was taken by the Cooper-Hewitt lamps used for general lighting.

FIG. 1—West Point power station steel frame.

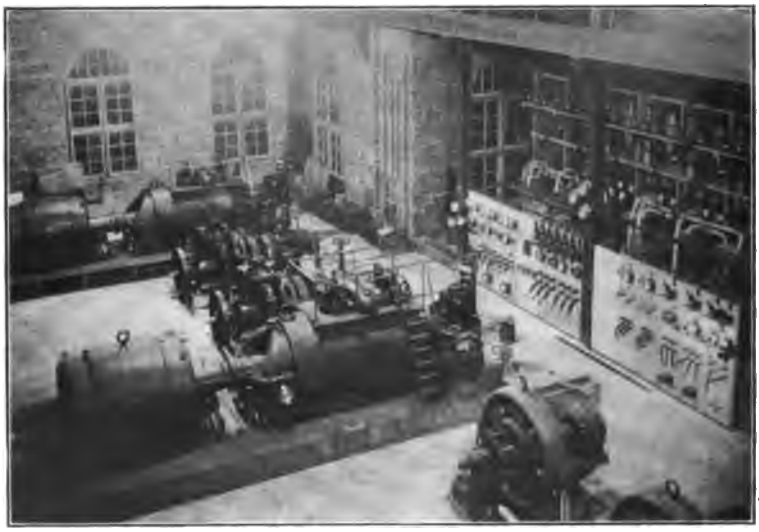


FIG. 2—West Point power station from top of crane.

A portable box camera mounted on a light tripod was set up on top of the crane, which is about 33 ft. above the station floor, in a position to experience the maximum vibration. A two-minute exposure was taken, and a perfectly sharp, clear picture resulted, which seems to show a degree of steadiness at least entirely sufficient for operating requirements.

J. R. Bibbins: I am exceedingly gratified in Mr. Ricker's presentation of the cost of the West Point station. The total figure he has quoted, approximately $82 per kilowatt for the plant complete, corroborates the figures noted in the paper for the cost of the Ft. Wayne station complete, $66.25 per kilowatt. Considering the largely increased expense in transporting erecting and building materials to a comparatively inaccessible location, it is safe to say that a station of this kind could be duplicated in a city like Ft. Wayne for even lower cost. For a complete power station of high-grade construction throughout and only 3000 kw. capacity to total as low as $82 per kw., emphasizes the principal theme in this study of power station construction. With these two excellent examples before us, the double-decked type station should, in future station work, evidently receive the thoughtful consideration of every power plant designer.

This type is applicable not only to the turbine, but also to producer gas plants of considerable capacity where the relative location of power machinery may be reversed to good advantage —the engines upon the ground floor and producers, scrubbers, and all auxiliaries on the second floor. . Recently a 16,000-kw. plant was designed for a large city railway system, in which the ground floor area was only 2.25 sq. ft. per kilowatt, accommodating eight 2000 kw., tandem, horizontal gas-engine units and 24,000 h.p. in producers, all in a building 336 ft. long, 107 ft. wide and 82 ft. high, to the lower chords of the roof trusses. In this type of plant the building structure is comparatively light by reason of exclusively static loading and the absence of extremely heavy parts in producer-gas apparatus.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

WORKING RESULTS, GAS-ELECTRIC POWER PLANT

REPORT OF 30-DAY TEST ON SERVICE PLANT, RICHMOND WORKS AMERICAN LOCOMOTIVE CO., RICHMOND, VA.

BY J. R. BIBBINS

In this paper an attempt is made to present a general study of a problem upon which only very limited data seem to be available—power generation by gas-engine-driven plants, its commercial efficiency, and ultimate cost. It seems best to approach the subject from the operating standpoint, basing calculations upon actual working results from a specific case, one of demonstrated fact. The Richmond tests* are conclusive in this respect, and although somewhat limited in scope (being only a commercial test), nevertheless provide a starting point for further deductions. While of a somewhat cursory nature, it is hoped the data presented will serve at least to emphasize the wide field of the modern gas power plant.

The Richmond plant. The equipment consists of a 23.5 by 33 in. horizontal, tandem gas engine, with a direct connected direct-current generator, operating on producer gas generated by a pair of 9-ft. (shells) bituminous producers. The gas is purified by means of wooden slat scrubbers and centrifugal tar extractor, motor-driven. A 15,000 cu. ft. holder serves to equalize its quality and to start the engine, which is necessary to bring the motor-driven auxiliaries into service. The engine is of the modern double-acting type, giving two impulses per revolution, and is governed by a sensitive oil relay system designed to relieve the governor of all valve work. Employing the *constant qual-*

* Presented by courtesy of Mr. Wm. Dalton, chief engineer, American Locomotive Co.

ity method, the ratio of air to gas is constant except when the gas itself changes, for which event hand-regulating dampers are provided. Duplicate igniters work together for increased security. For the same reason duplicate sources of igniter current are available—motor-generator and lighting mains. An important feature of the producer is that it is designed for continuous operation, having a water-sealed bottom instead of a closed ash pit, to permit the removal of ash at any time. It generates its own steam, requiring no auxiliary boilers, so that the only auxiliaries required for the entire plant are a motor-driven fan, tar-extractor, and igniter set. In the aggregate these auxiliaries absorb about 5 per cent. of the station capacity.

The Richmond works employ motor-drives to a considerable extent, and for such diversified purposes that the individual demands superpose to a large degree and yield a fairly constant total. This is a condition conducive to the most efficient working of a gas power plant, and influenced to some extent the choice of this form of motive power in this particular location. Constituting the main service plant of the Richmond works, it is called upon for continuous 24-hour service, except on Sundays and holidays.

Herein lies one essential requirement for a producer plant test. Owing to the comparatively long period required for a pound of fresh coal to work through a fuel bed several feet deep, a long test is necessary for any degree of accuracy. With the rapid combustion on a boiler grate, this is a different matter, and an 8- or 10-hour test will suffice. But in producer work, 24 hours, or 48 hours, is the minimum. In this case, the test was continued for practically four weeks, half of the time on a full load run and the remaining two weeks to three-fourths and one-half load respectively, with a rate of gasification of 0.25 tons per hour. A continuous run of 223 hours' duration evidently provides an excellent guarantee of accurate results.

The test. From the great mass of data recorded during this test, it will suffice to extract only such as pertain to the operation of a plant of this nature. The test was conducted entirely by the company's engineering and operating staff, and for the purpose of determining the fulfilment of guarantees. The electrical output was measured by direct-current recording wattmeters calibrated from a laboratory standard previous to the test. The water consumption was also metered, no great accuracy being desired. The coal was weighed by scales checked from time to time by standard weights.

Samples from such weighings, accumulated during a day's run, were quartered down and sealed for analysis, Table I.

It should be noted that at full load, 312 kw., the engine was running somewhat below full rating, but the true full-load coal consumption may readily be found. For this purpose, these data have been plotted in Fig. 1 in the form of three related

TABLE I.
GENERAL RESULTS OF TEST.

Nominal Load	Full	Three-Quarters	One-Half
Length of run, hours	223	125	136
Average load, kw	312.3	228.3	159.6
Average load, computed brake h.p.	455.0	333.0	238.0
Load, per cent. engine rating	91.0	67.6	47.5
Load, per cent. generator rating	104.0	77.2	53.2
Coal gasified, lbs	115,289	54,143	47,775
Coal gasified per hr	517.0	433.0	351.0
Output, kw-hr	69,650	28,540	21,710
Lbs. coal per kw-hr	1.654	1.697	2.20
Lbs. coal per kw-hr., guaranteed	1.93	2.10	2.64
Lbs. coal per brake h.p-hr	1.14	1.31	1.56
Avg. heat value of coal, B.t.u	14,392	14,392	14,392
B.t.u. per kw-hr	23,700	27,280	31,650
B.t.u. per brake h.p-hr	16,415	18,710	21,670
Per cent. thermal efficiency, brake	15.51	13.6	11.75
Per cent. thermal efficiency, elec	14.35	12.65	10.78

Coal.—Pocohontas run-of-mine; avg. heat value dry sample, 14,703, as fired, 14,392; volatile matter, 22.8%, ash 4.5%, sulphur 1%.

Test.—August 12, 7 a.m., to September 7, 12 m.

curves: (a) Rate of gasification in pounds per hour; (b) Pounds per unit of output per hour, and (c) Corresponding thermal efficiency. The first mentioned curve is important in all analyses of steam or gas engines to determine the rationality of the results, for the line of heat input to the engine as well as to the producer should be practically straight, as developed during the test of a similar engine at the works of The Norton Company.* The second curve, showing the relative fuel

* Transactions A. S. M. E., December, 1907.

economy at variable loads, necessarily follows the form of an equilateral hyperbola, being derived from the line of total coal input. Finally, the third curve, efficiency, represents that part of the heat in the coal that has been converted into useful work. This is absolute or kinetic efficiency, and not akin to the "efficiency ratio" employed in steam-engine practice. It covers all losses between coal pile and switchboard.† From the curve, Fig. 1, the gross coal consumption of the plant at full engine load, is 550 lb. per hr., or 1.59 lb. per kw-hr., equivalent to approximately 1.09 lb. per brake h.p-hr. And with 400-kw.

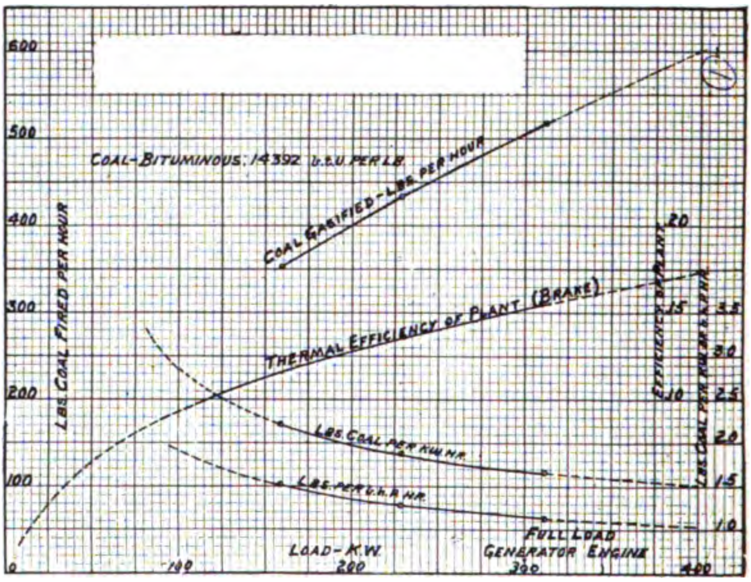


FIG. 1.—Gas power plant economy at various loads.

load, which is easily within the limits of this particular plant, the coal consumption would be roughly 1.5 lb. per kw-hr., or 1.0 per brake h.p-hr.

This characteristic of constantly increasing economy shown by the gas engine plant up to the point of maximum load illustrates an essential difference from steam engine economy which

† Taking the mechanical equivalent of heat as 776 B.t.u., the thermal value of one horse power is 2545 B.t.u., or of a kilowatt, 3412 B.t.u. Hence, absolute thermal efficiency = 2545 ÷ B.t.u. per b.h.p., per hour, or upon an electrical basis, 3412 ÷ B.t.u. per kw-hr.

usually is best at loads 70 to 80 per cent. rating, according to the cylinder ratios employed. Although the gas economy curve is quite flat beyond half-load, yet the plant evidently does its best work well loaded. By projecting backward the line of total coal gasified, we find that it requires over one-third of the full-load coal to run the plant unloaded. This represents a standing charge against its operation, becoming less and less important as the load increases. Barring this constant loss, this plant would be capable of generating power at the rate of one pound of coal per kilowatt-hour, which rate would, moreover, be *constant at all loads*. In other words, additional load might be acquired at this rate by providing for the constant losses as a fixed charge. This method is actually employed by some central stations in determining power rates.

During the four weeks' test, there were several days during which the plant was allowed to stand idle, as the works are not operated Sundays. During this period, some coal was required to keep the producer fires at their normal level and in good condition. It would, of course, have been possible to starve the fires during standby periods, but this would simply have resulted in a higher coal consumption during the succeeding run. According to the judgment of the producer operator, an average of 1700 lb. of coal were required to compensate for standby losses from 6 p.m. Saturday until 7 a.m. Monday, a period of 37 hours. Herein lies a remarkable feature of the producer plant—the low rate of standby losses, averaging only 46 lb. per hour, or 2.5 per cent. of the weekly coal consumption.

Carrying this analysis one step further, Fig. 2 shows a graphical method of determining the producer plant efficiency. Here are plotted three economy tests and the standby loss. Not knowing the heat input to the engine at various loads by actual test, the guaranteed efficiencies have been plotted extending the line backward to a point of zero input. This corresponds to 166 kw. total losses in the engine unit. Assuming that the producer is maintained ready for operation, there are four points to determine the curve or line of producer input. And the ratio between engine and plant input gives directly the gross efficiency of the gas generating plant from coal pile to engine throttle. This curve shows that the producer maintains remarkably uniform efficiency throughout the range of plant load. This results from the fact that at zero output of station the producer is operating at over one-third its full-load rate.

A careful measure of oil consumption was made during the two weeks' run at full load, comprising 223 hours' operation, also the water consumption of the engine. During this period, the average rate of cylinder oil consumption was 0.09 gal. per hour, or, on the basis of a ten-hour working day, 4.9 gal. per week. This confirms to a considerable degree, the experience at the Norton works, where an engine of the same size uses 3.5 to 4 gal. per week, ten-hour day—a rate so low as to have excited

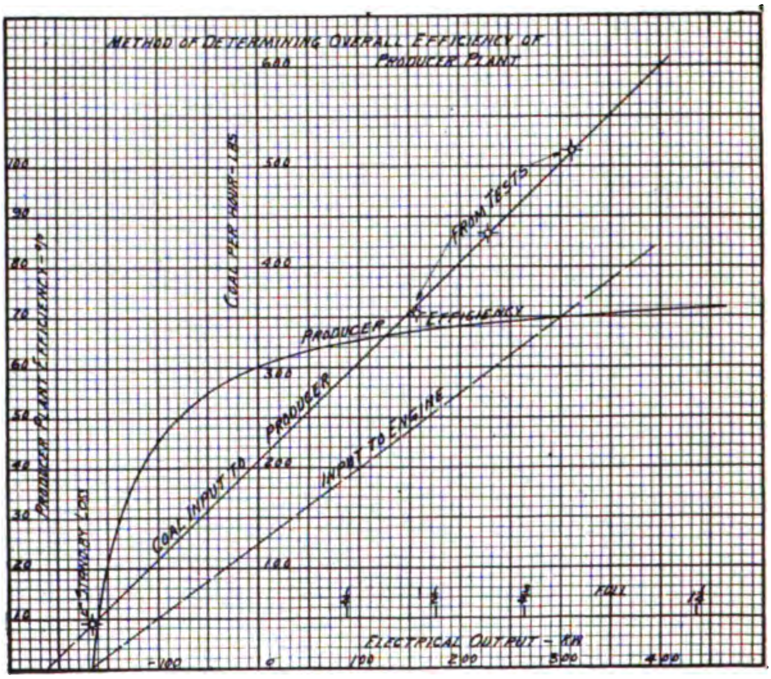


FIG. 2.—Graphical method of determining over-all efficiency of producer plant.

suspicion of its correctness. The high economy of oil is, however, due to the system of timed, forced circulation employed.

The average quantity of cooling water used was 6.56 gal. per boiler h.p-hr. at heavy load with inlet temperature varying from 75 to 80° fahr. and outlet from 140 to 150° fahr. This quantity is quite reasonable for the temperature rise. Inasmuch as a definite quantity of heat must be removed from the engine at a given load, the volume of water necessary evidently varies with the rise, so that in winter even less cooling water would be required. Economy of water may be carried to any reasonable point provided the outlet temperature of any part does not exceed 150° fahr. The upper limit only is important.

Operating results. It seems to be the general impression that a gas-power plant must be handled gingerly in order to obtain good results. The accompanying operating data from Richmond should dispel this impression. Fig. 3 shows typical results for two months' operation, representing, if anything, an output somewhat below normal. It is worth while to note the continuous operation throughout the week, 24 hours per day, and particularly the high average load sustained. The normal run at Richmond is six days without stopping. In fact the pro-

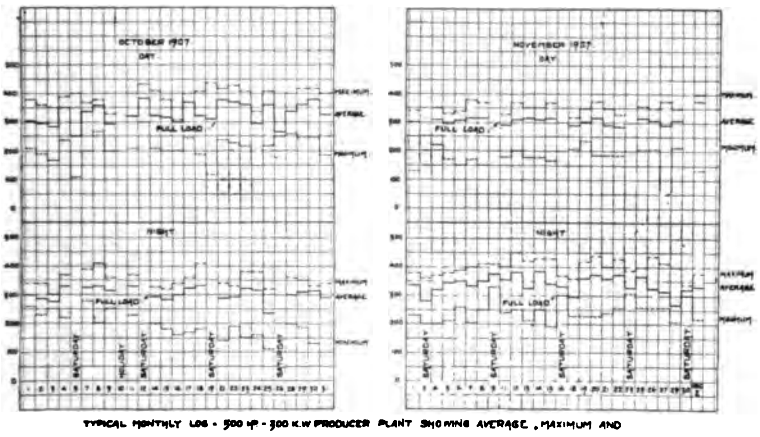


FIG. 3.—Electrical log for two months, showing average maximum and minimum loads carried.

ducer fires remained undisturbed from July 4, 1907, until early in the present year, when an opportunity occurred to make careful inspection of the entire equipment.

The plant has sustained a load of 410 kw. for three hours, 19 per cent. overload on the engines, and even higher overloads for short periods. With rich gas, the power of the engine is correspondingly increased in a manner exactly analogous to the effect of variation in boiler pressure in a steam plant. How far this conservative practice may be carried depends upon the operator; it is for him to decide the limitations of his plant.

Operating costs. From the preceding data and the other items of cost, the cost of power may be estimated at various load-factors. Owing to the fact that the present power plant was

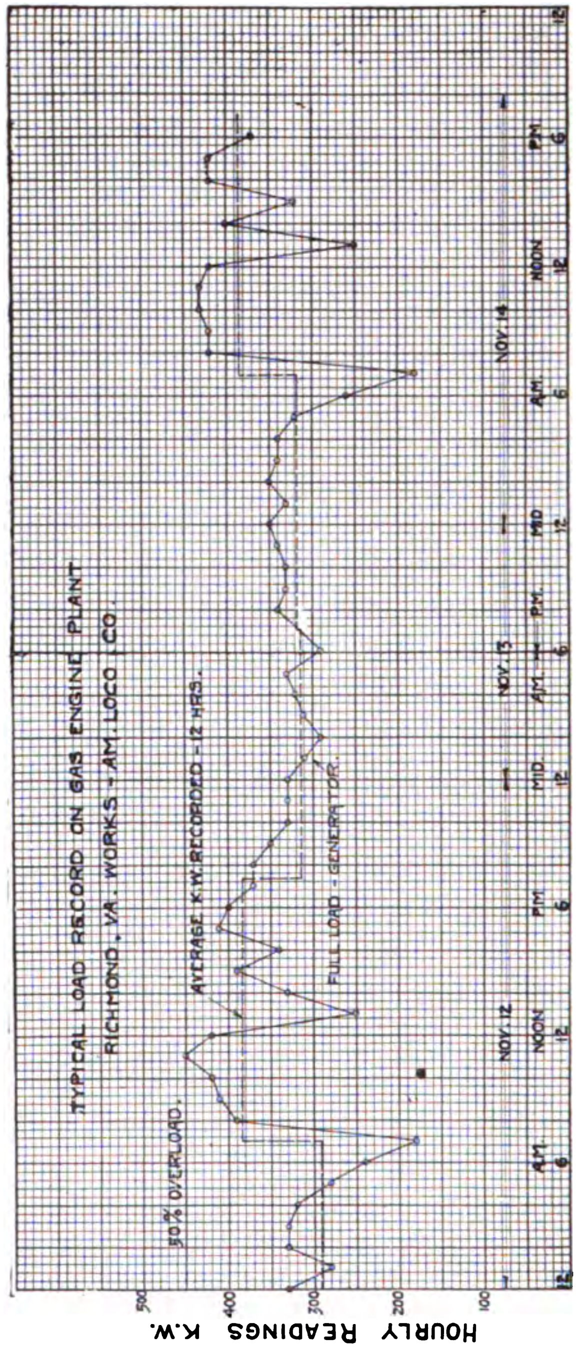


FIG. 3A.—Typical hourly load record, showing overload capacity of plant.

made to accommodate a two-unit plant, the cost has been computed upon this basis; and in view of the excellent results that have been obtained the electrical rating of the plant has been placed at 700 kw., or 1000 h.p., at which rating the investment cost becomes about $138 per kw., or $96 per brake h.p. complete, including machinery, buildings, foundations,

TABLE II.

COST OF POWER, 700 KW. GAS POWER PLANT.

ASSUMPTIONS.

Equipment cost:

Building and machinery $96,600

Cost per kilowatt, $138, per brake h.p., $96.60.

Fixed charges:

Interest 5%, taxes and insurance 1.5%, depreciation (sinking fund 15 years 5%), 4.63%, running repairs 1.5% on investment. Total 12.63% per year 12,220

Operation:

300 days, 7200 hr. per year, 5,040,000 kw-hr.

Input to auxiliaries, 5.4% full, 10.8% half-load.

Standby losses, producer plant, 1600 lb. per week, 2.1% full, 3.1% half-load.

Fuel rate, full load, 1.59 lb. + 2.1% = 1.62 lb. per kw-hr.; ½-load 2.1 lb. + 3.1% = 2.17 lb. per kw-hr.

COST OF POWER, 700-KW. TURBINE PLANT.

ASSUMPTIONS.

Equipment:

Building and machinery, $100 per kilowatt $70,000

Fixed charges:

Interest 5%, taxes and insurance 1.5%; depreciation (sinking fund 16⅔ yrs. at 5%) 4%; repairs 1%; total 11.5% 8,050

Operation:

300-day year, 7200 hr.

Average water rate, full-load 21.5 lb. per kw-hr.

Average water rate, one-half load 25.5 lb. per kw-hr.

Gross evaporation, 7.5 to 8.0 lb.

Standby, banking, 10 to 15%.

Gross coal consumption, full, 2.96 lb. per kw-hr.; half, 3.9 lb. per kw-hr.

Wages and supplies—Same as gas.

piping, erection—in fact all items except the value of the land occupied. Considering the limited size of the plant, this does not represent an excessive cost, which in large plants would probably be as low as $100 to $125 per kw.

In this total cost of power are included the usual fixed charges —interest at 5 per cent., taxes and insurance 1.5 per cent. In the absence of any data on repairs (there having been none on the engine up to the present time, and very small repairs on generator and producer), this item has for convenience been included in the fixed charges at 1.5 per cent. on the plant investment. Further, depreciation has been included in the form of a *sinking fund,* based upon 15 years' life of plant at

TABLE III.

COST PER KILOWATT-HOUR AND HORSE-POWER-YEAR, GAS PLANT.

Cost items		Full load	Half load
		Cents per kw-hr.	
Coal	at $1.00	0.081	0.109
	2.00	0.162	0.217
	4.00	0.324	0.434
	6.00	0.486	0.651
Wages	per year, $6,160	0.121	0.242
Supplies	" 3,850	0.076	0.143
Fixed charges	" 12,200	0.242	0.484
Total costs—coal at $1.00		0.520	0.978
2.00		0.601	1.086
Richmond coal....2.70		0.658	1.163
4.00		0.763	1.303
6.00		0.925	1.520
		Dollars per electric h.p. year	
Equivalent power rate:			
300-day year, coal at $1.00		$27.90	$52.40
2.00		32.30	58.20
2.70		35.60	62.40
4.00		41.00	69.90
6.00		49.60	81.50
Charges for auxiliaries if motor-driven		2.7%	7.4%
Saving gas over steam:			
Coal at	1.00	—3% loss	—8.5% loss
	2.00	+8% gain	+0.9% gain
	2.70	12.9% "	4.7% "
	4.00	19.6% "	12.4% "
	6.00	33.7% "	19.0% "

5 per cent. interest. This latter method may be contrary to general practice, but considering the nature of the depreciation fund and its earning capacity, the annuity method seems most logical.

These costs are shown in Table III. Based on 300-day operation, 7200 hours per year, the fixed costs are distributed only over the operating period, not over the entire year. And, assuming the price of coal largely determined by location, the efficiencies at various loads are considered constant, irrespective of the price of coal. Both standby losses and power consumption charges have been included. As we are considering an industrial plant in which the load averages practically uniform throughout the day, it suffices to estimate power costs at full and half loads respectively. This has been done in Table III for various coal prices. Coal prices are based upon net ton.

The results show that at the price of fuel at Richmond, $2.70,

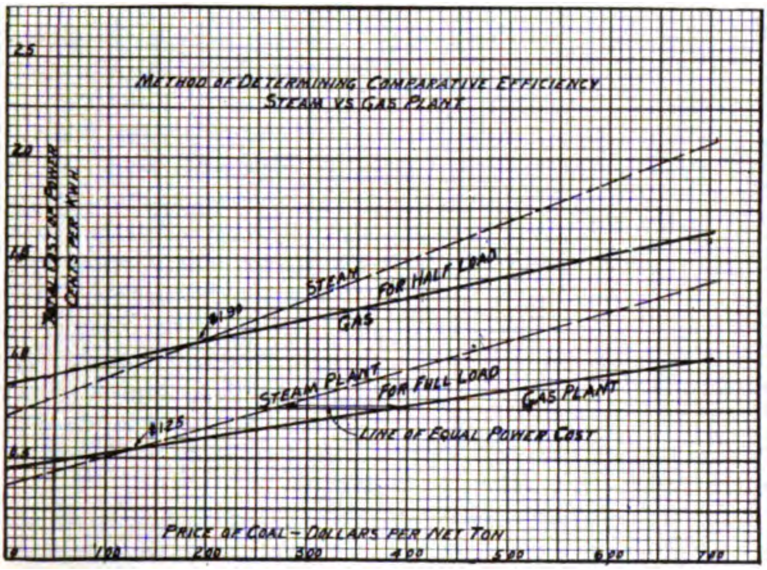


Fig. 4.—Graphical method of determining comparative cost efficiency producer gas versus steam turbine plant for Richmond conditions.

power can be delivered at switchboard at a cost of two-thirds of a cent per kilowatt-hour for a fully loaded plant operating 7200 hours per year, or less than 1.25 cents operating at half-load, and this taking into account fixed charges which range around 40 per cent. of the total cost. Even with coal as high as $6 per ton, the total cost of power would be under one cent per kilowatt-hour, obviously an excellent result.

Relative cost of gas and steam power. That there may be no misunderstanding of the preceding cost results, a comparison has been drawn up for a steam turbine plant operating under the same load. This comparison covers only two variable factors, coal cost and loading. And, while not intended to be of more than tentative interest, it will serve to show the general range of working in which the gas and the turbine plant will respectively predominate. The assumptions upon which this comparison is based, are given in Table II, the costs are embodied in Fig. 4, and the relative saving in Table III.

At the price of coal prevailing in Richmond, the gas plant shows 13 per cent. gain over steam at full load, and 5 per cent. at half load. Owing to the handicap of higher fixed charges, the turbine plant naturally shows a lower power cost with low-priced coal; that is, where freight rates are low. But even a mine operator is not justified in charging himself for power fuel *less than the prevailing market price* (f.o.b. mines). For the high-grade fuels, the mine rates are about $1.25 for run-of-mine and $0.90 for slack and screenings. Hence, in reality, this gas power system influences an extensive field. It is evident from the diagram that a less efficient prime mover than the steam turbine would show a cost-line lying not only above but at a greater inclination than the turbine line, which would bring the point of equal cost between gas and steam to a still lower price of coal, and largely increase the saving. In Fig. 4 the crossing of the steam and gas lines indicates the starting point of gas power work—$1.25 coal for a fully loaded plant, and $1.90 at half-load. Or, considering the matter in another light, with steam coal at $2.70, we could afford to pay about $4 for gas coal for the same cost of power.

On the other hand, with light loads or fluctuating loads averaging but a fraction of the generating capacity, the gas plant is evidently at a disadvantage. And for this reason the turbine plant still finds exclusive application in all service subject to extreme overloads of variations. In fact, with the extension of a large electric power system, such as at the Richmond works, it is possible that a combined gas-engine-turbine plant may best meet load requirements with the highest resultant economy, the turbine unit carrying fluctuating peaks and the gas engines the uniform load, as suggested by Mr. H. G. Stott. At present, hot gas-engine jacket-water is utilized directly for boiler feed in an auxiliary air-compressing plant, although charged for in the cost table. This arrangement would, of course, be even more applicable to the combined plant, permitting most of the auxiliaries to be motor-driven without loss of heat for feed water.

DISCUSSION ON "WORKING RESULTS FROM GAS-ELECTRIC POWER PLANT. REPORT OF 30-DAY TEST ON SERVICE PLANT, RICHMOND WORKS, AMERICAN LOCOMOTIVE COMPANY." ATLANTIC CITY, N. J., JULY 1, 1908

J. P. Jackson: I have found, as I presume all who have dealt with gas power-plants have found, that the first question to be determined is that of reliability. Is it possible to have a plant that will operate continuously with good service under all conditions, a plant that will not shut down on account of trouble of one kind or another, as gas plants have a habit of doing?

The first cost of gas plants has been referred to, and it is admittedly high. A steam plant can usually carry a heavy overload but it is difficult to equal such service with a gas plant unless the gas is exceedingly rich. If the gas is poor even less than rated load is sometimes obtained.

Flexibility and reliability of the auxiliaries are essential. Can a spark system be made reliable: a system that can be fixed, in case it gets out of order, without shutting down the whole unit? Probably, with proper duplication, that can be done. Compressed air is usually used to start gas engines. I think many gas plants have had trouble because the compressed air apparatus has not been independent, so that in case the air is lost in the tanks, they can be pumped full in a few minutes. Circulating pumps and coolers are needed, unless water is inexpensive. Such water is for cooling the engines and for use in the scrubbers and vaporizers. If a suction producer plant is under consideration, a blower is necessary to start the fire and also suitable arrangements must be made for cleaning fires, if it is desired to run twenty-four hours a day.

So far as I am able to observe, if a small gas plant is in good running order the labor is less expensive than in the case of steam plants, though in most cases, such plants being still in the experimental stage, there is apt to be extra expense which materially raises the labor account. I believe, however, that when we get the gas engine to the same point of perfection, the labor will probably be a less expensive item than in the case of the steam plant. In a plant with a large load-factor the fuel cost will be less than that in a steam plant of small unit or size. The saving in coal has to be offset against other items, however. The cost of water is important, inasmuch as a water cooler and condenser must be used.

Deterioration and repairs have been enormous items of expense, which can be eliminated to a large extent. There is no reason why a well-designed gas engine cannot maintain its brasses, instead of having them eaten out by sulphuric acid; why it cannot continue to run with about the same stability as a steam engine.

So far as making the gas goes, producers for gas purposes are undoubtedly less expensive to keep up, and they need less attention than steam boilers. They cost less originally. In gas plants there are needed gas testers that continuously record the character of the gas; these are as necessary as gauges in a steam plant to record the steam pressure. Suction gauges are scarcely sufficient.

I have spoken of troubles, because they have come forcibly to my notice. But I believe that so far as reliability of service goes, it should be possible to build a producer plant which will give just as reliable service as a steam plant, especially for small units in locations where the price of fuel is high. The annual cost, including depreciation on the higher first cost, labor, and fuel, and every other item of expense, should be in many cases less for the gas plant than for the steam plant.

J. R. Bibbins: The ability of a gas-power plant to carry overload is entirely a question for the designing engineer to decide. The maximum capacity of a gas engine is reached when the cylinder is full of mixture of maximum density; this is analogous to a steam engine taking full-stroke steam. At the present time engines of good construction are rated with a continuous overload capacity of from 10 to 15 per cent. In the plant at Richmond the engine has sustained a one-hour overload of over 30 per cent, or a three-hour overload of 19 per cent. These results were obtained with good gas, but conditions are not always so favorable, and conservative builders lower their ratings in order to cover contingencies entirely beyond their control. In fact, this particular engine has about 20 per cent excess capacity. A knowledge of future operating conditions would undoubtedly have enabled the designer of this plant to take advantage of this in electrical ratings.

The steam turbine, as built to-day in America, is exerting an embarrassing influence on the question of gas-power ratings, as American practice is extremely conservative. Compared with German practice, for example, it will be found that foreigners are much less conservative in overload ratings, as determined by temperature guarantees. If the same basis of rating existed for domestic machinery as for foreign generating machinery, even turbines, American practice in gas-engine ratings would not be so generally criticized, and the prime mover would not be called upon to compensate for deficiencies in plant equipment or operation. Standardization of gas-power practice is now being considered by the American Society of Mechanical Engineers. A committee was appointed last December, and within the next year it is probable that American practice will be well defined and placed upon a more logical basis than at present exists, where nearly every manufacturer has his own standards.

The principal object of the paper is to present some definite data on the operation of the modern plant, properly equipped and operated, data which clearly refutes the general charge of unreliability and costliness so often encountered. Such charges are usually based upon individual experience or general misinformation. There is nothing unusual about the Richmond installation—a standard type of gas engine—a simple form of bituminous producer with independent auxiliaries for tar removal. The plant is capable of continuous operation through the use of water-seals at the producers; the lubrication and cooling of the engine are automatic; the ignition system is in duplicate; and the governing fully as sensitive as in standard steam practice—2 to 2.5 per cent variation of mean speed. Although a direct-current installation in this instance, the regulation obtained from this type of engine is sufficient to permit alternating current parallel operation without the use of spring couplings.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

FROM STEAM TO ELECTRICITY ON A SINGLE TRACK ROAD

J. B. WHITEHEAD

As bearing on the general problem of conversion from steam to electricity for railroad operation, attention may be drawn to the probable influence of the methods followed and results obtained in equipping single-track roads. Of the total mileage in this country, about 60% is single track. The conditions obtaining on such roads are necessarily widely different from those on the large trunk-line sections which have been recently electrically equipped. While the several notable papers which have recently appeared on this and kindred subjects have taken quick advantage of the data available from the short periods of operation of several large installations, it cannot be said that the conditions in those cases are typical of more than a part of the entire problem.

Between the interurban road of comparatively light units and the large trunk-line installation with locomotive equipment, there is an intermediate class of steam road which, if equipped with electricity, should furnish excellent indication as to the results to be expected in the larger projects. These roads have usually single track and operate combined passenger and freight traffic over moderate distances. Unfortunately, the change in this type of road cannot often be made attractive on the ground of increased economy of operation; the transfer is usually accompanied by such an increase in fixed charges as to more than offset the economies possible under electrical operation. The road described in this paper is no exception in this respect, but, as is often the case, other considerations were deemed important enough to justify the change to electricity. The road has a long steam history which should permit excellent opportunities for comparison of results under steam and electricity. The considerations leading to the adoption of the 6600-volt, single-phase, alternating-current system are given at some length; opportunity is taken to review the authority for several engineering constants commonly accepted; and an estimate is made as to the probable effect of the change on the cost of operation.

The road. The Annapolis Short Line, a property of the Maryland Electric Railways Company, is a single-track, standard gauge steam road between Baltimore and Annapolis. Its length is 25.25 miles and in addition there is a branch 4 miles long extending south of Annapolis to Bay Ridge, a summer resort on Chesapeake Bay. The Baltimore terminal is Camden Station, the main terminal of the Baltimore & Ohio Railroad. At Annapolis the entrance is direct into the Short Line's own terminal, at which point are located its shops, roundhouse, car barns, etc. The maximum gradient is 1.5%, the longest stretch at this figure being 1.5 miles. There is one curve of 8°, one of 6°, several of 4° 30′, and many of easier figure. There are three bridges, one of them, over the Severn River being 3700 ft. long and two have draws. The rail is 80 lb.; the ballast gravel and cinder. The present normal service comprises seven trains per day, averaging three coaches each, in each direction. The approximate monthly car-mileage is 30,000. The fastest time is 45 minutes between terminals with 5 intermediate stops, a schedule speed of 33.7 miles per hour. There is one train per day of this class in each direction. The local running time is 1 hour with an average of 15 stops, a schedule speed of 25.25 miles per hour. There is one freight train each way daily and there are occasional large excursion loads.

Aside from the demands of a constant high-class patronage between the two cities, the conditions imposed by operation over more than a mile of the crowded B. & O. R.R. tracks and into its terminal, have resulted in excellent operating efficiency and schedule maintenance.

Reasons for change to electricity. The objects sought in changing to electrical operation were increased car-mileage, more frequent service, express service at least as fast, cleaner service, and the sentimental and indefinable attraction inherent in electrical operation. It is evident that the first three of these objects could be attained with much less initial expenditure by increasing the rolling stock, double tracking, installing block signals, etc. Difficulty might be met in the Baltimore terminal of the B. & O. R.R., where the number of train movements in and out attendant upon turning a locomotive and the time consumed, are already limiting factors. It would appear, however, that this difficulty also might be met by the use of double-end locomotives.

Elsewhere in this paper a comparison is made of operating expenses, maintenance, and interest charges for steam and electric operation of the present schedule. The comparison indicates that the operating and maintenance expenses alone would be about the same in the two cases; with interest charges included, however, the total of these items combined would be increased about 16% in changing from steam to electricity. While a part of this increase is due to the high price paid for power, it will be evident, notwithstanding, that the conditions are not such as to offer any prospect of increased economy of operation by electricity. Reasons are present, however, which in the eyes of the controlling interests are sufficiently cogent to more than overcome this objection, and to justify the abandonment of steam.

Owing to the fixed nature of the traffic to and from Annapolis, and to the fact that the local business is only 25% of the total, the direction in which increased business may be looked for is in the development of the region between the two cities. As the road runs for about 8 miles along the north side of Round Bay and Severn River, through high and attractive country, the prospect is unusually good and already attested by the rise in real estate values. It is believed that this development, slow in the past under steam, will be an early result of frequent electric service. Further, the aim is to develop excursion business to Bay Ridge to a degree not possible with steam. The superiority of electricity for this class of service is well known. Finally the Washington, Baltimore & Annapolis Railroad, to a certain extent a parallel line, and formerly a steam road, is also changing to electricity and is extending its tracks to city streets at each end. The distance from Annapolis to Baltimore by this route is about 15% greater than by the Short Line, and the latter has at present about 80% of the through business. The two roads are not competitive in local business, as they lie on opposite sides of the Severn River. The practical necessity for electrification on the part of the Short Line is evident. This, therefore, is one further instance in which the adoption of electricity is caused not by any prospect of better economy of operation, and not because it was more attractive in first cost than steam for increasing service. The reasons are found in special local conditions, and in the confidence that electric service will increase business to a degree not possible with steam.

Power supply. There are in Baltimore two 25-cycle, 13,200-volt, three-phase power plants. One, operating the street railways, has 35,000 kw. installed; the other supplies the larger part of the city's lighting and power and has 13,000 kw. installed. This latter plant is on the Annapolis Short Line about 1.5 miles from Baltimore. Each of these companies has an arrangement for receiving power from the Susquehanna development. Under these circumstances it was decided to purchase power. A contract, several features of which will be mentioned later, was entered into with the latter company whose plant lies on the line of the road.

Type of car. To meet the present daily passenger car mileage, single-car trains on one-half hour headway in the busier morning and afternoon hours, and on hourly headway at other times are proposed; through expresses and locals will alternate. A description of the car adopted is not necessary here, and may be had elsewhere. It may be stated, however, that it is of a type and design specially chosen to meet the demands of the required high-speed service. The center, intermediate and side sills are of steel. Wheels, trucks and couplers are according to M. C. B. standards. Train control is provided for both air brake and electric equipment. The car is of the Pullman type. The electric control is on one side of the vestibule and is enclosed at the rear end by a door, which when operating in the forward direction serves to seclude the motorman. The express car seats 62 passengers, is 55 ft. long, and the body alone weighs about 32,000 pounds. By multiple unit train control two-car trains on the above mentioned schedule will double the present car mileage. Moderate excursion loads may be handled with the equipment as installed. For possible concentrated loads steam will be relied on.

Available systems. In estimating the relative advantages of direct current and alternating current systems for the service, it was considered that the single-phase system is the only available alternating current system which has shown itself commercially successful in this country, and that the direct current system has no advantages so far as actual operating conditions are concerned. On the other hand the 1200-volt direct current system was not considered, since it does not offer a sufficient number of examples from which deductions as to operative success may be drawn. The choice of system was therefore based on a comparison of the available single-phase 25-cycle system and the 600-volt direct current system. This comparison was made from the standpoints of first cost and operating expenses; under the latter are included power consumption, station attendance and maintenance of equipment. As the physical differences in the two systems are not limited to the car and station equipments, but extend also to the distributing and return conductors, a somewhat detailed study of the effects in these two portions of both systems was made with special reference to the values of constants as given by several authorities. The constants for rail return and alternating current trolley drop vary rather widely in the literature of the subject. The following comparative figures are tnerefore not without interest. A note on the train resistance of single cars is also given.

Distributing Conductors

Direct-current rail return. Owing to indeterminate values of the resistance of the paths of currents actually passing to earth, and of increased conductivity due to rail joints, the results of tests on contacts between bonds and rails and the values of bond resistance do not lead to reliable results when applied to the calculation of the resistance of rail return. Data from existing tracks are available however and show a rather wide difference of opinion as to permissible resistance of joint. Parshall and Hobart in " Electric Railway Engineering " place the resistance of "well-bonded track" at 5% greater than calculated continuous rail. One well-known road of this country replaces a bond if it shows a resistance higher than that of 3 ft. of rail, *i.e.*, about 10% of continuous rail, and tests on its track show the bond resistance to vary between 24 inches and 14 inches, or in the neighborhood of 6% of rail. Another conspicuous road places the defective limit as high as 5 ft. or 15% of continuous rail, with rails bonded to full capacity. The Electric Railway Test Commission Reports show that for single track laid in cinders and bonded to 17% of its conductivity the measured resistance is about 36% greater than the value for continuous rail. In this case, however, the bond was outside the angle bar and 30 inches long. The measured value is less by 3% than the calculated sum of rail and bond, even with contact between rail and bond assumed as zero. This indicates leakage to earth and increased conductivity due to rail joints. To estimate the resistance of the same track with bonds 12 inches long, a calculation was made assuming the 30-inch bond as one branch of a divided circuit, the other branch being the joint plates and stray paths. The resistance of the stray path was thus determined. For this track, then, bonded to 17% of its capacity with bonds 12 inches long, the resistance would be that of continuous rail plus 11%. Bonded to 34% of equivalent copper and with bonds 12 inches long, the resistance would be that of continuous rail plus 6.6%.

In view of the above we may take as attainable values of track return with 12 inch bonds the resistance of continuous rail plus 15% when bonded to 17% of the equivalent copper; the resistance of continuous rail plus 10% when bonded to 34% of equivalent copper; and when bonded to full capacity and under best conditions of maintenance the value may be as low as that of continuous rail plus 5%.

Alternating-current distributing conductors. We may conveniently consider the effects in trolley and track at the same time. When carrying alternating current the rails are the seat of reactance as well as resistance. Numerous tests have been made for determining their values. These tests differ somewhat, as the phenomena depend largely on the current density and on the shape of section and material of the rail. Perhaps the most pretentious tests are those of the Electric Railway Test Commission at St. Louis. It is to be regretted that these tests were not made on a track which conformed more nearly to present standards and were not more carefully prepared for publication. The report on this portion of the work presents many inconsistences and omissions; and a 30-inch 2/0 bond can scarcely be said to be common in practice. There is, however, much of value in the published account of the tests.

The drop in a single track of 56-lb. rails, and with the trolley 18 ft. above the rails at 25 cycles and 200 amps. is given by the Railway Test Commission as 122 volts per mile, with a total power factor of .60. The total reactance volts are about 97.6, of which 38.6 are due to the rails and ground, (as shown in a separate test) leaving 59 reactance volts per mile due to the field between trolley and track. The current density of 200 amperes has been chosen here for comparison of values since it represents approximately the maximum alternating trolley current which will be reached on the road under investigation.

Parshall and Hobart in " Electric Railway Engineering " give the following figures on page 285.

At 25 cycles, 4/0 trolley, 20 ft. above track

Impedance volts per 100 ampere per mile, 80 lb. rails = 76.5
" " " " " " " 60 " " = 75.3

These figures are higher than those of the Railway Test Commission. This is largely due to the large value of the ratio of alternating current to direct current drop in the track, Parshall and Hobart giving 8.1 and the Test Commission Report giving 5.5 for the same weight of rail. The sections of rails, however, are different and the current density is not given by Parshall and Hobart although this ratio depends largely on current density. That these large values of this ratio do not have a greater effect on the impedance of trolley and track is due to the comparatively high ohmic resistance of the trolley. Thus in the calculation given hereafter for 80 lb. rails the total drop at 160 amperes per mile at 25 cycles is about 64 volts, the direct current drop in the trolley would be 33 volts, and in the track 3.2 volts, giving as the ratio of alternating current to direct current drop for trolley and track about 1.85. Parshall and Hobart's figures indicate the small influence of the difference in weight of rail on the total impedance, although the impedance of the rails alone increases markedly with the weight.

One of the manufacturing companies places the drop per 100 amperes per mile in 3/0 trolley and 80 lb. track at 62 volts, height of trolley not stated. This figure is practically the same as that given by the Railway Test Commission for very different weight of rail and type of bond.

The above figures are at considerable variance. This is probably due in some measure to the difference in conditions of the several tests on which the figures are based. It is possible that the conditions in practice as dependent on the proportion of trolley current returned by the rails, may vary with the locality to a degree sufficient to account for the differences in the values of the voltage drop in trolley and track. It may not be without interest therefore to investigate by calculation the values to be expected.

The inductive component of the voltage drop is due to the aerial reactance and the reactance in the rail itself. The watt component is due to the resistance of trolley and track and to iron losses in the rails. The calculation is simple if all the return current be assumed to be in the rails and the values for rail iron loss be taken from published tests. The reactive volts at 25 cycles per mile of line of two wires of radius r and distance apart d, carrying current i in amperes are given by the expression $0.101\, i \log \frac{d}{r}$. One half of this value is due to each wire. We may take this expression as a basis for calculating the aerial reactance electromotive forces. Owing to the wide difference in nature of the overhead and return conductors we may consider them independently both as regards energy and reactance electromotive forces.

The most common type of trolley suspension for roads of the class here discussed is the single catenary. It is necessary to consider the influence of the suspension cable. Within the limits of accuracy of this calculation it will be sufficient to consider this cable as parallel to and at its average distance from the trolley wire. Let R_1 be the resistance per mile of the suspension cable, r_1 its radius, d_1 its distance above the rails, and i_1 the current flowing in it. The same letters with subscript 2 refer to the trolley wire. If i is the total overhead current, we have two equations for determining i_1 and i_2. We may conveniently use Steinmetz's symbolic method of notation

$$\dot{i}_1 R_1 - j\,.05 \left(\dot{i}_1 \log \frac{d_1}{r_1} + \dot{i}_2 \log \frac{d_2}{r_2} - \dot{i}_2 \log \frac{d_1 - d_2}{r_2}\right)$$

$$= \dot{i}_2 R_2 - j\,.05 \left[\dot{i}_2 \log \frac{d_2}{r_2} + \dot{i}_1 \left(\log \frac{d_1}{r_1} - \log \frac{d_1 - d_2}{r_1}\right)\right] \qquad (1)$$

$$\dot{i}_1 + \dot{i}_2 = \dot{i} \qquad (2)$$

The former equation expresses the equality in messenger cable and trolley of the total drop between station and car. Equation (2) expresses the total current as the vector sum of the currents in the two conductors. Solving we have:

$$\dot{i}_2 = \frac{\dot{i}\left(R_1 - j \log \frac{d_1 - d_2}{r_1}\right)}{(R_1 + R_2) - j \left(\frac{\log (d_1 - d_2)^2}{r_1 r_2}\right)}$$

This expression does not contain either d_1 or d_2 explicitly, only the difference of these quantities appearing. Thus the division of current between trolley and messenger is independent of the height above the track. The effect of the field of the track current on this division is negligible for the values of d_1 and d_2 met in practice. For 3/0 trolley wire, and $\frac{7}{16}$ in. steel suspension cable we have $R_1 = 3.7$, $r_1 = 0.218$ in. $d_1 = 22.875$ ft., $R_2 = 0.326$, $r_2 = 0.205$ in., $d_2 = 22$ ft.
For $i = 100$ we have

$$\dot{i}_2 = 91.5 + j\,4.04;\; i_2 = 91.6$$

$$\dot{i}_1 = 8.5 - j\,4.04;\; i_1 = 9.4$$

Substituting these values in either side of equation (1) we find the drop per 100 amperes per mile in trolley and messenger cable is

$$\dot{E} = 30.58 - j\,32.76;\; E = 44.75.$$

This value will vary with the height of the trolley. The variation is small, however, if d_2 is above 20 ft, and formula (1) readily permits its calculation.

If the suspension cable is neglected as a conductor the value of the above drop is

$$\dot{E} = 32.6 - j\,36;\; E = 48.5$$

In considering the track return we face the uncertain distribution and value of its magnetic field, consequent aerial reactance, and the proportion of total current carried by the track. Assuming 100 amperes in each 80 lb. rail and assuming that the magnetic field outside the rail is the same as though all the current were concentrated at the center of the web, and applying the above expression for aerial reactance, we have as the reactance volts due to the field set up between track and trolley by the track current about 32 volts. This value takes due account of the mutual induction between the two track rails. The reactance volts within the two rails, as deduced from the results of the Railway Test Commission's tests on 80 lb. rails are 21. The total reactance volts in the track are thus 53, for trolley 22 ft. above track and total current 200 amperes. The ohmic drop in the rails and bonds is 6.45 volts; the component of rail drop due to iron loss is 22.4 volts (Cf. Report Rwy. Test. Comm.). The total watt component of drop at 200 amperes in the track is thus 28.85 volts. The electromotive force per mile per 100 amperes in the track is, therefore expressed by $\dot{E}_t = 14.42 - j\ 26.5$. Adding this to the complex expression for the drop in the overhead conductors we have as the total drop $\dot{E}_0 = 45 - j\ 59.26$, or $E_0 = 74.5$ volts per 100 amperes per mile. If all the current were carried by the trolley wire the value would be $\dot{E}_0 = 47 - j\ 62.5$, or $E_0 = 78$. These figures also assume all of the return current to be in the track. This latter figure is remarkably close to that given by Parshall and Hobart, although the several components differ widely. Their value for the track is $7.7 - j\ 27$ and for the trolley $34 - j\ 37$. They have apparently not considered the secondary losses in the rails, nor the effect of suspension cable in reducing trolley reactance and resistance.

The numerous tables given by Parshall and Hobart showing the properties of rails and track as conductors for alternating current, indicate a very unfavorable comparison with the same properties when conductors of direct current. The basis of the tables is not definitely stated, however, and the values are not always in accord with published tests from other sources. To mention only one case, the drop per mile in 100 lb. rail carrying 100 amperes is given as 59 volts. The results of the Railway Test Commission give 41.5 volts for 80 lb. rail.

It is evident that in the particular case which we have considered the value arrived at is too high, for, as is well known, all of the return current does not flow in the rails. The results of an unpublished test of which the author has knowledge indicated the track current to be in places as low as 40% of that in the trolley, the average proportion being 50% or 60%. The conditions affecting this proportion vary so widely as to make it difficult to estimate the effect of this current dispersion on the reactance of the circuit. Let us assume, however, that p is the fraction of the trolley current in the rails, and that the iron losses and reactance within the body of the rail vary directly as the current at the low densities here used. (Cf. Report Railway Test Commission). Then the only indeterminate quantities entering into the value of the total drop in trolley and track are the reactance volts due to the trolley current as affected by the position of the mean path of the stray current in earth, the reactance volts due to the track current, and those due to the stray current. If for the purpose of investigation we make the not too violent assumption that the earth current distributes itself so as to be of circular cross-section tangent to the surface at the track, the first of these quantities increases and the last decreases with increasing radius of the circular section. The values of increased trolley reactance may be readily calculated from the right hand side of formula (1) by adding to d_1 and d_2 the value of the radius, R, of the assumed circular section of the stray current. The reactance volts due to the stray current are given by the expression:

$$\frac{0.101\, i\,(1-p)}{2} \log \frac{(22+R)}{R}$$

in which i is the total current and p the proportion of i carried by the rails. The other components of the total drop are either unchanged or directly proportional to p. The following table of values of total drop indicates how far the drop is affected by variations in the value of R and p.

	Values of p.				
Values of R	1	0.8	0.6	0.5	0.4
1	74.5	71.1	67.8	66.	64.4
2	74.5	70.9	67.	64.9	62.9
5	74.5	70.6	66.1	63.7	61.4
10	74.5	71.	65.9	63.3	60.8
50	74.5	72.9	67.8	64.9	62.5
100	74.5	76.	69.7	66.7	63.8

The effect of diminishing values of p, as was to be expected, is to decrease the total reactance. Increasing values of R first decrease, then increase the reactance. The stray current, if concentrated in a small section, has a magnetic field of appreciable value; consequently when the radius of section is small, increasing the radius causes rapid decrease in the field. For large values of R, however, the increased distance between trolley and resultant center of return conductor causes an increase in the total flux more than sufficient to compensate the above decrease. For all the values of p given the reactance volts are at a minimum in the neighborhood of $R = 10$. So far as the writer is aware there is no evidence as to the distribution of the stray earth currents. This distribution is in all probability never the same in any two cases. The amount of the stray current is obviously also a variable, not only with the particular case, but also with the proximity to feeding points. What is of interest, however, is that the reactance values vary very little within wide changes of both R and p, as is evident from the above table. Assuming the stray current to have a section of any radius between 10 and 50 ft., and p to be .5, the reactance volts per 100 amperes per mile are practically constant at 64 with a power factor .60. An increase from .5 to .8 as the proportion of trolley current in the rails changes this value scarcely 10%, while for smaller values of R the change is still less. With the assumptions stated the voltage drop per 100 amperes per mile at 25 cycles in 3/0 catenary trolley 22 ft. above track of 80 lb. rails is between 65 and 70 volts.

Train Resistance

There is wide diversity of statement and figures on the magnitude of train resistance. Two recent conspicuous comments on this subject also are found in the Report of the Railway Test Commission and in Parshall and Hobart's " Electric Railway Engineering ". In the following table the figures of these authorities are compared with those of Blood as given before the American Society of Mechanical Engineers, June 1903, and those of Armstrong (Trans. A.I.E.E., June 1903). The values given are for single cars.

Train Resistance—Pounds per Ton.

Speed miles per hour....	0	5	20	30	40	50	60	Single cars
Elect. Rwy. Test Comm..			12	15	20	26.8	35	38-ton car
Berlin-Zossen...........	15		3.3	7.7	17	15.4	18.3	90-ton and 77-ton car
Aspinall...............				9.4	12.7	17	22	90-ton and 77-ton car
Blood.................	5		9.7	13.5	18	21.9	29	38-ton car
Blood.................			9	12.3	16.1	20.5	25.2	50-ton car
Armstrong............	4	4.5	7.8	11.5	17	23	29	45-ton car

Parshall and Hobart conclude that Aspinall's formula gives reliable values for single car operation owing to their agreement with the results of the Berlin-Zossen tests. Aspinall's values for lighter cars do not change materially; for a 22-ton car at 50 miles per hour the figure is 16 lb. per ton. The figures are markedly less than those of the Railway Test. Commission. The heavy cars and excellent track conditions of the Berlin-

Zossen tests undoubtedly account for the low values, and values for lighter cars and average track may not be deduced from them.

The values given by the Railway Test Commission are distinctly higher than those in general favor, and should, therefore, receive attention. Without further corroboration, however, they must be accepted with some reserve owing to the variations in the figures in the several tests and to the unseasoned nature of the track.

It is evident that shortly after starting, the value of train resistance reaches a minimum somewhere about 5 lb. per ton. With 91.1 lb. per ton for a uniform frictionless rectilinear acceleration of one mile per hour per second, the necessary tractive effort after the actual moment of starting is about 96 lbs. per ton. The familiar and convenient figure of 100 lb. per ton thus allows about 4 lb. per ton for rotational acceleration, head wind and defective track conditions. It is evident also that during the acceleration period train resistance consumes only a small part of the total tractive effort and that errors in the values chosen assume no great importance for short runs. In this paper Blood's formula has been used for speeds of 20 miles per hour and upward and 100 lb. per ton has been assumed necessary for an acceleration of one mile per hour per second.

The Systems Compared

The basis upon which the choice of system was made has been already indicated. The proper capacity of available motor for the service was first determined for each type of equipment. To this end typical speed-time, current-time, and power-time curves for the average local run were plotted for 4-motor equipments with available motors of 75, 100 and 125 h.p. capacity as based on their one hour temperature ratings. The gearing in each case was made as low as consistent with the desired express schedule, since the conditions offered no special advantage in adpoting different gear ratios for express and local service. The methods of plotting these curves are familiar and require no comment for the direct current equipment. The treatment for the alternating current equipment during the period of acceleration is necessarily somewhat different and an outline of the method followed is given below. The typical curves for 100 h.p. alternating current and 90 h.p. direct current equipments were plotted, these motors showing themselves suitable on the basis of motor heating already mentioned. Speed-time, current-time, and power-time curves for each type of equipment were then plotted for every local run in each direction, due account being taken of curvature, grades, limited speed on bridges, etc. The results of this series of curves are given in the accompanying tables, together with their extension to the distributing system and power station. The value of this rather tedious process has been questioned. In the present case it not only afforded by comparison an indication as to the value of the typical curve for the purpose of computing an entire project, but it also enabled a run sheet to be constructed which developed passing points at rather different places than would have been indicated by the usual straight lines of schedule speed. The latter point is of particular value in a single track problem as determining which sidings should be equipped with trolley. After fixing the regular schedule sidings, a drop-back siding was provided in each case so as to prevent a loss of time being communicated to opposite traffic. This resulted in the fixing of eleven sidings, one of which is one mile long and another a half mile long, and there are three miles of double track at the Baltimore end. The total length of track thus becomes about 33 miles.

In the case of the alternating current equipment the construction of the speed-time and motor current-time curves is according to the usual methods, for the portion of the run after the period of uniform acceleration, *i.e.*, after the full voltage is on the motors. The initial portion of these and other derived curves does not, however, permit the simple determination possible with direct current equipments. As the literature of the subject presents little or no treatment at this point the following is offered as a simple method, which, from the data available, appears to approximate the actual conditions sufficiently closely for the purposes of calculation.

The motor current may be assumed as constant during the period of uniform acceleration at the value corresponding to the speed at which the acceleration begins to fall off. This conclusion is based on a study of the curves given by Bright (*Elect. Jour.* Vol. II, No. 11, p. 651, Nov. 1905) which, so far as the writer is aware, are the only curves taken from tests heretofore published. The motor current curves indicate a current rising slightly by steps during the period of constant acceleration to a maximum and beginning to decrease before the period of constant acceleration ends. An inspection of these curves shows that the average value of the motor current during constant acceleration is slightly less than the value corresponding to the termination of the initial straight portion of the speed-time curve. Thus, by area integration the average current per motor from start to stop in one of the runs measured by Bright is found to be 332.2 amperes. The value of this average, if the current during uniform acceleration is assumed constant at its value at the end of that period, is 335.6. The two corresponding values of the root mean square motor current from start to start are 412 and 416.

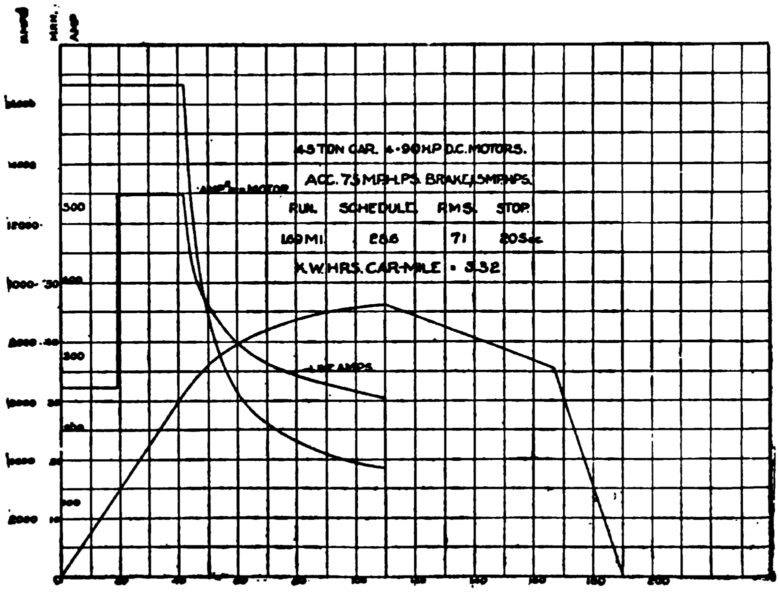
The initial portion of the line current curve will have a rising series of sharp changes corresponding to the voltage values of the several controller notches. The exact shape of this portion of the curve has no particular importance. The two aspects in which the line current must be considered are its maximum value as affecting line regulation, and the average value as determining the average apparent watts and the power factor. The maximum value is deduced from the maximum motor current and the ratio of line and auto-transformer voltages; it occurs then at the instant when full voltage is applied to the motors. The average value is found by the usual integration over its area, the average being taken for the period from start to cut-off. The shape of the initial portion should thus be considered in its effect on the area included by the whole curve. If, as already stated, the motor current is constant during this period, the starting value of the line current may be assumed as one-half its maximum value, since the motor voltage on the first notch is about one-half that on the last, a resistance automatically cut out limiting the first rush of current. The line current would then decrease slightly with increasing speed, increasing sharply at each controller notch. It may be safely assumed in calculation that a straight line between the above starting value and the maximum value gives the same area as the irregular curve just described. This suggestion is based on an inspection of the curves given by Bright and is sufficiently accurate, as the shape of this part of the curve affects the area only slightly in comparison with the remainder. The curve of motor kilovolt amperes is now determined. The line kw. at any instant is the sum of the useful mechanical output and the various losses. The mechanical output is the product of total tractive effort and speed, and when full voltage is on the motors is given directly by the motor curve of brake horse power with gears.

The losses to be considered are those in the gears and those due to copper and iron in motors, auto-transformer and car wiring. the losses in gears and motors are given by the curve of efficiency with gears. Those in the auto-transformer can only be determined from a knowledge of its characteristics. In the present case regulation and loss curves for the auto-transformer were furnished by the manufacturers. The values of the losses in the car wiring are here assumed as 1% of the total output. Following this method the portion of the curve for line kilowatts after full voltage is on the motors is readily plotted. The rising portion from the instant of starting, as in the case of the curves for motor current and line current, cannot be accurately predetermined owing to obvious indeterminate factors. Here, too, for the purposes of calculation it is sufficient to estimate its approximate form. Obviously the motor curve may not be used for the interval during which the motors are on the low voltage notches. The motor current, however, being approximately constant there, so is the net tractive effort, and the rise in speed being assumed to be uniform, the values of mechanical output are at once deduced and are seen to increase uniformly. At the instant of starting, the mechanical output is zero and the power drawn from the line is entirely dissipated in the transformer, wiring and motor. The voltage of the first transformer point is impressed on the circuit consisting of the starting resistance, the motor field and compensating winding and the stationary motor armature. The armature being stationary, there is a heavy loss in the short-circuit consisting of armature coil resistance leads and brush; this loss is of course greatest at standstill and decreases with increasing speed. The starting resistance also introduces a loss. In addition there are the normal copper losses in the remainder of the motor winding, the motor iron loss, and the losses of both types in the auto-transformer. The sum of these losses gives the initial value of the rising portion of the line power curve. It is manifestly impossible, and fortunately unnecessary, to determine this value accurately. We may note, however, that the value of the acceleration gives a value of the motor current, which as already stated, may with fair accuracy be taken as constant during this period. This current given, the motor and transformer curves give the normal losses when the motor is at speed, and thus a minimum value below which the initial line power cannot be. The increase due to starting resistance and armature coil short-circuit can only be known by a greater knowledge of the motor than is generally available. The line power curves in this investigation have their initial value as twice the minimum referred to. From this point a straight line has been drawn to the maximum value of power at the instant where the acceleration begins to fall off. Here, as before, a study of the curves of Bright is found to be corroborative. It may be emphasized again that the area comprised by this rising portion of the curve is for the usual type of run a sufficiently small part of the total to render error in the above assumptions of small effect. The curve of line kw. is thus derived, and with that of kilovolt-amperes determines the line power factor.

The results of the complete series of speed-time and other curves as given in the tables apply to a 37-ton car for the direct current and a 42-ton car for the alternating current equipments. There are also added in each case the figures taken from typical curves for the same cars. These typical curves were plotted for the same schedule speed derived from the complete set of curves. The results indicate that at the direct current car the average power consumption per car mile for the complete run is about 3.2% greater than shown in the typical curves. For the alternating current car the values of increase of complete run over typical run are 3.5% for kw-hr. per car mile. The typical run sheet therefore agrees very closely with the averages as deduced from the complete set. This is an interesting result, as the road is neither exceptionally straight nor level. The grades are generally up and down, and the two terminal cities are at virtually the same elevation. In the run which includes the Severn River bridge allowance was made for a reduction of speed to 10 miles per hour at the draw.

A continuous speed curve for the single-phase equipment was constructed for the express run between the terminals, with frequent coasts on grades, and a reduction of speed on the Severn bridge to 10 miles per hour. The average schedule speed was indicated as 42 miles per hour, the power consumption as 2.64 kw-hr. per car mile, and the average power-factor as 0.96.

In the case of neither of the cars above mentioned were the motors loaded to their continuous capacity. A later desire on the part of the management to increase the size of cars could, therefore, be met with the same motors, as is indicated by the figures in the tables for a 45-ton direct current car and a 50-ton


45 TON CAR. 4-90 H.P. D.C. MOTORS.
ACC. 7.5 MPH.PS. BRAKE 1.5 MPHPS.
RUN. SCHEDULE. RMS. STOP.
1.69 MI. 28.6 71 20 Sec.
K.W. HRS. CAR-MILE = 3.32

FIG, 1

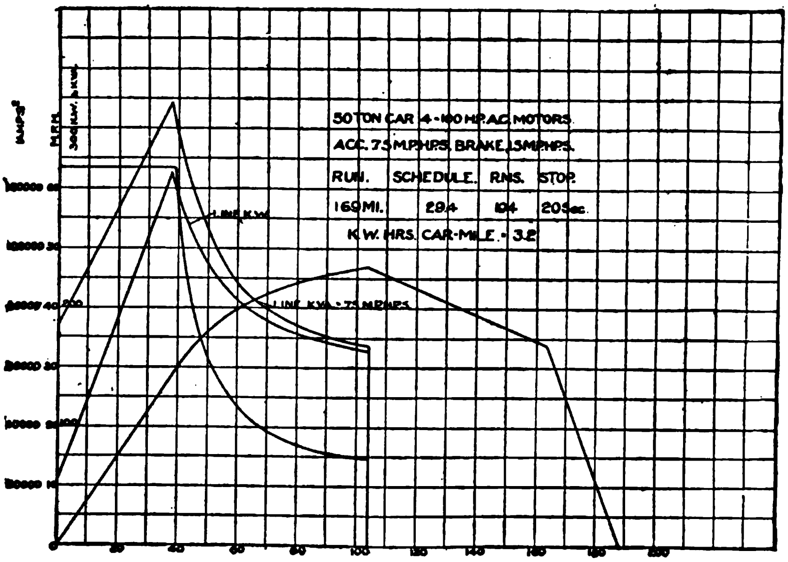

50 TON CAR 4-100 H.P. A.C. MOTORS.
ACC. 7.5 M.P.H.P.S. BRAKE 1.5 M.P.H.P.S.
RUN. SCHEDULE. RMS. STOP.
1.69 MI. 29.4 104 20 Sec.
K.W. HRS. CAR-MILE = 3.2

FIG. 2

TABLE I

37 Ton Car 4-60 h.p. direct curre

Station	Distance Feet	Distance Miles	Time sec.	r.m.s. motor cur.	Line cur. Start	Line cur. Av.	Line kw. Start	Line kw. Av.	Kw Run
A–B	10,960	2.07	260	69.7	600	124	330	68.3	4.55
B–C	7,960	1.5	213	98	772	243	425	140.5	7.85
C–D	16,900	3.2	316	88.5	650	101	358	87.5	4.09
D–E	3,800	.72	121	87.8	710	186.6	390	102.7	2.98
E–F	7,900	1.49	180	72.4	710	144.6	390	79.5	3.72
F–G	12,400	2.35	274	72.3	710	165	390	91	6.42
G–H	5,200	.98	142	78.4	650	150	358	82.5	2.9
H–I	14,800	2.8	303	64.2	710	143	390	78.7	6.2
I–J	10,200	1.93	224	73	600	210	330	115.4	6.85
J–K	7,200	1.36	172	75.3	650	181	358	90	4.27
K–L	5,200	.98	128	66.1	555	130	306	71.4	2.84
L–M	9,110	1.72	241	54.2	519	75	282	41.2	2.64
M–N	3,960	.75	112	81	600	162	330	60	2.52
Average	Sched. speed 29.6 m.p.h.			72				85.9	4.88
N–M	3,960	.75	110	87	604	218	332	126.5	3.34
M–L	9,110	1.72	267	81.7	604	194	332	111.5	7.33
L–K	5,200	.98	142	79.6	660	192	363	105.5	3.55
K–J	7,200	1.36	172	67	604	141	332	89.5	3.09
J–I	10,200	1.93	212	54	450	88	306	48.25	2.58
I–H	14,800	2.8	270	66.6	604	206	332	118	7.81
H–G	5,200	.98	142	75.3	604	173	332	96	3.10
G–F	12,400	2.35	240	66	604	163	332	80.9	5.48
F–E	7,900	1.49	172	72.6	604	180	332	99	4.18
E–D	3,800	.72	119	75.6	652	149	369	89	2.86
D–C	16,900	3.2	319	71.6	604	233	332	129	10.59
C–B	7,960	1.5	183	66.5	719	101	392	59.9	2.46
B–A	10,960	2.07	225	57.6	518	138	282	75.9	4.38
Average	Sched. speed 30.3 m.p.h.			70.7				93.6	4.67

alternating current car. These figures are taken from the typical curves shown in Figs. 1 and 2. In view of the reliability of the typical curve as above indicated, the comparison of the two systems is based on these curves, due regard being had to the small percentage increases already mentioned. The comparative figures are given in the following table.

Type of car	Schedule speed	Kilowatt-hours car-mile at car	Kilowatt-hours car-mile at sub-station
37-ton direct current from complete runs	30.3	2.85	3.58
37-ton direct current from typical curves	30.3	2.76	
42-ton alternating current from complete runs	30.3	3.27	3.31
42-ton alternating current from typical curves	30.3	3.16	
45-ton direct current from typical curves	28.6	3.32	
50-ton alternating current from typical curves	29.4	3.2	

A further fact to be noted from a comparison of the two typical curves is that the alternating current car requires less power consumption for the run in spite of its greater weight. The two values of kw-hr. per car-mile are 3.2 and 3.32. This is due to the series resistance losses in the direct current car; owing to the short length of the run the motors are on resistance more than one-third the time during which power is applied. The maximum power reached for the single-phase car is greater, and so also is the power throughout the period when the motors are on full voltage. The former is, however, applied for a comparatively short time, and the direct current power is applied longer owing to the lesser value of maximum attainable speed. This difference does not obtain in the runs for the lighter cars. The direct current equipment on a 37-ton car accelerates with less demand and cuts off earlier than the alternating current equipment on the 42-ton car. Increasing the sizes of car to 45 tons and 50 tons imposes less additional demand on the single-phase motor than on the direct current motor of lower rating. The latter motor on the 45-ton car is worked well up to its figure for continuous rating, and it is questionable whether or not it is sufficient for the service. If the conclusions of the investigation pointed less strongly to the single-phase system, it would be necessary to consider a larger direct current motor before reaching a conclusion.

Increasing the figures for power by the percentages given above we have for values of kilowatt-hour per car-mile at the car, for the 45-ton direct current car 3.43 and for the 50-ton alternating current car 3.32.

The weights of the two cars are distributed as follows:

	Alternating current.	Direct current
Car body	32,000 lb	32,000 lb.
Trucks	21,000	21,000
Motors and control	32,000	22,500
Air brakes	3,000	3,000
Live load	9,000	9,000
	97,000	87,500

Station Equipment and Power Consumption

Single-phase system. In the alternating current system the starting current per car, assuming that the rate of acceleration is constant, varies with the track conditions. On level track the line current per car at 6600 volts is 57 amperes at 0.75 miles per hr. per sec. and 70 amperes at 1 mile per hr. per sec. at a power factor of 0.80 or better. Assuming the use of the available 3/0 catenary construction, and the voltage drop per 100 amperes per mile as 70 with a power factor 0.60 and a maximum permissible drop of 10%, it is evident that a two-car train may accelerate at 1 mile per hr. per sec. when fed by a single trolley at a distance of about 6.75 miles. The schedule is based on an acceleration of 0.75 miles per hr. per sec. and the equipments chosen will operate satisfactorily at considerably greater values of drop. The bunching of cars in such a section should therefore offer no difficulty on this score. A substation was located at a convenient station 6.25 miles from Annapolis and 17.4 miles from Westport. Using the figures already given, two cars starting at Annapolis at 0.75 miles per hr. per sec. will cause a maximum drop of 7.6% when fed from this substation. The line impedance at power factor 0.60 increases the reactive component of the car demand but slightly. If the car power factor be taken as low as 0.80, the load at the substation has a power factor 0.78.

Considering the resort Bay Ridge, 4 miles from Annapolis, the substation might with better advantage be located nearer Annapolis. This would, however, increase the starting drop in the region between power house and substation. As single cars may be readily started at Bay Ridge when fed over the above distance, and as the equipment of this branch is as yet undetermined, this question is of little importance. Future extension to Bay Ridge may be accomplished by an extension of the transmission line and the installation of a temporary or portable transformer substation. The above location of the substation as one trolley feeding point and the powerhouse as another meets the requirements of regulation satisfactorily.

The power house at Westport, 1.6 miles from Baltimore, as the feeding point for the north half of the line, although not located to the best advantage from the standpoint of symmetrical feeding, saves the building of a second substation. An effort was made with the power company to permit the load to go directly on the station bus through two-phase three-phase transformer connection. This would result in an average uniform distribution of load on the three-phases, but also in possible short applications of large fractions of the total load on one phase. The form of contract permits this, but the power company's distaste for unbalancing is shown by a further stipulation that the maximum demand shall be reckoned as three times that on the most heavily loaded phase. As the total installed capacity in the powerhouse is 13,000 kw. and as the maximum starting demand of the normal daily schedule may reach 1800 kilovolt-amperes, this attitude is not to be wondered at. It is increasingly evident that there are few cases where a single-phase railway, with the usual erratic combinations of starting peaks, may be supplied from a system furnishing at the same time light and power for other purposes. None of the polyphase-single-phase transformations recently reviewed by Armstrong (PROC. A.I.E.E.) will prevent occasional excess demands on one phase. Also some of these methods have the additional disadvantage of largely increased reactance drops with unbalanced loads. The problem narrows down to isolated generators for the railway load, or motor-generators for distributing the load uniformly, whatever its value. The former alternative was out of the question in the present instance, the units in the powerhouse being 2000 kw. each. It is, therefore, evident that the adoption of the single-phase system for this road meant the building of its own power plant or the installation of motor generators.

Aside from the question of efficiency, the motor generator is not attractive when power is purchased, since power contracts always have their limit of life. In Baltimore, however, as already mentioned, there are two large plants of similar voltage and frequency characteristics, and the Susquehanna development will also deliver power there in the same form in the near future. The likelihood of having the motor-generators left without a proper source of power is therefore remote.

The power and distributing system for single-phase equipment then resolves itself as follows: synchronous 13,000-volt 25-cycle 3-phase motors direct-coupled to 6600-volt 25-cycle single-phase generators. The trolley is fed directly from the generators, as are also step-up transformers for the 17.5 mile transmission to the substation.

The starting demand of a car varies between 375 and 475 kilovolt-amperes depending on the grade. Normal half hourly traffic requires four cars on the line at one time, with the possible bunching of three cars on one substation, or about 100% overload for two 300 kw. transformers. Three such units in the substation, and space and wiring for a fourth, will provide reserve capacity and provision for future expansion. The same transformer capacity is installed at the powerhouse. Excursion traffic should be handled by two car trains at lessened headway. Such trains would accelerate at a low rate. The running of nine trains on a 15-minute headway means a minimum of six on the line at once to preserve opposite traffic. The sustained demand if these trains are single cars will not exceed 1600 kilovolt-amperes. Two two-car trains and a single car starting at the same instant would impose 150% overload on three 300 kw. transformers. The motor generators obviously take the whole load. The sustained excursion demand above mentioned would be about 25% overload for 1200 kw. in motor generators. Three 600 kw. units will provide ample reserve and provision for considerable future growth.

For the comparatively short transmission distance there is no decided advantage in increasing the voltage above 22,000. If 33,000 volts is chosen, the size of wire is reduced to a point inadvisable from considerations of tensile strength and the difficulties and cost of insulation are also increased. Two transmission lines each of two No. 2 B. & S. wires 30 in. apart, and both lines carried on the same cross-arm, provide interchangeability and reserve capacity. With a load of 800 kilovolt-amperes at power factor 0.85 on two substation transformers, the drop between the power house 6600-volt bus and the substation trolley connection will be about 15%; the power factor at the station will be about 0.82. From the tabulated results of the complete local runs, the average efficiency of trolley and track is better than 98%, as shown by the values of kw-hr. per car mile at the car and at the substation. The loss was calculated by assuming that the average line current during the period while power is applied, is fed to a point one-third through the particular run in question. If the value of root mean square line current for the typical run is assumed to be delivered at the average distance of a car from a substation, the trolley and track loss is somewhat over 1.5%. In calculating these losses the energy component of track and trolley impedance as already given has been used. Assuming the average efficiency of the transformers at 96.5% and calculating that of the transmission line at 98%, we have half the total power supplied through the substation at about 89% and the other half fed directly to the trolley from the generator bus-bar at 98%. For the normal schedule already described the daily power consumption would be as follows:

22 locals at 3.32 kw. hr. per car mi.	1840 kw-hr.
20 expresses at 2.85 " " " "	1401 " "
	3240

One half of this at 98% and one half of 89% gives 3475 kw. hr. at the generator bus-bar. At an average motor generator efficiency of 80% we have 4350 kw-hr. daily consumption for the proposed normal schedule under the single-phase system.

Direct-current system. The normal starting current of the car described above for the direct current system at an acceleration 0.75 miles per hr. per sec. is 516 amperes; for a two-car train approximately 1000 amperes will be taken at starting. With two cars starting and one running the demand may easily go to 1200 amperes. Assuming a maximum permissible drop of 25% and 600 volts at converters and supposing that the single 80 lb. track is bonded to full capacity (0.028 ohms per mile), something more than 2,000,000 cm. of copper would be necessary to supply 1000 amperes to a two-car train starting from Annapolis from a substation 3 miles away. Aside from the consideration of the cost of copper, however, the question of collecting the large current values, and the high speed to be reached, points at once to the third rail as the proper method of feeding. Assuming 80 lb. third rail bonded to 10% of equivalent copper as having a resistance of 0.044 ohms per mile, we have as the resistance of the third rail and single track 0.076 ohms per mile. With 600 volts on the third rail and a 30% drop, this will permit a distance of somewhere between two and three miles as the length of the end of the feeding system beyond the last substation. Assuming that two cars in starting take 1000 amperes, and that they will make the only demand on the end beyond the substation, it would be possible in the present instance to locate a substation at a convenient passing point, indicated by the proposed schedule, 2.8 miles from Annapolis. The northernmost feeding point would naturally be the Westport power house. The distance between these two stations would thus be 20.85 miles. Considering the possibility of operating with only one intermediate substation, it is to be noted that the distance between substations would then be approximately 10.5 miles. With three cars starting midway between two substations, which would be a common occurrence, the drop would be approximately 300 volts. This figure assumes also very low values for track resistance. On the above basis it is therefore inadvisable to attempt to operate with less than four substations. These would be located as follows: one at Westport, one at Winchester (2.8 miles from Annapolis) and two others between these. The greatest distance between substations would thus be 7 miles. With three cars starting midway between two substations the drop would be about 200 volts. It is worth noting that if the road were double tracked it would be possible to obtain satisfactory regulation with two substations.

In the matter of substation capacity, when the demand on one substation is that of two cars starting and one running, the demand is in the neighborhood of 760 kw. With three starting on one substation, the demand is 920 kw. At 100% overload the latter figure would necessitate 460 kw. capacity. Thus one 300-kw. converter would take care of the normal single car schedule, and two 300-kw. units in a substation would handle a schedule involving two-car trains in one direction, and would, therefore, constitute a certain spare capacity when considered from the standpoint of the normal single-car schedule. This reserve is, however, not so great as that allowed for the single-phase system. For the transmission line, if a resistance drop of 5% be allowed, the average normal load can be handled by a 22,000-volt, three-phase line of three No. 3 B. & S. wires.

From an inspection of the results of the complete set of run sheets it will be noticed that the efficiency of trolley and track has an average value of about 80%. This was, however, based on three substations and a smaller value of conductivity in the distributing system, thus obtaining less first cost at the expense of regulation. Taking the mean distance of a car from a substation as one quarter the distance between substations, and assuming the resistance of third rail and track as 0.086 ohms per mile, the car will be fed over a resistance of 0.114 ohms. The R M S line current for the typical run is 351 amperes. Using these figures the efficiency of trolley and track is in the neighborhood of 92%. The single ends beyond the extreme substations are not included in this value which may be considered high. Taking the efficiency of converters and transformers at 90%, of the transmission line at 95%, and the average efficiency of powerhouse transformers at 96.5%, we would have a resultant efficiency somewhat less than 76%. About three quarters of the total load would be supplied at this figure—one quarter would be supplied directly from the powerhouse at 82.8%.

On the above basis the average daily consumption of power for the normal schedule would be as follows:

22 locals, or 555.5 car miles, at 3.43 kw-hr.	= 1900 kw-hr.
20 expresses, or 505 car miles, at 2.7 kw-hr.	= 1363 kw-hr.

Of this total of 3263 kw. hrs., figuring three quarters at 76% and one quarter at 83%, we would have about 4200 kw. hr. as the average daily consumption under the direct current system.

Choice of System

For the purpose of making a choice of system it is only necessary to consider those particulars in which the two systems differ. The most important of these are first cost and operating expenses, the latter including power consumption, station attendance and maintenance of equipment. Following is a comparison of the cost of the essential parts of the two systems, as outlined above, and for the most part based on prices asked from manufacturers to cover the specific cases. The copper was bought during the high market of 1907. This fact has no particular weight in throwing the advantage to either system, if the direct current system includes the third rail as conductor. The cost of the third rail is based on a comparison of the several estimates compiled by Gottshall in " Electric Railway Economics ". The length of track, trolley and third rail is taken as 33 miles to include the double tracks at the north end and middle of the line and the sidings. The transmission line is about 3.4 miles longer for the direct current than for the single-phase system, and the figures for this item include poles. Converters with transformers are valued at $17.50 per kilowatt.

	Direct current	Alternating current
9 cars completely equipped	$107,300	$140,300
Catenary trolley, poles, wire and guys		75,000
Third rail	132,000	
Transmission line	65,000	36,000
Powerhouse apparatus	21,000	62,000
Substation "	39,000	8,000
Substation buildings	15,000	3,000
Bonding	18,000	11, 00
	$397,300	$344,300

In power consumption the two systems appear to be on an equal footing. This is rather surprising since the motor generators reduce the efficiency of the single-phase system very markedly. In this case this reduction is offset by the losses in rotaries, the excess loss in distributing system, and the increased power consumption at the car, due to the smaller motors and short runs. For longer local runs the single-phase power would go to higher values than that for the direct current system. The price of power is based on the maximum demand and the total consumption, and for the normal schedule would average about 2.3 c per kw-hr. The monthly excess of single-phase power would thus be about $100.

In the matter of station attendance the direct current system would require that of three substations, which have no counterpart in the single-phase system. Two men per station at $75 per month each would make this item $5400 per year.

The cost of maintenance and repairs of car equipments would be less in the direct current system. Recent figures indicate that 0.5 cent for direct current and 0.75 cent per car-mile for alternating current is a fair basis for comparison. At 30,000 car miles per month the excess of this item for alternating current operation would be $900 per year. These items represent the principal particulars in which the two systems would differ. Other less conspicuous differences may be embraced for the purposes of this comparison in a figure of 10.5% for fixed charges on the initial cost. This item presents then an advantage on the part of the alternating current system of $5565 per year.

Summarizing the above items, we find a total of $8,865 per year in favor of the alternating current system. The figures for cost of the alternating current equipment have been taken for the most part from prices actually paid for construction and apparatus, while those for the direct current system are estimated. The result is that there are several advantages possessed by the alternating equipment as installed which would represent a considerable increase in the figures for first cost and operating cost, if obtained under the direct current system. The equipment as installed provides considerably more substation and power station reserve capacity than was estimated for the direct current system. It provides for the possibility of feeding moderate loads at Bay Ridge, and increased feeding capacity to that point at moderate cost. To accomplish this with the direct current system would mean a considerable addition to the equipment as outlined. The regulation of the system as installed is better than that available with direct current. The schedule speed is somewhat better, or to express it differently, the initial cost of the direct current equipment should be increased by an amount necessary to cover a motor of capacity equal to the alternating current motor. No attempt has been made to evaluate the cost to the alternating current system of these several advantages.

Comparative Costs of Steam and Electrical Operation

The company's statements of operating expense under steam were available for a period of several years. They are sufficiently detailed to invite an attempt at a comparative estimate of the cost of electrical and steam operation. The difficulties in the way of such an estimate are well known. In the present case the nature of the service is to be changed both in frequency and the size of units. The extent of these changes cannot be predetermined. The exact proportion of old rolling stock necessary to maintain the present schedule, and the proportion of new equipment, both in rolling stock and station apparatus, necessary to operate on an equivalent new schedule can only be roughly estimated.

In the following comparison the present normal steam schedule is assumed to require four locomotives and fifteen coaches. Under this schedule the maximum number of trains on the line at one time is two and a train averages three coaches; there are, however, occasional special trains. Under electricity the proposed maximum number of cars on the line at once for equivalent normal schedule is four, and the necessary charges are based on these assumptions. The operating expenses for a typical nine months of steam operation are given below; the only items discussed are those peculiar to steam operation.

Conducting transportation	$43,366.49
Maintenance of way	22,129.46
Maintenance of equipment	12,159.08
General expenses	12,196.91
	89,851.94

The car-mileage during this period was 277,754 or 30,861 car-miles per month. The cost per car mile is thus 32.36 cents. The proportions of the above figures peculiar to steam operation are indicated as follows:

Conducting transportation.

Fuel	$13,692	
Waste, water, and oil	1,114	
Trainmen	11,776	
Roundhouse men	354	
	$26,993, or	9.71 cent per car-mile

Maintenance of Equipment.

Repairs to locomotives	2.13 cent per car-mile
" " coaches	1.76 " " " "
Maintenance of way	7.97 " " "
Interest on equipment peculiar to steam operation, including locomotives, coaches, 2 water plants turn-table, coal plant, etc	1.52 " " " "
The total of these items is	23.1 " " " "

Each of the above items will be modified under electrical operation approximately as follows, the same car mileage being assumed:

Conducting Transportation.

Power, 1060 car miles per day	7.86 cent per car-mile
Trainmen, 7 crews of 2 men	3.90 " " " "
Signalmen and dispatchers	.5 " " " "

Maintenance of Equipment.	
Repairs to cars and equipments	2.76 cent per car-mile
Transmission line, 3% of cost	.3 " " " "
Trolley 5% of cost	1.02 " " "
Station equipment 3% of cost	.4 " " "
Maintenance of way	7.25 " " "
The proportionate change in this item is based on the figures given by Stillwell and Putnam; as given here, bonds are included but not overhead construction.	
Interest on Equipment	
Eight cars, power apparatus, trolley and transmission line and bonding	4.48 cent per car-mile

The total of these items is 28.47 cent per car-mile, or an increase of 5.37 per car mile over the present figure for steam operation. This increase therefore represents an increased total operating cost of electricity over steam of about 16.6%.

Considering the possibility of compensating for the above increase in operating cost, it may be noted that during the last ten years the passenger traffic of the road has shown a consistent increase, amounting on the average to about 5.5% for the past year. There was therefore a large rate of increase in the earlier years of this period. Much of this increase must be attributed to the legitimate results of the facility of travel afforded by the increasing efficiency of the steam road. In spite of the persistence of the increased receipts from year to year a discussion with the management reveals a decided idea that the through traffic is about fixed in value. Notwithstanding an increase due to improved service the reduction in rates consequent upon competition is regarded as an offset such as to maintain the through traffic at a constant value.

As suggested before in this paper, it is to the local traffic that an increase of earnings is looked for. At present the earnings from local traffic represent about 25%. The ratio of operating expense to earnings is about 65%. With the above increase of operating expense and assuming no increase of traffic, the ratio of operating expense to earnings is increased to about 76%. It is only necessary, however, to assume that the local traffic will double, to reduce the ratio of operating expense to earnings to less than its present value. As there is practically no competition in local traffic, the results of frequent electric service in other localities indicate that such an increase in local traffic in this instance is not improbable.

It would seem, therefore, that considering passenger traffic alone, notwithstanding the fixed charges on the investment, on a road with traffic of the nature indicated in this case, the substitution of electricity for steam is well worth considering.

Conclusion

Outlining briefly the results of the discussion in this paper, there may be mentioned besides the general description of the road and the considerations leading to the adoption of electricity the following:

1. A review of the values of impedance of the circuit consisting of trolley and track, and a method for calculating the same.

2. Several suggestions as to the methods of constructing current and power curves for single-phase equipments.

3. A discussion as to the accuracy of the typical run curve in its application to the results to be met in the entire series of runs.

4. An indication that in some cases the use of motor generators with the single-phase alternating current system does not preclude its comparing favorably with the direct current system of 600 volts.

5. While no general conclusions may be drawn from individual cases in the substitution of electricity for steam, the indications in the road here described are that, on the basis of passenger traffic alone, the undertaking will prove a profitable one.

DISCUSSION ON "FROM STEAM TO ELECTRICITY ON A SINGLE-TRACK ROAD." ATLANTIC CITY, N. J., JULY 1, 1908

J. B. Whitehead: This paper was suggested by some of the questions arising in undertaking to change a 30-mile, single-track road from steam to electricity. It is understood, of course, that every project of this nature must be considered in the light of the particular conditions involved, so that it is rather to the methods which have been followed in the present case than to the actual results and values which were finally adopted, that I ask attention.

W. I. Slichter: In the equipment of a single-phase railway system one of the most important points is the design of the conductor system, since there is not the synchronous converter to fall back upon as a booster to make up for any line losses by its compounding.

The drop in voltage in the rails with alternating current has been investigated by various persons, but as Mr. Whitehead has pointed out, there seems to be considerable difference in the results obtained. It is probable that these results differ because of the numerous conditions which enter with important effects into the phenomenon.

The drop in the rail varies with the perimeter of the rail, because it is a skin effect; it also varies with the chemical composition of the rail, which affects the permeability; with the bonding which affects the true or ohmic resistance of the circuit, and with the quantity of current in the rail, as this affects the permeability; and with the leakage of current to ground.

While the results of experimental tests will differ greatly on this account, conditions found in practice do not vary so greatly and we do get quite reliable and uniform values.

The general results of a series of tests made under operating conditions show that at 25 cycles the impedance drop in the rail itself is approximately eight times as great as the ohmic drop with direct current, that the power-factor of this drop is about 80 per cent. The impedance drop in the trolley is approximately 50 per cent greater than that with direct current. The result is that the total drop with alternating current is approximately twice as great per ampere as the drop with direct current.

An interesting feature of this voltage drop is that it has a power-factor greater than the power-factor of the input to the car at free running and less than that at starting, but more closely approximating the power-factor of free running. Thus under free-running conditions this drop is almost algebraically subtracted from the sub-station voltage.

At starting, when the input to the car has a value about twice that of free running, the drop is out of phase with the line voltage and is subtracted geometrically at a large angle, so that at starting there is a loss of voltage only about 50 per cent greater than that at free running, although the ratio of line current and actual voltage drops is as two to one.

Wm. McClellan: In regard to the choice of systems. Of the various systems proposed for electrification, none of us knows which one is going to be the best, if there is a best. I am perfectly willing to grant that there may be a heavy grade at some place where a 1200-volt, direct-current system may seem the best; there may be a long grade, perhaps, where a three-phase system may seem very advantageous; there are other places where the 11,000-volt, single-phase system may seem the right one; and still other places where the 650-volt system may seem best; and so on.

But we are not electrifying heavy grades or tunnels. We are not electrifying small portions of systems. We are beginning to electrify whole systems. Therefore, while we cannot at present choose any one system and electrify by means of that system, yet we must constantly realize that the great problem we have is the electrification of the large system. If we are called upon to work with a tunnel or grade, or something which needs immediate electrification, let us keep in mind the whole problem of the general electrification of the whole road at some future day.

I know of one place where they want to make a forty-mile extension on a part of the system, where a certain system of electrification is used on the original part of the road. There is no question whatever that another system of electric propulsion is more advisable for the extension; the problem is to know what to do, whether to throw away the part now in use, and remake it, or go ahead with the old system and install that on the extension, which is the easier thing to do, and trust to the future.

It is fortunate that one large system decided to solve their problem on a general basis. As a result they have been subjected to criticism, because some one can show where a few dollars might have been saved now by adopting some other system. They have done wisely, however, in keeping in mind the electrification of the whole system rather than to adopt the method that is perhaps more immediately applicable.

A. H. Babcock: The method given herein is not the only accepted one to be followed in discussing problems of this character. If it were the only method, in at least one case with which I have to deal, we would be hopelessly at sea. That particular case involves the electrification of about 135 miles of trans-continental railroad over the Sierra Nevadas, with a grade rising 7,000 ft. in 83 miles.

Have there been any tests made on the road, as in operation, to show how much drop was encountered in the rails and in the overhead system, as compared with the calculated figures?

J. B. Whitehead: There have been no tests of these figures. The best I was able to do was to take the literature as I found it in the matter and lay the system out. The starting conditions are excellent, so far as I can observe. As for actual figures of drop, I have not been able to get any.

A. W. Copley (by letter): The subject of the constants of alternating-current railway circuits is one to which I have given considerable attention. During the last few years I have had opportunity to make a number of tests for the determination of such constants, the most important of which were made on the New York, New Haven & Hartford Railroad, under the general direction of C. F. Scott and W. S. Murray. The data obtained in these tests is summarized in the following tables. The values given for double-track and single-track are calculated from the data obtained on the four-track road; the values obtained in this manner for single-track check closely with the results of tests made elsewhere on single-track roads.

The impedances given in the table are the electromotive forces required for sending current through the circuit, and are practically equal to the difference between the electromotive forces at the power house and at the load. The values in the table are the volts required per mile per hundred amperes in the trolley.

The figures showing the division of current between the track and the earth refer to intermediate portions of long sections; the division is different near the power house and near the load.

The figures in the tables apply to average practical conditions and are suitable for the calculation of commercial circuits.

FOUR-TRACK ROAD

Four 0000 trolley wires	Eight 100-lb. rails.	
Electromotive force	25 cycles	15 cycles
Resistance		
Trolley	6.8 volts	6.7 volts
Rail	1.8 "	1.5 "
Total	8.6 "	8.2 "
Reactance		
Trolley wires (internal)	0.35	0.2
Rails (internal)	0.85	0.5
Between trolley and return circuit	15.6	9.4
Total	16.8	10.1
Impedance, total	18.9	13.
Division of current		
Each outside trolley wire	26.6 amperes	26.3 amperes
Each inside trolley wire	23.4 "	23.7 "
Rails	75 "	
Earth	25 "	

FOUR TRACK WITH AUXILIARY FEEDERS

Four 0000 trolley wires	Two 00 feeders	Eight 100-lb. rails
Electromotive force	25 cycles	15 cycles
Resistance		
Overhead wires	5.7 volts	5.6 volts
Rail	1.8 "	1.5 "
Total	7.5 "	7.1 "

Reactance		
Overhead wires (internal)	0.2	0.1
Rails (internal)	0.9	0.6
Between overhead wires and return circuit	13.3	8.0
Total	14.4 volts	8.7 volts
Impedance, total	16.5 "	11.2 "
Division of current		
Each feeder wire	14.2 amperes	
Each outside trolley	18.8 "	
Each inside trolley	17. "	
Rails	75 "	
Earth	25 "	

Double-Track Road

Two 0000 trolley wires	Four 100-lb. rails	
Electromotive force	25 cycles	15 cycles
Resistance		
Trolley	13 volts	13 volts
Rail	2.5 "	2 "
Total	15.5 "	15 "
Reactance		
Trolley wires (internal)	0.6	0.4
Rails (internal)	1.2	0.7
Between trolley wires and return circuit	25.1	15
Total	26.9	16.1
Impedance, total	31.	22.
Division of current		
Rails	58 amperes	
Earth	42 "	

Single-Track Road

One 0000 trolley wire	Two 100-lb. rails	
Electromotive force	25 cycles	15 cycles
Resistance		
Trolley	26 volts	26 volts
Rail	3 "	2 "
Total	29 "	28 "
Reactance		
Trolley wire (internal)	1.3 volts	.8 volts
Rails (internal)	1.5 "	.9 "
Between trolley and return circuit	44.2 "	26.5 "
Total	47. "	28.2 "
Impedance, total	55.3 "	39.6 "
Division of current		
Rails	40 amperes	
Earth	60 "	

Note.—With 000 trolley wire instead of 0000 for single-track the total impedance in volts per 100 amperes per mile is 60.

The earth losses are left out of the above tables. Such losses are comparatively small and are not easily measured; moreover they undoubtedly vary greatly with different kinds of soil and the ballast of the roadbed, the largest part of the loss occurring in the region where the current is leaking from the rails and going into the earth.

Mr. Whitehead has given figures for various losses in railway circuits published by Parshall & Hobart and by the Railway Test Commission and has made calculations of such constants from theoretical considerations. A comparison of the values given in his paper with values obtained from the tests on the New Haven road brings out several interesting points.

Rail impedance. The values for the ratio between the impedance of the rails at 25 cycles and the direct-current resistance, are given from Parshall & Hobart as 8.1 and from the Test Commission Report as 5.5. These ratios are presumably for an 80-lb. rail. Measurements made on the New York, New Haven & Hartford Railroad with 100-lb. rails with 0000 bonds gave a value of between 2.5 and 3 for the ratio. This ratio was obtained with current densities of from 40 to 200 amperes per rail, and the results show only a very slight increase in the ratio with increased density. The resistance of the rail showed no noticeable change with the density, but the impedance rose slightly as the density increased.

Rail resistance. The resistance of the rails to alternating current at 25 cycles as measured on the New Haven road was 0.16 ohm per mile of single rails. Multiplying this by the rail current gives the ohmic drop of voltage in the rails. With 100 amperes per rail, this would amount to 16 volts. This value is little more than half of the value given by the Railway Test Commission (28.8 volts for 80-lb. rail) which is used in Mr. Whitehead's calculations.

Rail reactance. The reactance within the rail was found in the New Haven tests to be about 8 volts per 100 amperes per mile for a single rail. The value used by Mr. Whitehead (21 volts for 80-lb. rail) in his calculations is that taken from the Railway Test Commission's tests and is over double the value given above for a 100-lb. rail.

Rail current. The proportion of the trolley current which returns by rails is seen from the tables to vary according to the number of tracks. A single-track road has only 40 per cent of the current returning by the rails while a four-track road has 75 per cent.

At the point where current enters the rails on a single-track road 70 per cent of the current starts toward the power house and 30 per cent in the opposite direction. The 30 per cent leaks from the rails into the earth in from two to three miles. Coming toward the power house the 70 per cent decreases to 40 per cent in about the same distance, and the rail current then stays at this value until near the power house. Similar results were found on the four-track road, 87 per cent starting toward the power house and decreasing to 75 per cent in two miles.

Position of earth current. The position of the earth current varies, no doubt, under different conditions of soil etc. On the New Haven road it was found that if the earth conductor was assumed as 1000 ft. in diameter and tangent to the surface at the rails, the constants of the circuit calculated theoretically would be substantially the same as those determined experimentally. For a single-track road an assumed diameter of 600 ft. gave theoretical constants the same as the measured values.

Impedance volts due to trolley current. In the calculation given for the division of current between a 000 copper trolley wire and a $\frac{7}{16}$ in. steel messenger wire, it does not appear that any allowance has been made in the value of R_1 (the resistance of the messenger wire) for the fact that the current is alternating. The value 3.7 ohms is about the value of the resistance to direct current, but 9 or 10 ohms is probably nearer the value for the resistance at 25 cycles. By making this change from 3.7 to 10, the values of i_2 and i_1 (the currents in the trolley wire and messenger respectively) become 97 and 3.5 amperes, respectively and E (the impedance voltage in the circuit due to the trolley current) becomes 47.2 instead of 44.75. This value of E (47.2 volts) is close to the value found by neglecting the messenger wire as a conductor.

Impedance volts due to rail current. The resistance voltage due to rail current as given above is 16.2 per 100 amperes in each rail per mile of 8.1 volts per 100 amperes in a pair of rails per mile.

The reactance volts due to the field inside the rails is 4 volts per mile of single track (two rails). The reactance voltage in the trolley-rail circuit due to rail current is calculated from the distance between rails and trolley wire, the rail current, and the diameter of the rails. If this last is taken as 2.5 in., the value of reactance volts is about 32 as is given by Mr. Whitehead for 100 amperes in each rail or 16 for 100 amperes in the track. The total impedance due to track current of 100 amperes is then $8.1 - j\,20$. Adding this to the value deduced for the impedance due to the trolley current, gives E_0, the total impedance, as 67.8 volts per mile per 100 amperes for 000 trolley wire and 100 lb. rails. Taking into account that the rail current is only 40 per cent of the trolley current drops the value of E_0 to about 60 volts.

Chas. F. Scott: Mr. Copley is, I believe, substantially correct in his measurements, and what he has contributed is a valuable addition to our knowledge on this subject. I might add that the height of the trolley wire is 22 ft.

J. B. Whitehead: I am particularly glad to see these figures. This is one of the objects which I had in mind in bringing the calculation of the impedance volts into the paper. I have only seen Mr. Copley's figures for the first time, in the last few minutes, and am not prepared to make specific comments upon them.

The value of the impedance volts for the single-track road is what I was most concerned with in my paper. I notice that the total impedance in Mr. Copley's discussion at 100 amperes per mile, is 60 volts as against 65 volts given in my paper.

The difference between 80 and 100 lb. rails at 100 amperes per mile is appreciable; maybe it will make up for the difference in impedance values.

I did not mention the matter of rail impedance at the time I was abstracting the paper, but I called attention at the time to the difference in the ratios of impedance of rails and direct-current resistances, as given by the several authorities which I quoted. I brought out in the substance of the paper the point which Mr. Copley has reported here. He has touched on the subject of rail resistance, and I would be glad to see more figures on that particular point. They are all over the place, if one looks up the measurements which have been taken, and doubtless the variations are caused by the different conditions of track under which the measurements are taken; that is the only way I can account for them.

As to the matter of rail current, these are the first figures I have seen bearing on the subject. I should like to ask Mr. Scott whether there were any single-track measurements taken at the time of these tests? Do I understand these figures for single-track are calculated from the results of four-track tests?

Chas. F. Scott: Yes; calculated, and also the result of direct measurement. The figures relating to three single-track measurements taken at three places are from actual tests.

J. B. Whitehead: I should like to see the record of these single-track tests, because there are remarkably few at present available in the literature. The resistance of the catenary cable at 25 cycles is placed here at from 9 to 10 ohms, but by conjecture only. The value appears excessive.

S. H. Clarkson: In figuring out the cost per car-mile, I should like to know whether Mr. Whitehead took the locomotive into consideration, and if so to what extent? It seems to me that the cost per ton-mile would be a much more equitable basis for comparison, not only in the present instance where a steam and a single-phase road are being considered, but in all cases where it may be necessary to compare trains having different equipment or motive power.

J. B. Whitehead: I shall answer the question last asked. The figures given at the end of the paper on the relative costs of operation are based entirely on the passenger traffic of the road. The freight traffic will be handled by steam, certainly for the present. The comparison here given is entirely on the basis of passenger traffic, and it has been estimated that a certain car mileage was necessary to handle the passenger traffic. That is the reason for the basis of car-mileage.

As to the maintenance of locomotives, the figures for the cost of maintenance are at once available from the company's books, and it was a simple matter to put them on the car-mileage basis.

A paper to be presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

INDUCTION MOTORS FOR MULTISPEED SERVICE WITH PARTICULAR REFERENCE TO CASCADE OPERATION

BY H. C. SPECHT

During the last ten or fifteen years a great deal of work has been done toward perfecting methods for varying the speed of induction motors. Each of these methods has certain disadvantages, which have precluded the general adoption of any one of them for practical use. The method most generally applied for varying the speed is that of inserting resistance in the secondary circuit. This method of speed regulation, however, has the one great disadvantage that, for a certain load and speed a certain amount of resistance is required; and as soon as the load changes, the resistance must be changed in order to maintain the same speed. When the load is taken off the motor will return to its synchronous speed. It is also evident that such speed regulation can be obtained only by a great sacrifice of efficiency, due to the high ohmic losses in the secondary circuit.

For work which does not require large size motors, the above method might be satisfactory. For motors of large size, there are as a rule only two or three different speeds required; in such cases good efficiency and power-factor, and a good speed regulation for all loads and speeds are wanted. Motors which fulfil these conditions and which are best known are the following:

1. Independent motors with different numbers of poles, the rotors of which are mounted on the same shaft.

2. A single motor with separate windings, each of which is connected for a certain number of poles.

3. A single motor with one winding, which can be connected for different number of poles.

4. An induction motor, the secondary of which is connected to a synchronous motor.

5. Two induction motors connected either in direct or in differential concatenation or as single motors.

It is the intention in this paper to discuss mainly the characteristics and the operation of motor sets as mentioned in the last paragraph.

The motors of the cascade set are ordinary induction motors with wound secondaries, the rotors of both motors being mounted on the same shaft or interconnected mechanically by other means. The primary of the first motor is connected to the line circuit and its secondary winding to the primary winding

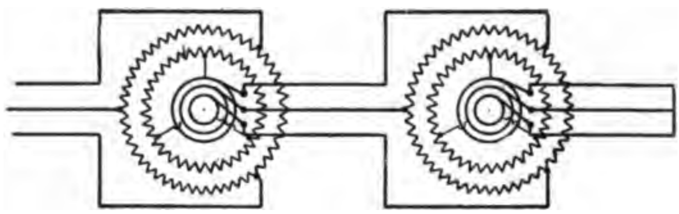


Fig. 1.—Concatenation diagram of motors arranged in direct concatenation

of the second motor, which may be the stator or the rotor. The secondary of this last motor is short-circuited direct or through an external resistance.

The two motors are connected in direct concatenation if they have a tendency to start up in the same direction, the synchronous speed being then $= \frac{\text{cycles} \times 120}{p_1 + p_2}$, where p_1 is the number of poles of motor I and p_2 the number of poles of motor II. (See Fig. 1).

When the two motors tend to start in directions opposite to each other, they are connected in differential concatenation and have a synchronous speed of $\frac{\text{cycles} \times 120}{p_1 - p_2}$ (Fig. 2).

We shall first investigate the changes of slip, frequencies, voltages, and magnetizing current in each motor when they are connected either in direct or differential concatenation and running at different speeds.

NOTATIONS

n_1 = Synchronous speed of motor I, running as single motor.
n_2 = Synchronous speed of motor II, running on secondary circuit of motor I.
n = Speed of the motor set.
c_1 = Frequency in the line circuit.
c_2 = Frequency in secondary of motor I and primary of motor II.
c_2' = Frequency in secondary of motor II.
e_1 = Voltage on primary of motor I.
e_2 = Voltage induced in secondary of motor I.
e_2' = Voltage induced in secondary of motor II.
p_1 = Number of poles in motor I.

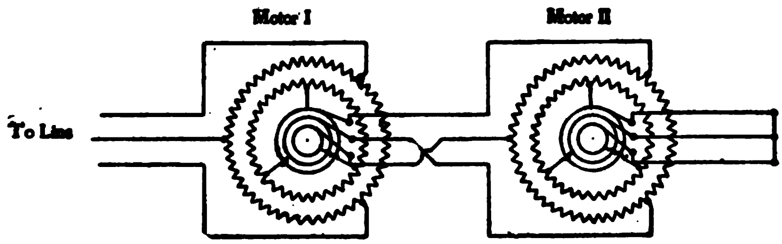


FIG. 2.—Concatenation diagram of motors arranged in differential concatenation

p_2 = Number of poles in motor II.
t_1 = Winding turns in primary of motor I.
t_2 = Winding turns in secondary of motor I.
t_1' = Winding turns in primary of motor II.
t_2' = Winding turns in secondary of motor II.
s_1 = Slip of motor I.
s_2 = Slip of motor II.
s = Slip of motor set.

The slips of motor I and motor II may be expressed by:

$$s_1 = \frac{n_1 - n}{n_1} = 1 - \frac{n}{n_1} \tag{1}$$

$$s_2 = \frac{n_2 \mp n}{n_2} = 1 \mp \frac{n}{n_2} \tag{2}$$

The minus sign in the last equation is to be used when motors are connected in direct concatenation and the plus sign when motors are connected in differential concatenation.

The synchronous speeds of motor I and motor II may be determined from

$$n_1 = \frac{c_1 \times 120}{p_1} \tag{3}$$

$$n_2 = \frac{c_2 \times 120}{p}$$

In the latter formula the frequency (c_2) in the primary of motor II is equal to $c_1\, s_1$ and n_2 may then be expressed by:

$$n_2 = \frac{c_1\ .\ s_1\ .\ 120}{p_2} \tag{4}$$

Formulas (1) and (2) combined, give

$$s_2 = \frac{n_2 \mp n_1\,(1-s_1)}{n_2} = 1 \mp \frac{n_1}{n_2}\,(1-s_1) \tag{5}$$

Formulas (3) and (4) combined, give

$$\frac{n_1}{n_2} = \frac{1}{s_1} \cdot \frac{p_2}{p_1}$$

and this latter equation placed into formula (5) gives

$$s_2 = 1 \mp \frac{1-s_1}{s_1} \cdot \frac{p_2}{p_1} \tag{6}$$

or

$$s_1 = \frac{1}{1 \mp (s_2 - 1)\,\frac{p_1}{p_2}} \tag{7}$$

These last two equations, (6) and (7), show how the slip of one motor depends on the slip of the other motor. If, for instance, the slip of motor I equals 1, the slip of motor II will also equal 1; if $s_1 = 0$ then s_2 will equal infinity.

Since the primary of motor II is connected to the secondary of motor I, the frequency in both these members is the same, and equals:

$$c_2 = s_1 c_1 \tag{8}$$

The frequency in the secondary of motor II is

$$c_2' = s_2 c_2 = s_1 s_2 c_1$$

Replacing s_2 by its value in formula (6), we obtain

$$c_2' = s_1 c_1 \left(1 \mp \frac{1-s_1}{s_1} \cdot \frac{p_2}{p_1}\right)$$

or

$$c_2' = c_1 \left[s_1 \mp (1-s_1) \frac{p_2}{p_1}\right] \tag{9}$$

Assuming that the drop of voltage in the two motors is comparatively small and may, therefore, be neglected, the induced voltages in the windings of the two motors at different speeds may be expressed as follows:

$$e_2 = s_1 \cdot \frac{t_2}{t_1} \cdot e_1 \tag{10}$$

$$e_2' = s_2 \cdot \frac{t_2'}{t_1'} \cdot e_2 = s_1 s_2 \cdot \frac{t_2}{t_1} \cdot \frac{t_2'}{t_1'} \cdot e_1$$

The value of s_2 from formula (6) inserted, gives

$$e_2' = \frac{t_2}{t_1} \cdot \frac{t_2'}{t_1'} \cdot s_1 \cdot \left(1 \mp \frac{1-s_1}{s_1} \cdot \frac{p_2}{p_1}\right) \cdot e$$

or

$$e_2' = \frac{t_2}{t_1} \cdot \frac{t_2'}{t_1'} \cdot \left[s_1 \mp (1-s_1) \frac{p_2}{p_1}\right] \cdot e_1 \tag{11}$$

The above equations, (10) and (11), give the values of e_2 and e_2'. To obtain the exact values of the voltages e_2 and e_2', it would be necessary to know the magnitude as well as the direction of the current vectors in the different members and the ohmic as well as the inductive resistance of the windings. This would mean a simple geometrical summation of voltages.

Let us now ascertain how the magnetizing currents change with the speed. When motor I is running as a single motor, the line circuit has to excite this motor only, and as is generally known, the magnetizing current for a given motor will be constant if the induced voltage and frequency remain constant, independent of the speed. Since in direct or differential concatenation the second motor is connected to the first, the line circuit must also furnish the exciting current for this second motor. From formulas, (8) and (10), we know that the induced voltage in the secondary of motor I (which at the same time is the voltage on the primary of motor II) changes in the same ratio as the frequency, the drop in the windings being ignored as before. Under no-load condition, to neglect this drop will not lead to any appreciable error.

Since the voltage e_2, on the primary of motor II changes directly with the frequency c_2, the magnetizing or no-load current in this motor will also remain constant at the different speeds. The exception to this is, that when motor I is running at its synchronous speed, the frequency and voltage of the primary of motor II become zero, with the result that the magnetizing current also becomes zero. And, as the vectors of the magnetizing currents of the two motors are practically in the same phase, they may be added numerically after being reduced to equal voltage. This sum would represent the total magnetizing or no-load current, and would be the same whether motors were connected in direct or differential concatenation.

To prove the correctness of the above formulas, tests as follows could be made:

(A) Motors Connected in Direct Concatenation

Secondary of the second motor open-circuited, and the set driven mechanically by a third motor of variable speed characteristics. At different speeds the frequencies, voltages, and amperes to be taken in all members, the voltage and frequency on the primary of motor I to be held constant.

Example of actual test and calculation. Both motors wound 3-phase, of which motor I was designed for 8 poles, 50 cycles, 400 volts and motor II for 4 poles, 16⅔ cycles. 77 volts. The ratio of turns in motor I was

$$\frac{t_2}{t_1} = 0.58$$

and that of motor II was

$$\frac{t_2'}{t_1'} = 0.66.$$

Fig. 3 represents the different values when motors are connected

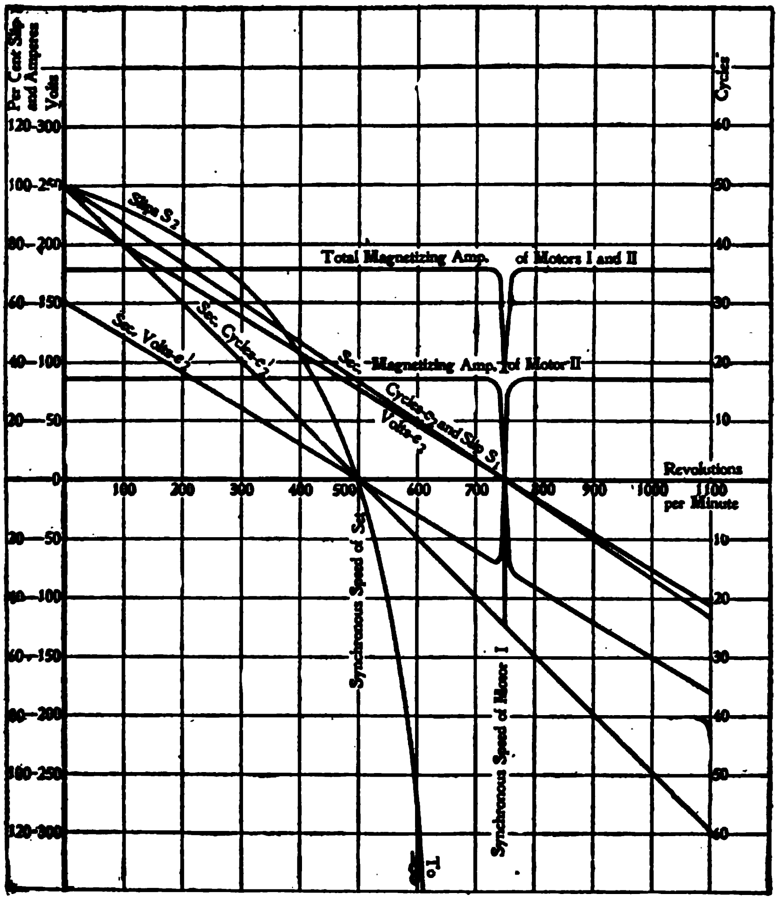


FIG. 3.—Motors connected in direct concatenation. Secondary of motor II open-circuited

in direct concatenation. These values are derived from the previous formulas and have been proved by actual test.

The conditions at zero speed are practically identical with those of a stationary transformer connected in cascade. With increase of speed the induced secondary voltages (e_2; e_2') and the frequencies (c_2; c_2'), of motors I and II, the slip (s_1) of motor I and slip (s_2) of motor II decrease. At the synchronous speed of the cascade set (500 rev.per min.) e_2', c_2', and s_2 drop to zero and above that speed they become negative, which indicates that motor II acts as an induction generator.

At 750 rev.per min., motor I reaches its synchronous speed and it follows that e_2, c_2, and s_1 become zero. Consequently the secondary frequency, the voltage, and the magnetizing current of motor II should drop suddenly to zero, which, in fact, they do as is found by actual test. (In Fig. 3 the magnetizing current of motor II is reduced to primary voltage of motor I).

Formulas (9) and (11), however, do not show this, because the drop is not taken into consideration and because our previous assumption is correct only for relatively small drops. In the vicinity of the synchronous speed of motor I, (750 rev.per min.) the induced voltage in its secondary is very small and as the magnetizing current of motor II remains constant, if the voltage increases in the same ratio as the frequency, the drop in voltage amounts here to quite a percentage. Even if there were no losses in the motors, it might be of interest to note that our mathematical formulas do not agree fully with the physical laws.

As the magnetizing current of motor II at 750 rev.per min. drops abruptly to zero, the primary current of motor I is equal to the magnetizing current of motor I. This was also proved by test.

By increasing the speed of the cascade set still further, *i.e.*, above 750 rev.per min., motor I becomes a generator and motor II again a motor.

(B) Motors Connected in Differential Concatenation

Tests to be made in the same manner as in (A). The results of test and calculations are shown in Fig. 4. We learn from the curves in Fig. 4, that at zero speed the voltages (e_2; e_2') and the frequencies (c_2; c_2') are of full value and that the slips (s_1; s_2) of both motors are equal to 1 or 100 per cent. All these values, except the slip s_2 and the magnetizing currents, decrease in a certain relation to the speed; at 750 rev.per min., e_2, c_1, s_1 have become zero. The secondary voltage (e_2') and the frequency (c_2') at this speed drop abruptly from half their full values to zero, owing to the zero voltage and frequency in the primary of motor II.

The slip (s_2) of motor II increases from 1, slowly at first, but gradually faster and faster until infinity. Taking into consideration the several rotations of the fields as well as data given above, we find that up to this speed, 750 rev.per min., both machines act as motors. At 750 rev.per min., motor I runs at its synchronous speed; and since the cycles in the primary of motor II have dropped to zero and further, since motor II formerly ran in the direction opposite to its normal rotation, the speed of motor II corresponds to the infinite value.

Owing to the fact that in motor II the frequency and volts are zero, it is obvious that at the synchronous speed of motor I there is no tendency in the motor set itself to rise to the synchronous speed of the differential concatenated connection. Other means must be resorted to in order to obtain this speed. After the set has been speeded up it will stay there and will carry load. One way to speed up the set is to start it by using another motor; another way is to connect the motor having the smaller number of poles with the line and when the set reaches synchronous speed switch over to the normal operating connection.

It is not advisable to leave the motor having the smaller number of poles on the line because the frequency in the circuit connecting the motors will be of double magnitude. The total iron losses of the set would, therefore, be considerably greater, causing a drop in the efficiency and also causing a higher temperature rise.

Considering now the curves of the pass from 750 to 1500 rev.per min. (see Fig. 4), we observe that the secondary voltage (e_2) and the frequency (c_2) of motor I become negative and at 1500 rev. per min. reach their full negative value compared with the positive values at standstill.

The voltage (e_2') and frequency (c_2') in the secondary of motor II drop gradually to zero. The slip (s_2) of motor II has changed at 750 rev.per min. from + infinity to –infinity and has at 1500 rev.per min. the final value of zero.

In reviewing this data, bearing in mind the field rotations in both motors, we observe that at a speed above 750 rev.per min., motor II becomes an induction generator which forces a frequency into the secondary of motor I. This frequency has a field rotation opposite to that of the primary of motor I. The result is that the rotor of motor I must run at a speed corresponding to the sum of the primary and secondary frequencies (c_1 and c_2) in order to be in synchronism.

Since the primary voltage (e_2) of motor II varies directly with its frequency (c_{2},) the magnetizing current of motor II remains constant, ignoring the drop in the windings.

At 750 rev.per min., however, the magnetizing current of this motor will suddenly drop to zero owing to absence of voltage and frequency. As the magnetizing current of motor I is constant and

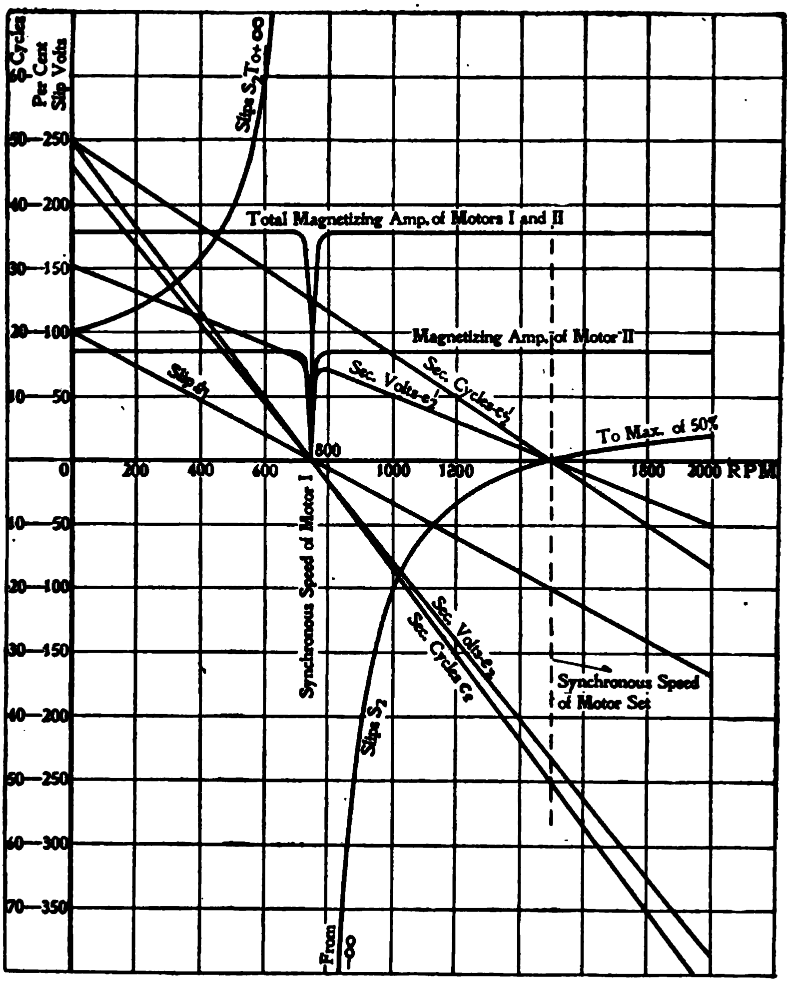


Fig. 4.—Motors connected in differential concatenation. Secondary of motor II open-circuited

has practically the same power factor as the magnetizing current of motor II, these currents can be added numerically (when reduced to the same voltage as is done in Fig. 4), to get the total magnetizing current of the set. This current will be the same as in direct concatenation.

Conditions with secondary of motor II closed. In the foregoing it has been shown that with motors connected either in direct or differential concatenation, running at their synchronous speed, the frequency and voltage in the secondary winding of motor II is zero. Therefore, the short-circuiting of this winding would not affect conditions in the other members. When the motor set is running on load the speed will be somewhat lower. In each winding there will be a certain frequency and voltage and, therefore, some current. The voltages, magnetizing amperes, and frequencies of all members may be taken from Figs. 3 and 4

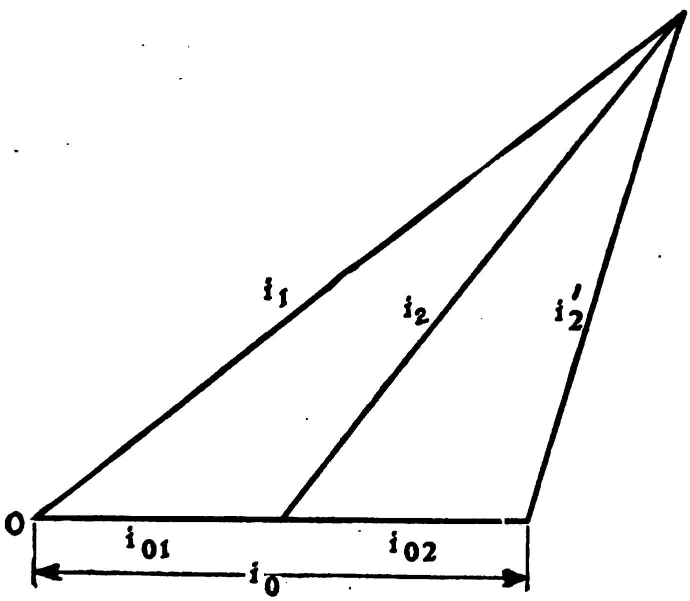


FIG. 5.—Vector diagram of currents in motor set

respectively, and by taking into consideration the drop in the winding, it is possible to determine the actual currents which flow. The primary current (i_1) in motor I is made up of its magnetizing current (i_{01}) and its secondary current (i_2) which form a triangle. The secondary current of this motor is at the same time the primary current of motor II and this current again is made up of the magnetizing current (i_{02}) and secondary current (i_2') of motor II, which also form a triangle. (See Fig. 5.)

The two vectors of the magnetizing current (i_{01}) and (i_{02}) are practically in line as mentioned before.

The inductive resistances and the corresponding induced voltages vary directly with the frequency, the ohmic resistances being generally small in comparison with the inductive resistances. Therefore, considering the voltages as constant, the inductive resistances, according to this theory, must also be considered constant. An induction motor is equivalent to a circuit with ohmic and inductive resistances, providing the winding turns are reduced to the same voltage. To a certain extent, this is also true of a motor set connected either in direct or differential concatenation as shown in Fig. 6.

In the above:

r_1 = Ohmic resistance in primary of motor I.
x_1 = Inductive resistance in primary of motor I
r_2 = Ohmic resistance in secondary of motor I.
x_2 = Inductive resistance in secondary of motor I.
R_1 = Ohmic resistance in secondary representing resistance corresponding to load of motor I.

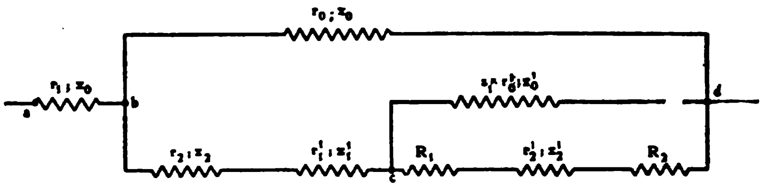


FIG. 6.—Schematic diagram of motor set

r_1' = Ohmic resistance in primary of motor II.
x_1' = Inductive resistance in primary of motor II.
r_2' = Ohmic resistance in secondary of motor II.
x_2' = Inductive resistance in secondary of motor II.
R_2 = Ohmic resistance in secondary representing resistance corresponding to load of motor II.
r_0 = Ohmic resistance representing the factor of iron loss in motor I.
$s_1 r_0'$ = Ohmic resistance representing the factor of iron loss in motor II.
x_0 = Inductive resistance representing the factor of the magnetizing current in motor I.
x_0' = Inductive resistance representing the factor of the magnetizing current in motor II.

In Fig. 6 everything remains constant except $s_1' r_0'$, R_1 and R_2. The resistance representing iron loss in motor II varies directly with the slip of motor I approximately. R_1 and R_2 vary inversely with the slip (s) of the set approximately.

It is apparent in Fig. 6, that with increase of load or increase of amperes, the voltage across the points ($c\,d$) decreases on account of the drop in voltage in the section ($a\,c$). This results in a decrease of magnetizing current in motor II. The question now is, what will this change amount to. According to formula (10), the voltage (e_2) of the secondary of motor I, (which is also the primary voltage of motor II), depends mainly on the slip (s_1) of motor I and this slip depends in turn on the ratio of the number of poles in both motors and on slip (s_2) of motor II. If for instance, the number of poles in motor II is small in comparison with the number of poles in motor I, the slip (s_1) will be small, as will be also the voltage (e_2). Therefore, the drop in the windings will be proportionately large ; *i.e.*, the change in the magnetizing current (i_{02}) will be great with the variations of load. In most cases, however, the ratio of the number of poles in motor II to those of motor I is not greater than 1 to 3, and if motors are prooerly designed the magnetizing current in motor II will remain practically constant within those limits which are of interest in determining performances. Assuming that our conclusions are as nearly correct as commercial use requires, and providing that the slip of the motor set is not too large, we can consider the two motors as a single one; *i.e.*, at different loads the factors in Fig. 6 will remain constant with the exception of R_1 and R_2.

It would now be an easy matter to demonstrate that the extremities of the current vectors for the different loads travel on a circle.[1]

To draw up the diagram circle the following test data are required:

1. Primary and secondary amperes and primary watts of motor I, running in direct concatenation (or differential concatenation as the case may be) at full voltage.

2. Primary amperes and watts with motor set locked, to be taken at any voltage, preferably at approximately half voltage, these tested values to be reduced to full voltage.

3. Secondary voltages of motor I and II, when standing still and secondary of motor II open-circuited.

4. Ohmic resistance of primary and secondary windings of

1 *Electrical World & Engineer*, Feb. 23, 1905, "Practical Diagram for Induction Motors."

motor I and of primary winding of motor II. All resistances to be reduced to the primary of motor I.

To show how the diagram circle is obtained from few test values and how the performances are derived from the same, the following example is made use of:

Motor I, 60 h.p., 400 volts, 50 cycles, 8 poles, primary as well as secondary wound, three-phase in star.

Motor II, 60 h.p., 77 volts, 16⅔ cycles, 4 poles, primary as well as secondary wound, three-phase in star.

These two motors were connected in direct concatenation, the primary of motor II being connected to the secondary of motor I, and the primary of motor I to a line circuit of three-phase, 400 volts, 50 cycles, and finally the secondary of motor II short-circuited.

(1) No-Load Readings

i_0 = 71.4 amperes, total in primary of motor I.

i_{02} = 34 amperes, total in secondary of motor I, reduced to primary winding.

P_0 = 3.1 kw. no-load losses.

(2) Locked Readings

i_L = 412 amperes in primary of motor I.

P_L = 64.6 kw. in primary of motor I.

(3) Motors at Rest, Secondary of Motor II Open-circuited

Voltage $e_2 = 232$ and $e_2' = 153$.

(4) Resistance per leg Reduced to Primary Winding of Motor I and to a Temperature Corresponding to Continuous Full-Load Run

r_1 = 0.11 ohms in primary of motor I.

r_2 = 0.104 ohms in secondary of motor I.

r_1' = 0.064 ohms in primary of motor II.

r_2' = 0.046 ohms in secondary of motor II.

In Fig. 7, the vertical axis represents the direction of the voltage impressed into primary of motor I; the quadrant *A B* with its center *O* and with a radius of 100 parts represents the power-factor circle.

To determine the center of the diagram circle, first lay off to a convenient scale the no-load and locked current vectors

$$O\,C_0 = 71.4 \text{ amperes},$$

and

$$O\,C_L = 412 \text{ amperes},$$

in the direction of their respective power-factors.

$$\cos \phi_0 = \frac{P_0 - [i_0^2\, r_1 + i_{02}^2\, (r_2 + r_1')]}{i_0 \cdot e_1} = 0.0825$$

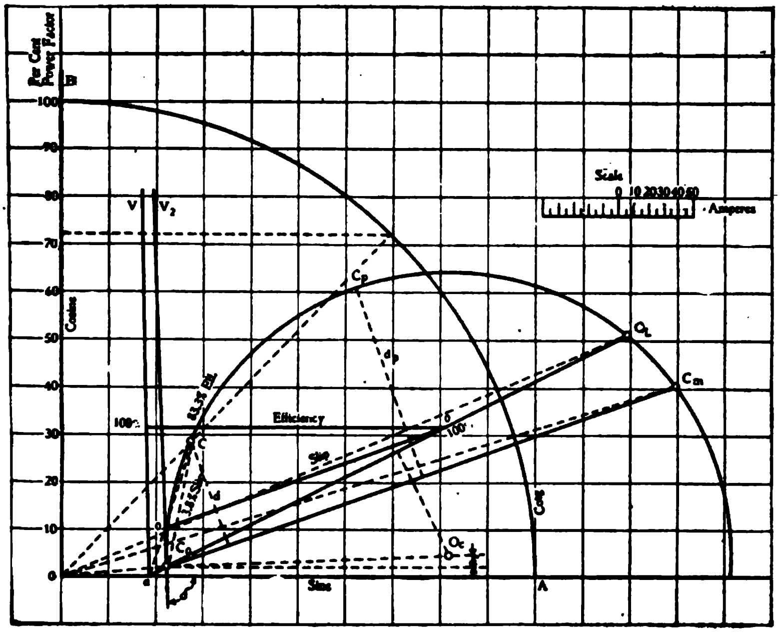


FIG. 7.—Circle diagram

$$\cos \phi_L = \frac{P_L}{i_L \cdot e_1} = 0.392$$

Then from C_0 draw the line $C_0\,O_c$ making an angle ϵ with the base line

$$\tan \epsilon = \frac{i_0\, r_1 + i_{02}\, (r_2 + r_1')}{e_1} = 0.0339$$

The intersection of the line perpendicular to the middle of $\overline{C_0\, C_L}$ with the line $\overline{C_0\, O_c}$ is the center of the diagram circle. The line $\overline{C_0\, C_L}$ represents the base line for the output; *i.e.*, the distance from any point on the circle above $\overline{C_0\, C_L}$ to this line $(\overline{C_0\, C_L})$ multiplied by a certain constant gives the horse power output for the corresponding current vector.

The line which connects the points C_0 and C_m represents the base line for torque; the point C_0 is already known and point C_m can be determined by

$$\text{cotan}\, \phi_m = \frac{r_1 + r_2 + r_1'}{X_1 + X_2 + X_1' + X_2'}$$

Herein is:

$$X_1 + X_2 + X_1' + X_2' = \frac{\sin \phi_L \,.\, e_1}{i_L} = \frac{0.92 \times 400}{412} = 0.893 \text{ ohms}$$

$$\text{and } r_1 + r_2 + r_1' = 0.278 \text{ ohms}$$

Therefore:

$$\text{cotan}\, \phi_m = \frac{0.278}{0.893} = 0.311$$

The torque for any current vector is equal to the distance from the corresponding point on the circle to the line $(C_0\, C_m)$ multiplied by a certain constant which may be easily calculated from:

$$\frac{5250 \times \text{horse power output}}{\text{speed}} = \text{lb. torque at 1 foot radius.}$$

Tangent $(C_0\, V_2)$ to the current circle at C_0 is the base line for the secondary copper losses. Therefore a line with a convenient scale of 100 parts drawn between $\overline{C_0\, V_2}$ and $\overline{C_0\, C_L}$ and parallel with $(\overline{C_0\, C_m})$ gives the slip scale. A line from C_0 through any current point on the circle indicates the percentage of slip on the latter scale, reading from left to right.

A line $(a\, V)$ drawn through the intersection (a) of line $\overline{C_0\, C_L}$ with $\overline{O\, A}$ and which makes approximately an angle with $\overline{O\, A}$ equal to $\frac{1}{2}$ $(90° + < \delta)$, (δ being the angle of $\overline{V\, C_0}$ with $\overline{O\, A}$) represents the base line for the total losses. Therefore, a convenient scale between this line and $\overline{C_0\, C_L}$ and parrallel to $\overline{O\, A}$ represents the efficiency scale, *i.e.*, a line from (a) through any current point of the circle will indicate on the scale the percentage of efficiency. As an example Fig. 7 shows the results for point C only, which are as follows:

$\overline{OC}$ = 124 amp. (total); efficiency = 83.3 per cent.; slip = 3.8 per cent.; power factor = 72 per cent. From these we calculate

$$\text{Horse power output} = \frac{\text{amperes} \times \text{volts} \times \text{efficiency} \times \text{power-factor}}{746}$$

$$\text{Horse power output} = \frac{124 \times 400 \times 0.833 \times 0.72}{746} = 40$$

$$\text{Horse power input} = \frac{\text{amperes} \times \text{volts} \times \text{power-factor}}{746} = 48.2$$

Speed of motor set = 500 (1—0.038) = 481 rev. per min.

$$\text{Lb. torque at 1 ft. radius} = \frac{5250 \times \text{horse power output}}{\text{speed}}$$

$$\text{Lb. torque at 1 ft. radius} = \frac{5250 \times 40}{481} = 437$$

Data for all other loads may be obtained in the same way as in above. The maximum torque; *i.e.*, the pull-out torque of the motor set, is equal to the maximum distance d_p multiplied by a constant, which might be determined from the above load of

point C constant $= \frac{\text{lb. torque}}{d} = \frac{437}{d}$. Consequently maximum

torque $= \frac{d_p}{d} \times 437$ = 788 lb. at 1 ft. radius.

All the data derived from this diagram checked up to within a fraction of 1 per cent. with the results obtained by test and by calculation.

In the same way as described above we may obtain the data for motors connected in differential concatenation.

Comparing now these results with those which might have been obtained from two independent motors, one having 12 poles and the other 8 poles, and together having the same amount of material as the complete cascade set, we find the following: The efficiency and power-factor will be somewhat higher if each motor has the same maximum torque as the cascade set. Nevertheless, the cascade set has certain advantages over a set composed of two independent motors. A few of the more important points of advantage are: greater latitude in design, a more flexible and a simpler speed control, and safer operation.

For instance, in a cascade set the individual motors may be designed so as to have greater width and smaller diameter, thus keeping the peripheral speed lower than would be the case with a single motor for the speed of direct concatenation. In cascade connection each motor tends to balance the other, consequently the speed regulation is more smooth; that is, the change from slow to high speed or from high to slow speed can be made gradually without any mechanical jarring or electrical choking effects. For example, a change from slow to high speed can be accomplished by inserting a high resistance across the circuits connecting motors I and II and then cutting out resistance gradually with the increase in speed until the set reaches the normal speed of motor I, or any other between that of synchronism of motor I and that of cascade connection as may be desired; all other connections to remain unchanged. In a case where three or four different speeds are wanted, it is obvious that a cascade set can be built very much more cheaply than three or four independent motors. For instance with two motors, motor I having 10 poles and motor II 6 poles and at a line frequency of 60 cycles, the following synchronous speeds can be obtained:

1. Motor 1 and II in direct concatenation 450 rev. per min.
2. Motor I runs single 720 rev. per min.
3. Motor II runs single 1200 rev. per min.
4. Motor I and II in differential concatenation 1800 rev. per min.

It should be noted that the above speeds are possible with a set consisting of a 10-pole motor and a 6-pole motor. Other combinations of speeds may, of course, be obtained by changing the number of poles. However, in any combination of motors the speeds have a certain arbitrary relation to each other. For this reason the four speeds obtained might in some cases not be the ones desired; that is, some of them could be the ones wanted while the others would not be.

In comparing the different methods of varying speeds of induction motors we find that each method has some advantage over the others when considered in connection with certain classes of work. But at the same time, there are certain disadvantages to be found in all these methods. The operation of a cascade set is simple and safe even if the efficiency and power factor are not altogether the best, on account of the increased inductance.

The elimination of this objection depends, it would seem, on the development of an efficient and practical phase compensator.

DISCUSSION ON "INDUCTION MOTORS FOR MULTISPEED SERVICE WITH PARTICULAR REFERENCE TO CASCADE OPERATION."
ATLANTIC CITY, N. J., JULY 1, 1908

W. I. Slichter: There is a point in the subject of cascade connection of induction motors which the speaker would like to hear discussed, as it is particularly important in connection with railway engineering. The usual and first idea of the result of the cascade connection of induction motors is that it will make it possible to obtain from a given pair of motors a greater torque at a low speed than is obtained in normal operation. But a closer study of this subject shows that the maximum torque of two motors connected in cascade is always less than that of either one of the individual motors. This is a proved fact with regard to the operation of motors with like numbers of poles, but it would be interesting to know what the relation is with motors having different numbers of poles.

A. E. Averrett: I have found that, in general, concatenated coupling, when used for a very small range of speed, will work out quite nicely, as it is comparable to a relatively small change of voltage, with a compensator or autotransformer, but as the range of speed increases, the effectiveness of the concatenated set decreases. The second motor has all its power carried through the first motor, and the first motor will have a great impedance drop, so that the effective voltage of the second motor, after the current has passed through the impedance of the first motor, is low. Furthermore, the concatenated motor acts similarly to a single two-speed motor. The lower the speed of any induction motor, the lower the power-factor; that is, in case the speed reduction is by a change of poles. A constant-speed, collector-ring, induction motor will maintain approximately constant torque with an efficiency in proportion to the speed; a concatenated or multispeed motor, where the change of speed is made by shortening the pole pitch, will maintain fairly constant efficiency, but the power factor goes down with the speed reduction. In concatenated motors the problem is nearly as much a mechanical as an electrical problem; the diameters become small, distances between bearings centers increase, and with the slow speed which we are really trying to maintain, the question of ventilation becomes important. There was recently built a concatenated set for a very peculiar service, rolling mill work, where the concatenated motors were 20 and 16 poles, and the combination 36 poles. It happened that the power required for the actual concatenation work, at 36 poles, was very slight, the main power being at the 20- and 16-pole speeds.

Most motor work will require constant torque or perhaps torque increasing as the speed drops (constant power). In these cases, the low power-factor of the concatenated motor renders it extremely difficult to make a good combination at the low speed; but for a close range to the normal speed, as before stated, the concatenated motor works very well.

Elmer A. Sperry: Can anybody here throw light on the subject of returning power to the line when this class of motor is employed in traction service, coupled as described?

H. C. Specht: It has been asked if two motors connected in cascade would give any more torque than one of the motors. This certainly is not possible unless auxiliary devices are used. The motors connected in cascade will have somewhat less, or

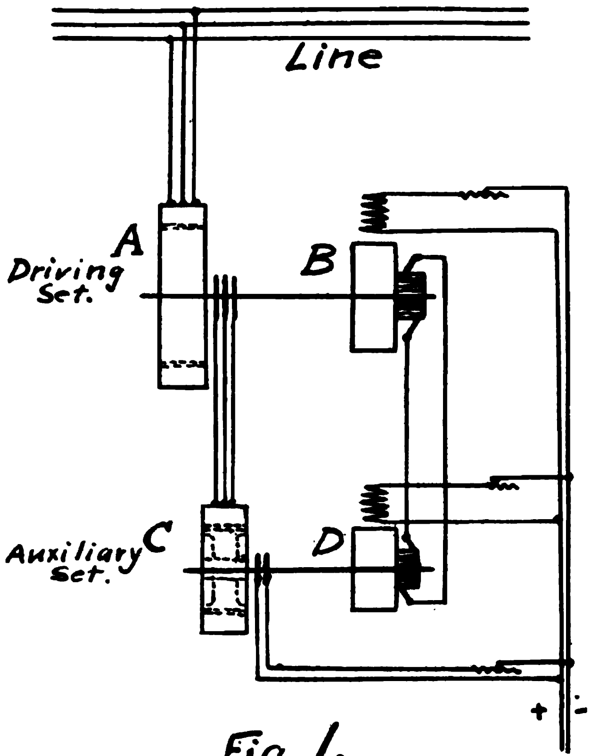


Fig. 1.

not more torque, than the average of the two motors, and the reason is simply that the current has to pass through both motors, which increases the leakage. As was mentioned at the end of my paper, it is a problem of finding a good phase compensator, to eliminate the most objectionable features of cascade connections.

In regard to compensated cascade sets, I would like to mention that several schemes have been suggested in the last few years. Whether or not any of these schemes will be adopted for general use and give satisfactory results, is yet an open question.

The method generally used for compensation is that of connecting a synchronous motor or a compensator induction motor in the secondary circuit. The motors are then regulated so as to produce a leading current. Since this current, with the primary current and the magnetizing current of the main motor, form a triangle, it is possible to raise the power-factor not only to one but to obtain an over compensation or leading current.

The following will serve as examples of the methods referred to and as described recently by Heyland, Willner, and Kubler in German technical periodicals.

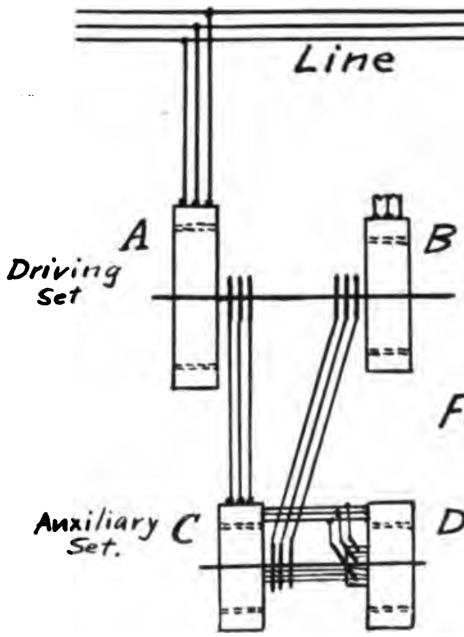


Fig. 2.

Fig. 1 shows an arrangement with a synchronous motor (C) in the secondary circuit. On the same shaft with the latter motor is a direct-current generator which supplies current to the second driving motor (B), which is, in this case, a direct-current motor. The direct-current machines and the synchronous motor are excited from a separate direct-current circuit.

In Fig. 2, compensation is accomplished by means of a compensated induction motor (D) connected to the secondary circuit of the main driving motor and to the other induction motor (C), which latter is mounted on the same shaft as the compensated motor, and serves as a frequency changer.

The second driving motor (*B*) is also an induction motor and is connected to the secondary circuit of motor (*C*).

In Fig. 3 is shown a method similar to that in Fig. 1, with the exception that the synchronous motor (*C*) is replaced by an induction motor, the secondary of which is connected through the collector rings to the armature winding of the driving motor (*B*).

The foregoing methods serve not only to compensate the power-factor, but also allow of starting up the tandem set

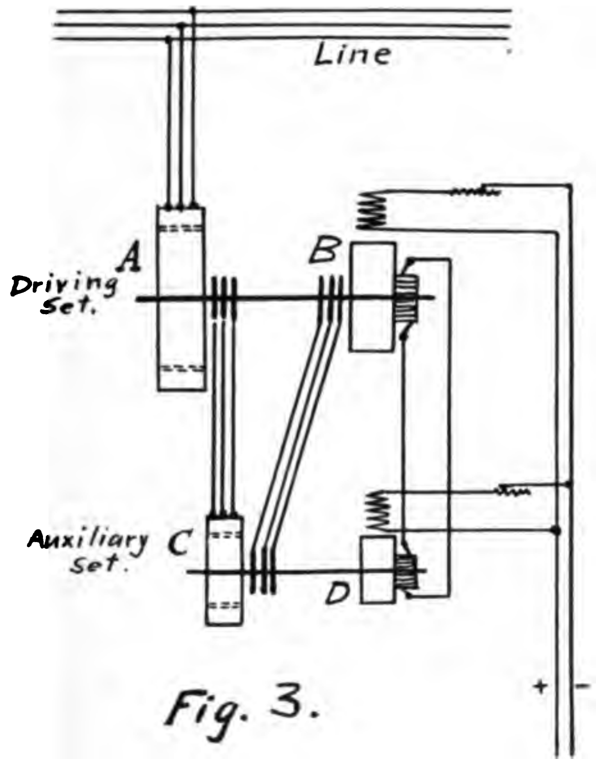


Fig. 3.

with a great deal less kilowatt input than would be required in the case of ordinary induction motors, on which a great deal of power is wasted on the secondary resistance. It likewise increases the maximum running torque over that of an ordinary cascade set.

In addition, the schemes as shown in Figs. 2 and 3, permit of reversing the set in a comparatively short time and with no sudden rush of current, as the inertia in the two driving motors is kept small. The flywheel on the auxiliary motors serves as a storage for kinetic energy. For obvious reasons these latter should be designed for high speed.

Further, we see that the methods above described allow of running tandem sets at variable speeds with the minimum waste of energy. In some respects they are similar to the Ilgner & Leonard reversing systems.

The method as shown in Fig. 3 would seem to contain possibilities in connection with hoisting, rolling-mill, and traction service.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

THE DETERMINATION OF THE ECONOMIC LOCATION OF SUB-STATIONS IN ELECTRIC RAILWAYS

BY GERARD B. WERNER

The method of attacking the problem of the location of substations in electric railways is governed by the physical layout of the road. The problem involved in the case of networks serving a limited territory, such as urban or suburban lines, may be solved from a study of the magnitude of the various relatively fixed load-centers which are created by the configuration of the lines; and the sub-stations may be placed at the different centroids of the system. In the case of long single roads connecting distant communities, such as interurban and trunk lines, there is presented the study of more or less uniformly loaded stretches, in which there are no distinct centers of load, except those that result from the characteristics of the line profile or the traffic movement. The latter problem is more susceptible to mathematical treatment, so that the contents of this paper will be confined to the consideration of interurban and trunk line projects, or, in other words, to relatively long roads.

Of the various considerations that govern the layout of the secondary distribution of electric railways, the question of economy is generally the preponderant influence. It is obvious that the economic condition should be determined as a basis of reference, and approximated as far as is consistent by the actual design. The theoretical economic solution, however, may be difficult or impossible of attainment, if certain operating considerations of a mechanical or an electrical nature intervene and dictate the use of a cross-section of secondary copper other than that determined by Kelvin's law. For example, the economic drop may exceed the limiting drop permitted for the acceleration of the motors, under which conditions economy would give precedence to regulation. In the present problem, as will appear shortly, for a given mean square current per car, a given cost of copper, and a given unit cost of energy, the drop, according to Kelvin's law, is inversely proportional to the square root of the number of annual car-hours. The cross-section of the secondary copper is, therefore, dependent on the number of cars or trains in service, the length of line, and the schedule speeds. Thus for infrequent service, the economic drop may be excessive and result in stalling the cars; similarly, for dense traffic movements, the economic drop may be insufficient and render the resulting investment prohibitive. Kelvin's law is based on the mean power lost, while the operating criterion is based on the maximum drop.

Besides these technical considerations, which may be decisive in the design of the distributing copper, a number of commercial considerations exert a pronounced influence on the location of the sub-stations. In this category may be included the question of placing the sub-station at or near a railway station, in order to utilize the station employes for emergency switchboard operation, or to provide accessibility for periodical inspection, in the absence of continuous attendance. Thus, the sub-station may be somewhat removed from the site determined by purely economic reasons.

The specific purpose of this paper is to develop an equation for the number of sub-stations, or the distance between sub-stations, which will render the total annual charges on the installation a minimum. This equation is to be in terms of the various constants fixed by the length of line, time-table, the weight of cars or trains, the motor characteristics, the cost of energy, and the equipment charges.

The initial step in the treatment of this problem is the determination of the analytical expressions of the various annual charges as functions of a single variable, the number of sub-stations. All those items of the annual charges which may be regarded as constant and independent of the number of sub-stations, may be omitted in the analysis. Thus, assuming that it is not necessary to feed the sub-stations with separate high-tension lines, the weight of copper and the losses in the primary distribution are dependent only on the energy to be transmitted and the mean distance, so that the primary transmission may be eliminated from consideration. Obviously, the charges on the trolley suspension and insulation do not enter. The following algebraic expressions are therefore required:

1. Annual charges on sub-stations,
2. Annual charges on overhead copper,
3. Annual cost of sub-station losses,
4. Annual cost of secondary conductor losses.

These four items are present whether the system employed is continuous, single-phase, or three-phase current. For the purposes of this paper, it will be sufficient to develop the formula for the simplest layout. The single-phase system has been elected, therefore, to illustrate the general method of deducing the equation, although by the introduction of the proper modifications in the analytical expressions (to cover such features as the third-rail, synchronous converters, attendance, etc., in the continuous-current system, and the two overhead conductors, three-phase transformation, etc., in the three-phase system) the results of this analysis will be equally applicable to all.

Annual charges on sub-stations. The first point to be determined is the sub-station capacity. In general, if a line contemplates electrification we may assume that the frequency of the various traffic movements dictated by the present or prospective time-table results in a more or less uniformly distributed load along the line, and consequently the load between adjacent sub-stations varies more or less directly with the length of the sub-station section. Thus, neglecting for the moment the question of reserve, the capacity of each sub-station will be proportional to the distance between sub-stations, the aggregate sub-station capacity remaining constant, and being governed by the total maximum load occurring on the line at one time.

In those classes of service where the traffic density through the varying exigencies of the train-weights, time-table, or profile, is not uniform along the entire line, it is necessary to consider individual *load-sections* rather than general *distance-sections*. When this condition exists, the line may be broken up into divisions where the load per mile is approximately constant, each particular division segregated, and treated separately for the determination of the sub-station capacity and the distance between sub-stations.

The question of spare capacity is influenced by the kind of transformation between the transmission and the distribution.

A three-phase primary may be incumbent on account of one or more of the following reasons: 1. The railroad may desire to sell surplus power to various industrial establishments where single-phase current is not acceptable. 2. The additional cost of single-phase generation may not offset the advantages accruing from the simplicity of the switching gear. 3. The railroad may, from compelling commercial reasons, desire to purchase its power from an outside company.

With a three-phase primary transmission we are practically limited to two methods of transformation to the secondary distribution. 1. Sub-stations transforming to single phase and feeding successive sections with separate phases, or 2, sub-stations transforming to two-phase and feeding adjacent sections with either phase. In the first case, two transformers are required in each sub-station, so that the same phase may be supplied to the common section extending between them. In the second case, two transformers are also required, so that the sub-station capacity will be divided between at least two units, with a possible third as a spare unit.

With the assumption made above of the uniformity of the distribution of load along the line either (*a*) for the entire stretch between terminal, or (*b*) for the particular load-section in question, and further assuming that the transformers are capable of withstanding momentary overloads of 150 per cent., the following expression results:

$$K\,W = \frac{0.4\,P\,(1+q)}{s} \tag{1}$$

where

$K\,W$ = required capacity of one sub-station in kilowatts,

P = total maximum power input in kilowatts required at one time on the line,

q = spare capacity in terms of the capacity actually required

s = number of sub-stations

One transformer will have a capacity of

$$K\,W' = \frac{0.2\,P}{s} \tag{2}$$

The cost of a single sub-station consists of the cost of real estate (building and ground) which may be taken as independent of the capacity, and the cost of equipment, which is a function of the capacity. The equipment cost comprises the cost of transformers, switchboard, wiring and protective apparatus. The cost of the transformers for a given type, voltage and frequency in sizes between 150 kw. and 500 kw. may be taken as a linear function of the output as follows:

$$F_t = 2\,(K_1 + K_2\,K\,W')\,(1+q) \qquad (3)$$

where K_1 and K_2 are constants fixed by the manufacturer.

The remainder of the equipment may be grouped as auxiliary, and, without appreciable error, taken at a fixed constant value irrespective of the kilowatt capacity. If it is further assumed that one average fixed charge will uniformly cover the different equipment items, and another the building and ground, the annual charges on the total sub-stations may be written down:

$$a_s = s\left[f_r\,F_r + f_e\left\{F_a + (1+q)\left(2\,K_1 + \frac{0.4\,K_2\,P}{s}\right)\right\}\right] \qquad (4)$$

in which

F_r = cost of building and ground
F_a = cost of switchboard, wiring, protective apparatus, etc.
f_r = fixed charges on building and ground as decimal
f_e = fixed charges on equipment as decimal

or, putting,

$$M_1 = f_r\,F_r + f_e\,\{F_a + (1+q)\,2\,K_1\} \qquad (5)$$

and

$$M_2 = f_e\,(1+q)\,0.4\,K_2\,P \qquad (6)$$

$$a_s = M_1 s + M_2 \qquad (7)$$

Annual charges on overhead copper. The overhead conductors may be proportioned according to Kelvin's law, if the losses therein can be computed without too much labor. To this end it is consistent to assume that the mean load on each sub-station is fixed at a certain distance, one-third of the length of the section, from the sub-station. Since the cross-section of the copper by Kelvin's law, for a given transmitted power, depends only on the cost of copper and the cost of the energy, the trolley can be determined for the economic condition independent of the distance that the power is transmitted from the sub-station to the car or train. However the cost of copper becomes a function of the distance between sub-stations through the fact that the losses in the overhead conductors per section vary as the car-hours per section or inversely as the number of sub-stations.

Letting

w = root of mean square load per car or train in apparent watts

v = trolley voltage

h = car or locomotive hours per year

s = number of sub-stations

r = resistance per mile of trolley wire

$D = T/s$ = length of one section in miles, the end-sections being one-half the length of the others.

T = total length of line in miles

k = cost of one kilowatt-hour at high-tension side of sub-station in dollars

ε = per cent. all-day efficiency of sub-stations

F_c = cost of copper in cents per pound

f_c = annual charges on copper as decimal

the following expression of Kelvin's law results:

$$\frac{w^2\, h\, D\, k\, r}{v^2\, 1000 \times 3\, s\, \varepsilon} = \frac{876\, D\, F_c\, f_c}{r\, 100} \tag{8}$$

the first member representing the annual cost of the energy lost section, and the second member the annual charges on the investment.

Solving for r, to obtain the cross-section of copper,

$$r = \frac{162\ v}{w} \times \sqrt{\frac{F_c\, f_c\, s\, \varepsilon}{h\, k}} \tag{9}$$

Substituting in the preceding equation the value for r, we obtain the total annual charges on the overhead conductors,

$$a_c = \frac{8.76 \times D\, F_c\, f_c\, s}{\dfrac{162\ v}{w} \times \sqrt{\dfrac{F_c\, f_c\, h\, k}{s\, \varepsilon}}} \tag{10}$$

or

$$a_c = \frac{0.0541\ T\ w}{v} \times \sqrt{\frac{F_c\ f_c\ h\ k}{s\ \varepsilon}} \tag{11}$$

or, placing

$$M_s = \frac{0.0541\ T\ w}{v} \times \sqrt{\frac{F_c\ f_c\ h\ k}{\varepsilon}} \tag{12}$$

$$a_c = \frac{M_s}{\sqrt{s}} \tag{13}$$

Annual cost of sub-station losses. Reverting again to the premises of the practical uniformity of line loading and the equal distribution of the total sub-station capacity along the line, it is apparent that under the assumption that the per cent. of iron and copper losses is constant for all sizes of transformers under consideration, the transformation of the given amount of energy required at the overhead conductor will entail transformer losses that will be constant and independent of the capacity or number of the individual sub-stations.

As a corollary to this conclusion, the sub-station all-day efficiency will be the same irrespective of the spacing of the stations. The error accruing from the assumption of constant transformer losses is almost negligible, and, furthermore, the omission of the cost of sub-station losses simplifies the algebraic operations in the determination of the condition for the minimum annual charges.

Annual cost of secondary conductor losses. The annual cost of the losses occurring in the overhead copper was computed from the expression of Kelvin's law. The secondary conductor losses should contain, to be rigorously exact, the losses occurring in the track-return, but the calculations show that these amount to only a small per cent. of the total yearly cost. This is readily seen from a comparison of the trolley and track resistances. To take the most unfavorable combination likely to occur in practice, assume an overhead wire of No. 0000 and 60-lb. rails. The ratio of track resistance to trolley resistance is 0.0422 : 0.259, or 0.163. To take the most favorable combination, No. 00 wire and 100-lb. rails, the ratio is 0.0253 : 0.412, or 0.0615. So that the track losses lie between about 6% and 16% of the copper losses.

The track losses will take a form similar to the first member of equation (8), all the factors entering being constants except the car-hours per section, and, as will appear farther on, the retention of a term with s in the denominator will render the expression $d\,a/d\,s$ an equation of the third degree. The resulting value of s, after the algebraic solution, would be rather cumbersome to evaluate from the numerical constants, and the sacrifice of mathematical precision will not materially vitiate the practical result.

Solution. With the algebraic expressions for the several items constituting the total annual charges on the sub-station and overhead copper investments, we may proceed with the solution. The total annual charges,

$$a = a_s + a_c + a_c \tag{14}$$

since the annual charges on the copper are equai to the annual cost of the losses.

$$a = s\left[f_r F_r + f_e\left\{F_a + (1+q)\left(2\,K_1 + \frac{0.4\,K_2 P}{s}\right)\right\}\right] + 2\left[\frac{0.0541\,T\,w}{v} \times \sqrt{\frac{F_c\,f_c\,h\,k}{s\,\varepsilon}}\right] \tag{15}$$

Or, using the corresponding expressions,

$$a = M_1\,s + M_2 + 2\,\frac{M_3}{\sqrt{s}} \tag{16}$$

From this equation it is seen that the curve of the total annual charges is the result of a curve superposed on a straight line. Differentiating,

$$\frac{da}{ds} = M_1 - M_3\,s^{-\frac{3}{2}} \tag{17}$$

Placing the differential equal to zero and solving for s

$$s^{\frac{3}{2}} = \frac{M_3}{M_1} \tag{18}$$

or

$$\log s = 0.667\,(\log M_3 - \log M_1) \tag{19}$$

Hence

$$\log D = \log T/s = \log T - 0.667\,(\log M_3 - \log M_1) \tag{20}$$

It will be seen on inspection that $\frac{d^2 a}{d s^2}$ will be greater than zero, so that the value of s given in equation (19) will render a a minimum.

The equation $\frac{d a}{d s}$ when multiplied through by s, shows that *for a minimum value of the annual charges, the cost of copper is equal to that part of the cost of the sub-stations which is independent of the output.*

Example: To illustrate the use of this equation, there is submitted herewith a certain interurban railway project. The line is single-tracked, 110 miles in length laid with 80-lb. rails. The traffic is a mixed passenger and freight, the former being handled by 40-ton motor cars, and the latter by 60-ton locomotives. The preliminary study of the calculated run-sheets shows the following:

Service	No.	Weight tons	Schedule speed m.p.h.	Train hours per day	Average kilowatt input
Through freight	2	900	15	14.6	275
Local freight	4	480	21	21.0	221
Express passenger	8	40	40	22.0	104
Local passenger	16	40	32	55.0	97
Baggage	4	50	34	12.9	120

Total train-hours per day, 126.
Average apparent kilowatt input per train, 176.

The energy is transmitted from a power company's plant at 66,000 volts, and is transformed to 3300 volts at the sub-stations, where it is sold to the railway company at the rate of one cent per kilowatt-hour.

The cost of each sub-station building and ground was estimated uniformly at $2000, the cost of the switchboard, wiring, and protective apparatus at $1800 per sub-station, the price of copper at 15 cents per pound, and the fixed charges on these three items at 7 per cent., 15 per cent., and 8 per cent., respectively. The cost of oil-insulated self-cooling transformers in sizes of 150 kw. to 500 kw. for 25 cycles 60,000 to 3,300 volts was found to be

$$F_t^1 = 1080 + 2.9\, K\, W'$$

Hence the following numerical values result:

$$
\begin{aligned}
f_r &= 0.07 \\
F_r &= 2000 \\
f_e &= 0.15 \\
F_a &= 1800 \\
K_1 &= 1080 \\
T &= 110 \\
w &= 176{,}000 \\
v &= 3300 \\
F_c &= 15 \\
f_c &= 0.08 \\
h &= 46000 \\
k &= 0.01 \\
\varepsilon &= 0.96
\end{aligned}
$$

Then, assuming that there will be no spare transformers,

$$M_1 = 140+594 = 734$$

$$M_2 = \frac{0.0541\times 110\times 176000}{3300}\times\sqrt{\frac{15\times 0.08\times 46000\times 0.01}{0.96}}$$

$$
\begin{aligned}
M_2 &= 317\times 24.0 = 7608 \\
\log\ M_2 &= 3.8813 \\
\log\ M_1 &= \frac{2.8657}{1.0156} \\
\log\ s &= 0.6771 \\
s &= 4.754
\end{aligned}
$$

which indicates that the economic condition would be about 5 sub-stations about 22 miles apart.

The required trolley section has a resistance,

$$r = \frac{162\times 3300}{176000}\times\sqrt{\frac{15\times 0.08\times 5\times 0.96}{46000\times 0.01}}$$

$$
\begin{aligned}
r &= 3.04\times 0.1118 = 0.34 \text{ ohms per mile} \\
r' &= 0.0643 \text{ ohm per 1000 ft.}
\end{aligned}
$$

which corresponds to a No. 000 wire.

Having established the number of sub-stations and the cross-section of the overhead conductor, it remains to be seen whether the consequent drop between the sub-station and the train is prohibitive. The graphic time-table shows that the maximum number of trains on the line simultaneously is as follows: one through freight, two local freights, eight passenger cars, and two baggage cars. With these trains in the most unfavorable

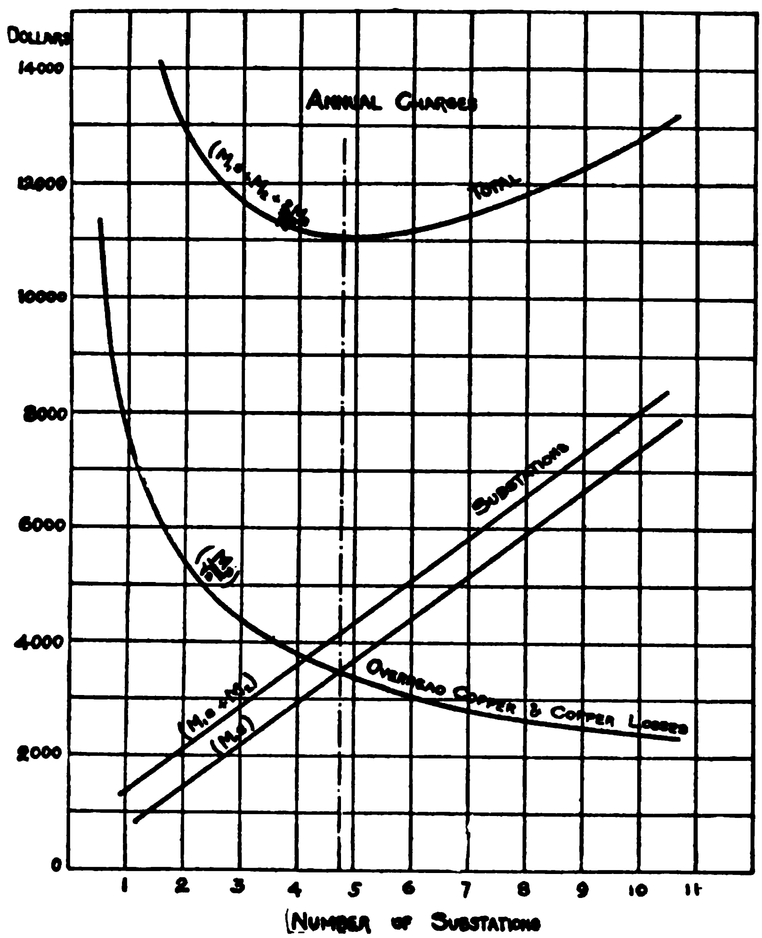


Fig. 1

position likely to occur, the total peak load is about 3700 kw. and the maximum input required between sub-stations is 1020 apparent kilowatts.

Assuming that this load is at the maximum distance from the sub-station, a single catenary construction and 80-lb. rails with a total drop (ohmic and inductive) of 11.7 volts per 100 amperes per 1000 ft. will give a momentary voltage-drop of

$$\frac{1020000 \times 11.7 \times 5.28 \times 22}{2 \times 3300 \times 100 \times 2} = 1050 \text{ volts}$$

or 32 per cent., which would be permissible, although it is somewhat beyond the usual working limit of 25 per cent.

With the numerical constants determined for the present problem, the loci of equations (7), (13), and (16) have been plotted in Fig. 1. These curves, within the approximations previously noted, represent the annual charges on the several items of the distribution. It will be seen that the lowest point of the curve of the total charge corresponds to that number of sub-stations at which the curve of sub-station charges ($M_1 s$) crosses the curve of charges on the secondary copper.

Conclusion. It is readily appreciated that certain approximations in the analytical expressions that make up the annual charges on the installation are of no great practical consequence. The introduction of complicated expressions would not enhance the accuracy of the actual result, since it is rarely possible to evaluate s into a whole number, to happen across a commercial transformer of the exact rating calculated, or to make the theoretical cross-section of the secondary copper conform exactly to the commercial sizes available for trolley wires and feeders. The combined influence of these considerations may more than offset the nicety attained by a more rigorous solution.

The use of the above formula for determining the economic location of sub-stations in single-phase railways constitutes a method which will obviate the necessity generally incumbent of solution by trial. The solution for the economic number of sub-stations, or the economic distance between adjacent sub-stations, is expressed in terms of those constants which are already available from the previous technical and commercial study of the service requirements. These numerical constants are evaluated for the particular problem at hand, and hence establish, within the scope of electric railroading problems defined at the beginning of this paper, the generality of the formula deduced.

DISCUSSION ON "THE DETERMINATION OF THE ECONOMIC LOCATION OF SUB-STATIONS IN ELECTRIC RAILWAYS." AT ATLANTIC CITY, N. J., JULY 1, 1908.

C. J. Hopkins: On December 15, 1905, a paper was read before this Institute on the most economical location of sub-stations for direct-current systems.* Mr. Werner's paper only differs from that, as far as I understand it, in its particular reference to single-phase systems.

Of course, we are all familiar with the form of curves given in Fig. 1. I would not criticise the accuracy of the formulas presented by Mr. Werner, but I question their general usefulness. As simple as it may sound, it appears to me that the most important feature in working out an electrification proposition is the proper consideration and formation of a train diagram, to represent what the service will be when the electrification is completed.

In the last paragraph Mr. Werner refers to the use of his formulas as obviating the necessity of solution by trial. This seems to imply that there might be a large number of solutions by trial necessary if that were the method resorted to, but in general there are only two or three practical solutions, and as a rule, they have to be worked out in detail and carefully considered.

The specific purpose of this paper is to develop an equation for the number of sub-stations, but I do not believe that such a determination is simplified by resorting to any formula of a general nature, but rather is the result of practical consideration and sound judgment.

Gerard B. Werner: The paper to which Mr. Hopkins refers was typical of the method of solution by trial, by evaluating the annual charges for a different number of sub-stations. Now the adjustment of the cost of sub-stations and the cost of copper, considered as an economical problem, is quite definite, and my paper is an attempt to develop some formula to obviate the trial method.

The economic condition of the secondary distribution, whether it can be executed in the actual layout of the road or not, should be determined as a basis of reference. This consideration is the primary intent of the paper, and not the substitution of a formula for judgment, for the handling of engineering problems of this character.

To the best of my recollection, Mr. Lincoln made a contribution to the proceedings of the International Electrical Congress in St. Louis in 1904 on a similar subject, and I have no doubt that he has some comment to make on this topic.

* "Some Considerations Determining the Location of Electric Railway Sub-Stations," by C. W. Ricker, *Transactions* A. I. E. E., Vol. XXIV p. 1097

P. M. Lincoln: I have not much to add to the discussion, although I have given this matter considerable thought. I am rather inclined to agree with Mr. Hopkins in his suggestion, that after having worked over this matter with a whole lot of formulas, the actual locating of sub-stations is not so much a matter of manipulation by formulas as it is a matter of judgment. It is safe to use the location obtained by formulas only when no other location is dictated by common sense.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 1, 1908.

CONDUCTOR RAIL MEASUREMENTS

BY S. B. FORTENBAUGH

General. The Underground Electric Railways Company of London, Ltd., control and operate electrically the Metropolitan District Railway and contingent lines, 56 miles of main-line double track, and three deep level " tube " railways, with connections, aggregating approximately 28 miles of double track. A detailed description of the electrification of the system, by the writer, appeared in the *Street Railway Journal* of March 4, 1905.

This company installed a " third " and " fourth " conductor rail on all of the above lines, both the conductor rails being of low carbon steel and supported on brown stoneware insulators. The insulators used in supporting the positive (outside) and negative (center) conductor rails are essentially of the same design on the individual roads, the top of the positive insulator being 1.5 in. higher than the negative. A malleable-iron cap and base is used on the District Railway insulators, the difference in height of the positive and negative insulator being partly in the depth of the insulation and partly in the design of the base.

No base or cap is used with any of the insulators on the Tube railways. The top of the positive conductor rail is 10 in. and 10.5 in. above the top of the sleepers on the District and tube railways respectively. Figs. 1 to 5 inclusive show the type of insulators, conductor rails, spacing, etc., as installed on the district and tube railways.

The direct-current track-circuit signal system has been installed on these roads and hence the desirability of not using the running rails as part of the main power circuit.

LEAKAGE AND INSULATION MEASUREMENTS

Metropolitan District Railway. An extended series of preliminary measurements and tests were made on the Hounslow and Putney section of the District Railway early in 1905 with a view of trying to account for the comparatively low insulation resistance of the negative conductor rail. These tests were made during the constructional period; that is, before the introduction of commercial electric trains but with the regular steam trains in daily commercial service, and the insulation of the conductor rails was therefore more or less of a variable quantity and subject to the temporary changes and conditions incidental to such work.

The positive (outside) conductor rail, particularly at the

FIG 1 FIG. 2
Positive and negative insulators—District Railway

stations, was heavily coated with grease from the steam trains and this, together with loose ballast, dirt, etc., made it very difficult and virtually impossible to maintain good insulation during these tests.

The results of these preliminary measurements were very erratic, for the reasons just given, and are therefore not included in this paper. They plainly indicated, however, the existence of the peculiar phenomena observed on the tube railways where the conditions were much more favorable for the perfectly definite and consistent observations given herewith.

The observations given in Table 1 were made in January, 1906, and show the variation in potential between conductor rails and earth from day to day. These measurements were made on the main line of the District Railway at Earls Court, the electric trains having been in commercial service since July 1, 1905.

All sections of the road are tied together and cross-bonded through the sub-station bus-bars under normal operating conditions, and these results are therefore representative of the conditions existing over the entire District Railway system, collectively considered. The continuously bonded running rail was used as the earth connection for these measurements.

TABLE I.—LINE VOLTAGE 580

Date—1906	Time	Volts between earth and Positive	Negative	Remarks
Jan. 8	2.15 p.m.	530	50	Ground very damp.
" 8	4.35 "	512	68	" " "
" 9	4.50 "	500	80	Ground very wet.
" 10	10.00 a.m.	529	51	Fine.
" 10	5.30 p.m.	512	68	Fine and dry.
" 12	11.00 a.m.	510	70	Fine, ground damp.
" 12	5.50 p.m.	512	68	" " "
" 13	11.00 a.m.	490	90	Ground very wet.
" 15	12.30 p.m.	540	40	Very fine.
" 15	6.00 "	520	60	Very fine and dry.
" 16	5.00 "	512	68	Showery.
" 18	12.45 "	478	102	Very wet.
" 22	5.00 "	520	60	Fine.
" 23	6.00 "	532	48	Fine, frosty.

Baker Street and Waterloo Railway. The tests on this railway like those on the District Railway, were made about the end of the constructional period and were therefore subject, but in a much lesser degree, to the possibility of the same general disturbances.

The insulators were of brown stoneware with a good glaze, free from grease, and reasonably clean, with the exception of the usual dust from the small granite ballast and construction work. A rectangular section of conductor rail is used on this line, there being no intervening metal cap or support between the rails and insulators. These measurements were not subject to the usual outdoor variation of London weather and temperature, the section of track on which the measurements were made being entirely below the surface and far enough removed from the tunnel entrance so as to be, at the most, only possibly indirectly affected:

Up-road, January 2 *to* 4, 1906. The following measurements were made on the "up-road," between the cable passageway beneath the London Road sub-station and the far end of Baker street station platform, a distance of 18,317 ft. or 3.47 miles. All measurements were made from the London Road sub-station.

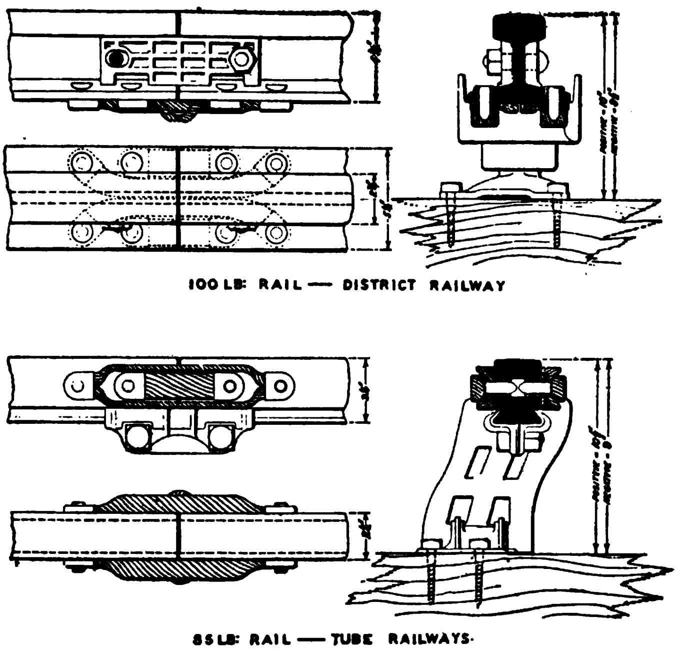


FIG. 3—Bonding and supports for conductor rails

Polarity of conductor rails, normal.

Between positive (outer) and negative	570 volts
" " and earth	510 "
" negative and earth	60 "
Leakage, positive to negative	0.5 amperes
Leakage current per mile of single track	0.144 "
Leakage, positive earthed	5.0 "
" negative earthed	0.57 "

Current was on from 2 to 5.30 p.m. on January 1, and from 7 to 10 a.m. January 2; that is, 2.5 hours on the preceding

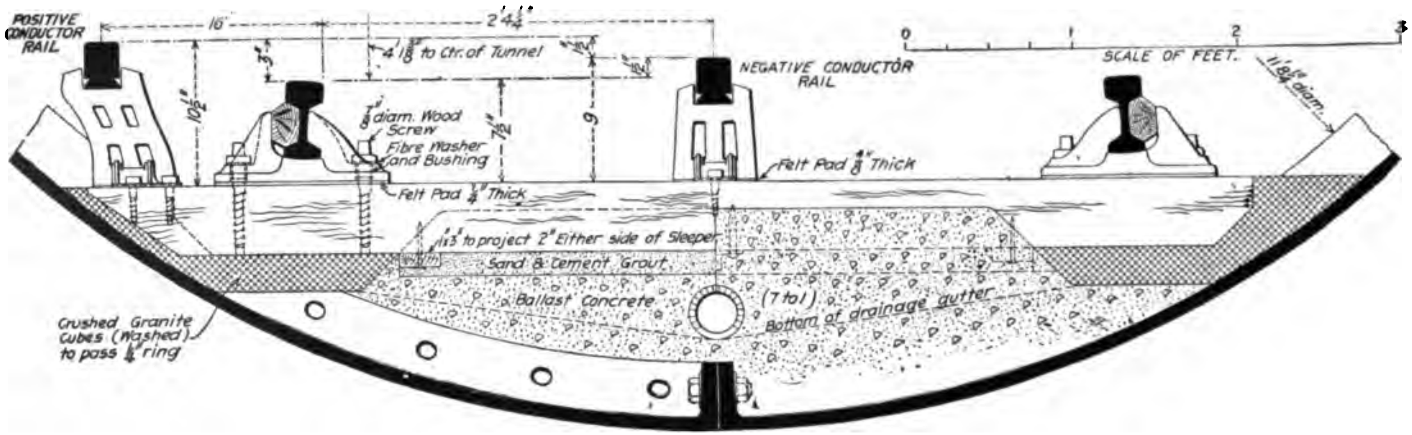


FIG. 4—Section illustrating type of construction, Tube Railways

afternoon and for 3 hours continuously and immediately preceding the above measurements. The insulators were supposed to have been cleaned prior to above measurements. Current had also been on these rails on several previous occasions for testing purposes and the running of trains.

Polarity of conductor rails, reversed. The polarity of the conductor rails was reversed immediately after the preceding readings and the pressure applied continuously for 48 hours, all other conditions remaining the same. Readings were taken regularly between the outer rail (now negative polarity) and earth, the results being shown on Fig. 6.

At the end of the 48 hours the following readings were taken:

Between positive (centre) and negative	570 volts
" " and earth	455 "
" negative and earth	115 "
Leakage, positive to negative	0.5 amperes
Leakage current per mile of single track	0.144 "
Leakage, positive earthed	2.8 "
" negative earthed	0.7 "

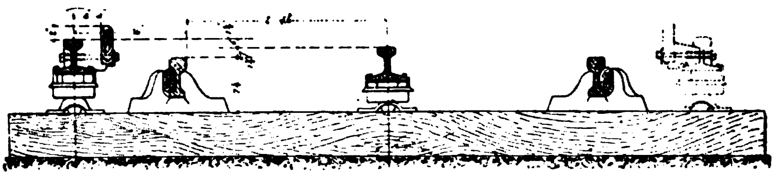


FIG. 5—Cross-section of single track, District Railway

These results show that the potential between the outer rail and earth was reduced from 510 (positive to earth) to 104 volts (negative to earth) in 24 hours. Some local disturbance then caused a sudden rise to about 118 volts, after which it again slowly but steadily fell to 115 volts at the end of 48 hours. During the first half-hour immediately following the reversal, the leakage current increased steadily from 0.5 to 1.27 amperes and then gradually became less. It reached the normal value of 0.5 of an ampere on or *before* the end of the 48 hours. The low-reading ammeter was damaged by a temporary short-circuit about the end of the first hour and was therefore not immediately available for current measurements.

Polarity of conductor rails, normal. The polarity of the conductor rails was again made normal—immediately after the 48-hour run with the polarity reversed—the increase in leakage

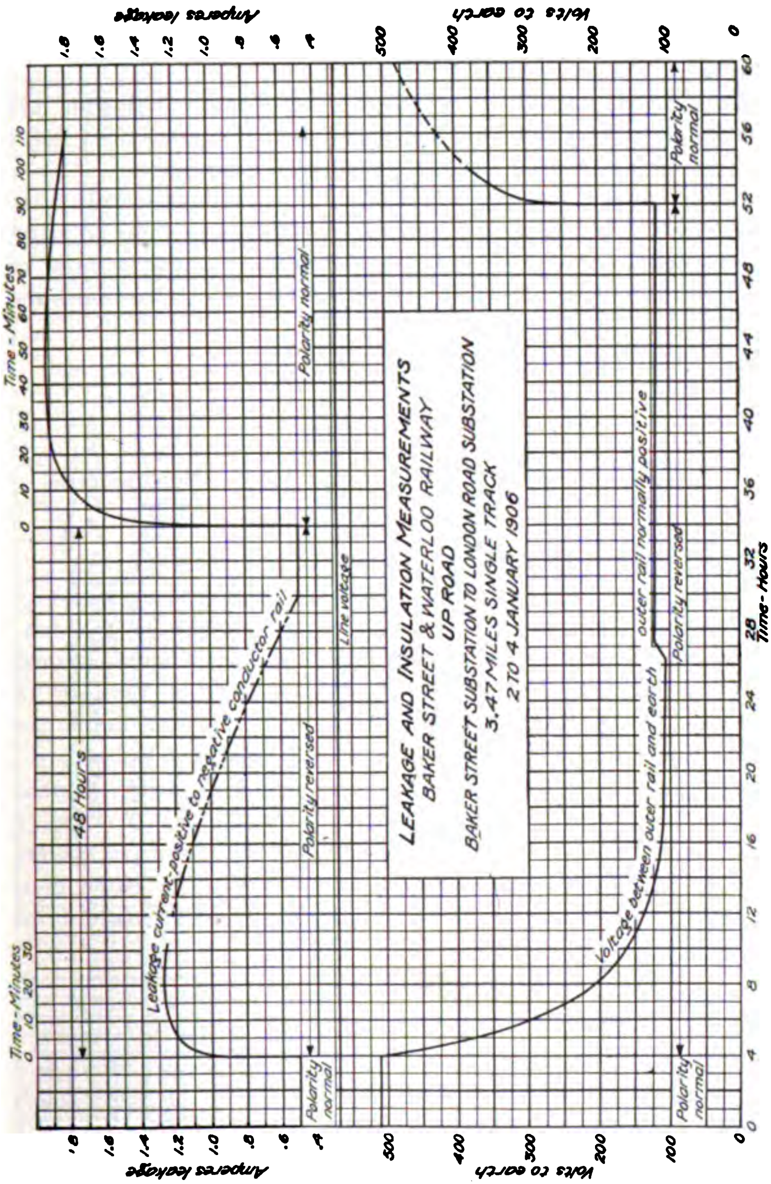

LEAKAGE AND INSULATION MEASUREMENTS
BAKER STREET & WATERLOO RAILWAY
UP ROAD
BAKER STREET SUBSTATION TO LONDON ROAD SUBSTATION
3.47 MILES SINGLE TRACK
2 TO 4 JANUARY 1906
Amperes leakage
Volts to earth
Time - Minutes
Time - Hours
48 Hours
Leakage current-positive to negative conductor rail
Polarity normal
Polarity reversed
Line voltage
Voltage between outer rail and earth outer rail normally positive

FIG. 6

current and the variation in potential between the outer rail and earth for the first two hours being also shown in Fig. 6.

These latter measurements unfortunately could not be continued any longer as the road was needed for the running of trains.

The leakage current rose rapidly immediately following the change from reversed to normal polarity, the maximum value of 1.92 amperes being reached in about 35 minutes.

" Up " road, January 16, 1906. The conductor rails were alive an average of about nine hours per day, normal polarity, from January 4 to 16 inclusive for running trains a total of 117 hours between the readings on January 4 and the following readings on January 16, 1906.

Polarity of conductor rails, normal.

Between positive (outer) and negative	565 volts
" " and earth	530 "
" negative and earth	35 "
Leakage, positive to negative	0.32 amperes
Leakage current per mile of single track	0.092 "
Leakage, positive earthed	5.70 "
" negative earthed	0.35 "

" Up " road, January 19 *to* 22, 1906. The following measurements show the effects of reversing the polarity of the conductor rails, the rails having been alive about 24 hours, normal polarity, for running trains between the tests of January 16 and 19. The rails were alive continuously for 43.5 hours between the readings of January 19 and 21 with the polarity reversed; that is, outer rail negative—and for 21 hours, normal polarity, between the readings of January 21 and 22. Fig. 7 gives full details of these tests and Fig. 6 the details of similar tests on the same road and under practically the same conditions. All the " Up " road measurements were made from the London Road sub-station.

General exhibit	Date of tests—1906		
	Jan. 19	Jan. 21	Jan. 22
Between positive and negative, volts	575	575	575
Between outer rail and earth, volts	531	112	530
Between centre rail and earth, volts	44	463	45
Leakage, positive to negative, amperes	0.29	0.35	0.3
Amperes leakage per mile single track	0.084	0.101	0.087
Leakage, outer rail earthed, amperes	4.3		
Leakage, centre rail earthed, amperes	0.3		
Approximate distance, miles	3.47	3.47	3.47

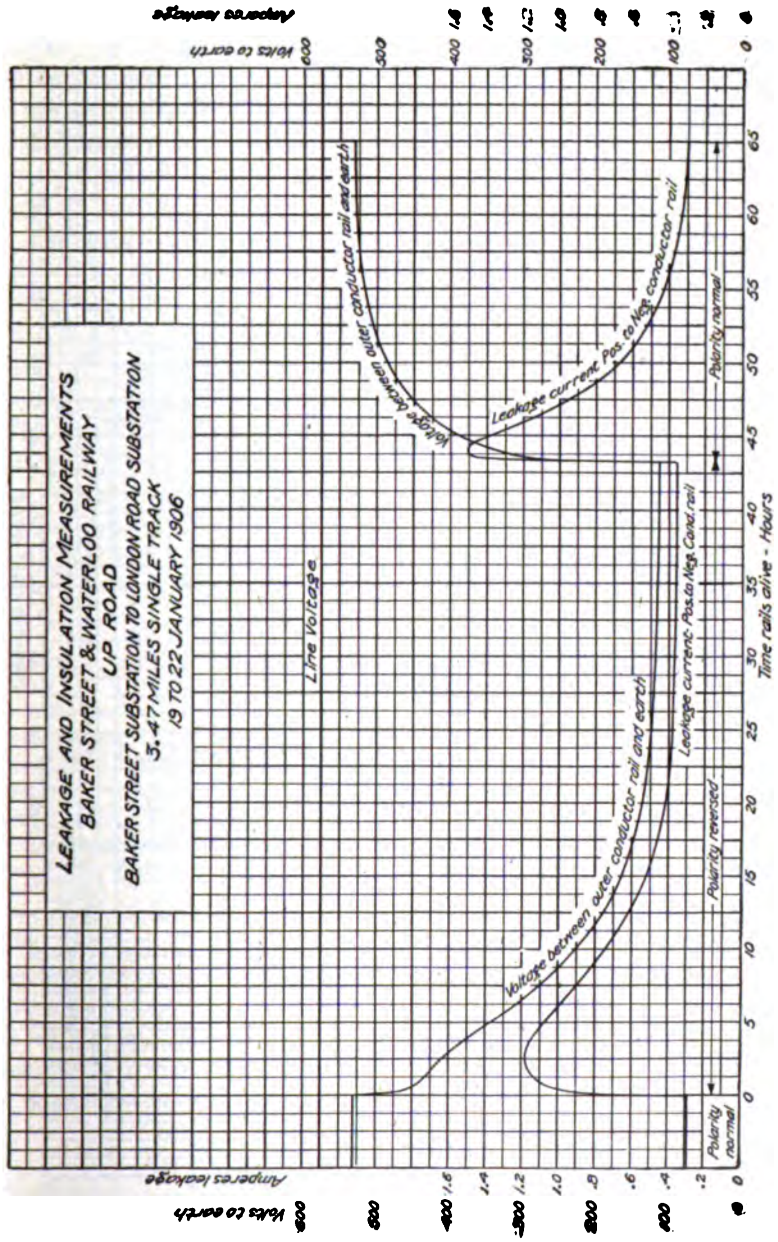

LEAKAGE AND INSULATION MEASUREMENTS
BAKER STREET & WATERLOO RAILWAY
UP ROAD
BAKER STREET SUBSTATION TO LONDON ROAD SUBSTATION
3.47 MILES SINGLE TRACK
19 TO 22 JANUARY 1906
Line Voltage
Voltage between outer conductor rail and earth
Leakage current Pos. to Neg. conductor rail
Voltage between outer conductor rail and earth
Leakage current Pos. to Neg. Cond. rail
Polarity normal
Polarity reversed
Polarity normal
Time rails alive - Hours
Volts to earth
Amperes leakage

FIG. 7

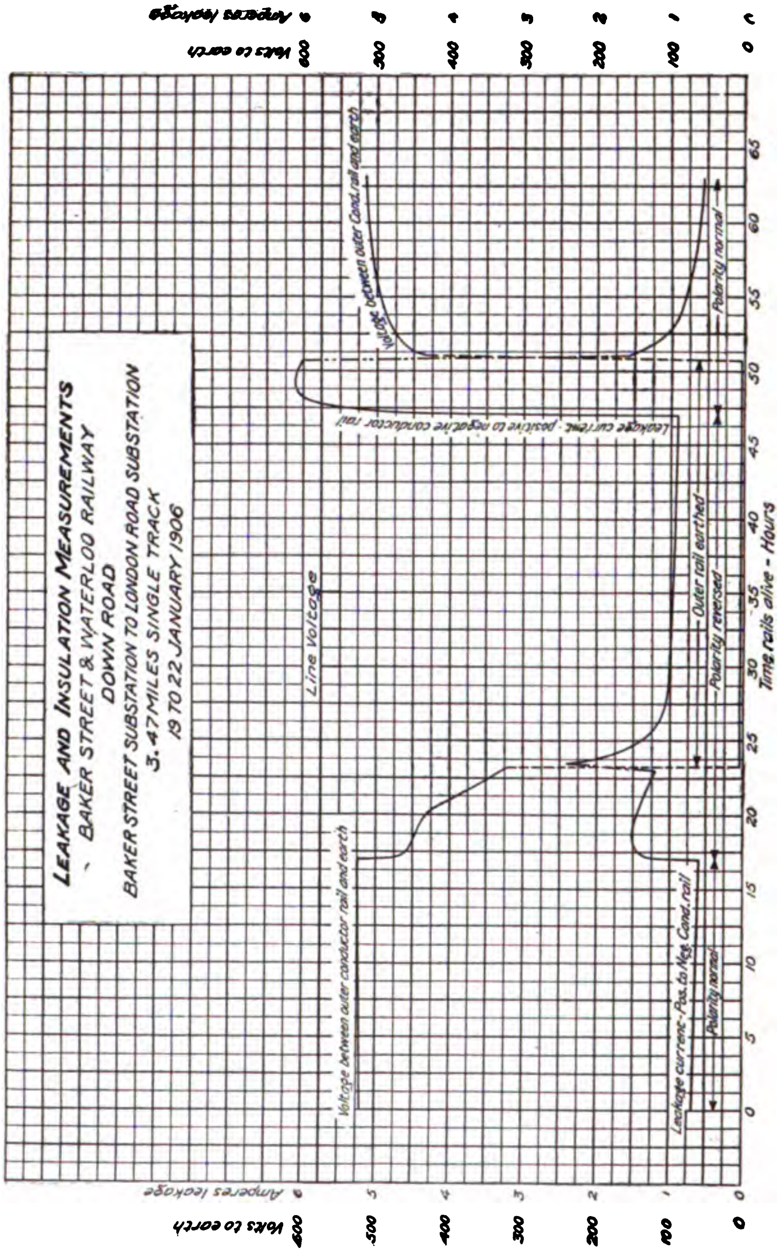

LEAKAGE AND INSULATION MEASUREMENTS
BAKER STREET & WATERLOO RAILWAY
DOWN ROAD
BAKER STREET SUBSTATION TO LONDON ROAD SUBSTATION
3.47 MILES SINGLE TRACK
19 TO 22 JANUARY 1906
Line Voltage
Voltage between outer conductor rail and earth
Voltage between outer Cond. rail and earth
Leakage current - positive to negative conductor rail
Leakage current - Pos. to Neg. Cond. rail
Polarity normal
Polarity reversed
Outer rail earthed
Polarity normal
Time rails alive - Hours
0 5 10 15 20 25 30 35 40 45 50 55 60 65
Volts to earth
600 500 400 300 200 100 0
Amperes leakage
6 5 4 3 2 1 0

FIG. 8

"Down" road, January 19 *to* 22 *and February* 12, 1906. The initial measurements for this road were made on January 19, 1906. The rails were alive continuously for about 8 hours previously to the initial measurements and for 17 hours, normal polarity, between the measurements on January 19 and 20. The normal polarity was reversed immediately after the readings on January 20 for 30 hours and again made normal for 15 hours, a total of 45 hours between the readings of January 20 and 22.

A temporary but effective "earth" mysteriously appeared on the outer rail about six hours after the reversal on January 20, this condition of affairs being maintained continuously for about 28 hours, four hours after the polarity was again made normal on January 21. The effect of this earth on the leakage current, together with the other details of this test, is shown in Fig. 8. The rails were alive daily, normal polarity, for the running of trains between the readings of January 22 and February 12, a total of about 260 hours between these readings. The readings for the "down" road, made at the Baker Street sub-station, are as follows:

General Exhibit	Date of tests—1906			
	Jan. 19	Jan. 20	Jan. 22	Feb. 12
Between positive and negative, volts..	575	575	575	575
Between outer rail and earth, volts...	520	525	515	499
Between centre rail and earth, volts...	55	50	60	76
Leakage, positive to negative, amperes	0.68	0.58	0.55	0.2
Amperes leakage per mile single track.	0.196	0.167	0.158	0.058
Leakage, outer rail earthed, amperes..	6.75			3.0
Leakage, centre rail earthed, amperes.	0.74	0.62		0.25
Approximate distance, miles..........	3.47	3.47	3.47	3.47

The following observations, Table II, show the variations in potential between the positive conductor rail and earth—both roads—from day to day as well as the variation between the time of switching "on" and "off" on the same day. All measurements made by the sub-station attendant at the London Road sub-station, normal polarity.

Discussion of Results

The following results are corroborated by many other measurements made during the years of 1903-1907, all of which plainly and unmistakably show the existence of the phenomena substantially as illustrated in Figs. 6, 7 and 8. There has been considerable study and investigation as to what actually takes place at the time of reversal, the general concensus of opinion being that it is an ordinary case of electrolytic action combined with ordinary insulation resistance conductivity. These results seem to depend upon one form of phenomenon which has long been

TABLE II.—LINE VOLTAGE 575

Date—1906	Time	Hours rails alive to-day	Volts between positive and earth	
			Up-road	Down-road
Jan. 24	7.00 p.m.	19	525	512
" 25	10.30 a.m.		500	510
" 25	8.00 p.m.	9.5	540	512
" 26	10.00 a.m.		500	495
" 26	7.30 p.m.	9.5	540	506
" 27	10.00 a.m.		510	445
" 27	3.30 p.m.	5.5	535	503
" 29	10.00 a.m.		465	410
" 29	7.00 p.m.	9.0	560	575
" 30	10.00 a.m.		530	452
" 30	8.30 p.m.	10.5	565	510
" 31	8.00 a.m.		522	462
" 31	8.30 p.m.	12.5	530	480
Feb. 1	9.00 a.m.		490	430
" 1	7.30 p.m.	10.5	540	495
" 2	8.30 a.m.		498	452
" 2	7.30 p.m.	11.0	500	465
" 3	8.00 a.m.		484	444
" 3	4.30 p.m.	8.5		
" 4	9.00 a.m.	15	463	440
" 5	8.00 p.m.	20	493	469
" 7	9.00 a.m.		505	480
" 7	9.00 p.m.	12	505	495
" 9	9.30 a.m.	14.5	486	444
" 10	5.30 p.m.	17.5	520	492

known in connection with static charges; namely, that a negatively charged conductor leaks away its charge with much greater facility than does a positive; or virtually that under the same conditions a positively electrified conductor is better insulated. A negatively electrified conductor appears to be continuously discharging negative ions, which have the effect of conferring conductivity on the surrounding gas and of causing deposition of moisture from air more or less saturated. It is perfectly conceivable, therefore, that this negative discharge should favor the depositing of moisture between the surfaces of insulators progressively in such a manner that a slight conductivity might be conferred upon the insulator surfaces and that this effect might be expected to be absent with the positive side of the system. The film of liquid existing between the electrodes on a glass surface will gradually move from the positive to the negative electrode, and the negative insulators may therefore lose their insulating properties by having moisture drawn upon them, an action which does not take place on the positive side.

The reducing or chemical action of the ions liberated at the positive rail insulator may result in the formation of compounds which lower the resistance of the negative insulator, while the oxidation which takes place at the positive insulator may improve the insulation.

Conclusions. These results, whatever the immediate cause or the reason, have clearly demonstrated certain facts which are probably applicable to all similar installations and may be briefly summarized as follows:

1. That the difference of potential between the positive conductor and earth is always normally considerably greater than the potential existing between the negative conductor and earth.

2. That this difference between the positive and negative insulation becomes more marked the longer the conductors are subjected continuously to a difference of potential in the same direction.

3. That a reversal of the polarity is always instantly accompanied by a considerable increase in the normal leakage current between the positive and negative conductor.

4. The above phenomena can be repeated indefinitely and are independent of the length of time that the pressure has been previously applied to the conductors in either direction.

5. That the insulation of the negative conductor to earth can not be proportionately maintained.

Resistance of conductor rails. The resistance measurements given in Table III show about what may be expected commercially with a soft conductor rail containing an unusually small amount of manganese and carbon. The resistance of these rails was about 6.4 times that of an equivalent area of copper and the chemical composition substantially as follows:

Carbon	0.05
Manganese	0.19
Sulphur	0.06
Phosphorus	0.05
Silicon	0.03

It is interesting to note that the cost of this special conductor rail was no more than the standard track rail.

TABLE III.—RESISTANCE MEASUREMENTS

General exhibit	Section of railway			
	Hounslow	Putney	Wimbledon	B. St. & W.
General Data:				
Date of measurements	5/9/05	5/27/05	5/27/05	12/30/05
Net length measured, feet	88,300	37,700	53,500	36,600
Weight of rail per yd., lb	100	100	100	85
Area of rail, sq. in	9.86	9.86	9.86	8.3
Temp. of rails, deg. cent	25	25	25	11.5
No. of switchboard contacts	0	4	4	2
No. of terminal post contacts	20	20	32	6
No. of rail contacts	20	24	44	36
No. of bonds per joint	4	4	4	4
Total no. of bonded joints	2,107	834	1,160	1,095
Area bonds per joint, sq. in	1.57	1.57	1.57	1.33
Contact area per joint, sq. in	9.4	9.4	9.4	12.56
Measurements.				
A, Total resistance	0.55309	0.24405	0.36412	0.24088
B, Total resistance	0.54856	0.22992	0.35013	0.23353
A, Res. per 1000 ft. of rail—ohms	0.006264	0.006473	0.006806	0.006581
B, Res. per 1000 ft. of rail—ohms	0.006213	0.006099	0.006544	0.006381
A. Res. per mile-ohm	0.033070	0.034180	0.035936	0.03475
B, Res. per mile-ohm	0.032805	0.032203	0.034552	0.03369
A, Equiv. cu. area, sq. in	1.328	1.285	1.222	1.2
B, Equiv. cu. area, sq. in	1.339	1.364	1.271	1.236
A, Rel. resistance, cu. at 1	7.42	7.67	8.07	6.92
B, Rel. resistance, cu. at 1	7.36	7.22	7.75	6.72
B, Contact res. per jt., ohm	0.0000341	0.0000316	0.0000528	0.00001044
B, Contact res. per bond terminal, ohm	0.0000681	0.0000632	0.0001056	0.00002088

A includes the resistance of all feeder, track and jumper cables and the terminal post, rail and switchboard contact resistance; that is, the resistance of the bonded conductor rails, cables and contacts as installed and ready for commercial service.

B same as " A " less the calculated resistance of all feeder, track and jumper cables; that is, includes the resistance of the bonded conductor rails and auxiliary contacts.

Remarks. All calculated resistances are based on the temperature at which the measurements were made.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 2, 1908.

THE GENERAL EQUATIONS OF THE ELECTRIC CIRCUIT

BY CHARLES P. STEINMETZ

Introduction. The following investigation of the general equations of the electric circuit was carried out during the last year, and necessarily is still very incomplete, and in some instances may require modification. As, however, our incomplete knowledge of the phenomena resulting from the stored energy of the electric field is at present the main hindrance to a practically unlimited extension of high potential transmission and distribution, the study of these phenomena represents the most important problem of electrical engineering, and the following outline is given in the hope of inducing other investigators to take up and continue the study of the subject.

As in all theoretical investigations, the most important part is the practical interpretation of the mathematical equations. Some such interpretations have been attempted throughout the following and may be read even without following the equations from which they are derived.

This paper represents an attempt to investigate mathematically the phenomena which may occur in the most general case of an electric circuit.

The general equations of the electric circuit, *i.e.*, equations which represent current and voltage in any circuit or circuit section irrespective of the character of power, voltage etc., can be derived under the condition that the constants of the electric circuit: resistance, inductance, conductance, and capacity are constant.

These equations are given by a sum of terms or groups containing trigonometrical and exponential functions of time and distance, and the differences between all the phenomena which may occur in electric circuits merely result from different numerical values of the integration constants of this general equation.

Each group or term comprises four waves of the same frequency—two propagate in the one, the other two in the opposite direction—and can be considered as comprising two component waves, each consisting of a wave and its reflected wave, or return wave.

One of the component waves increases in the direction of propagation, the other decreases. The former, however, decreases with the time more rapidly than the latter.

A standing wave is a wave in which the component waves coincide. A standing wave has the same intensity throughout the circuit. It starts at maximum value and dies out with the time at a rate corresponding to the dissipation of energy in the circuit, as given by the time constant of the circuit.

The general wave, comprising two component waves (each consisting of main wave and return wave), gradually builds up to a maximum and then dies out again. One of the component waves dies out slower, the other faster than corresponds to the energy dissipation in the circuit, and the former decreases while the latter increases with the distance.

A free oscillation of a circuit is a wave in which no energy is supplied to or abstracted from the circuit.

The free oscillation of a uniform circuit is a standing wave, but a standing wave is not necessarily a free oscillation.

The electric wave in a uniform circuit is oscillatory in time if its frequency is high and the circuit of moderate length. With very low frequency or extreme length, as in submarine cables, or long distance telephone lines, waves may occur which are not oscillatory, but logarithmic.

In overhead transmission lines and underground cable systems all phenomena which may result in excessive voltages or currents are oscillatory in nature.

In a traveling wave each component wave traverses the circuit at constant velocity and wave shape, and an amplitude which decreases with the distance traveled, but at any point rises from zero to a maximum, and then gradually decreases again to zero. It is not a free oscillation, but energy is transferred along the circuit by it.

The phenomena of alternating currents are those of a general wave in which the time decrement of the first component wave vanishes, that is, the energy transfer constant f equals the time constant w.

In a complex circuit, that is, circuit containing a number of sections of different constants, the same general equation can be extended throughout the entire circuit and across the transition points between different sections of the circuit of different constants, by using as distance variable the velocity of propagation of the wave in each section of the circuit.

The free oscillation of a complex circuit is not a standing wave, and the equations of the standing wave do not apply to it, but is a general wave in which the intensity of the wave increases or decreases with the distance in each section, and energy transfer occurs between the different sections of the circuit.

The free oscillation of a complex circuit dies out at a rate given by the mean time decrement, or mean time constant of the entire circuit.

Those sections having an individual time constant greater than the mean time constant of the entire circuit receive energy from the adjoining sections, while those sections having an individual time constant less than the mean time constant of the entire circuit supply energy to the rest of the circuit during a free oscillation.

During a free oscillation of a complex circuit each section of the circuit does not perform a free oscillation.

In a section of high energy dissipation, as a transmission line, the oscillation may last very much longer, and be of lower frequency, if this section is a part of a complex oscillating circuit, than if the section oscillates alone.

In the free oscillation of a complex circuit the entire length of the circuit, when expressed in velocity measure, represents a complete wave, or a half wave, or a quarter wave, or a multiple thereof.

In the free oscillation of a complex circuit the energy of the electric field of each section divides between the energy dissipated in the section and the energy transferred to other sections (or received) in a proportion depending upon the circuit constants.

At a transition point between sections of a complex circuit, the general equations remain the same when expressed with the propagation velocity as distance unit, but the numerical values of the integration constants change.

At a transition point between sections of a complex circuit transformation of current and voltage, reflection and refraction occurs.

A single wave at a transition point is partly reflected, partly transmitted, but without change of phase angle, that is, the reflected and the transmitted wave have at the transition point the same phase angle as the incident wave.

A single wave at a transition point divides between transmitted and reflected wave in a proportion depending upon the circuit constants.

The greater the change of constants at the transition point, the greater is the reflected and less the transmitted wave.

In a general wave, at a transition point, a transformation of current and voltage occurs by a ratio proportional to the circuit constant $\sqrt{\frac{L}{C}}$.

When traversing a transition point a refraction occurs, so that the ratio of the tangents of the distance phase angle of the wave at the two sides of the transition point is constant and equal to the voltage transformation ratio.

I. Derivation of General Equations

1. The energy relations of an electric circuit are characterized by the four constants:

r = effective resistance, representing the power consumption depending upon the current, $i^2 r$, or the energy voltage, $i r$, consumed in the circuit.

L = effective inductance, representing the energy storage depending upon the current, $\frac{i^2 L}{2}$, as electromagnetic component, $i L$, of the electric field; or the voltage induced by a change of the current, $L \frac{d i}{d t}$.

g = effective shunted conductance, representing the power consumption depending upon the voltage, $e^2 g$, or the energy current, $e g$, consumed in the circuit.

C = effective capacity, representing the energy storage depending upon the voltage, $\frac{e^2 C}{2}$, as electrostatic component, $e C$, of the electric field, or the current consumed by a change of the voltage, $C \frac{d e}{d t}$.

As every circuit element consumes and stores energy, and so contains all four circuit constants, the assumption usually made in the investigation of electric circuits, that these four constants r, L, g, C, are localized, is permissible only in cases dealing with the usual phenomena of direct and of alternating currents, but fails when dealing with the transient phenomena resulting from the readjustment of the circuit to changes of circuit conditions, to external influences such as lightning, or when investigating high frequency phenomena, voltage and current distribution in long distance high potential circuits, cables, telephone lines, etc., and r, L, g, C, must then be treated as distributed throughout the circuit.

As r, L, g, C, are denoted the effective resistance, inductance, conductance, and capacity, per unit length of circuit, choosing as unit of length of the circuit whatever may be suitable, a centimeter in the high frequency oscillation over the multigap lightning-arrester circuit, or a mile in a long distance transmission circuit, or high potential cable.

In the most general case of an electric circuit or circuit section, it is possible to derive from the constants of the circuit, r, L, g, C, and without any assumption whatever regarding current, voltage, etc., general equations of the electric circuit, and some results and conclusions from such equations, which are based on the single assumption, that the constants r, L, g, C, remain constant with the time, t, and distance, u; *i.e.*, are the same for every unit length of the circuit or section of the circuit to which the equations apply. Where the circuit constants change, as where another circuit joins it, the integration constants in the equations also change correspondingly.

Special cases of these general equations then are all the phenomena of direct currents, alternating currents, discharges of reactive coils, high frequency oscillations, traveling waves, etc., and the differences between these different circuits are due merely to different values of the integration constants.

2. In a circuit, or a section of a circuit, containing distributed resistance, inductance, conductance, and capacity, as a transmission line, cable, high potential coil of transformer, telephone or telegraph circuit, etc., let:

r = effective resistance, per unit length of circuit.
L = effective inductance, per unit length of circuit.
g = effective shunted conductance, per unit length of circuit.
C = effective capacity, per unit length of circuit.

Let the time be denoted by t, and the distance from some starting point by u.

Let e = voltage, and i = current, at any point u and at any time t.

e and i are functions of time t and distance u.

For any element $d\,u$ of the circuit, the differential equations then apply:

$$\frac{d\,e}{d\,u} = r\,i + L\,\frac{d\,i}{d\,t} \tag{1}$$

$$\frac{d\,i}{d\,u} = g\,e + C\,\frac{d\,e}{d\,t} \tag{2}$$

Differentiating (1) after t, and (2) after u, and substituting (1) into (2), gives:

$$\frac{d^2\,i}{d\,u^2} = r\,g\,i + (r\,C + g\,L)\,\frac{d\,i}{d\,t} + L\,C\,\frac{d^2\,i}{d\,t^2} \tag{3}$$

and in the same manner:

$$\frac{d^2\,e}{d\,u^2} = r\,g\,e + (r\,C + g\,L)\,\frac{d\,e}{d\,t} + L\,C\,\frac{d^2\,e}{d\,t^2} \tag{4}$$

These differential equations, of second order, of current i and voltage e, are identical. That is:

In an electric circuit, current and electromotive force are represented by the same equations, differing by the integration constants only, which are derived from the terminal conditions of the problem.

Equation (3) is integrated by terms of the form:

$$i = A\,\varepsilon^{-vu-st} \tag{5}$$

Substituting (5) in (3) gives the identity:

$$\begin{aligned} v^2 &= r\,g - (r\,C + g\,L)\,s + L\,C\,s^2 \\ &= (s\,L - r)\,(s\,C - g) \end{aligned} \tag{6}$$

In the terms of the form (5), the relation (6) must exist between the coefficients of u and t.

Substituting (6) into (1) gives:

$$\frac{d e}{d u} = (r - s L) A \epsilon^{-vu-st} \tag{7}$$

and, integrated:

$$e = \frac{s L - r}{v} A \epsilon^{-vu-st} \tag{8}$$

In their most general form, the *Equations of the Electric Circuit* are:

$$i = \sum^{n} \{A_n \epsilon^{-v_n u - s_n t}\} \tag{9}$$

$$e = \sum^{n} \left\{\frac{s_n L - r}{v_n} A_n \epsilon^{-v_n u - s_n t}\right\} \tag{10}$$

$$v_n^2 - (s_n L - r)(s_n C - g) = 0 \tag{11}$$

where A_n, v_n, and s_n, are integration constants, the latter two being related to each other by the equation (11).

3. These pairs of integration constants: A_n, v_n and s_n, are determinated by the terminal conditions of the problem, as the current and voltage at fixed points of the circuit as function of time, or the distribution of current and voltage at a fixed time as function of the distance, etc.

While i and e must always be real quantities; v_n and s_n as exponents may be complex quantities, and in this case, the integration constants, A_n, are such complex quantities, that by combining the different exponential terms of the same index n, the imaginary terms in A_n and $\frac{s_n L - r}{v_n} A_n$ cancel.

Denoting, in the exponential function:

$$\epsilon^{-vu-st}$$

$$v = h + j k \tag{12}$$
$$s = p + j q$$

it i

$$\epsilon^{-vu-st} = \epsilon^{-hu-pt} \ \epsilon^{-j(ku+qt)}$$

and the latter term resolves into trigonometric functions of the angle:

$$k\,u+q\,t$$

$$k\,u+q\,t = \text{constant} \tag{13}$$

so gives the relation between u and t for constant phase of the oscillation or alternation of the current or voltage.

With change of time t, the phase changes in position u, in the circuit, *i.e.*, moves along the circuit, and the velocity of this motion is given by differentiating (13) after t as

$$V = \frac{d\,u}{d\,t} = -\frac{q}{k} \tag{14}$$

that is:

$$V = -\frac{q}{k} \tag{15}$$

is the velocity of *propagation of the electric phenomena in the circuit.*

(If no energy losses occur, $r = 0$, $g = 0$, in a straight conductor in a medium of unit magnetic and dielectric constant, that is, unit permeability and unit inductive capacity; V is the velocity of light.)

4. Since (11) is a quadratic equation, several pairs or corresponding values of v and s exist, which correspond to the same equation (11), and can be combined with each other into a group.

Such a group of terms, of the same index n is defined by the equation (11):

$$v^2 = (s\,L - r)\,(s\,C - g) \tag{16}$$

and substituting:

$$v = v_1 \sqrt{L\,C} \tag{17}$$

and also:

$$\begin{aligned} v &= h + j\,k \\ v_1 &= h_1 + j\,k_1 \\ s &= p + j\,q \end{aligned} \tag{18}$$

Hence:

$$\left.\begin{aligned} h &= h_1 \sqrt{L\,C} \\ k &= k_1 \sqrt{L\,C} \end{aligned}\right\} \qquad (19)$$

it is, substituting (18) in (16):

$$(h_1 + j\,k_1)^2 = \left[(p + j\,q) - \frac{r}{L}\right] \left[(p + j\,q) - \frac{g}{C}\right] \qquad (20)$$

resolved, and substituting:

$$w = \tfrac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right) \qquad (21)$$

$$m = \tfrac{1}{2}\left(\frac{r}{L} - \frac{g}{C}\right) \qquad (22)$$

$$p = f + w \qquad (23)$$

gives:

$$\left.\begin{aligned} h_1^2 - k_1^2 &= f^2 - q^2 - m^2 \\ h_1\,k_1 &= f\,q \end{aligned}\right\} \qquad (24)$$

and:

$$\left.\begin{aligned} f^2 - q^2 &= h_1^2 - k_1^2 + m^2 \\ f\,q &= h_1\,k_1 \end{aligned}\right\} \qquad (25)$$

hence:

$$\left.\begin{aligned} h &= \sqrt{L\,C}\,\sqrt{\tfrac{1}{2}\left\{R_1^2 + f^2 - q^2 - m^2\right\}} \\ k &= \sqrt{L\,C}\,\sqrt{\tfrac{1}{2}\left\{R_1^2 - f^2 + q^2 + m^2\right\}} \\ R_1^2 &= \sqrt{(f^2 + q^2 - m^2)^2 + 4\,q^2\,m^2} = \frac{h^2 + k^2}{L\,C} \end{aligned}\right\} \qquad (26)$$

or:

$$\left.\begin{aligned} f &= \frac{1}{\sqrt{L\,C}}\sqrt{\tfrac{1}{2}\left\{R_2^2 + h^2 - k^2 + L\,C\,m^2\right\}} \\ q &= \frac{1}{\sqrt{L\,C}}\sqrt{\tfrac{1}{2}\left\{R_2^2 - h^2 + k^2 - L\,C\,m^2\right\}} \end{aligned}\right\} \qquad (27)$$

$$R_2^2 = \sqrt{(h^2+k^2+LCm^2)^2 - 4LCk^2m^2} = LC(f^2+q^2)$$

This gives eight pairs of corresponding values of v and s:

$$\left.\begin{array}{lll} 1. & v = +h+jk & s = w-f-jq \\ & \quad\;\; +h-jk & \quad\;\; w-f+jq \\ 2. & v = -h-jk & s = w-f-jq \\ & \quad\;\; -h+jk & \quad\;\; w-f+jq \\ 3. & v = -h+jk & s = w+f-jq \\ & \quad\;\; -h-jk & \quad\;\; w+f+jq \\ 4 & v = +h-jk & s = w+f-jq \\ & \quad\;\; +h+jk & \quad\;\; w+f+jq \end{array}\right\} \quad (28)$$

Substituting these values in the general equations (9) and (10), and in the usual manner transforming the exponential functions of imaginary exponents, into trigonometric functions, and substituting for the integration constants, A, the new constants:

$$C = A + A'$$

$$C' = j(A - A')$$

then gives as the general expression of the

Equations of the Electric Circuit:

$$\left.\begin{array}{ll} i = \Sigma[\epsilon^{-hu-(w-f)t}\{C_1\cos(qt-ku)+C_1'\sin(qt-ku)\} & (i_1) \\ \quad -\epsilon^{+hu-(w-f)t}\{C_2\cos(qt+ku)+C_2'\sin(qt+ku)\} & (i_2) \\ \quad +\epsilon^{+hu-(w+f)t}\{C_3\cos(qt-ku)+C_3'\sin(qt-ku)\} & (i_3) \\ \quad -\epsilon^{-hu-(w+f)t}\{C_4\cos(qt+ku)+C_4'\sin(qt+ku)\}] & (i_4) \end{array}\right\} \quad (29)$$

$$\begin{array}{l} e = \Sigma[\epsilon^{-hu-(w-f)t}\{(c_1'C_1'-c_1C_1)\cos(qt-ku)-(c_1'C_1+c_1C_1') \\ \qquad \sin(qt-ku)\} \qquad (e_1) \\ \quad +\epsilon^{+hu-(w-f)t}\{(c_1'C_2'-c_1C_2)\cos(qt+ku)-(c_1'C_2+c_1C_2') \\ \qquad \sin(qt+ku)\} \qquad (e_2) \\ \quad +\epsilon^{+hu-(w+f)t}\{(c_2'C_3'-c_2C_3)\cos(qt-ku)-(c_2'C_3+c_2C_3') \\ \qquad \sin(qt-ku)\} \qquad (e_3) \qquad (30) \end{array}$$

$$+\epsilon^{-hu-(w+f)t}\,\{(c_2' C_4' - c_2 C_4)\cos(qt+ku) - (c_2' C_4 + c_2 C_4') \sin(qt+ku)\}] \qquad (e_4)$$

where C_1, C_1', C_2, C_2', C_3, C_3', C_4, C_4', and two of the four values, f, q, h, k, are integration constants, depending on the terminal conditions, and:

$$\left.\begin{aligned} c_1 &= \frac{q\,k + h\,(m+f)}{h^2+k^2} L \\ c_1' &= \frac{k\,(m+f) - q\,h}{h^2+k^2} L \\ c_2 &= \frac{q\,k - h\,(m-f)}{h^2+k^2} L \\ c_2' &= \frac{k\,(m-f) + q\,h}{h^2+k^2} L \end{aligned}\right\} \qquad (31)$$

and:

$$\left.\begin{aligned} w &= \tfrac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right) \\ m &= \tfrac{1}{2}\left(\frac{r}{L} - \frac{g}{C}\right) \end{aligned}\right\} \qquad (32)$$

and h, k, and f, q, are related by the equations:

$$\left.\begin{aligned} h &= \sqrt{L\,C}\sqrt{\tfrac{1}{2}\,\{R_1^2 + f^2 - q^2 - m^2\}} \\ k &= \sqrt{L\,C}\sqrt{\tfrac{1}{2}\,\{R_1^2 - f^2 + q^2 + m^2\}} \\ R_1^2 &= \sqrt{(f^2+q^2-m^2)^2 + 4\,q^2\,m^2} \end{aligned}\right\} \qquad (33)$$

hence:

$$h^2+k^2 = L\,C\,R_1^2 \qquad (34)$$

or:

$$\left.\begin{aligned} f &= \frac{1}{\sqrt{LC}}\sqrt{\tfrac{1}{2}\,\{R_2^2 + h^2 - k^2 + L\,C\,m^2\}} \\ q &= \frac{1}{\sqrt{LC}}\sqrt{\tfrac{1}{2}\,\{R_2^2 - h^2 + k^2 - L\,C\,m^2\}} \\ R_2^2 &= \sqrt{(h^2+k^2+L\,C\,m^2)^2 - 4\,L\,C\,k^2\,m^2} \end{aligned}\right\} \qquad (35)$$

hence

$$f^2+q^2=\frac{R_2^2}{L\,C} \tag{36}$$

II. Discussion of General Equations.

5. The general equations of current and voltage of a circuit or section of circuit having uniformly distributed and constant values of r, L, g, C, appear as a sum of groups of four terms each, characterized by the feature, that the four terms of each group have the same values of f, q, h, k.

Of the four terms of each group, i_1, i_2, i_3 i_4, or e_1, e_2, e_3, e_4, respectively (equations (29) and (30)), two contain the angles $(qt-ku)$: i_1, e_1, and i_3, e_3; and two contain the angles $(qt+ku)$: i_2, e_2, and i_4, e_4.

In the former, the velocity of propagation is:

$$\frac{d\,u}{d\,t}=+\frac{q}{k}$$

hence positive, that is, towards raising u.

In the latter, the velocity of propagation is:

$$\frac{d\,u}{d\,t}=-\frac{q}{k}$$

or negative, that is, towards decreasing u.

Each group consists of two waves and their reflected waves.

$i'=i_1-i_2$ and $e'=e_1+e_2$ is the first wave and its reflected wave, and

$i''=i_3-i_4$ and $e''=e_3+e_4$ is the second wave and its reflected wave.

In the first wave, i', e', the amplitude decreases in the direction of propagation, ε^{-hu} for rising, ε^{+hu} for decreasing u, and the wave dies out with increasing time t, by $\varepsilon^{-(w-f)t}=\varepsilon^{-wt}\,\varepsilon^{+ft}$.

In the second wave, i'', e'', the amplitude increases in the direction of propagation, ε^{+hu} for rising, ε^{-hu} for decreasing u, but the wave dies out with the increasing time t, by $\varepsilon^{-(w+f)t}=\varepsilon^{-wt}\,\varepsilon^{-ft}$, that is, faster than the first wave.

If the amplitude of the wave remained constant throughout the circuit—as would be the case in a free oscillation of the circuit, in which the stored energy of the circuit is dissipated, but no power transferred one way or the other—that is, if $h=0$, it also is, by (56), $f=0$, that is, both waves coincide into one. which dies out with the time by the decrement ϵ^{-wt}.

It thus follows:

In general, two waves, with their reflected waves, traverse the circuit, of which the one i'', e'', increases in amplitude in the direction of propagation, but dies out correspondingly more rapidly in time, that is, faster than a wave of constant amplitude; while the other i', e', decreases in amplitude, but lasts a longer time, that is, dies out slower than a wave of constant amplitude. That is, in the one wave, i'', e'', a decrease of amplitude takes place at a sacrifice of duration in time, while in the other wave, i', e', a slower dying out of the wave with the time is produced at the expense of a decrease of amplitude during its propagation.

It is seen herefrom, that the equations usually given for an oscillating circuit, which do not contain an exponential function of the distance, are not the most general equations, but correspond only to the special case of a free oscillation in which no energy transfer takes place, and so do not apply to the traveling wave or impulse, nor to the oscillation of a complex circuit.

6. In the equations (29) and (30):

$$q\,t = 2\,\pi$$

gives the time of a complete cycle, that is, the *period of the wave*:

$$T = \frac{2\pi}{q}$$

and:

$$N = \frac{q}{2\pi}$$

the frequency of the wave.

$$k\,u = 2\,\pi$$

gives the distance of a complete cycle, that is, the *wave length*:

$$l = \frac{2\pi}{k}$$

$$(w - f)\,t = 1 \text{ and}$$
$$(w + f)\,t = 1$$

gives the time:

$$T_1' = \frac{1}{w - f} \text{ and}$$

$$T_1'' = \frac{1}{w + f}$$

during which the wave decreases to $1/\epsilon = .3679$ of its value, and

$$h\,u = 1$$

gives the distance:

$$U_1 = 1/h$$

over which the wave decreases to $1/\epsilon = .3679$ of its value.

That is:

q is the *frequency constant* of the wave:
k is the *wave length constant*:
$(w - f)$ and $(w + f)$ are the *time attenuation constants* of the wave:
h is the *distance attenuation constant* of the wave:

7. If the frequency of the current and voltage is very high, thousands of cycles and more, as with traveling waves, lightning disturbances, high frequency oscillations, etc., q is a very large quantity compared with f, w, m, h, k, and k is a large quantity compared with h, and, by dropping terms of secondary order, the preceding equations can be simplified, and become:

$$\left.\begin{aligned} h &= f\sqrt{L\,C} \\ k &= q\sqrt{L\,C} \end{aligned}\right\} \quad (37)$$

and:

$$\left.\begin{aligned} c_1 = c_2 &= \sqrt{\frac{L}{C}} \\ c_1' = c_2' &= \frac{m}{q}\sqrt{\frac{L}{C}} = 0 \end{aligned}\right\} \quad (38)$$

Denoting:

$$\left.\begin{aligned} \sigma &= \sqrt{L\,C} \\ c &= \sqrt{\frac{L}{C}} \end{aligned}\right\} \quad (39)$$

where σ is the reciprocal of the frequency of propagation (velocity of light), it is:

$$\left.\begin{aligned} h &= \sigma f \\ k &= \sigma q \end{aligned}\right\} \quad (40)$$

$$\left.\begin{aligned} c_1 &= c_2 = c \\ c_1' &= c_2' = \frac{m}{q} c = 0 \end{aligned}\right\} \quad (41)$$

and, introducing the new independent variable, as distance:

$$\lambda = \sigma u \quad (42)$$

it is:

$$\left.\begin{aligned} k u &= q \lambda \\ h u &= f \lambda \end{aligned}\right\} \quad (43)$$

hence, the wave length is given by:

$$q \lambda = 2 \pi$$

as:

$$\lambda_0 = \frac{2\pi}{q} \quad (44)$$

and since the period is:

$$T_0 = \frac{2\pi}{q}$$

it follows, that by the introduction of the denotation (42), distances are measured with the velocity of propagation as unit length, and wave length U and period T have the same numerical values.

Substituting now in equations (29) and (30), gives:

$$\left.\begin{aligned} i = \varepsilon^{-ut} \Sigma \{ & \varepsilon^{+f(t-\lambda)} D_1 [q(t-\lambda)] - \varepsilon^{+f(t+\lambda)} D_2 [q(t+\lambda)] & (i') \\ & + \varepsilon^{-f(t-\lambda)} D_3 [q(t-\lambda)] - \varepsilon^{-f(t+\lambda)} D_4 [q(t+\lambda)] \} & (i'') \end{aligned}\right. \quad (45)$$

$$\left.\begin{aligned} e = \sqrt{\frac{L}{C}}\, \varepsilon^{-ut} \Sigma \{ & \varepsilon^{+f(t-\lambda)} D_1 [q(t-\lambda)] + \varepsilon^{+f(t+\lambda)} D_2 [q(t+\lambda)] & (e') \\ & + \varepsilon^{-f(t-\lambda)} D_3 [q(t-\lambda)] + \varepsilon^{-f(t+\lambda)} D_4 [q(t+\lambda)] \} & (e'') \end{aligned}\right. \quad (46)$$

where:

$$D[q(t \pm \lambda)] = C \cos q (t \pm \lambda) + C' \sin q (t \pm \lambda) \tag{47}$$

8. As seen from equations (45) and (46), the waves are products of ϵ^{-wt} and a function of $(t-\lambda)$ for the main wave, $(t+\lambda)$ for the reflected wave:

$$\left.\begin{aligned} i_1 + i_3 &= \epsilon^{-wt} f_1 (t - \lambda) \\ i_2 + i_4 &= \epsilon^{-wt} f_2 (t + \lambda) \end{aligned}\right\} \tag{48}$$

Hence, for constant $(t-\lambda)$ on the main waves, and for constant $(t+\lambda)$ on the reflected waves, it is:

$$\left.\begin{aligned} i_1 + i_3 &= B\ \epsilon^{-wt} \\ i_2 + i_4 &= B' \epsilon^{-wt} \end{aligned}\right\} \tag{49}$$

That is, during its passage along the circuit, the wave decreases by the decrement ϵ^{-wt}, or at a constant rate, independent of frequency, wave length, etc., and depending merely on the circuit constants r, L, g, C.

That is, the decrement of the traveling wave in the direction of its motion is independent of the character of the wave, as its frequency, etc.

In other words, an impulse traveling along the circuit retains its wave shape, provided that r, L, g, C, are constant.

If r and g increase with the frequency, as is often the case, the higher frequency waves die out faster, and the wave shape gradually smooths out in the direction of travel.

The physical meaning of the two waves i' and i'' can best be appreciated by observing the effect of the wave when traversing a fixed point λ of the circuit.

Consider as instance the main wave only; $i' = i_1 + i_3$; the same applies for the reflected wave.

Assuming then, that at time $t=0$, the current $I'=0$, it is, for constant λ:

$$I = D\left(\epsilon^{-(w-f)t} - \epsilon^{-(w+f)t}\right)$$

the amplitude of current I at point λ.

As this is the difference of two exponential functions of different decrement, it follows, that as function of time t, I' rises from 0 to a maximum and then decreases again to zero, as shown in Fig. 1, and the actual current i is the oscillatory wave with I as envelope.

The combination of the two waves thus represents the passage

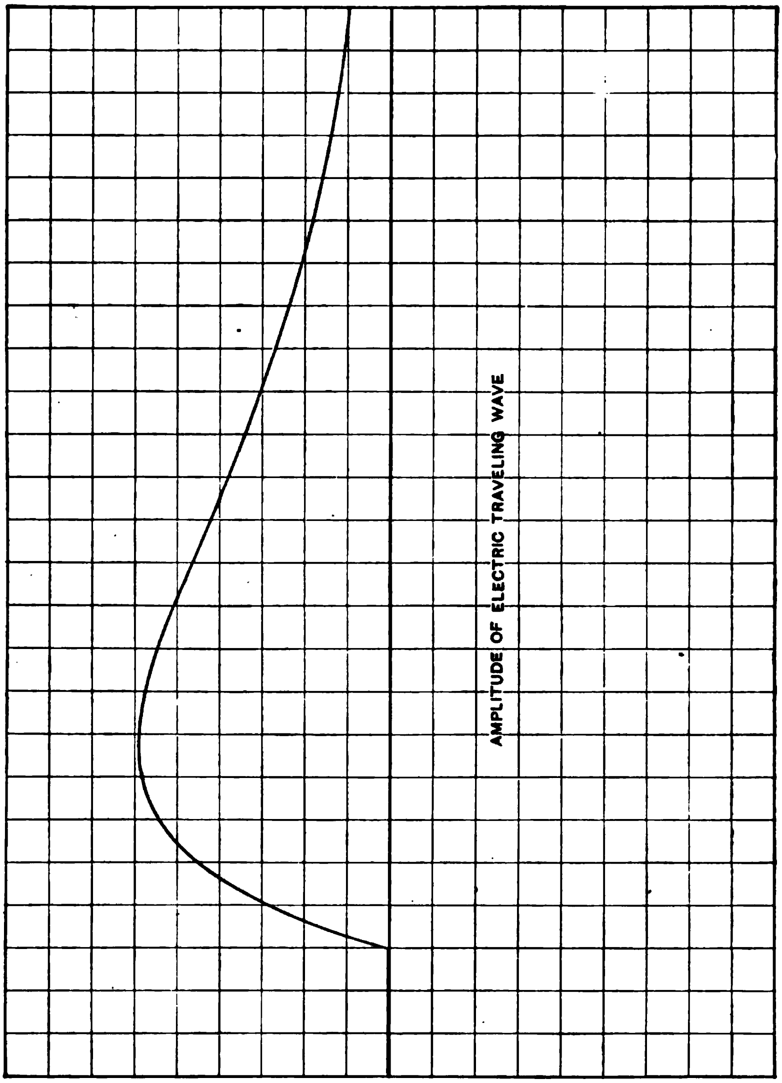


FIG. 1

of a wave across a given point, the amplitude rising during the arrival, and decreasing again after the passage of the wave.

9. If h and also f equal zero, i', e', and i'', e'', coincide in

equations (45) and (46), and the equations (45) (46) assume the form:

$$i = \epsilon^{-wt} \Sigma \{[B_1 \cos q (t-\lambda) + B_1' \sin q (t-\lambda)] - [B_2 \cos q (t+\lambda) + B_2' \sin q (t+\lambda)]\} \qquad (50)$$

$$e = \sqrt{\frac{L}{C}} \epsilon^{-wt} \Sigma \{[B_1 \cos q (t-\lambda) + B_1' \sin q (t-\lambda)] + [B_2 \cos q (t+\lambda) + B_2' \sin q (t+\lambda)]\} \qquad (51)$$

These equations contain the distance λ only in the trigonometric, but not in the exponential function, that is, i and e vary in phase throughout the circuit, but not in amplitude, or in other words, the oscillation is of uniform intensity throughout the circuit, dying out uniformly with the time, from an initial maximum value, but the wave does not travel along the circuit, but is a stationary or standing wave. It is an oscillatory discharge of a circuit containing a distributed r, L, g, C, and so analogous to the oscillating condenser discharge through an inductive circuit, except that, due to the distributed capacity, the phase changes along the circuit. The free oscillations of a uniform circuit, as a transmission line, are of this character.

For $\lambda = 0$, that is, assuming the wave length of the oscillation is so great, hence the circuit as such a small fraction of the wave length, that the phase of i and e can be assumed as uniform throughout the circuit, the equations (50) and (51) assume the form:

$$i = \epsilon^{-wt} \{B_0 \cos q t - B_0' \sin q t\}$$

$$e = \sqrt{\frac{L}{C}} \epsilon^{-wt} \{B_0 \cos q t + B_0' \sin q t\}$$

and these are the usual equations of the condenser discharge through an inductive circuit, which here appear as a special case of a special case of the general circuit equations.

If q equals zero, the functions D in equations (45) and (46) become constant, and these equations assume the form:

$$i = \epsilon^{-wt} \Sigma \{[C_1 \epsilon^{+f(t-\lambda)} + C_3 \epsilon^{-f(t-\lambda)}] - [C_2 \epsilon^{+f(t+\lambda)} + C_4 \epsilon^{-f(t+\lambda)}]\}$$

$$e = \sqrt{\frac{L}{C}} \epsilon^{-wt} \Sigma \{[C_1 \epsilon^{+f(t-\lambda)} + C_3 \epsilon^{-f(t-\lambda)}] + [C_2 \epsilon^{+f(t+\lambda)} + C_4 \epsilon^{-f(t+\lambda)}]\}$$

This gives expressions of current and voltage, which are not oscillatory any more, but exponential, representing a gradual change of i and e as functions of time and distance, corresponding to the gradual or logarithmic oonderiser discharge. For $\lambda = 0$, these equations change to the equations of the logarithmic condenser discharge.

These equations are only approximate however, since in them the quantities f, w, h, have been neglected compared with q, assuming the latter as very large, while now it is assumed as zero.

It thus follows:

If the constant h in equations (29) and (30) differs from zero, the oscillation (using the term *oscillation* here in the most general sense, that is, including also *alternation*, as an oscillation of zero attenuation) travels along the circuit; but it becomes stationary, as standing wave, for $h = 0$. That is:

The distance attenuation constant h may also be called the *propagation* constant of the wave.

$h = 0$ thus represents a wave which does not propagate or move along the circuit, but stands still, that is, a stationary or standing wave.

III. Standing Waves.

10. If the propagation constant of the wave vanishes:

$$h = 0$$

the wave becomes a stationary or standing wave, and the equations of the standing wave so are derived from the general equations, by substituting thereon: $h = 0$.

This gives:

$$R_2^2 = \sqrt{(k^2 - L\,C\,m^2)^2}$$

hence, if:

$$k^2 > L\,C\,m^2:\ R_2^2 = k^2 - L\,C\,m^2,$$

and, if:

$$k^2 < L\,C\,m^2:\ R_2^2 = L\,C\,m^2 - k^2$$

Two different cases exist, depending upon the relative values of k^2 and $L\,C\,m^2$, and in addition thereto the intermediary or critical case, in which $k^2 = L\,C\,m^2$.

These three cases require separate consideration.

$$L\,C\,m^2 = \left\{L\,C\,\tfrac{1}{2}\left(\frac{r}{L} - \frac{g}{C}\right)\right\}^2$$

is a circuit constant, while k is the wave length constant, that is, the higher k, the shorter is the wave length.

A. *Short Waves*:

$$k^2 > L\,C\,m^2$$

hence:

$$R_2^2 = k^2 - L\,C\,m^2$$

and,

$$= 0$$

and approximately,

$$= \frac{k}{\sqrt{L\,C}}$$

Substituting in equations (29), (30), the two waves i', e', and i'', e'', coincide, and all the exponential terms reduce to ϵ^{-wt}, and rearranged, give,

$$i = \epsilon^{-wt}\,\{[B_1 \cos(qt - ku) + B_1' \sin(qt - ku)] - [B_2 \cos(qt + ku) + B_2' \sin(qt + ku)]\} \tag{52}$$

$$e = \sqrt{\frac{L}{C}}\,\epsilon^{-wt}\,\{[B_1 \cos(qt - ku) + B_1' \sin(qt - ku)] + [B_2 \cos(qt + ku) + B_2' \sin(qt + ku)]\} \tag{53}$$

These equations represent a *stationary electrical oscillation* or standing wave in the circuit.

B. *Long Waves*:

$$k^2 < L\,C\,m^2$$

hence:

$$R_2^2 = L\,C\,m^2 - k^2$$

and:

$$f = \sqrt{m^2 - \frac{k^2}{LC}}$$

$$q = 0$$

or approximately, for very small values of k:

$$f = m = \frac{1}{2}\left(\frac{r}{L} - \frac{g}{C}\right)$$

Substituting now into (29) and (30), the two waves i', e', and i'', e'', remain separate, having different exponential terms, $\varepsilon^{-(w-f)t}$ and $\varepsilon^{-(w+f)t}$, but in each of the two waves, the main wave and the reflected wave coincide, due to the vanishing of q.

$$i = \varepsilon^{-wt}\left\{(B_1\,\varepsilon^{+ft} + B_2\,\varepsilon^{-ft})\cos ku - (B_1'\,\varepsilon^{+ft} + B_2'\,\varepsilon^{-ft})\sin ku\right\} \tag{54}$$

$$e = \sqrt{\frac{L}{C}}\,\varepsilon^{-wt}\left\{[(m+f)\,B_1'\,\varepsilon^{+ft} + (m-f)\,B_2'\,\varepsilon^{-ft}]\cos ku + [(m+f)\,B_1\,\varepsilon^{+ft} + (m-f)\,B_2\,\varepsilon^{-ft}]\sin ku\right\}$$

$$= \sqrt{\frac{L}{C}}\,\varepsilon^{-wt}\left\{m\,[(B_1'\,\varepsilon^{+ft} + B_2'\,\varepsilon^{-ft})\cos ku + (B_1\varepsilon^{+ft} + B_2\varepsilon^{-ft})\sin ku] + f\,[(B_1'\varepsilon^{+ft} - B_2'\varepsilon^{-ft})\cos ku + (B_1\varepsilon^{+ft} - B_2\varepsilon^{-ft})\sin ku]\right\}$$

These equations represent a *gradual or exponential circuit discharge*. The distribution is a trigonometric function of the distance, that is, a wave distribution, but dies out gradually in time, or without oscillation.

C. *Critical Case*:

$$k^2 = L\,C\,m^2$$

This gives:

$$i = \varepsilon^{-wt}\left\{B\cos ku - B'\sin ku\right\}$$

$$e = \sqrt{\frac{L}{C}}\,\varepsilon^{-wt}\left\{B'\cos ku + B\sin ku\right\}$$

In the critical case, the wave is distributed as trigonometric function of the distance, but dies out as simple exponential function of the time.

11. An electrical standing wave can have two different forms: it can be either oscillatory in time, or exponential in time, that is, gradually changing. It is interesting to investigate the conditions under which these two different cases occur.

The transition from gradual to oscillatory, takes place at:

$$k_0^2 = m^2 L C$$

for larger values of k, the phenomenon is oscillatory; for smaller, exponential or gradual:

As k is the wave length constant, the wave length, at which the phenomenon ceases to be oscillatory in time, and becomes a gradual dying out, is given by:

$$U_0 = \frac{2\pi}{k_0}$$

$$= \frac{2\,\pi}{m\sqrt{L\,C}}$$

In an undamped wave, that is, in a circuit of zero r and zero g, in which no energy losses occur, the velocity of propagation is:

$$v = \frac{q}{k} = \frac{1}{\sqrt{L\,C}}$$

and is the velocity of light:

$$v_0 = 3 \times 10^{10}$$

if the medium has unit permeability and unit inductivity.

In an undamped circuit, this wave length U_0 would correspond to the frequency:

$$q_0 = \frac{k_0}{\sqrt{L\,C}} = m$$

hence:

$$N_0 = \frac{q_0}{2\pi} = \frac{m}{2\pi}$$

The frequency, at the wave length U_0, is however, zero, as at this wave length the phenomenon ceases to be oscillatory. That is:

Due to the energy losses in the circuit, by the effective resistance r and effective conductance g, the frequency N of the wave is reduced below the value corresponding to the wave length U, the more, the greater the wave length, until at the wave length U_0 the frequency becomes zero, and the phenomenon thereby non-oscillatory. This means, with increasing wave length, the velocity of propagation of the phenomenon decreases, and becomes zero at wave length U_0.

$$\text{If } m^2 L C = 0, \ k_0 = 0, \text{ and } \ U_0 = \infty,$$

that is, the standing wave is always oscillatory.

$$\text{If } m^2 L C = \infty, \ k_0 = \infty, \text{ and } U_0 = 0,$$

that is, the standing wave is always non-oscillatory, or gradually dying out.

In the former case:

$$m^2 L C = 0, \text{ or oscillatory phenomenon,}$$

it is, substituting for m^2 and expanding:

$$\frac{r}{g} = \frac{L}{C}$$

or:

$$r C - g L = 0$$

In the latter case:

$m^2 L C = \infty$, or non-oscillatory or exponential standing wave, it is either:

$$L = 0, \ \text{or } C = 0.$$

That is:

The standing wave in a circuit is always oscillatory, regardless of its wave length, if

$$\frac{r}{g} = \frac{L}{C},$$

that is, the ratio of the energy coefficients equals the ratio of the reactive coefficients of the circuit.

The standing wave can never be oscillatory, but is always exponential, or gradually dying out, if either the inductance L, or the capacity C, vanish, that is, the circuit contains no capacity or contains no inductance. In all other cases the standing wave is oscillatory for waves shorter than the critical value $U_0 = \frac{2\pi}{k_0}$, where

$$k_0^2 = m^2 L C = \tfrac{1}{4} \left\{ r \sqrt{\frac{C}{L}} - g \sqrt{\frac{L}{C}} \right\}^2 \tag{56}$$

and is exponential or gradual, for standing waves longer than the critical wave length U_0. Or, for $k < k_0$, the standing wave is exponential; for $k > k_0$, it is oscillatory.

The term $k_0 = m \sqrt{L C}$ takes a similar part in the theory of standing waves as the term: $r_0^2 - 4 L_0 C_0$ in the condenser discharge through an inductive circuit, that is, it separates the exponential or gradual, from trigonometric or oscillatory conditions.

The difference is, that the condenser discharge through an inductive circuit is gradual, or oscillatory, depending on the circuit constants, while in a general circuit, with the same circuit constants, in general gradual as well as oscillatory standing waves exist, the former at greater wave length, or

$$m \sqrt{L C} > k,$$

the latter for shorter wave length, or

$$m \sqrt{L C} < k,$$

An idea of the quantity k_0, and so the wave length U_0, at which the frequency of the standing wave becomes zero, or the wave non-oscillatory, and of the frequency N_0, which in an undamped circuit will correspond to this critical wave length N_0, can best be derived by considering some representative numerical instance.

1. *High Power High Potential Overhead Transmission Line.*

12. Assuming power to be transmitted 120 miles, at 40,000 volts between line and ground, by a three-phase system with grounded neutral, over a line consisting of copper conductors, wire No. 00 B. & S. gauge with 5 feet between conductors.

Choosing as unit length the mile, it is, approximately,

$$r = .41 \text{ ohms}$$
$$L = 1.95 \times 10^{-3}\, h$$
$$g = .25 \times 10^{-6} \text{ mho}$$
$$C = .0162 \times 10^{-6}\, f.$$

Herefrom then follows:

$$w = 113$$
$$m = 97$$
$$\sigma = \sqrt{L\,C} = 5.62 \times 10^{-6}$$
$$k_0 = m\,L\,C = 545 \times 10^{-6}$$

hence, the critical wave length:

$$U_0 = \frac{2\pi}{k_0} = 11{,}500 \text{ miles}$$

and, in an undamped circuit, this wave length would correspond to the frequency of oscillation:

$$N_0 = \frac{m}{2\pi} = 15.7 \text{ cycles per second.}$$

Since the shortest wave at which the phenomenon ceases to be oscillatory, is 11,500 miles in length, and the longest wave, which can originate in the circuit, is 4 times the length of the circuit, or 480 miles, it follows that whatever waves may originate in this circuit, are by necessity oscillatory, and non-oscillatory currents or voltages can exist in this circuit only when impressed upon it by some outside source, and then are of such great wave length, that the circuit is only an insignificant fraction of the wave, and great differences of voltage and current of non-oscillatory nature cannot exist.

Since the difference in length, between the shortest non-oscillatory wave, and the longest wave which can originate in the circuit, is so very great, it follows, that in high potential long distance transmission circuits, all phenomena which may result in considerable potential differences and differences of current throughout the circuit, are oscillatory in nature.

With a length of circuit of 120 miles, the longest standing wave, which can originate in the circuit, has the wave length:

$$U = 480 \text{ miles.}$$

and herefrom follows:

$$k = \frac{2\pi}{U} = .0134$$

$$q = \frac{k}{\sqrt{L\,C}} = 2380$$

$$N = \frac{q}{2\pi} = 380 \text{ cycles.}$$

Hence, even for the longest standing wave, which may originate in this transmission line, and still more so for shorter waves, the overtones of the fundamental wave, the approximations in paragraph 7 are justified, and it thus is:

General Equations of Standing Waves in Long Distance Power Transmission Lines:

$$i = \Sigma\, \epsilon^{-wt} \{[B_1 \cos (qt - ku) + B_1' \sin (qt - ku)] - [B_2 \cos (qt + ku) + B_2' \sin (qt + ku)]\} \tag{57}$$

$$e = + \sqrt{\frac{L}{C}}\, \Sigma \epsilon^{-wt} \{[B_1 \cos (qt - ku) + B_1' \sin (qt - ku)] + [B_2 \cos (qt + ku) + B_2' \sin (qt + ku)]\} \tag{58}$$

or:

$$e = \Sigma \epsilon^{-wt} \{[A_1 \cos (qt + ku) + A_1' \sin (qt + ku)] + [A_2 \cos (qt - ku) + A_2' \sin (qt - ku)]\} \tag{59}$$

$$i = \sqrt{\frac{C}{L}}\, \Sigma \epsilon^{-wt} \{[A_1 \cos (qt + ku) + A_1' \sin (qt + ku)] - [A_2 \cos (qt - ku) + A_2' \sin (qt - ku)]\} \tag{60}$$

where:

$$A_1 = -\sqrt{\frac{L}{C}}B_2$$

$$A_1' = -\sqrt{\frac{L}{C}}B_2'$$

etc.

2. *High Potential Underground Power Cable.*

13. Assuming, per mile of single conductor:

r = .41 ohms.

$L = .4 \times 10^{-3} h.$

$g = 1 \times 10^{-6}$ mho, corresponding to a power factor of the cable charging current, at 25 cycles, of 1 per cent.

$C = .6 \times 10^{-6} f.$

Herefrom then follows:

$$w = 513$$

$$m = 512$$

$$\sigma = \sqrt{LC} = 15.5 \times 10^{-6}$$

$$k_0 = m\sqrt{LC} = 7.95 \times 10^{-3}$$

and, critical wave length:

$$U_0 = 790 \text{ miles,}$$

and, frequency of an undamped oscillation, corresponding to U_0:

$$N_0 = 81.5 \text{ cycles.}$$

As seen, in an underground high potential cable, the critical wave length is very much shorter than in the overhead long distance transmission line. At the same time, however, the length of an underground cable circuit is very much shorter than that of a long distance transmission line, so that the critical wave length still is very large compared with the greatest wave length of an oscillation originating in the cable; about 10 times as great.

That is, the discussion of the possible phenomena in any overhead line applies also to the underground high potential cable circuit.

3. *Submarine Telegraph Cable.*

Choosing the values:

Length of cable, 4000 miles.

Constants per mile of conductor:

$$r = 3 \text{ ohms}$$
$$L = 1 \times 10^{-3}\, h$$
$$g = 1 \times 10^{-6} \text{ mho}$$
$$C = .1 \times 10^{-6}\, f$$

hence:

$$w = 1500$$
$$m = 1500$$
$$\sigma = L\,C = 10 \times 10^{-6}$$
$$k_0 = m\sqrt{L\,C} = 15 \times 10^{-3}$$

and:

$$U_0 = 418 \text{ miles}$$

thus:

$$N_0 = 477 \text{ cycles.}$$

That is, in a submarine cable, the critical wave length U_0 is relatively short, so that in long submarine cables, standing waves may appear, which are not oscillatory in time, but die out gradually. In such cables, due to their relative high resistance, the damping effect is very great, $w = 1500$, and standing waves rapidly die out.

In the investigation of the submarine cable, the complete equations must therefore be used, and q cannot always be assumed as large compared with m and w, except when dealing with local oscillations.

4. *Long Distance Overhead Telephone Circuit.*

14. Considering a telephone circuit of 1000 miles length, metallic return, consisting of two wires No. 4 B. & S. gauge, 24 in. distant from each other.

Per mile of conductor:

$$r = 1.31 \text{ ohms}$$
$$L = 1.84 \times 10^{-3}\, h$$
$$C = .0172 \times 10^{-6}\, f$$

as conductance g we may assume:

(*a*) $g = 0$, that is, very perfect insulation, as in dry weather.

(*b*) $q = 2.5 \times 10^{-6}$, that is, slightly leaky lines.

(*c*) $g = 12 \times 10^{-6}$, that is, poor insulation, or a leaky line.

(*d*) $g = 40 \times 10^{-6}$, that is, extremely poor insulation, as during heavy rain.

The condition may also be investigated where the line is loaded with inductance coils, spaced so close together, that in their effect we can consider this additional inductance as uniformly distributed. Let the total inductance, per unit length, be increased by the loading coils to:

$$L_1 = 9 \times 10^{-3} h$$

or about 5 times the normal value.

Denoting then the constants of the loaded line by the index 1, it is:

				(a)	(b)	(c)	(d)
w	=	=		356	429	706	1518
w_1	=	=		73	146	423	1236
m	=	=		356	283	6	−806
m_1	=	=		73	0	−277	−1090
σ	$=\sqrt{LC}$	=	5.63×10^{-6}				
σ_1	$=\sqrt{L_1C}$	=	12.45×10^{-6}				
k_0	$=m\sqrt{LC}$	=		2×10^{-3}	1.6×10^{-3}	33.7×10^{-6}	4.56×10^{-3}
k_{01}	$=m_1\sqrt{L_1C}$	=		91×10^{-6}	0	3.45×10^{-3}	13.5×10^{-3}
U_0	=	=		3140	3920	187,000	1380
U_{01}	=	=		69,000	∞	1820	464
N_0	=	=		55.6	45	.96	128
N_{01}	=	=		11.6	0	44	173

Hence, in a long distance telephone line, distributed leakage up to a certain amount, increases the critical wave length, and so makes even the long wave oscillatory. Beyond this amount, leakage again decreases the wave length. Distributed inductance, as by loading the line, increases the critical wave length, if the leakage is small, but in a badly leaky line, it decreases the critical wave length, and the amount of leakage, up to which an increase of the critical wave length occurs, is less, in a loaded line, that is, in a line of higher inductance.

In other words, a moderate amount of distributed leakage improves a long distance telephone line, an excessive amount of leakage spoils it. An increase of inductance, by loading the line, improves the line, if the leakage is small, but spoils the line, if the leakage is considerable. The amount of leakage up to which improvement in the telephone line occurs, is less in a loaded than in an unloaded line, that is, a loaded telephone line requires a far better insulation than an unloaded line.

IV. Traveling Waves.

15. As seen, especially in electric power circuits, overhead or underground, the longest existing standing wave has a wave length which is so small compared with the critical wave length —where the frequency becomes zero—that the effect of the damping constant on frequency and wave length is negligible. The same obviously applies also to traveling waves, generally to a greater extent still, since traveling waves commonly are of wave lengths, which are only a small part of the length of the circuit. Usually therefore in the discussion of traveling waves, the effect of the damping constants on the frequency constant q and the wave length constant k can be neglected, that is, frequency and wave length assumed as independent of the energy loss in the circuit, and the equations (45) and (46) applied in dealing with the traveling wave.

In these equations, the distance traveled by the wave per second, is used as unit length, by the substitution:

$$\lambda = \sigma u$$

where:

$$\sigma = \sqrt{L C}$$

as this brings t and λ into direct comparison, and eliminates h and k from the equations, by the equation (43).

This gives, under the limiting conditions discussed before, the

General Equations of the Traveling Wave:

$$i = \epsilon^{-ut} \{\epsilon^{+f(t-\lambda)} [B_1 \cos q (t-\lambda) + B_1' \sin q (t-\lambda)] - \epsilon^{+f(t+\lambda)} [B_2 \cos q (t+\lambda) + B_2' \sin q (t+\lambda)] + \epsilon^{-f(t-\lambda)} [B_3 \cos q (t-\lambda) + B_3' \sin q (t-\lambda)] - \epsilon^{-f(t+\lambda)} [B_4 \cos q (t+\lambda) + B_4' \sin q (t+\lambda)]\} \tag{61}$$

$$e = \sqrt{\frac{L}{C}}\,\epsilon^{-wt}\,\{\epsilon^{+f(t-\lambda)}\,[B_1 \cos q\,(t-\lambda) + B_1{}' \sin q\,(t-\lambda)] + \epsilon^{+f(t+\lambda)}$$
$$[B_2 \cos q(t+\lambda) + B_2{}' \sin q(t+\lambda)] + \epsilon^{-f(t-\lambda)}[B_3 \cos q(t-\lambda)$$
$$+ B_3{}' \sin q\,(t-\lambda)] + \epsilon^{-f(t+\lambda)}\,[B_4 \cos q\,(t+\lambda) + B_4{}' \sin q$$
$$(t+\lambda)]\} \tag{62}$$

or:

$$e = \epsilon^{-wt}\,\{\epsilon^{+f(t-\lambda)}\,[A_1 \cos q\,(t-\lambda) + A_1{}' \sin q\,(t-\lambda)] + \epsilon^{+f(t+\lambda)}[A_2 \cos$$
$$q\,(t+\lambda) + A_2{}' \sin q(t+\lambda)] + \epsilon - {}^{f(t-\lambda)}\,[A_3 \cos q\,(t-\lambda) + A_3{}'$$
$$\sin q\,(t-\lambda)] + \epsilon^{-f(t+\lambda)}\,[A_4 \cos q\,(t+\lambda) + A_4{}' \sin q\,(t+\lambda)]\} \tag{63}$$

$$i = \sqrt{\frac{C}{L}}\,\epsilon^{-wt}\,\{\epsilon^{+f(t-\lambda)}\,[A_1 \cos q\,(t-\lambda) + A_1{}' \sin q\,(t-\lambda)] - \epsilon^{+f(t+\lambda)}$$
$$[A_2 \cos q\,(t+\lambda) + A_2{}' \sin q\,(t+\lambda)] + \epsilon^{-f(t-\lambda)}\,[A_3 \cos q$$
$$(t-\lambda) + A_3{}' \sin q\,(t-\lambda)] - \epsilon^{-f(t-\lambda)}\,[A_4 \cos q\,(t+\lambda) + A_4{}'$$
$$\sin q\,(t+\lambda)]\} \tag{64}$$

where:

$$w = \tfrac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right) \tag{65}$$

In these equations the values A, B, are integration constants, which are determined by the terminal conditions of the problem.

The terms with $(t-\lambda)$ may be considered as the main wave, the terms with $(t+\lambda)$ as the reflected wave, or inversely, depending on the direction of propagation of the wave.

16. As the traveling wave consists of a main wave, with variable $(t-\lambda)$, and a reflected wave of the same character, but moving in opposite direction, thus with the variable $(t+\lambda)$, these waves may be studied separately, and afterwards the effect of their combination investigated.

Thus, considering at first one of the waves only, that with the variable $(t-\lambda)$, it is, from equations (63) to (64):

$$e = \epsilon^{-wt}\,\{\epsilon^{+f(t-\lambda)}\,[A_1 \cos q\,(t-\lambda) + A_1{}' \sin q\,(t-\lambda)] + \epsilon^{-f(t-\lambda)}$$
$$[A_3 \cos q\,(t-\lambda) + A_3{}' \sin q\,(t-\lambda)]\} \tag{66}$$
$$= \epsilon^{-wt}\,\{(A_1\,\epsilon^{+f(t-\lambda)} + A_3\,\epsilon^{-f(t-\lambda)}) \cos q\,(t-\lambda) + (A_1{}'\,\epsilon^{+f(t-\lambda)} + A_3{}'$$
$$\epsilon^{-f(t-\lambda)}) \sin q\,(t-\lambda)\}$$

and:

$$i = \sqrt{\frac{C}{L}}\, e$$

That is, in a single traveling wave, current and voltage are in phase with each other, and proportional to each other, with an *effective impedance*:

$$z = \frac{e}{i} = \sqrt{\frac{L}{C}}$$

This proportionality between e and i, and coincidence of phase, obviously does not exist any more in the combination of main waves and reflected waves, since in reflection, the current reverses with the reversal of the direction of propagation, while the voltage remains in the same direction.

In equation (66), the time t appears only in the term $(t-\lambda)$, except in the factor ϵ^{-wt}, while the distance λ appears only in the term $(t-\lambda)$. Substituting therefore:

$$\vartheta = t - \lambda$$

hence:

$$t = \vartheta + \lambda$$

that is, counting the time differently at any point λ, and counting it at every point of the circuit from the same point in the phase of the wave, from which the time t is counted at the starting point of the wave, $\lambda = 0$, or, in other words, shifting the starting point of the counting of time, with the distance λ, it is, substituted in (66):

$$\left.\begin{aligned} e &= \epsilon^{-wt}\, \{\epsilon^{+j\vartheta} (A_1 \cos q\,\vartheta + A_1' \sin q\,\vartheta) + \epsilon^{-j\vartheta} (A_3 \cos q\,\vartheta \\ &\qquad + A_3' \sin q\,\vartheta)\} \\ &= \epsilon^{-w\lambda-}\epsilon^{w\vartheta}\, \{\epsilon^{+j\vartheta} (A_1 \cos q\,\vartheta + A_1' \sin q\,\vartheta) + \epsilon^{-j\vartheta} (A_3 \cos q \\ &\qquad \vartheta + A_3 \sin q\,\vartheta)\} \\ &= \epsilon^{-w\lambda}\, \epsilon^{-w\vartheta}\, \{(A_1\, \epsilon^{+j\vartheta} + A_3\, \epsilon^{-j\vartheta}) \cos q\,\vartheta + (A_1' \epsilon^{+j\vartheta} + A_3' \\ &\qquad \epsilon^{-j\vartheta}) \sin q\,\vartheta)\} \end{aligned}\right\} \quad (67)$$

The latter form of the equation is best suited to represent the variation of the wave, at a fixed point λ in space, as function of the local time ϑ.

The wave is the product of a term: $\varepsilon^{-w\lambda}$ which decreases with increasing distance λ, and a term:

$$e_0 = \varepsilon^{-w\vartheta}\{\varepsilon^{+f\vartheta}(A_1 \cos q\,\vartheta + A_1' \sin q\,\vartheta) + \varepsilon^{-f\vartheta}(A_2 \cos q\,\vartheta + A_2' \sin q\,\vartheta)\}$$

$$= \varepsilon^{-w\vartheta}\{(A_1\varepsilon^{+f\vartheta} + A_2\varepsilon^{-f\vartheta}) \cos q\,\vartheta + (A_1'\varepsilon^{+f\vartheta} + A_2'\varepsilon^{-f\vartheta}) \sin q\,\vartheta)\} \tag{68}$$

which latter term is independent of the distance, but merely a function of the time ϑ, when counting the time at any point of the line from the moment of the passage of the same phase of the wave.

Since the coefficient in the exponent of the distance decrement $\varepsilon^{-w\lambda}$, contains only the circuit constants:

$$w = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right)$$

but does not contain f and q, or the other integration constants.

It follows herefrom:

The attenuation constant of a traveling wave, and therefore the distance decrement of the wave depends upon the circuit constants r, L, g, C, only, but does not depend upon the wave length, frequency, voltage or current; hence all traveling waves in the same circuit die out at the same rate, regardless of their frequency and of their wave shape, or in other words, a complex traveling wave retains its wave shape, when traversing a circuit, and merely decreases in amplitude by the distance decrement $\varepsilon^{-w\lambda}$.

The wave attenuation is a constant of the circuit.

This obviously applies only for waves of constant velocity, that is, such waves in which q is large compared with f, w, and m, and does not strictly apply to extremely long waves, as discussed in 13.

17. By changing the line constants, as by inserting inductance in such a manner as to give the effect of uniform distribution (loading the line), the attenuation of the wave can be reduced, that is, the wave caused to travel a greater distance u with the same decrease of amplitude.

As function of the inductance L, the attenuation constant is a minimum for:

$$\frac{d\,w_0}{d\,L} = 0$$

hence:

$$r\,C - g\,L = 0$$

or:

$$\frac{L}{C} = \frac{r}{g}$$

and, if the conductance:

$$g = 0$$

it is:

$$L = \infty$$

Hence, in a perfectly insulated circuit, or circuit having no energy losses depending on the voltage, the attenuation decreases with increase of the inductance, that is, by " loading the line ", and the more inductance is inserted, the better the telephonic transmission is.

In a leaky telephone line, increase of inductance decreases the attenuation, and so improves the telephonic transmission, up to the value of inductance,

$$L = \frac{r\,C}{g}$$

and beyond this value, inductance is harmful, by again increasing the attenuation.

For instance, if in a long distance telephone circuit per mile:

$$\begin{aligned} r &= 1.31 \text{ ohms} \\ L &= 1.84 \times 10^{-3}\, h \\ g &= 1.0 \times 10^{-6} \text{ mho} \\ C &= 0.0172 \times 10^{-6}\, f \end{aligned}$$

the attenuation of a traveling wave or impulse is:

$$w_0 = 0.00217$$

hence, for a distance or length of line of $l = 2000$ miles:

$$\begin{aligned} \varepsilon^{-w_0 l} &= \varepsilon^{-4.34} \\ &= 0.0129 \end{aligned}$$

that is, the wave is reduced to 1.29 per cent. of its original value.

The best value of inductance is, by

$$L = \frac{r}{g} C = 0.0225\, h$$

and in this case, the attenuation constant becomes:

$$w_0 = 0.00114$$

and thus:

$$\begin{aligned} \varepsilon^{-w_0 l} &= \varepsilon^{-2.24} \\ &= 0.1055 \end{aligned}$$

or 10.55 per cent of the original value of the wave.

In this telephone circuit, by adding per mile an additional inductance of:

$$22.5 - 1.84 = 20.7 \text{ mh.}$$

the intensity of the arriving wave is increased from 1.29 per cent. to 10.55 per cent. or more than 8 times.

If however, in wet weather, the leakage increases to the value:

$$g = 5 \times 10^{-6}$$

in the unloaded line, it is:

$$\begin{aligned} w_0 &= 0.00282 \\ \varepsilon^{-w_0 l} &= 0.0035 \end{aligned}$$

while in the loaded line, it is:

$$\begin{aligned} w_0 &= 0.00341 \\ \varepsilon^{-w_0 l} &= 0.0011 \end{aligned}$$

and while with the unloaded line, the arriving wave is still 0.35 per cent. of the outgoing wave, in the loaded line, it is only 0.11 per cent.; that is, in this case, loading the line with inductance has badly spoiled telephonic communication and increased the decay of the wave more than three-fold. A loaded telephone line is much more sensitive to changes of leakage g, that is, to meteorological conditions.

18. The equation of the traveling wave (67):

$$e = \varepsilon^{-w\lambda}\varepsilon^{-w\vartheta}\{\varepsilon^{+f\vartheta}(A_1 \cos q\vartheta + A_1 \sin q\vartheta) + \varepsilon^{-f\vartheta}(A_2 \cos q\vartheta + A_2' \sin q\vartheta)\} \tag{67}$$

can be reduced to the form:

$$e = \varepsilon^{-w\lambda}\{E_1 \varepsilon^{-w\vartheta_1}(\varepsilon^{+f\vartheta_1} - \varepsilon^{-f\vartheta_1}) \sin q\vartheta_1 + E_2 \varepsilon^{-w\vartheta_2}(\varepsilon^{+f\vartheta_2} - \varepsilon^{-f\vartheta_2}) \cos q\vartheta_2\} \tag{69}$$

where:

$$\vartheta_1 = \vartheta + \gamma_1 = t - \lambda - \gamma_1$$

$$\vartheta_2 = \vartheta + \gamma_2 = t - \lambda - \gamma_2$$

Any traveling wave can be resolved into, and considered as consisting of, a combination of 2 waves: the

Traveling Sine Wave:

$$e_1 = E_1 \varepsilon^{-w\lambda}\varepsilon^{-w\vartheta}(\varepsilon^{+f\vartheta_1} - \varepsilon^{-f\vartheta_1}) \sin q\vartheta_1 \tag{70}$$

Traveling Cosine Wave:

$$e_2 = E_2 \varepsilon^{-w\lambda}\varepsilon^{-w\vartheta_2}(\varepsilon^{+f\vartheta_2} - \varepsilon^{-f\vartheta_2}) \cos q\vartheta_2 \tag{71}$$

Since q is a large quantity compared with w and f, the two component traveling waves, (70) and (71), differ appreciably from each other in appearance only for very small values of ϑ, that is, near $\vartheta_1 = 0$ and $\vartheta_2 = 0$. The traveling sine wave rises in the first half cycle very slightly, while the traveling cosine wave rises rapidly, *i.e.*, the tangent of the angle which the wave makes with the horizontal, or $\frac{d\,e}{d\,t}$, equals 0 with the sine wave, and has a definite value with the cosine wave.

All traveling waves in an electric circuit can be resolved into constituent elements, traveling sine waves and traveling cosine waves, and the general traveling wave consists of four component waves, a sine wave, its reflected wave, a cosine wave, and its reflected wave.

The elements of the traveling wave, the traveling sine wave e_1, and the traveling cosine wave e_2, contain 4 constants:

The intensity constant: E

The attenuation constant: w respectively w_0

The frequency constant: q

And the constant: f

The wave starts from zero, builds up to a maximum, and then gradually dies out, to zero at infinite time.

The absolute term of the wave, that is, the term which represents the values between which the wave oscillates, is:

$$e_0 = E\,\varepsilon^{-w\lambda}\,\varepsilon^{-w\vartheta}\,(\varepsilon^{+f\vartheta} - \varepsilon^{-f\vartheta}) \tag{72}$$

This term e_0 may be called the *amplitude* of the wave. The amplitude e_0 is a maximum for the value of ϑ, given by:

$$\frac{d\,e_0}{d\,\vartheta} = 0$$

$$\vartheta_0 = \frac{1}{2f}\log\frac{w+f}{w-f} \tag{73}$$

and, substituting this value into the equation of the absolute term of the wave, (72), gives:

$$e_m = E\,\varepsilon^{-w\lambda}\frac{2f}{\sqrt{w^2-f^2}}\left(\frac{w+f}{w-f}\right)^{-\frac{w}{2f}}$$

The rate of building up of the wave, or the steepness of the wave front, is given by:

$$G_0 = \left|\frac{d\,e_0}{d\,\vartheta}\right|_{\vartheta=0}$$

$$= 2\,f\,E\,\varepsilon^{-w\lambda}$$

It is a maximum, that is, ϑ_0 a minimum, for the value of f, in equation (73), given by:

$$\frac{d\,\vartheta_0}{d\,f} = 0$$

or:

$$f = 0$$

or the standing wave, which rises infinitely fast, that is, appears instantly.

The smaller therefore f is, the more rapidly is the rise of the traveling wave, and f may be called the *acceleration constant* of the traveling wave.

ϑ is the time counted from the beginning of the wave.

Since:

$$\vartheta = t - \lambda - \gamma$$

if we change the zero point of the distance, that is, count the distance λ from that point of the line, at which the wave starts at time $t = 0$, or in other words, count time t and distance λ from the origin of the wave, it is:

$$\vartheta = t - \gamma$$

and the traveling wave so may be represented by:

The amplitude:

$$e_0 = E\,\varepsilon^{-wt}\,(\varepsilon^{+f\vartheta} - \varepsilon^{-f\vartheta})$$

The sine wave:

$$e_1 = E\,\varepsilon^{-wt}\,(\varepsilon^{+f\vartheta} - \varepsilon^{-f\vartheta})\sin q\,\vartheta = e_0 \sin q\,\vartheta \qquad (74)$$

The cosine wave:

$$e_2 = E\,\varepsilon^{-wt}\,(\varepsilon^{+f\vartheta} - \varepsilon^{-f\vartheta})\cos q\,\vartheta = e_0 \cos q\,\vartheta$$

and $\vartheta = t - \lambda$ can be considered as the distance, counting backwards from the wave front, or the distance counted with the point λ, which the wave has just reached, as zero point, and in opposite direction to λ, or as the time, counting from the moment of the arrival of the wave front.

Equation (74) represents the distribution of the wave along the line, at the moment t, or the variation of the wave with the time, at a point λ.

As seen, the wave maintains its shape, but progresses along the line, and at the same time dies out, by the time decrement ε^{-wt}.

19. As an instance may be considered a traveling wave of the constants:

$$w = 115$$
$$f = 45$$
$$q = 2620$$
$$E = 100$$

hence:

$$e_0 = 100\, \varepsilon^{-115t}\, (\varepsilon^{+45\vartheta} - \varepsilon^{-45\vartheta})$$

$$= 100\, \varepsilon^{-115\lambda}\, (\varepsilon^{-70\vartheta} - \varepsilon^{-160\vartheta})$$

where:

$$\vartheta = t - \lambda$$

In Fig. 2 is shown the amplitude e_0, as function of the distance λ, for the different values of time:

$$t = 2,\ 4,\ 8,\ 12,\ 16,\ 20,\ 24 \text{ and } 32 \times 10^{-3}$$

with the maximum amplitude e_m, in dotted line, as envelope of the curves of e_0.

As seen, the amplitude of the wave gradually rises, and at the same time spreads over the line, reaching the maximum at the starting point $\lambda = 0$ at the time $t_0 = 9.2 \times 10^{-3}$ sec., and then decreases again while continuing to spread over the line, until it gradually dies out.

It is interesting to note that the distribution curves of the amplitude are nearly straight lines, but also, that even in the longest power transmission line the wave has reached the end of the line, and reflection occurs, before the maximum of the wave is reached: the unit of length λ is the distance traveled by the wave per second, or 188,000 miles, and during the rise of the wave, at the origin, from its start to the maximum, or 9.2×10^{-3} sec., the wave has traveled 1760 miles, thus the reflected wave would have returned to the origin before the maximum of the wave is reached, if the circuit is shorter than 880 miles.

Fig. 3 shows the passage of the traveling wave: $e_1 = e_0 \sin q\, \vartheta$, across a point λ of the line, with the local time ϑ as abscissas, and the instantaneous values of e_1 as ordinates. The values are given for $\lambda = 0$, where $\vartheta = t$; for any other point of the line, λ, the wave shape is the same, but all the ordinates reduced by the factor $\varepsilon^{-115\lambda}$, in the proportion as shown in the dotted curve in Fig. 2.

Fig. 4 shows the beginning of the passage of the traveling wave across a point $\lambda = 0$ of the line, that is, the starting of a

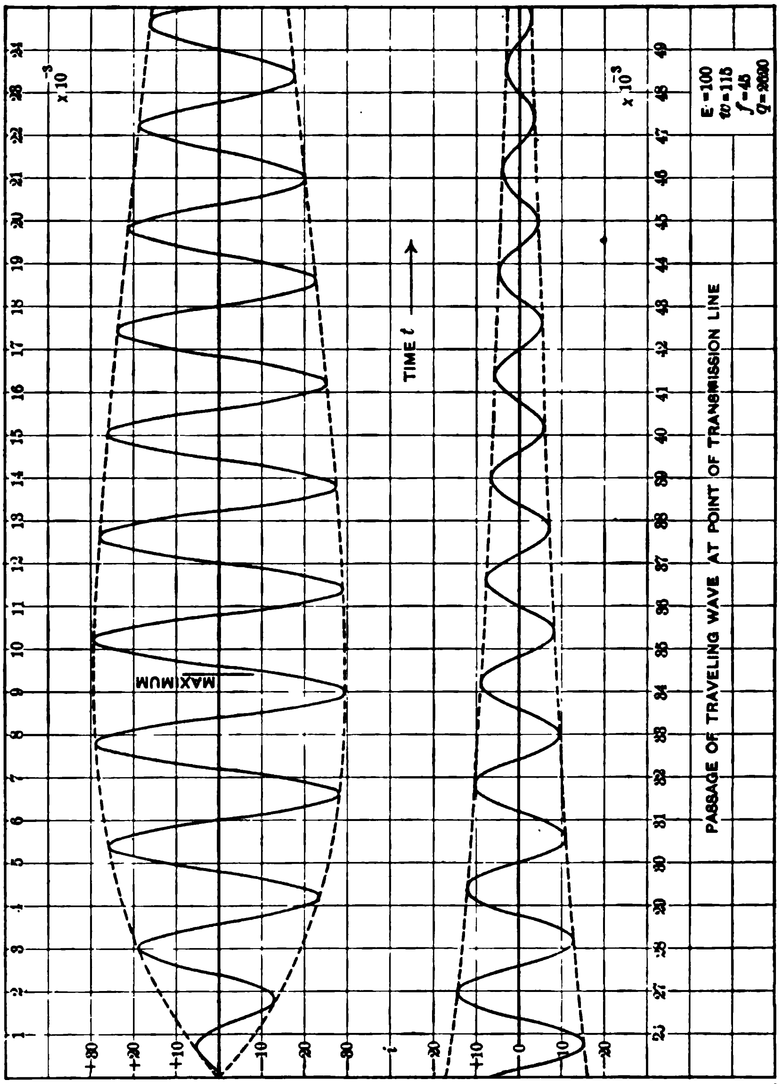

PASSAGE OF TRAVELING WAVE AT POINT OF TRANSMISSION LINE
TIME t
MAXIMUM
E=100

FIG. 3

gives:

$$i = [\epsilon^{-hu} \{C_1 \cos (qt - ku) + C_1' \sin (qt - ku)\} - \epsilon^{+hu} \{C_2 \cos (qt+ku) + C_2' \sin (qt+ku)\}] - \epsilon^{-2wt} [\epsilon^{-hu} \{C_4 \cos (qt+ku) + C_4' \sin (qt+ku)\} - \epsilon^{+hu} \{C_3 \cos (qt-ku) + C_3' \sin (qt-ku)\}] \quad (75)$$

and:

$$e = [\epsilon^{-hu} \{(c_1' C_1' - c_1 C_1) \cos (qt-ku) - (c_1' C_1 + c_1 C_1') \sin (qt - ku)\} + \epsilon^{+hu} \{(c_1' C_2' - c_1 C_2) \cos (qt+ku) - (c_1' C_2 + c_1 C_2') \sin (qt+ku)\}] + \epsilon^{-2wt} [\epsilon^{-hu} \{(c_2' C_4' - c_2 C_4) \cos (qt+ku) - (c_2' C_4 + c_2 C_4') \sin (qt+ku)\} + \epsilon^{+hu} \{(c_2' C_3' - c_2 C_3) \cos (qt-ku) - (c_2' C_3 + c_2 C_3') \sin (qt-ku)\}] \quad (76)$$

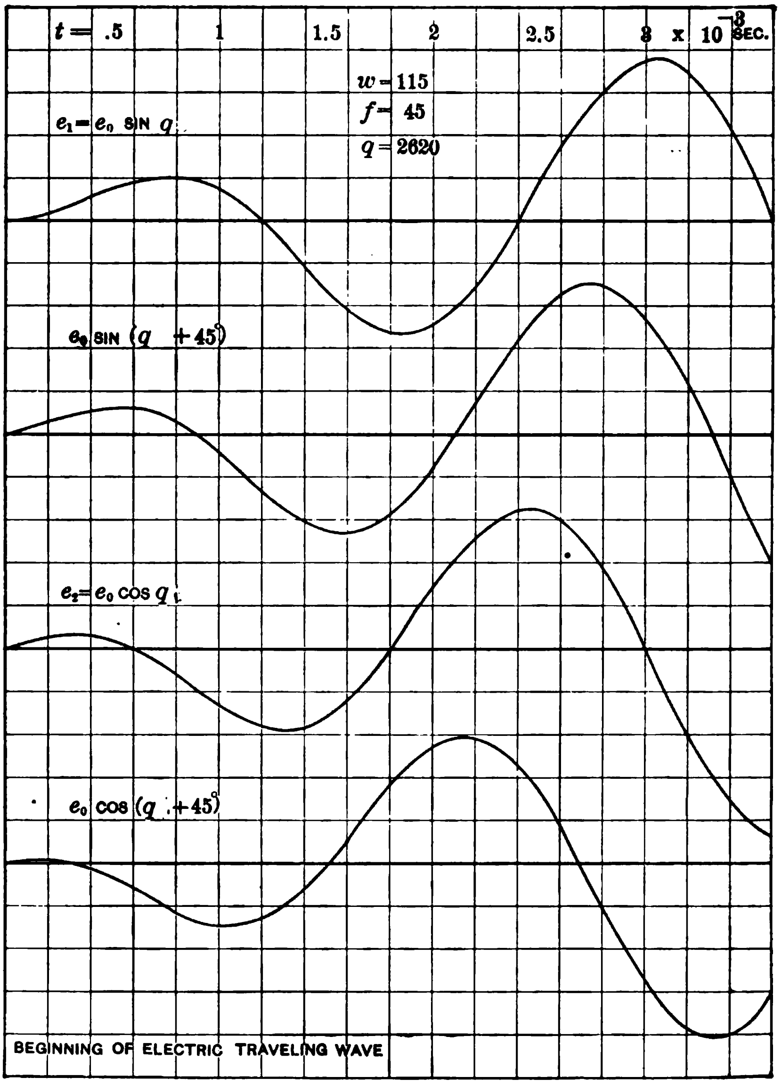
n these equations of current i and voltage e, the first term represents the usual equations of the distribution of alternating current and voltage in a long distance transmission line, and can by the substitution of complex quantities be reduced to the form usually given.

The second term is a transient term of the same frequency. That is:

In a long distance transmission line or other circuits of distributed r, L, g, C, when traversed by alternating current under an alternating impressed voltage, at a change of circuit conditions a transient term of fundamental frequency may appear, which has the time decrement, that is, dies out at the rate:

$$\epsilon^{-2wt} = \epsilon^{-\left(\frac{r}{L}+\frac{g}{C}\right)t}$$

It is interesting to note, that here the general equations of alternating-current long distance transmission appear as a special case of the equation of the traveling wave, and indeed can be considered as a section of a traveling wave, in which the acceleration constant f equals the exponential decrement w, and the attenuating constant α of the usual transmission line equation is the constant h; the wave length constant β is the constant k.



$t =$ .5 1 1.5 2 2.5 3 x 10^{-8} SEC.
$w = 115$
$f = 45$
$q = 2620$
$e_1 = e_0$ SIN q
e_0 SIN $(q + 45°)$
$e_2 = e_0$ COS q
e_0 COS $(q + 45°)$
BEGINNING OF ELECTRIC TRAVELING WAVE

FIG. 4

V. Free Oscillations.

21. The general equations of the electric circuit (29) and (30), contain 8 terms: 4 waves, 2 main waves and their reflected waves, and each wave consists of a sine term and a cosine term.

The equations contain 5 constants:

The frequency constant q
The wave length constant k
The time attenuation constant w
The distance attenuation constant k
The time acceleration constant f

of which the time attenuation w is a constant of the circuit, independent of the character of the wave.

By the value of the acceleration constant f, waves may be subdivided into:

$f = 0$: standing waves,

$w > f > 0$: traveling waves

$f = w$: alternating, current and voltage waves.

The general equations contain 8 integration constants C and C', which have to be determined by the terminal conditions of the problem.

Upon the values of these integration constants C and C' largely depends the difference between the phenomena occurring in electric circuits, as those due to direct currents or pulsating currents, alternating currents, oscillating currents, inductive discharges, etc., and the study of the *terminal conditions* is of the foremost importance.

Free oscillations are understood as the transient phenomena occurring in an electric circuit or part of the circuit, to which neither electric energy is supplied by some outside source, nor electric energy abstracted therefrom.

Free oscillations thus are the transient phenomena resulting from the dissipation of the energy stored in the electric field of the circuit, or inversely, the accumulation of the energy of the electric field, and their appearance so presupposes the possibility of energy storage in more than one form, so as to allow an interchange or surge of energy between its different forms, electromagnetic and electrostatic energy. Free oscillations occur only in circuits containing capacity C as well as inductance L.

The absence of energy supply or abstraction so defines the free oscillations by the conditions, that the power $p = e\,i$ at the two ends of the circuit or section of the circuit must be zero at all times, or the circuit must be closed upon itself.

With a circuit of uniformly distributed constants, the latter case is not realized, but is of foremost importance in a complex circuit, that is, a circuit comprising sections of different constants.

Representing then the two ends of the electric circuit by $u = 0$ and $u = l$, that is, counting the distance from one end of the circuit, the condition of a free oscillation is:

$$u = 0: \qquad p = 0:$$
$$u = l: \qquad p = 0.$$

since $p = e\,i$, this means that at $u = 0$ and $u = l$, either e or i must be zero.

This gives 4 sets of terminal conditions:

1. $e = 0$ at $u = 0$; $\quad i = 0$ at $u = l$.
2. $i = 0$ at $u = 0$; $\quad e = 0$ at $u = l$.
3. $e = 0$ at $u = 0$; $\quad e = 0$ at $u = l$.
4. $i = 0$ at $u = 0$; $\quad i = 0$ at $u = l$.

Case 2 represents the same condition as 1, merely with the distance u counting from the other end of the circuit: a line open at one end, grounded at the other end. Case 3 represents a circuit open at both ends, and Case 4 a circuit grounded at both ends.

Substituting these terminal conditions into the general equations it follows:

$$h = 0$$

that is, the *free oscillation of a circuit is a standing wave.*

And also:

$$k\,l = \frac{(2\,n+1)\,\pi}{2} \tag{77}$$

if $e = 0$ at one, $i = 0$ at the other end of the circuit, and

$$k\,l = n\,\pi \tag{78}$$

if either $e = 0$ at both ends of the circuit, or $i = 0$ at both ends of the circuit.

From (77) it follows, that

$$k\,l = \frac{\pi}{2},$$

or an *odd* multiple thereof. That is, the longest wave which can exist in the circuit, is that which makes the circuit a quarter wave length. Besides this fundamental wave, all its odd multiples can exist. Such an oscillation may be called a *quarter wave oscillation.*

The oscillation of a circuit, which is open at one end, grounded at the other end, is a quarter wave oscillation, which can contain only the odd harmonics of the fundamental wave of oscillation.

From (78) it follows, that

$$k\,l = \pi$$

or a multiple thereof. That is, the longest wave, which can exist in such a circuit, is that wave, which makes the circuit a half wave length. Besides this fundamental wave, all its multiples, odd as well as even, can exist. Such an oscillation may be called a *half wave oscillation.*

The oscillations of a circuit which is open at both ends, or grounded at both ends, is a half wave oscillation, and a half wave oscillation can also contain the even harmonics of the fundamental wave of oscillation, and so also the constant term, for $n = 0$ in (78).

It is interesting to note, that in the half wave oscillation of a circuit, we have a case of a circuit, in which *even* higher harmonics exist, and the electromotive force and current wave are not symmetrical.

22. From $h = 0$ follows, by equation (35):

$$s = 0, \quad \text{if:} \quad k^2 > L\,C\,m^2,$$

and:

$$q = 0, \quad \text{if:} \quad k^2 < L\,C\,m^2,$$

The smallest value of k, which can exist, is, by equation (77):

$$k = \frac{\pi}{2\,l}$$

and, as discussed in paragraph (15), this value in high potential high power circuits usually is very much larger than $L\,C\,m^2$, so that the case: $q = 0$, is realized only in extremely long circuits, as long distance telephone or submarine cable, but not in transmission lines, and the first case: $f = 0$, is mainly of importance.

Substituting therefore: $h = 0$ and $f = 0$, dropping terms of higher order, and rearranging, finally gives the

Equations of the Free Oscillation:

$$\left.\begin{aligned} i &= \Sigma A\,\epsilon^{-wt}\,\{\cos(qt - ku - \gamma) \pm \cos(qt + ku - \gamma)\} \\ e &= \sqrt{\frac{L}{C}}\,\Sigma\,A\,\epsilon^{-wt}\,\{\cos(qt - ku - \gamma) \mp \cos(qt + ku - \gamma)\} \end{aligned}\right\} \quad (79)$$

While the free oscillation of a circuit is a standing wave, the general standing wave, as represented by equations (59) and (60), with 4 integration constants A_1, A_1', A_2, A_2', is not necessarily a free oscillation.

To be a free oscillation, the power $e\,i$, that is, either e or i, must be zero at two points of the circuit, the ends of the circuit or section of circuit, which oscillates.

This gives:

$$A_1^2 + A_1'^2 = A_2^2 + A_2'^2$$

as the condition, which must be fulfilled between the integration constants, to make a standing wave a free oscillation, and as can easily be shown:

if the integration constants of a standing wave fulfil the condition:

$$A_1^2 + A_1'^2 = A_2^2 + A_2'^2 = B^2$$

the circuit of this wave contains points u_1, distant from each other by half a wave length, at which $e = 0$, and points u_2, distant from each other by half a wave length, at which $i = 0$, and the points u_2 are intermediate between the points u_1, that is, distant therefrom by one-quarter wave length. Any section of the circuit, from a point u_1 or u_2 to any other point u_1 or u_2 then is a free oscillating circuit.

In the free oscillation of the circuit, either the circuit is bounded by one point u_1 and one point u_2, that is, the voltage is zero at one end, and the current zero at the other end of the circuit, and the circuit then is a quarter wave or an odd multiple thereof, or the circuit is bounded by two points u_1, or by two points u_2, and then the voltage is zero at both ends of the circuit, or the current is zero at both ends of the circuit, and in either case the circuit is one half wave or a multiple thereof.

23. Choosing one of the points u_1 or u_2 as starting point of the distance, and introducing again the velocity of propagation as unit distance:

$$\lambda = \sigma u$$

$$\sigma = \sqrt{L C}$$

and denoting:

$$\left.\begin{aligned} \phi &= \frac{2\pi}{\lambda_0} t \\ \omega &= \frac{2\pi}{\lambda_0} \lambda = \frac{2 \pi \sqrt{L C}}{\lambda_0} u \end{aligned}\right\} \quad (80)$$

That is, representing a complete cycle of the fundamental frequency, or complete wave in time, by $\phi = 2 \pi$, and a complete wave in space, by $\omega = 2 \pi$.

A. *Quarter Wave Oscillation*:

(*a*) $e = 0$ at $u = 0$ (or: $\omega = 0$):

$$\left.\begin{aligned} e &= \epsilon^{-wt} \sum_0^\infty {}_n B_n \sin (2 n+1) \omega \sin [(2 n+1) \phi + \gamma_n] \\ i &= \sqrt{\frac{C}{L}} \epsilon^{-wt} \sum_0^\infty {}_n B_n \cos (2 n+1) \omega \cos [(2 n+1) \phi+\gamma_n] \end{aligned}\right\} \quad (81)$$

(*b*) $i = 0$ at $u = 0$ (or: $\omega = 0$):

$$\left.\begin{aligned} e &= \epsilon^{-wt} \sum_0^\infty {}_n B_n \cos (2 n+1) \omega \cos [(2 n+1) \phi+\gamma_n] \\ i &= \sqrt{\frac{C}{L}} \epsilon^{-wt} \sum_0^\infty {}_n B_n \sin (2 n+1) \omega \sin [(2 n+1) \phi+\gamma_n] \end{aligned}\right\} \quad (82)$$

B. *Half-Wave Oscillation*:

(*a*) $e = 0$ at $u = 0$ (or: $\omega = 0$):

$$\left. \begin{aligned} e &= \varepsilon^{-wt} \sum_0^\infty {}^n B_n \sin n\,\omega \sin (n\,\phi + \gamma_n) \\ i &= \sqrt{\frac{C}{L}}\, \varepsilon^{-wt} \sum_0^\infty {}^n B_n \cos n\,\omega \cos (n\,\phi + \gamma_n) \end{aligned} \right\} \quad (83)$$

(*b*) $i = 0$ at $u = 0$ (or: $\omega = 0$):

$$\left. \begin{aligned} e &= \varepsilon^{-wt} \sum_0^\infty {}^n B_n \cos n\,\omega \cos (n\,\phi + \gamma_n) \\ i &= \sqrt{\frac{C}{L}}\, \varepsilon^{-wt} \sum_0^\infty {}^n B_n \sin n\,\omega \sin (n\,\phi + \gamma_n) \end{aligned} \right\} \quad (84)$$

where:

$$\left. \begin{aligned} \phi &= \frac{2\pi}{\lambda_0} t \\ \omega &= \frac{2\,\pi\,\sqrt{L\,C}}{\lambda_0} u \end{aligned} \right\} \quad (85)$$

$$\left. \begin{aligned} \lambda_0 &= 4\,l\,\sqrt{L\,C} \text{ in a quarter-wave,} \\ &= 2\,l\,\sqrt{L\,C} \text{ in a half-wave oscillation.} \end{aligned} \right\} \quad (86)$$

and:

$$\varepsilon^{-wt} = \varepsilon^{-\frac{\lambda_0 w}{2\pi}\phi} \quad (87)$$

λ_0 is the wave length, and thus l/λ_0 the frequency, of the fundamental wave, with the velocity of propagation as distance unit.

It is interesting to note, that the time decrement of the free oscillation, ε^{-wt}, is the same for all frequencies and wave lengths, and that the relative intensity of the different harmonic components of the oscillation, and thereby the wave shape of the oscillation, remains unchanged during the decay of the oscillation.

This result, analogous to that found in the discussion on traveling waves, obviously is based on the assumption, that the constants of the circuit do not change with the frequency. This, however, is not perfectly true; at very high frequencies, r increases, due to unequal current distribution in the conductor, and L slightly decreases hereby. g increases by the energy losses resulting from brush discharges and from electrostatic radiation, etc., so that in general, at very high frequency, an increase of r/L and g/C, and so of w may be expected, that is, a greater rapidity of the dying out of very high harmonics. This would result in smoothing out the wave shape with increasing decay, that is, making it more approach the fundamental and its lower harmonics.

24. The equations of a free oscillation of a circuit, as quarter wave or half wave, still contain the pairs of integration constants B_n and γ_n, representing respectively the intensity and the phase of the nth harmonic.

These pairs of integration constants are determined by the terminal conditions of time, that is, they depend upon the amount and the distribution of the stored energy of the circuit at the starting moment of the oscillation. or in other words, on the distribution of current and voltage at $t = 0$.

The voltage e_0 and current i_0, at time $t = 0$, can be expressed as an infinite series of trigonometric functions of the distance u, that is, the distance angle ω, or a Fourier series of such character as also to fulfil the terminal conditions in space, as discussed above, that is, $e = 0$ respectively $i = 0$ at the ends of the circuit.

The voltage and current distribution in the circuit, at the starting moment of the oscillation, $t = 0$ respectively $\phi = 0$, so determines the integration constants in the usual manner.

25. As an instance the discharge of a transmission line may be considered:

A circuit of length l is charged to a uniform voltage E, while no current passes in the circuit. This circuit then is grounded at one end, while the other end remains insulated.

Let the distance be counted from the grounded end, and the time from the moment of grounding.

The terminal conditions then are:

(a) $\omega = 0$: $e = 0$
$\omega = \pi/2$: $i = 0$

(*b*) at $\phi = 0$:
$e = 0$ for: $\omega = 0$; $e = E$ for: $\omega \pm 0$
$i = 0$ for: $\omega \pm 0$; $i =$ indefinite for: $\omega = 0$.

The distribution of voltage e_0 and current i_0 in the circuit, at the starting moment $\phi = 0$, can be expressed by the Fourier series, and it is,

$$e = \frac{4\,E}{\pi}\,\varepsilon^{-wt} \sum_{1}^{\infty}{}_{n} \frac{\sin\,(2\,n+1)\,\omega \cos\,(2\,n+1)\,\phi}{2\,n+1}$$

$$i = \frac{4\,E}{\pi}\sqrt{\frac{C}{L}}\,\varepsilon^{-wt} \sum_{1}^{\infty}{}_{n} \frac{\cos\,(2\,n+1)\,\omega \sin\,(2\,n+1)\,\phi}{2\,n+1}$$

Choosing the constants:

$$\begin{aligned} l &= 120 \text{ miles} \\ r &= 0.41 \text{ ohms} \\ L &= 1.95 \times 10^{-3}\,h \\ g &= 0.25 \times 10^{-6} \text{ mho} \\ C &= 0.0162 \times 10^{-6}\,f \\ E &= 40{,}000 \text{ volts.} \end{aligned}$$

$$e = 51{,}000\,\varepsilon^{-.0485\phi}\,\{\sin\omega\cos\phi + \tfrac{1}{3}\sin 3\,\omega\cos 3\,\phi + \tfrac{1}{5}\sin 5\,\omega\cos 5\,\phi + \;.\;.\;.\;\} \text{ volts}$$

$$i = 147.2\,\varepsilon^{-0.485\phi}\,\{\cos\omega\sin\phi + \tfrac{1}{3}\cos 3\,\omega\sin 3\,\phi + \tfrac{1}{5}\cos 5\,\omega\sin 5\,\phi + \;.\;.\;.\;\} \text{ amperes.}$$

VI. Transition Points and the Complex Circuit.

26. The discussions of standing waves and free oscillations, and traveling waves, in the preceding, directly apply only to simple circuits, that is, circuits comprising a conductor of uniformly distributed constants r, L, g, C. Industrial electric circuits, however, never are simple circuits, but are always complex circuits comprising sections of different constants, generator, transformer, transmission lines and load; and a simple circuit is realized only by a section of a circuit, as a transmission line or a high potential transformer coil, which is cut off at both ends from the rest of the circuit, either by open circuiting: $i = 0$, or by short circuiting: $e = 0$. Approximately, the simple circuit is realized by a section of a complex circuit, connecting to other sections of very different constants, so that the ends of the circuit can, approximately, be considered as reflection points. For instance, an underground cable, of low L and high C, when connected to a large reactive coil, of high L and low C, may, approximately, at its ends be considered as having reflec-

tion points: $i = 0$. A high potential transformer coil, of high L and low C, when connected to a cable, of low L and high C, may at its ends be considered as having reflection points: $e=0$. That is, in other words, in the first case the reactive coil may be considered as stopping the flow of current, in the latter case the cable considered as short circuiting the transformer. This approximation however, while frequently relied upon in engineering practice, and often permissible for the circuit section in which the transient phenomenon originates, is not permissible in considering the effect of the phenomenon on the adjacent sections of the circuit. For instance, in the first case above mentioned, a transient phenomenon in an underground cable connected to a high reactance: the current and voltage in the cable may approximately be represented by considering the reactive coil as a reflection point, that is, an open circuit, since only a small current enters the reactive coil. Such a small current, entering the reactive coil, may however, give a very high and destructive voltage in the reactive coil, due to its high L, and so in the circuit beyond the reactive coil. In the investigation of the effect of a transient phenomenon originating in one section of a complex circuit—as an oscillating arc on an underground cable—on other sections of the circuit—as the generating station—even a very great change of circuit constants, cannot be considered as a reflection point. Since this is the most important case met in industrial practice, as disturbances originating in one section of a complex circuit usually develop their destructive effects in other sections of the circuit, the investigation of the general problem of a complex circuit comprising sections of different constants becomes necessary. This requires the investigation of the changes occurring in an electric wave, and its equations, when passing over a transition point from one circuit or section of a circuit into another section of different constants.

The equations (29) and (30), while most general, are less convenient for studying the transition of a wave from one circuit to another circuit, of different constants, and since in industrial high voltage circuits at least for waves originating in the circuits, q and k are very large compared with f and h, as discussed before, f and h may be neglected compared with q and k. This gives the equations (45) and (46), or, substituting (47) and reversing the sign of λ, that is, counting the distance in opposite direction:

$$i = \varepsilon^{-wt} \{ \varepsilon^{-f(\lambda-t)} [C_1 \cos q (\lambda - t) + C_1' \sin q (\lambda - t)]$$
$$- \varepsilon^{+f(\lambda+t)} [C_2 \cos q (\lambda + t) + C_2' \sin q (\lambda + t)]$$
$$+ \varepsilon^{+f(\lambda-t)} [C_3 \cos q (\lambda - t) + C_3' \sin q (\lambda - t)]$$
$$- \varepsilon^{-f(\lambda+t)} [C_4 \cos q (\lambda + t) + C_4' \sin q (\lambda + t)] \} \quad (88)$$

$$e = \sqrt{\frac{L}{C}} \varepsilon^{-wt} \{ \varepsilon^{-f(\lambda-t)} [C_1 \cos q (\lambda - t) + C_1' \sin q (\lambda - t)]$$
$$+ \varepsilon^{+f(\lambda+t)} [C_2 \cos q (\lambda + t)) + C_2' \sin q (\lambda + t)]$$
$$+ \varepsilon^{+f(\lambda-t)} [C_3 \cos q (\lambda - t) + C_3' \sin q (\lambda - t)]$$
$$+ \varepsilon^{-f(\lambda+t)} [C_4 \cos q (\lambda + t) + C_4' \sin q (\lambda + t)] \} \quad (89)$$

where the constants are given by equations (37) to (43).

Substituting:

$$\left. \begin{aligned} C_1^2 + C_1'^2 &= A^2 \\ C_2^2 + C_2'^2 &= B^2 \\ C_3^2 + C_3'^2 &= C^2 \\ C_4^2 + C_4'^2 &= D^2 \end{aligned} \right\} \quad (90)$$

$$\left. \begin{aligned} \frac{C_1'}{C_1} &= \tan \alpha \\ \frac{C_2'}{C_2} &= \tan \beta \\ \frac{C_3'}{C_3} &= \tan \gamma \\ \frac{C_4'}{C_4} &= \tan \delta \end{aligned} \right\} \quad (91)$$

gives these equations in the form:

$$i = \varepsilon^{-wt} \{ A \varepsilon^{-f(\lambda-t)} \cos [q (\lambda - t) - \alpha] - B \varepsilon^{+f(\lambda+t)} \cos [q (\lambda + t) - \beta]$$
$$+ C \varepsilon^{+f(\lambda-t)} \cos [q (\lambda - t) - \gamma] - D \varepsilon^{-f(\lambda+t)} \cos [q(\lambda + t) - \delta] \} \quad (92)$$

$$e=\sqrt{\frac{L}{C}}\epsilon^{-ut}\{A\,\epsilon^{-f(\lambda-t)}\cos[q(\lambda-t)-\alpha]+B\,\epsilon^{+f(\lambda+t)}\cos[q(\lambda+t)-\beta]$$
$$+C\,\epsilon^{+f(\lambda-t)}\cos[q(\lambda-t)-\gamma]+D\,\epsilon^{-f(\lambda+t)}\cos[q(\lambda+t)-\delta]\} \tag{93}$$

In these equations, λ is the distance co-ordinate, using the velocity of propagation as unit distance, and at a transition point from one circuit to another, where the circuit constants change, the velocity of propagation also changes, and so, for the same time, constants f and q, h and k, also change, and so ku, but transformed to the distance variable λ, $ku = q\,\lambda$ remains the same. That is, by introducing the distance variable λ, the distance can be measured throughout the entire circuit, and across transition points, at which the circuit constants change, and the same equations apply throughout the entire circuit. In this case however, in any section of the circuit,

$$\left.\begin{aligned}\lambda &= \sigma_i\,u\\ &= u\sqrt{L_i\,C_i}\end{aligned}\right\} \tag{94}$$

where L_i and C_i are the inductance and the capacity, respectively, of the section i of the circuit, per unit length; for instance, per mile.

That is, in a complex circuit, the time variable t is the same throughout the entire circuit, or in other words, the frequency of oscillation, as represented by q, and the rate of decay of the oscillation, as represented by the exponential function of time must be the same throughout the entire circuit. Not so however with the distance variable u: the wave length of the oscillation, and its rate of building up or down, along the circuit, need not be the same, and usually are not, but in some sections of the circuit, the wave length may be far shorter, as in coiled circuits in transformers, due to the higher L, or in cables, due to the higher C. To extend the same equations over the entire complex circuit, it therefore becomes necessary to substitute for the distance variable u another distance variable λ of such character, that the wave length has the same value in all sections of the complex circuit. As the wave length of the section i is $\frac{1}{\sqrt{L_i\,C_i}}$, this is done by changing the unit distance by the factor: $\sigma_i = \sqrt{L_i\,C_i}$. The distance unit of the new distance variable λ then is the distance traversed by the wave in unit time, hence different in linear measure, for the different sections of the circuit, but offers the advantage of carrying the distance measurements across the entire circuit and over transition points, by the same distance variable λ.

This means, that the length u_i of any section i of the complex circuit is expressed by the length: $\lambda_i = \sigma_i u_i$.

The introduction of the distance variable λ also has the advantage, that in the determination of the constants r, L, g, C, of the different sections of the circuit, different linear distance measurements u may be used. For instance, in the transmission line, the constants may be given per mile, that is, the mile used as unit length, while in a high potential transformer coil as unit of length u may be used the turn, or the coil, or the total transformer, so that the actual linear length of conductor may be unknown. For instance, choosing the total length of conductor in the high potential transformer as unit length, then the length of the transformer winding in the velocity measure λ is, $\lambda_0 = \sqrt{L_0 C_0}$ where L_0 = total inductance, C_0 = total electrostatic capacity of transformer.

The introduction of the distant variable λ permits the representation, in the circuit, of apparatus as reactive coils, etc., in which one of the constants is very small compared with the other, and so is usually neglected, and the apparatus considered as " massed inductance " etc., and allows the investigation of the effect of the distributed capacity of reactive coils and similar matters, by representing the reactive coil as a finite (frequently quite long) section of the circuit.

27. Let λ_0, λ_1, λ_2, . . . λ_n be a number of transition points, at which the circuit constants change, and the quantities may be denoted by index 1 in the section from λ_0 to λ_1, by index 2 in the section from λ_1 to λ_2, etc.

At $\lambda = \lambda_1$, it then must be: $i_1 = i_2$, $e_1 = e_2$; thus, substituting $\lambda = \lambda_1$ into equations (92) and (93), and equating, gives two identities, from which it follows:

$$w_2 \pm f_2 = w_1 \pm f_1 \tag{95}$$

that is, since $w_2 \pm w_1$, only one of the two component waves can exist. Since these two waves differ from each other only by the sign of f, by assuming that f may be either positive or negative, we can select either one of the two waves, and the equations (92) and (93), or (88) and (89) so assume the simpler forms:

$$i = \varepsilon^{-(w+f)t} \{A\, \varepsilon^{+f\lambda} \cos [q\,(\lambda - t) - \alpha] - B\, \varepsilon^{-f\lambda} \cos [q\,(\lambda + t) - \beta]\}$$

$$e = \sqrt{\frac{L}{C}}\, \varepsilon^{-(w+f)t} \{A\, \varepsilon^{+f\lambda} \cos [q(\lambda - t) - \alpha] + B\, \varepsilon^{-f\lambda} \cos [q(\lambda + t) - \beta]\} \tag{96}$$

$$i = \varepsilon^{-(w+f)t} \{\varepsilon^{+f\lambda} [A \cos q\,(\lambda - t) + B \sin q\,(\lambda - t)] - \varepsilon^{-f\lambda} [C \cos q\,(\lambda + t) + D \sin q\,(\lambda + t)]\}$$

$$e = \sqrt{\frac{L}{C}}\, \varepsilon^{-(w+f)t} \{\varepsilon^{+f\lambda} [A \cos q\,(\lambda - t) + B \sin q\,(\lambda - t)] + \varepsilon^{-f\lambda} [C \cos q\,(\lambda + t) + D \sin q\,(\lambda + t)]\} \tag{97}$$

where f may be positive or negative.

It is then:

$$w_1 + f_1 = w_2 + f_2 = w_3 + f_3 = \ . \ . \ . \ = w_n + f_n = m_0 \tag{98}$$

where w_1, w_2, w_3, etc., w_n, are the time constants of the individual sections, $\frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right)$, and m_0 may be called the *resultant time decrement of the complex circuit.*

The identities: $i_1 = i_2$ and $e_1 = e_2$ for $\lambda = \lambda_1$, resolved for cos qt and sin qt, then give 4 equations, by which the integration constants A, B, α, β, or A, B, C, D, of one section of the circuit are determined by those of the adjoining section, by the expressions:

$$\left.\begin{aligned}
A_2 &= \varepsilon^{-f_2\lambda_1} \{a_1\, \varepsilon^{+f_1\lambda_1} A_1 + b_1\, \varepsilon^{-f_1\lambda_1} (C_1 \cos 2\,q\,\lambda_1 + D_1 \sin 2\,q\,\lambda_1)\} \\
B_2 &= \varepsilon^{-f_2\lambda_1} \{a_1\, \varepsilon^{+f_1\lambda_1} B_1 + b_1\, \varepsilon^{-f_1\lambda_1} (C_1 \sin 2\,q\,\lambda_1 - D_1 \cos 2\,q\,\lambda_1)\} \\
C_2 &= \varepsilon^{+f_2\lambda_1} \{a_1\, \varepsilon^{-f_1\lambda_1} C_1 + b_1\, \varepsilon^{+f_1\lambda_1} (A_1 \cos 2\,q\,\lambda_1 - B_1 \sin 2\,q\,\lambda_1)\} \\
D_2 &= \varepsilon^{+f_2\lambda_1} \{a_1\, \varepsilon^{-f_1\lambda_1} D_1 + b_1\, \varepsilon^{+f_1\lambda_1} (A_1 \sin 2\,q\,\lambda_1 - B_1 \cos 2\,q\,\lambda_1)\}
\end{aligned}\right\} \tag{99}$$

where:

$$\left.\begin{aligned} a_1 &= \frac{c_1 + c_2}{2\, c_2} \\ b_1 &= \frac{c_1 - c_2}{2\, c_2} \end{aligned}\right\} \quad (100)$$

$$\left.\begin{aligned} c_1 &= \sqrt{\frac{L_1}{C_1}} \\ c_2 &= \sqrt{\frac{L_2}{C_2}} \end{aligned}\right\} \quad (101)$$

28. The general equation of current and voltage in a complex circuit consists of two terms, a main wave and its reflected wave.

The factor $\epsilon^{-(w+f)t} = \epsilon^{-m_0 t}$ represents the time decrement, or the decrease of the intensity of the wave with the time, and as such is the same throughout the entire circuit. In an isolated section, of time constant w, the time decrement, was found however as ϵ^{-wt}. That is, with the decrement ϵ^{-wt}, the wave dies out in an isolated section at the rate at which its stored energy is dissipated by the power lost in resistance and conductance. In a section of the circuit connected to other sections, the time decrement $\epsilon^{-m_0 t}$ does not correspond to the power dissipation in the section, that is, the wave does not die out in each section at the rate as given by the power consumed in this section, or in other words, power transfer occurs from section to section, during the oscillation of a complex circuit.

If f is negative, m_0 is less than w, and the wave dies out in that particular section at a lesser rate than corresponds to the power consumed in the section, or in other words, in this section of the complex circuit, more power is consumed by r and g than is supplied by the decrease of the stored energy, and this section so must receive energy from adjoining sections. Inversely, if f is positive, $m_0 > w$, and the wave dies out more rapidly in that section than its stored energy is consumed by r and g; that is, a part of the stored energy of this section is transferred to the adjoining sections, and only a part—occasionally a very small part—dissipated in the section, and this section acts as a store of energy for supplying the other sections of the system.

The constant f of the circuit may be called *energy transfer constant*, and positive f means transfer of energy from the section to the rest of the circuit, and negative f means reception of energy from other sections.

This explains the vanishing of f in a standing wave of a uniform circuit, due to the absence of energy transfer, and its presence in the equations of the traveling wave, in which energy is transferred along the circuit, and in the alternating current equations.

It follows herefrom, that in a complex circuit, some of the f of the different sections must always be positive, some negative.

In addition to the time decrement, $\epsilon^{-m_0 t}$, the waves in equations (96) and (97) also contain the distance decrement $\epsilon^{+f\lambda}$ for the main wave, $\epsilon^{-f\lambda}$ for the reflected wave. Negative f means a decrease of the main wave for increasing λ, or in the direction of propagation, and a decrease of the reflected wave for decreasing λ, that is, also in the direction of propagation, while positive f means increase of main wave as well as reflected wave, in the direction of propagation along the circuit. That is if f is negative, and the section so consumes more power than is given by its stored energy, and therefore receives power from the adjoining sections, the electric wave decreases, in the direction of its propagation, showing the gradual dissipation of the power received from adjoining sections. Inversely, if f is positive and the section so supplies power to adjoining sections, the electric wave increases in this section, in the direction of its propagation.

In other words, in a complex circuit, in sections of low power dissipation, the wave increases, and transfers power to sections of high power dissipation in which the wave decreases.

29. Introducing the resultant time decrement m_0 of the complex circuit, the equations of any section can be expressed by the resultant time decrement of the entire complex circuit, m_0, and the energy transfer constant of the individual section:

$$f = m_0 - w \tag{102}$$

in the form:

$$i = \epsilon^{-m_0 t} \{\epsilon^{+f\lambda}[A \cos q (\lambda - t) + B \sin q (\lambda - t)] - \epsilon^{-f\lambda}[C \cos q (\lambda + t) + D \sin q (\lambda + t)]\} \tag{103}$$

where:

$$\left.\begin{aligned} a_1 &= \frac{c_1+c_2}{2\,c_2} \\ b_1 &= \frac{c_1-c_2}{2\,c_2} \end{aligned}\right\} \quad (100)$$

$$\left.\begin{aligned} c_1 &= \sqrt{\frac{L_1}{C_1}} \\ c_2 &= \sqrt{\frac{L_2}{C_2}} \end{aligned}\right\} \quad (101)$$

28. The general equation of current and voltage in a complex circuit consists of two terms, a main wave and its reflected wave.

The factor $\epsilon^{-(w+f)t} = \epsilon^{-m_0 t}$ represents the time decrement, or the decrease of the intensity of the wave with the time, and as such is the same throughout the entire circuit. In an isolated section, of time constant w, the time decrement, was found however as ϵ^{-wt}. That is, with the decrement ϵ^{-wt}, the wave dies out in an isolated section at the rate at which its stored energy is dissipated by the power lost in resistance and conductance. In a section of the circuit connected to other sections, the time decrement $\epsilon^{-m_0 t}$ does not correspond to the power dissipation in the section, that is, the wave does not die out in each section at the rate as given by the power consumed in this section, or in other words, power transfer occurs from section to section, during the oscillation of a complex circuit.

If f is negative, m_0 is less than w, and the wave dies out in that particular section at a lesser rate than corresponds to the power consumed in the section, or in other words, in this section of the complex circuit, more power is consumed by r and g than is supplied by the decrease of the stored energy, and this section so must receive energy from adjoining sections. Inversely, if f is positive, $m_0 > w$, and the wave dies out more rapidly in that section than its stored energy is consumed by r and g; that is, a part of the stored energy of this section is transferred to the adjoining sections, and only a part—occasionally a very small part—dissipated in the section, and this section acts as a store of energy for supplying the other sections of the system.

The constant f of the circuit may be called ***energy transfer constant***, and positive f means transfer of energy from the section to the rest of the circuit, and negative f means reception of energy from other sections.

This explains the vanishing of f in a standing wave of a uniform circuit, due to the absence of energy transfer, and its presence in the equations of the traveling wave, in which energy is transferred along the circuit, and in the alternating current equations.

It follows herefrom, that in a complex circuit, some of the f of the different sections must always be positive, some negative.

In addition to the time decrement, $\epsilon^{-m_0 t}$, the waves in equations (96) and (97) also contain the distance decrement $\epsilon^{+f\lambda}$ for the main wave, $\epsilon^{-f\lambda}$ for the reflected wave. Negative f means a decrease of the main wave for increasing λ, or in the direction of propagation, and a decrease of the reflected wave for decreasing λ, that is, also in the direction of propagation, while positive f means increase of main wave as well as reflected wave, in the direction of propagation along the circuit. That is if f is negative, and the section so consumes more power than is given by its stored energy, and therefore receives power from the adjoining sections, the electric wave decreases, in the direction of its propagation, showing the gradual dissipation of the power received from adjoining sections. Inversely, if f is positive and the section so supplies power to adjoining sections, the electric wave increases in this section, in the direction of its propagation.

In other words, in a complex circuit, in sections of low power dissipation, the wave increases, and transfers power to sections of high power dissipation in which the wave decreases.

29. Introducing the resultant time decrement m_0 of the complex circuit, the equations of any section can be expressed by the resultant time decrement of the entire complex circuit, m_0, and the energy transfer constant of the individual section:

$$f = m_0 - w \tag{102}$$

in the form:

$$i = \epsilon^{-m_0 t} \{\epsilon^{+f\lambda}[A \cos q (\lambda - t) + B \sin q (\lambda - t)] - \epsilon^{-f\lambda}[C \cos q (\lambda + t) + D \sin q (\lambda + t)]\} \tag{103}$$

$$e = \sqrt{\frac{L}{C}}\,\epsilon^{-m_0 t}\,\{\epsilon^{+f\lambda}[A\cos q(\lambda - t) + B\sin q(\lambda - t)] + \epsilon^{-f\lambda}[C\cos q(\lambda + t) + D\sin q(\lambda + t)]\}$$

The constants A, B, C, D, are the integration constants, and are such as given by the terminal conditions of the problem, as by the distribution of current and voltage in the circuit at the starting moment, for $t = 0$, or at one particular point, as $\lambda = 0$.

The constants m_0 and q are dependent upon the circuit conditions: if the circuit is closed upon itself—as usually is the case with an electrical transmission or distribution circuit—and Λ the total length of the closed circuit, the equations must for $\lambda = \Lambda$ give the same values as for $\lambda = 0$, and therefore $q\Lambda$ must be a complete cycle or a multiple thereof: $2\,n\,\pi$. That is:

$$q = \frac{2\,n\,\pi}{\Lambda} \tag{104}$$

and the least value of q, or the fundamental frequency of oscillation, is:

$$q_0 = \frac{2\pi}{\Lambda} \tag{105}$$

and:

$$q = n\,q_0 \tag{106}$$

If the complex circuit is open at both ends, or grounded at both ends, and so performs a half wave oscillation, and $\Lambda_1 =$ total length of the circuit, it is:

$$\left.\begin{aligned} q_0 &= \frac{\pi}{\Lambda_1} \\ q &= n\,q_0 \end{aligned}\right. \tag{107}$$

and if the circuit is open at one end, grounded at the other end, so performing a quarter wave oscillation, and $\Lambda_2 =$ total length of circuit, it is:

$$\left.\begin{aligned} q_0 &= \frac{\pi}{2\,\Lambda_2} \\ q &= (2\,n - 1)\,q_0 \end{aligned}\right\} \tag{108}$$

If the length of the complex circuit is very great compared with the frequency of the oscillation, q_0 may have any value. That is, if the wave length of the oscillation is very short compared with the length of the circuit, any wave length, and so any frequency, may occur. With uniform circuits, as transmission lines, this latter case, that is, the response of the line to any frequency, can occur only in the range of very high frequencies. That is, even in a transmission line of several hundred miles length, the lowest frequency of free oscillation is fairly high, and frequencies, which are so high, compared with the fundamental frequency of the circuit, that, considered as higher harmonics thereof, they overlap, must be extremely high, of the magnitude of a million cycles. In complex circuits however, the fundamental frequency may be very much lower, and below machine frequencies, as the velocity of propagation $\frac{1}{\sqrt{L C}}$ may be quite low in some sections of the circuit, as the high potential coils of large transformers, and the presence of iron increases the inconstancy of L for high frequencies, so that in such a complex circuit, even at fairly moderate frequencies, of the magnitude of 10,000 cycles, the circuit may respond to any frequency.

The constant m_0 also is determined by the circuit constants. Upon m_0 depends the energy transfer constant of the circuit section, and thereby the rate of building up, in a section of low power consumption, or building down, in a section of high power consumption. In a closed circuit, however, passing around the entire circuit, the same values of e and i must again be reached, and the rates of building up and building down of the wave in the different sections, must therefore be such as to neutralize each other when carried through the entire circuit, that is, the total building up, through the entire complex circuit, must be zero. This gives an equation from which m_0 is determined, as a mean value of the time constants w_i of the individual circuit sections.

VII. Power and Energy of the Complex Circuit.

30. The free oscillation of a complex circuit differs from that of the uniform circuit, in that the former contains exponential functions of the distance λ, which represent the shifting, or transfer of power between the sections of the circuit.

Thus the general expression of one term or frequency of current and voltage in a section of a complex circuit is given by equation:

$$i = \epsilon^{-m_0 t} \{\epsilon^{+f\lambda} [A \cos q (\lambda - t) + B \sin q (\lambda - t)] - \epsilon^{-f\lambda} [C \cos q (\lambda + t) + D \sin q (\lambda + t)]\} \quad (109)$$

$$e = \sqrt{\frac{L}{C}} \epsilon^{-m_0 t} \{\epsilon^{+f\lambda} [A \cos q (\lambda - t) + B \sin q (\lambda - t)] + \epsilon^{-f\lambda} [C \cos q (\lambda + t) + D \sin q (\lambda + t)]\}$$

where:

$$q = n q_0$$

$$q_0 = \frac{2\pi}{\Lambda}$$

Λ = total length of circuit. expressed in the distance co-ordinate:

$$\lambda = \sigma u,$$

u being the distance co-ordinate of the circuit section in any measure, as miles, turns, etc., and r, L, g, C, the circuit constants per unit length of u, and:

$$\sigma = \sqrt{L C}$$

$$c = \sqrt{\frac{L}{C}}$$

$$w = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right) = \text{time constant of circuit section,}$$

$m_0 = w + f$ = resultant time decrement of complex circuit,
$f = m_0 - w$ = energy transfer constant of circuit section.

The instantaneous value of power at any point of the circuit, at any time t, is:

$$p = e\, i$$

$$= \sqrt{\frac{L}{C}} \epsilon^{-2 m_0 t} \{\epsilon^{+2f\lambda} [A \cos q(\lambda - t) + B \sin q(\lambda - t)]^2 - \epsilon^{-2f\lambda} [C \cos q (\lambda + t) + D \sin q (\lambda + t)]^2\}$$

$$= \frac{1}{2}\sqrt{\frac{L}{C}} \epsilon^{-2 m_0 t} \{[\epsilon^{+2f\lambda} (A^2 + B^2) - \epsilon^{-2f\lambda} (C^2 + D^2)] + [\epsilon^{+2f\lambda} (A^2 - B^2) \cos 2 q (\lambda - t) - \epsilon^{-2f\lambda} (C^2 - D^2) \cos 2 q (\lambda + t)] + 2 [A B \epsilon^{+2f\lambda} \sin 2 q (\lambda - t) - C D \epsilon^{-2f\lambda} \sin 2 q (\lambda + t)]\} \quad (110)$$

that is, the instantaneous value of power consists of a constant term, and terms of double frequency in $(\lambda - t)$ and $(\lambda + t)$, that is, in distance λ and time t.

Integrating (110) over a complete period in time, gives the *effective* or *mean value of power at any point* λ, as:

$$P = \frac{1}{2}\sqrt{\frac{L}{C}}\epsilon^{-2m_0t}\left\{\epsilon^{+2f\lambda}(A^2+B^2) - \epsilon^{-2f\lambda}(C^2+D^2)\right\} \qquad (111)$$

That is, the instantaneous power at any time and any point of the circuit is the difference between the instantaneous power of the main wave, and that of the reflected wave.

The effective power at any point of the circuit is the difference between the effective power of the main wave and that of the reflected wave.

The effective power at any point of the circuit, gradually decreases in any section, with the resultant time decrement of the total circuit, ϵ^{-2m_0t}, and varies gradually or exponentially with the distance λ, the one wave increasing, the other decreasing, so that at one point of the circuit or circuit section the effective power is zero. That is, this point of the circuit is a power node, or point across which no power flows. It is given by:

$$\left.\begin{aligned} &\epsilon^{+2f\lambda}(A^2+B^2) = \epsilon^{-2f\lambda}(C^2+D^2) \\ &\text{or:} \\ &\lambda = \frac{1}{4f}\log\frac{C^2+D^2}{A^2+B^2} \end{aligned}\right\} \qquad (112)$$

The difference of power between two points of the circuit λ_1 and λ_2, that is, the power which is supplied or received (depending upon its sign) by a section $l = \lambda_2 - \lambda_1$ of the circuit, is given by equation (111) as:

$$P_0 = \frac{1}{2}\sqrt{\frac{L}{C}}\,\epsilon^{-2m_0t}\left\{(\epsilon^{+2f\lambda_2} - \epsilon^{+2f\lambda_1})(A^2+B^2) - (\epsilon^{-2f\lambda_2} - \epsilon^{-2f\lambda_1})(C^2+D^2)\right\} \qquad (113)$$

If $P_0 > 0$, this represents the power which is supplied by the section l to the adjoining sections of the circuit; if $P_0 < 0$, this is the power received by the section from the rest of the complex circuit.

If $f\lambda_2$ and $f\lambda_1$ are small quantities, the exponential function can be resolved in an infinite series, and all but the first term dropped, as of higher orders, or negligible, and this gives the approximate value:

$$\epsilon^{\pm 2f\lambda_2} - \epsilon^{\pm 2f\lambda_1} = \pm 2f(\lambda_2 - \lambda_1) = \pm 2fl$$

hence:

$$P_0 = f\,l\sqrt{\frac{L}{C}}\,\epsilon^{-2m_0t}\{A^2+B^2+C^2+D^2\} \tag{114}$$

That is, the power transferred from a section of length l, to the rest of the circuit, or received by the section from the rest of the circuit, is proportional to the length of the section, l, to its transfer constant f, and to the sum of the power of main wave and reflected wave.

31. The energy stored by the inductance, L, of a circuit element $d\lambda$, that is, in the magnetic field of the circuit, is:

$$d\eta_1 = \frac{L' i^2}{2} d\lambda$$

where L' = inductance per unit length of circuit, expressed by the distance co-ordinate λ.

Since L = inductance per unit length of circuit, of distance co-ordinate u, and $\lambda = \sigma u$, it is:

$$L' = \frac{L}{\sigma} = \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}}$$

hence:

$$d\eta_1 = \tfrac{1}{2}\sqrt{\frac{L}{C}}\,i^2\,d\lambda \tag{115}$$

In general, the circuit constants r, L, g, C, per unit length, $u = 1$, give, per unit length, $\lambda = 1$, the circuit constants:

$$\left.\begin{aligned}&\frac{r}{\sigma};\ \frac{L}{\sigma};\ \frac{g}{\sigma};\ \frac{C}{\sigma};\\ &\text{or:}\\ &\frac{r}{\sqrt{LC}};\ \frac{L}{\sqrt{LC}} = \sqrt{\frac{L}{C}};\ \frac{g}{\sqrt{LC}};\ \frac{C}{\sqrt{LC}} = \sqrt{\frac{C}{L}}\end{aligned}\right\} \tag{116}$$

Substituting (109) in equation (115), expanded, and integrated over a complete period in time, gives the effective power stored in the magnetic field at point λ:

$$\frac{d\,H_1}{d\,\lambda} = \frac{1}{4}\sqrt{\frac{L}{C}}\,\varepsilon^{-2 m_0 t}\,\{[\varepsilon^{+2 f\lambda}(A^2+B^2)+\varepsilon^{-2 f\lambda}(C^2+D^2)]-2\,[(AC-BD)\cos 2\,q\,\lambda+(A\,D-B\,C)\sin 2\,q\,\lambda]\} \tag{117}$$

and, integrated over one complete period of distance λ, or one complete wave length, this gives:

$$\frac{d\,H_1^0}{d\,\lambda} = \frac{1}{4}\sqrt{\frac{L}{C}}\,\varepsilon^{-2 m_0 t}\,\{[\varepsilon^{+2 f\lambda}(A^2+B^2)+\varepsilon^{-2 f\lambda}(C^2+D^2)\} \tag{118}$$

The energy stored by the inductance L, or in the magnetic field of the conductor, consists of:

A constant part:

$$\frac{d\,\eta_0}{d\,\lambda} = \frac{1}{4}\sqrt{\frac{L}{C}}\,\varepsilon^{-2 m_0 t}\,\{\varepsilon^{+2 f\lambda}(A^2+B^2)+\varepsilon^{-2 f\lambda}(C^2+D^2)\} \tag{119}$$

A part which is a function of $(\lambda - t)$ and $(\lambda + t)$:

$$\frac{d\,\eta'}{d\,\lambda} = \frac{1}{4}\sqrt{\frac{L}{C}}\,\varepsilon^{-2 m_0 t}\,\{[\varepsilon^{+2 f\lambda}(A^2-B^2)\cos 2\,q\,(\lambda - t)+\varepsilon^{-2 f\lambda}(C^2-D^2)\cos 2\,q\,(\lambda + t)]+2\,[A\,B\,\varepsilon^{+2 f\lambda}\sin 2\,q\,(\lambda - t)+C\,D\,\varepsilon^{-2 f\lambda}\sin 2\,q\,(\lambda + t)]\} \tag{120}$$

A part which is a function of the distance λ only, but not of time t:

$$-\frac{d\,\eta''}{d\,\lambda} = \frac{1}{2}\sqrt{\frac{L}{C}}\,\varepsilon^{-2 m_0 t}\,\{(A\,C-B\,D)\cos 2\,q\,\lambda+(A\,D+B\,C)\sin 2\,q\,\lambda\} \tag{121}$$

And a part which is a function of time t only, but not of the distance λ:

$$-\frac{d\,\eta'''}{d\,\lambda} = \frac{1}{2}\sqrt{\frac{L}{C}}\,\varepsilon^{-2 m_0 t}\,\{(A\,C+B\,D)\cos 2\,q\,t+(A\,D-B\,C)\sin 2\,q\,t\} \tag{122}$$

and the total energy of the electromagnetic field of circuit element $d\lambda$ at time t is:

$$\frac{d\,\eta_1}{d\,\lambda} = \frac{d\,\eta_0}{d\,\lambda} + \frac{d\,\eta'}{d\,\lambda} - \frac{d\,\eta''}{d\,\lambda} - \frac{d\,\eta'''}{d\,\lambda} \tag{123}$$

The energy stored in the electrostatic field of the conductor, or by the capacity C, is given by:

$$d\,\eta_2 = \frac{C'\,e^2}{2}\,d\,\lambda$$

or, substituting (116):

$$\frac{d\,\eta_2}{d\,\lambda} = \tfrac{1}{2}\sqrt{\frac{C}{L}}\,e^2 \tag{124}$$

and, substituting in (124) the value of e, from equation (109), gives the same expression as (123), except that the sign of the last two terms is reversed. That is:

The total energy of the electrostatic field of circuit element $d\lambda$ at time t is:

$$\frac{d\,\eta_2}{d\,\lambda} = \frac{d\,\eta_0}{d\,\lambda} + \frac{d\,\eta'}{d\,\lambda} + \frac{d\,\eta''}{d\,\lambda} + \frac{d\,\eta'''}{d\,\lambda} \tag{125}$$

and, adding (123) and (125), gives the total stored energy of the electric field of the conductor:

$$\frac{d\eta}{d\lambda} = 2\left(\frac{d\,\eta_0}{d\,\lambda} + \frac{d\,\eta'}{d\,\lambda}\right) \tag{126}$$

and, integrated over a complete period of time, this gives:

$$\frac{d\,H}{d\,\lambda} = 2\frac{d\,\eta_0}{d\,\lambda} = \tfrac{1}{2}\sqrt{\frac{L}{C}}\,\varepsilon^{-2m_0 t}\left\{\varepsilon^{+2l\lambda}(A^2+B^2) + \varepsilon^{-2l\lambda}(C^2+D^2)\right\} \tag{127}$$

The two last terms, $\frac{d\,\eta''}{d\,\lambda}$ and $\frac{d\,\eta'''}{d\,\lambda}$. represent the energy which is transferred, or pulsates, between the electromagnetic and the electrostatic field of the circuit; and the term $\frac{d\,\eta'}{d\,t}$ represents the alternating (or rather oscillating) component of stored energy.

The energy stored by the electric field in a circuit section l, between λ_1 and λ_2, is given by integrating $\frac{d\,H}{d\,\lambda}$ between λ_2 and λ_1, as:

$$H = \frac{1}{4\,f}\sqrt{\frac{L}{C}}\,\varepsilon^{-2\,m_o t}\,\{(\varepsilon^{+2f\lambda_2} - \varepsilon^{+2f\lambda_1})\,(A^2+B^2) - (\varepsilon^{-2f\lambda_2} - \varepsilon^{-2f\lambda_1})\,(C^2+D^2)\}$$

$$H = \tfrac{1}{2}\,l\sqrt{\frac{L}{C}}\,\varepsilon^{-2\,m_o t}\,\{A^2+B^2+C^2+D^2\} \qquad (128)$$

Differentiating (128) after t, gives the power supplied by the electric field of the circuit, as:

$$P = -\frac{d\,H}{d\,t}$$

or, more general:

$$= \frac{m_o}{2\,f}\sqrt{\frac{L}{C}}\,\varepsilon^{-2\,m_o t}\,\{(\varepsilon^{+2f\lambda_2} - \varepsilon^{+2f\lambda_1})\,(A^2+B^2) - (\varepsilon^{-2f\lambda_2} - \varepsilon^{-2f\lambda_1})\,(C^2+D^2)\}$$

$$= m_o\,l\sqrt{\frac{L}{C}}\,\varepsilon^{-2\,m_o t}\,\{A^2+B^2+C^2+D^2\} \qquad (129)$$

The power dissipated in the resistance: $r'\,d\,\lambda = \frac{r\,d\,\lambda}{\sqrt{L\,C}}$ and in the conductance: $g'\,d\,\lambda = \frac{g}{\sqrt{L\,C}}\,d\,\lambda$ of a conductor element $d\,\lambda$, and so the total power dissipated in the circuit element $d\,\lambda$ is, in the same manner:

$$\frac{d\,p_1}{d\,\lambda} = \frac{d\,p'}{d\,\lambda} + \frac{d\,p''}{d\,\lambda} = 4\,w\left(\frac{d\,\eta_o}{d\,\lambda} + \frac{d\,\eta'}{d\,\lambda}\right) - 4\,m\left(\frac{d\,\eta''}{d\,\lambda} + \frac{d\,\eta'''}{d\,\lambda}\right) \qquad (130)$$

where, as before:

$$w = \frac{1}{2}\left(\frac{r}{L} + \frac{g}{C}\right)$$

$$m = \frac{1}{2}\left(\frac{r}{L} - \frac{g}{C}\right)$$

and, integrated over a complete period:

$$\frac{d P_1}{d \lambda} = 4 w \frac{d \eta_0}{d \lambda} - 4 m \frac{d \eta''}{d \lambda} \qquad (131)$$

the power dissipated in the circuit contains a constant term.

$4 w \frac{d \eta_0}{d \lambda}$, and a term which is a periodic function of the distance λ:

$4 m \frac{d \eta''}{d \lambda}$, of double frequency.

Averaged over a half wave of the circuit, or a multiple thereof, the second term eliminates, and it is:

$$\frac{d P_1^0}{d \lambda} = 4 w \frac{d \eta_0}{d \lambda}$$

thus, the power dissipated in a section $l = \lambda_2 - \lambda_1$ of the circuit, is by integrating between limits λ_1 and λ_2:

$$P_1^0 = \frac{w_0}{2f} \sqrt{\frac{L}{C}}\, \varepsilon^{-2 m_0 t} \left\{ (\varepsilon^{+2f\lambda_2} - \varepsilon^{+2f\lambda_1})\ (A^2 + B^2) + (\varepsilon^{-2f\lambda_2} - \varepsilon^{-2f\lambda_1}) \right.$$
$$\left. (C^2 + D^2) \right\} \qquad (132)$$

$$= w \sqrt{\frac{L}{C}}\, \varepsilon^{-2 m_0 t} \left\{ A^2 + B^2 + C^2 + D^2 \right\}$$

Denoting therefore:

$$M^2 = (A^2 + B^2 + C^2 + D^2) \sqrt{\frac{L}{C}} \qquad (133)$$

it is:

Energy stored in the electric field of the circuit section of length l:

$$H = \tfrac{1}{2}\, l\, M^2\, \epsilon^{-2m_0 t} \tag{134}$$

Power supplied to the conductor by the decay of the electric field of the circuit:

$$P = m_0\, l\, M^2\, \epsilon^{-2m_0 t} \tag{135}$$

Power dissipated in the circuit section l, *by its effective resistance and conductance*:

$$P_1^0 = w\, l\, M^2\, \epsilon^{-2m_0 t} \tag{136}$$

Power transferred from the circuit section l, to the rest of circuit:

$$P_0 = f\, l\, M^2\, \epsilon^{-2m_0 t} \tag{137}$$

That is:

w/m_0 = ratio of power dissipated in the section, to that supplied to the section by the stored energy of its electric field.

f/m_0 = fraction of power supplied to the section by its electric field, which is transferred from the section to adjoining sections (or, if $f < 0$, received from them).

f/w = ratio of power transferred to other sections, to power dissipated in the section.

$m_0 \div w \div f$ is the ratio of the power supplied to the section by its electric field, dissipated in the section, and transferred from the section to adjoining sections.

These relations obviously are approximate only, and applicable to the case, where the wave length is short.

VIII. Reflection and Refraction at Transition Point.

23. The general equation of the current and voltage in a section of a complex circuit is given by equations (109).

At a transition point λ_1 between section 1 and section 2, the constants change by equation (99).

Choosing now the transition point as zero point of λ, so that $\lambda < 0$ is section 1, $\lambda > 0$ is section 2, equations (99) assume the form:

$$\left.\begin{aligned} A_2 &= a_1 A_1 + b_1 C_1 \\ B_2 &= a_1 B_1 - b_1 D_1 \\ C_2 &= a_1 C_1 + b_1 A_1 \\ D_2 &= a_1 D_1 - b_1 B_1 \end{aligned}\right\} \tag{138}$$

and herefrom follows:

$$\left.\begin{aligned} c_2 (A_2^2 - C_2^2) &= c_1 (A_1^2 - C_1^2) \\ c_2 (B_2^2 - D_2^2) &= c_1 (B_1^2 - D_1^2) \end{aligned}\right\} \quad (139)$$

If now a wave in section 1, A, B, travels towards transition point $\lambda = 0$, at this point, a part is reflected, giving rise to the reflected wave C, D, in section 1, while a part is transmitted and appears as main wave, A, B, in section 2. The wave C, D, in section 2 would not exist, as it would be a wave coming towards $\lambda = 0$, from section 2, and not a part of the wave coming from section 1. In other words, we can consider the circuit as comprising two waves moving in opposite directions:

1. A main wave $A_1 B_1$, giving a transmitted wave $A_2 B_2$ and reflected wave $C_1 D_1$.

2. A main wave $C_2 D_2$, giving a transmitted wave $C_1' D_1'$ and reflected wave $A_2' B_2'$.

The waves moving towards the transition point are single main waves, $A_1 B_1$ and $C_2 D_2$, and

The waves moving away from the transition point are combinations of waves reflected in the section, and waves transmitted from the other section.

Considering first the main wave moving towards rising λ: in this, $C_2 = 0 = D_2$, hence, by (138):

$$\left.\begin{aligned} A_2 &= \frac{2\, c_1}{c_1 + c_2} A_1 \\ B_2 &= \frac{2\, c_1}{c_1 + c_2} B_1 \end{aligned}\right\} \quad (140)$$

Herefrom it follows:

The *main wave* in section 1:

$$\left.\begin{aligned} i_1 &= \epsilon^{-m_0 t} \epsilon^{+f_1 \lambda} \{A_1 \cos q (\lambda - t) + B_1 \sin q (\lambda - t)\} \\ e_1 &= c_1 \epsilon^{-m_0 t} \epsilon^{+f_1 \lambda} \{A_1 \cos q (\lambda - t) + B_1 \sin q (\lambda - t)\} \end{aligned}\right\} \quad (141)$$

when reaching a transition point $\lambda = 0$, resolves into the *reflected wave*, turned back on section 1:

$$\left.\begin{aligned} i_1' &= -\frac{c_2 - c_1}{c_2 + c_1} \epsilon^{-m_0 t} \epsilon^{-f_1 \lambda} \{A_1 \cos q(\lambda + t) - B_1 \sin q(\lambda + t)\} \\ e_1' &= +\frac{c_2 - c_1}{c_2 + c_1} \epsilon^{-m_0 t} \epsilon^{-f_1 \lambda} \{A_1 \cos q(\lambda + t) - B_1 \sin q(\lambda + t)\} \end{aligned}\right\} \quad (142)$$

and the *transmitted wave*, which by passing over the transition point, enters section 2:

$$\left.\begin{aligned} i_2 &= \frac{2\, c_1}{c_1 + c_2} \epsilon^{-m_0 t} \epsilon^{+f_2 \lambda} \{A_1 \cos q\,(\lambda - t) + B_1 \sin q\,(\lambda - t)\} \\ e_2 &= c_2 \frac{2\, c_1}{c_1 + c_2} \epsilon^{-m_0 t} \epsilon^{+f_2 \lambda} \{A_1 \cos q(\lambda - t) + B_1 \sin q(\lambda - t)\} \end{aligned}\right\} \quad (143)$$

the reflection angle, $\tan (i_1') = -\frac{B_1}{A_1}$, is supplementary to the impact angle: $\tan (i_1) = +\frac{B_1}{A_1}$, and transmission angle: $\tan (i_2) = +\frac{B_1}{A_1}$.

Reversing the sign of λ in the equation (142) of the reflected wave, that is, counting the distance for the reflected wave also in the direction of its propagation, and so in opposite direction to the main wave and the transmitted wave, equation (142) becomes:

$$\left.\begin{aligned} i_1'' &= +\frac{c_2 - c_1}{c_2 + c_1} \epsilon^{-m_0 t} \epsilon^{+f_1 \lambda'} \{A_1 \cos q\,(\lambda' - t) + B_1 \sin q\,(\lambda' - t)\} \\ e_1'' &= +c_1 \frac{c_2 - c_1}{c_2 + c_1} \epsilon^{-m_0 t} \epsilon^{+f_1 \lambda'} \{A_1 \cos q\,(\lambda' - t) + B_1 \sin q\,(\lambda' - t)\} \end{aligned}\right\} \quad (144)$$

and it is then:

$$i_2 + i_1'' = i_1$$

or:

$$\left.\begin{array}{l} i_1 + i_1' = i_2 \\[1em] \frac{c_1}{c}\, e_2 + e_1'' = e_1 \\[1em] e_1 + e_1'' = e_2 \end{array}\right\} \quad (145)$$

33. That is:

1. In a single electric wave, current and electromotive force are in phase with each other. Phase displacements between current and electromotive force can occur only in resultant waves, that is, in the combination of the main and the reflected wave, and then are a function of the distance λ, as the two waves would travel in opposite directions.

2. When reaching a transition point, a wave splits up into a reflected wave and a transmitted wave, the former returning, in opposite direction, over the same section, the latter entering the adjoining section of the circuit.

3. Reflection and transmission occur without change of the phase angle. That is, the phase of the current, and of the voltage, in the reflected wave and in the transmitted wave, at the transition point is the same as the phase of the main wave or incoming wave. Reflection and transmission with a change of phase angle can occur only by the combination of two waves traveling in opposite direction, over a circuit, that is, in a resultant wave, but not in a single wave.

4. The sum of the transmitted and the reflected current equals the main current, when considering these currents in their respective direction of propagation.

The sum of the voltage of main wave and of reflected wave, equals the voltage of the transmitted wave.

The sum of the voltage of the reflected wave, and of the voltage of the transmitted wave, reduced to the first section by the ratio of voltage transformation $\frac{c_2}{c_1}$, equals the voltage of the main wave.

5. At the transition point, so a voltage transformation occurs,

by the factor $\frac{c_2}{c_1} = \sqrt{\frac{L_2}{C_2}\,\frac{C_1}{L_1}}$, that is, the transmitted wave of voltage equals the difference between main wave and reflected wave, multiplied by the transformation ratio $\frac{c_2}{c_1}$; $e_2 = \frac{c_1}{c_2}(e_1 - e_1'')$. As a result thereof, in passing from one section of a circuit to another section, the voltage of the wave may decrease or may increase. If $\frac{c_2}{c_1} > 1$, that is, when passing from a section of low inductance and high capacity, into a section of high inductance and low capacity, as from a transmission line into a transformer or a reactive coil, the voltage of the wave is increased; if $\frac{c_2}{c_1} < 1$, that is, when passing from a section of high inductance and low capacity, into a section of low inductance and high capacity, as from a transformer to a transmission line, the voltage of the wave is decreased.

This explains the frequent increase to destruction voltages, when entering a station from the transmission line or cable, of an impulse or a wave, which in the transmission line is of relatively harmless voltage.

34. The same equations also apply for a wave passing the transition point in opposite direction, and combining both waves gives the total expressions of current and voltage on the two sides of the transition point.

In the neighborhood of the transition point, that is, for values λ, which are sufficiently small, so that $\epsilon^{+f\lambda}$ and $\epsilon^{-f\lambda}$ can be dropped, this gives the equations:

$$\begin{aligned} i_1 = \epsilon^{-m_0 t} \Big[\{A_1 \cos q(\lambda - t) + B_1 \sin q(\lambda - t)\} - \frac{c_2 - c_1}{c_1 + c_2} \{A_1 \cos q \\ (\lambda + t) - B_1 \sin q(\lambda + t)\} - \frac{2\,c_2}{c_1 + c_2} \{C_2 \cos q(\lambda + t) + D_2 \\ \sin q(\lambda + t)\}\Big] \end{aligned}$$

$$\begin{aligned} e_1 = c_1 \epsilon^{-m_0 t} \Big[\{A_1 \cos q(\lambda - t) + B_1 \sin q(\lambda - t)\} + \frac{c_2 - c_1}{c_2 + c_1} \{A_1 \cos q \\ (\lambda + t) - B_1 \sin q(\lambda + t)\} + \frac{2\,c_2}{c_1 + c_2} \{C_2 \cos q(\lambda + t) + D_2 \\ \sin q(\lambda + t)\}\Big] \end{aligned} \qquad (146)$$

$$i_2 = -\epsilon^{-m_0 t}\left[\{C_2 \cos q\,(\lambda+t) + D_2 \sin q\,(\lambda+t)\} - \frac{c_1 - c_2}{c_1 + c_2}\{C_2 \cos q\,(\lambda - t) - D_2 \sin q\,(\lambda - t)\} - \frac{2\,c_1}{c_1 + c_2}\{A_1 \cos q\,(\lambda - t) + B_1 \sin q\,(\lambda - t)\}\right]$$

$$e_2 = c_2\,\epsilon^{-m_0 t}\left[\{C_2 \cos q\,(\lambda+t) + D_2 \sin q\,(\lambda+t)\} + \frac{c_1 - c_2}{c_1 + c_2}\{C_2 \cos q\,(\lambda - t) - D_2 \sin q\,(\lambda - t)\} + \frac{2\,c_1}{c_1 + c_2}\{A_1 \cos q\,(\lambda - t) + B_1 \sin q\,(\lambda - t)\}\right]$$

In these equations, the first term is the main wave, the second term its reflected wave, and the third term the wave transmitted from the adjoining section over the transition point.

Expanding and rearranging equations (146) gives as the distance phase angles of the waves:

$$\left.\begin{aligned} \tan i_1 &= \frac{c_2\,\{(B_1 - D_2)\cos qt + (A_1 + C_2)\sin qt\}}{(c_1 A_1 - c_2 C_2)\cos qt - (c_1 B_1 + c_2 D_2)\sin qt} \\ \tan i_2 &= \frac{c_1\,\{(B_1 - D_2)\cos qt + (A_1 + C_2)\sin qt\}}{(c_1 A_1 - c_2 C_2)\cos qt - (c_1 B_1 + c_2 D_2)\sin qt} \end{aligned}\right\} \qquad (147)$$

hence:

$$\frac{\tan i_2}{\tan i_1} = \frac{c_1}{c_2} = \sqrt{\frac{L_1 C_2}{L_2 C_1}} \qquad (148)$$

$$\left.\begin{aligned} \tan e_1 &= \frac{(c_1 B_1 + c_2 D_2)\cos qt + (c_1 A_1 - c_2 C_2)\sin qt}{c_2\,\{(A_1 + C_2)\cos qt - (B_1 - D_2)\sin qt\}} \\ \tan e_2 &= \frac{(c_1 B_1 + c_2 D_2)\cos qt + (c_1 A_1 - c_2 C_2)\sin qt}{c_1\,\{(A_1 + C_2)\cos qt - c_1\,(B_1 - D_2)\sin qt\}} \end{aligned}\right\} \qquad (149)$$

hence:

$$\frac{\tan e_2}{\tan e_1} = \frac{c_2}{c_1} = \sqrt{\frac{L_2 C_1}{L_1 C_2}} \qquad (150)$$

That is, at a transition point the distance phase angle of the wave changes, so that the ratio of the tangent functions of the phase angles is constant, and the ratio of the tangent functions of the phase angles of the voltages is proportional, of the currents inversely proportional to the ratio of the circuit constants $c = \sqrt{\frac{L}{C}}$.

That is, transition of an electric wave or impulse, from one section of a circuit to another, takes place at a constant ratio of the tangent functions of the phase angle, which ratio is a constant of the circuit sections, between which the transition occurs.

This law is analogous to the law of refraction in optics, except that in the electric wave it is the ratio of the tangent functions, while in optics it is the ratio of the sine functions, which is constant and a characteristic of the media, between which the transition occurs.

This law so may be called the

Law of refraction of a wave at the boundary between two circuits, or at a transition point.

The law of refraction of an electric wave at the boundary between two media, that is, at a transition point between two circuit sections, is given by the constancy of the ratio of the tangent functions of the incoming and reflected wave.

DISCUSSION ON "THE GENERAL EQUATIONS OF THE ELECTRIC CIRCUIT." ATLANTIC CITY, N. J., JULY 2, 1908

Frederick Bedell: It is unnecessary to dwell upon the great value of the work of Dr. Steinmetz in this and other papers. It might be proper that this paper be presented at one convention and discussed at another, a year or two later. The increasing value of the work of Dr. Steinmetz impresses me more and more, and looking back over his writings of ten or twelve years ago we find that to-day many of them are of even greater value than when they were written.

There is one thing to be noted particularly in this paper, and that is that Dr. Steinmetz has continually called attention to the physical significance and practical meaning of his results. We do not always find this careful rounding out of a paper. Not long ago, in looking over a mathematical paper on a subject in which I was interested, I was much disappointed to find that the paper stopped abruptly, and I was unable to find out what was the author's conclusion.

On account of the nature of this paper of Dr. Steinmetz, we can make only a few casual comments, which bring out nothing new. When we impress an electric impulse upon a circuit, we have a wave going in each direction; each wave keeps going around and around the circuit, until it dies out, and each time it goes around the circuit it gives us one of those terms in the series, which are added up, as Mr. Steinmetz has here done.

There is a physical feature to which attention might be called. We are most of us in the habit, when considering an ordinary alternating-current circuit, of thinking that the current in a series circuit is the same in all parts of the circuit, which is correct when there is no distributed capacity. In a circuit with distributed capacity, however, the current is not the same in all parts of the circuit; if we consider some point, P, in the circuit, the current may be going towards that point from both directions—a condition which we never have in a simple circuit without distributed capacity. This means that we are having electricity accumulated at that point. The point P, however, is not stationary, but travels along. We can think of this easier by analogy—by an hydraulic analogy. Considering first a pipe which is not flexible, an iron pipe, in which flows water which is not compressible; if we have a certain flow of water in one part of the pipe or circuit we have that same flow in all parts of the circuit. This construction illustrates the circuit without distributed capacity. Now let us consider, instead of a cast-iron pipe, a flexible rubber hose, and cause a series of impulses (as with a plunger) in the hose at one end; as a result of each impulse the hose will swell, and the swelling will travel along. Following the swelling there will be a contraction. Wherever there is a swelling, the water may be flowing from both directions towards the swelling;

and then it immediately flows away from the point and makes the hose contract. There is first a surplus and then a deficit of water (or electricity) and these are transmitted along the hose (or circuit).

Dr. Steinmetz called attention to the rates of decay, one with respect to time and the other with respect to distance. Suppose we have the impulses of different frequencies, or a complex wave which can be analyzed into the different component frequencies. These different frequencies, according to the equations of the paper, will be found to have different rates of decay; this is discussed in Chapters 12 and 13 of Bedell and Crehore's *Alternating Currents*. This means that if we have a complex wave impressed upon the circuit, the different components get to the receiving point at different times; one of them may get there a hundredth part of a second later than another, so that when they combine again into the resultant wave the wave form at the receiving end of the circuit and the wave form at the sending end of the circuit will be different. In power transmission this is ordinarily insignificant. When applied to telephony this change in wave form becomes significant, and these equations explain the distortion of speech on telephone lines, a case which has already been worked out in detail by Dr. Pupin.

Dr. Steinmetz speaks of the reflected wave. I would like to ask him whether he considers there is any reflected wave in a uniform circuit, in which there are the positive waves going around the circuit in one direction, and the negative waves going around the circuit in the other direction.

Dugald C. Jackson: The equations which Dr. Steinmetz has deduced are finely developed, comprehensively organized offsprings of the equation which was devised by Lord Kelvin in the first days of transatlantic telegraphy to analyze the problem of transmission of electric signal pulses through ocean cables of negligible self-induction. That equation was fertilized and hybridized by Heaviside until it could be applied to the problem of the transmission of telephonic speech-currents in circuits of uniformly distributed resistance, self-induction and capacity. Dr. Steinmetz has taken a third great and important step in developing equations which are applicable to complete circuits composed of sections of diverse characters and which also take adequate account of the terminal conditions or apparatus. The paper deals with the offspring of old friends in the electrical science, but the offspring are much more comprehensive and fully developed than the parents.

It has been usual to refer to a vibrating wire as a mechanical analogue of an electric circuit in either forced or natural electric oscillations,—that is, either when it is required to act under imposed pulsations or when stimulated into action and then left to carry out its own sweet will. A uniform wire however does not afford an illustration of the usual electric circuit of practice.

An electric circuit is ordinarily made up of sections of diverse electrical characters. Some of these sections may be long and some may be short, but they are almost inevitably found in the circuits of practice and their diversity of character robs the uniform wire of its usefulness as an analogue. We must therefore abandon the uniform wire for our analogue and take up a wire composed of sections of diverse character welded end to end. Sections of small resistance may be thought of as vibrating in air, others of higher resistance must be thought of as surrounded by molasses or other liquid of a density to correspond at each section with the desired resistance. In certain sections may be injected a large mass or inertia to represent high self-induction, and the elasticity from section to section must be adjusted to correspond with the desired electrostatic capacity. When this wire, made up of diverse sections of appropriate characters, is attached to abutments of suitably adjusted elasticity and inertia, it affords a true analogue of electric circuits of practice; but the analogue has become too complex to be illuminating. I have not found it possible to foresee the reactions that will occur in each section when a disturbance of particular character is put on such a wire by external means. Moreover, although uniform vibrating wires or plates, or loaded wires giving the semblance of uniform wires, have been prolifically treated in mathematical works, I have never observed a treatment adequate to predetermine the reactions in any wire of diverse sections. Dr. Steinmetz has now come to us with the formula for the electric circuit itself, which not only makes the analogue of less need but also probably at the same time gives the solution of the analogue.

The paper is reassuring and encouraging to the man who has only looked at it hastily, in that the simpler results set forth in it correspond with the phenomena that we know actually occur. For instance, as far as the telephone circuit is concerned, the effects of "loading" that Dr. Steinmetz sets down categorically as deductions from his equations, are those that are known to occur in the telephone circuit. Thus, it is known that leakage up to a certain degree and under certain conditions assists transmission of speech in a telephone circuit or the transmission of signals through an ocean cable, but that an excess of leakage beyond a point fixed by other conditions makes transmission worse. It is also well known that "loading" a telephone circuit under certain conditions materially improves the transmission of speech, but a line loaded to give admirable transmission with low leakage may afford poorer transmission than an unloaded line if the leakage becomes large. For instance, a particular loaded aerial open-wire telephone line may be good to talk over in clear weather, but let the weather get bad and the leakage become large and the loaded line is frightful to talk over; whereas 100 miles, or 150 miles of loaded underground line gives admirable transmission in all sorts of weather, because the leakage and other constants of such a line do not vary with the weather. That is the deduction Dr. Steinmetz draws from his equations. He doubtless knew these facts in advance, but they are demonstrated in his equations; and it is assuring to find that the equations do tell the truth in these simple matters, as this truthfulness supports a reliance upon their accuracy in more complex matters.

As an illustration of the fact that this paper carries us beyond what has been heretofore our frontier of knowledge, I will point out that the equations apparently enable us to understand the effects of terminal conditions on the action of circuits. As far as telephone analysis has heretofore gone, it has almost solely related to uniform circuits or to loaded circuits which are brought to the condition of substantially uniform circuits. The equations heretofore have not been able to cope successfully with the difficulties of the complete circuit including the complexities of terminal conditions, and the conditions of the telephone receiver and transmitter have been omitted from mathematical consideration or merely approximated. I believe that great advances in intercommunication by the telephone and telegraph are yet to come about from improvements of terminal conditions, and not so much from further improvements in the knowledge of line conditions. These equations of Dr. Steinmetz apparently lay the foundation for study of terminal conditions that may result in the improvements to which I refer.

I am now going to diverge for a moment and go back to a point to which I referred recently; and that is the tremendous increase in output of what is often called "pure" science, which now accompanies the advance of engineering. This paper illustrates the point. Formerly, engineers were followers of the men of scientific research, and the men of scientific research looked with some disdain upon men of the engineering industries. At the present time the engineers have caught up with and absorbed pretty much all of the accumulated output of the pure scientists, and they quickly absorb the current output as fast as it comes along. To keep the output in pure science (especially in physics and chemistry) abreast with the advance of the engineering industries, the engineers themselves have been forced to enter the field of pure research and their product must be recognized as unexcelled by that of the so-called pure scientists.

Dr. Steinmetz has set out in his paper a number of pronounced facts, and some of them illustrate how truth, when it is presented to one plainly, seems so common-sense and persuasive as to be almost axiomatic. For instance, Dr. Steinmetz calls attention to the fact that "The oscillation of a circuit, which is open at one end, grounded at the other end, is a quarter-wave oscillation, which can contain only the odd harmonics of the fundamental wave of oscillation." He further says "The oscillation of a circuit which is open at both ends, or grounded at both ends, is a half-wave oscillation, and a half-wave oscillation can also contain the even harmonics of the fundamental wave of oscillation, and so also the constant term, for $n = 0$ in (78)." That is to say, one can get a very erratic wave (compared with a sine alternating wave) when one sets up oscillations, and oscillatory waves need not be always of symmetrical half-pulses. The view that is commonly taught in the classroom explains the facts only partially. In another place there is stated a very interesting and very striking thing. After having studied some familiar equations we find the statement that " the wave shape of the oscillation remains unchanged during the decay of the oscillation." Then another very striking thing is found, where it is pointed out that " the wave does not die out in each section at the rate as given by the power consumed in this section, or in other words, power transfer occurs from section to section, during the oscillation of a complex circuit." I do not think that this had come to my mind before it was pointed out to me by Dr. Steinmetz. It seems a fact of great significance.

There are a number of other similar, clear-cut, categorical statements in the paper, which make it interesting to read even to one who does not undertake to go through the equations one by one for the purpose of getting more in the way of facts out of them.

Finally, I wish to congratulate Dr. Steinmetz upon the directness of the attack with which this paper goes at its object. The studies that have heretofore been made on this subject have been more or less of a roundabout siege. The direct assault, when it can be made, though sometimes imposing greater hardship on the attacking party, ordinarily brings greater results than a siege; or, at least it brings, as a rule, results that are more immediately and broadly useful. Dr. Steinmetz has been able to make successfully a direct attack upon the characteristics of a circuit that varies in character throughout its length. His processes are as novel as they are interesting, and I wish again to congratulate the Institute on receiving this demonstration of the great results that can follow a direct attack by mathematical analysis.

H. L. Wallau: I would like to ask Dr. Steinmetz one question. He mentions a voltage transformation at the transition point. If I am not mistaken he told us that at that transition point there could be but one frequency, the same on each side of the point, because the end of each section being the same point as the beginning of the next, the frequency was the same. If a voltage transformation occurs, can there be two voltages at the transition point? How does the transformation take place?

Chas. P. Steinmetz: As regards the reflected wave in a uniform circuit, the question Dr. Bedell brought up; if we had a perfectly uniform circuit without terminal points—a closed circuit—we could imagine a wave without a reflected wave, going around and around, provided we could produce such a wave. I cannot, however, at present see any possibility of producing such a wave, since any impulse or any action on the closed circuit would naturally produce two waves running in opposite directions from the starting point, and then either of these could be considered as the main wave, the other as the reflected wave. But in general when dealing with uniform circuits we have terminal points, because although the uniform circuit with terminal points, such as a line open at both ends, is not of very great practical importance, a perfectly uniform circuit without terminal points —that is, the line closed on itself at both ends—does not appear feasible from a practical standpoint.

As to the questions asked by Mr. Thomas, I think there is a misunderstanding. If we consider a circuit, with the generator on one end, the load on the other end, the main wave decreases from the generator toward the load, while the reflected wave decreases from the load toward the generator. That is, looking at the waves in the same direction of the circuit, from the generator toward the load, the one wave decreases, the other increases, with the distance. However, in treating of waves increasing or decreasing with the distance, I always consider them in the direction of propagation, which in the reflected wave is opposite to that of the main wave.

Considered in the direction of propagation, the character of the reflected wave thus is always the same as that of the main wave, either increasing with the distance or decreasing with the distance. The two component waves of the general equations, that which decreases with the distance, and that which increases with the distance, thus can not be a main wave and its reflected wave, but each of the two component waves comprises a main wave and its reflected wave.

Regarding the transformation of current and voltage at a transition point, this is a very interesting subject, deserving a far more extended discussion than was feasible in the paper. On the two sides of the transition point the resultant currents must be equal, and so also must be the resultant voltages. These however are the resultant of several waves, the main waves and the reflected waves, and the transformation ratio applies to the individual waves. Thus, at one side of the transition point the main wave and the reflected wave differ from those on the other side of the transition point by a constant factor, the transformation ratio, but at the same time the phase-angle on one side differs from that on the other side in such a manner that the resultant of main and reflected waves is the same at both sides of the transition point. As seen, this feature connects the phase-angles on the two sides of the transition point (hence the law of refraction) with the transformation ratio. For instance, if on one side the main wave and the reflected wave are much greater than on the other side, they are more nearly in opposition on the former side than on the latter, and therefore give the same resultant. However, as the phase-angle changes from point to point in the circuit, at some point, distant from the transition point, the main wave and the reflected wave are in phase with each other. The resultant is then their sum, and higher on the side where the individual waves are higher, Hence the maximum values of the wave-crests in the circuits on both sides of the transition point, have to each other the transformation ratio of the transition point (multiplied by the decrement of the circuit between transition point and wave crest). Thus, when transforming up at a transition point, at this point the voltage remains the same, but the higher voltage appears at the next wave-crest.

W. S. Franklin (by letter): I have found Mr. Steinmetz's paper on the General Equations of the Electric Circuit to be very instructive, and what I wish to say in discussing the paper relates chiefly to a method for discussing electric-wave motion which is due to Heaviside. Some time ago, in reading a treatise on optics by one of our leading specialists on that subject, I came across this statement: "To prove that any phenomenon is due to wave motion, it is sufficient to show (1) that it is periodic, and (2) that it is propagated with a finite velocity", and immediately there came into my mind the idea of bullets from a Gatling gun, which are periodic and which are propagated with a finite velocity. The fact of the matter is that the idea of periodicity has been too much used in the discussion of wave motion. The wave pulse involves all of the important actions of wave motion, except that action which serves as a basis for the theory of the dispersion of light,* and the physics of wave motion may be developed in the simplest possible manner by considering the behavior of wave pulses; for it is certainly true, as Heaviside says, that the physics of wave motion is presented in its simplest aspects in a wave pulse, whereas the mathematical formulation of the wave pulse is extremely complicated. On the other hand, the physics of the wave train (a periodic succession of similar waves) is extremely complicated, and its mathematical formulation is fairly simple.

I have recently been engaged in the preparation of a paper on electric-wave motion on transmission lines and on the electric oscillation of transmission lines, and a few abstracts from this paper will serve to illustrate the point of view of Heaviside.

Fig. 1 shows a transmission line, initially open at both ends, one line being positively charged, the other negatively charged—both uniformly. The end of the line is suddenly short-circuited, a current is suddenly established at the end of the line, the charges on the wires disappear, and a wave sheet W travels in the direction of the arrow V, wiping out the electric field and charges completely, and establishing the magnetic field and current as it travels along. The lines of force of the magnetic field

* The simple wave trains constituting the parts of the solar spectrum which is formed by a prism are produced by the prism and do not exist *as such* in the complex waves which enter the prism from the sun.

are represented by the dots and the magnetic field is directed away from the reader. When the wave sheet W reaches the open end of the line, the energy, which initially was electric, has been wholly converted into magnetic energy; the current, which has the same value over the whole line, is then suddenly reduced to zero at the open end, and, in being reduced to zero, it builds up a reversed charge and a reversed electric field at the extreme end of the line; and a wave sheet travels in a reversed direction along the line, wiping out magnetic field and electric current and laying down the reversed electric field and charge. When this wave sheet reaches the short-circuited end of the line, the line is uniformly charged (with zero current everywhere). The charge then suddenly disappears at the short-circuited end of the line, establishing a reversed current and a reversed magnetic field both of which are established by a wave sheet which travels towards the open end of the line. When this wave sheet reaches the open end of the line, the reversed current is suddenly reduced to zero at the open end, a wave sheet travels back to the short-circuited end and re-estab-

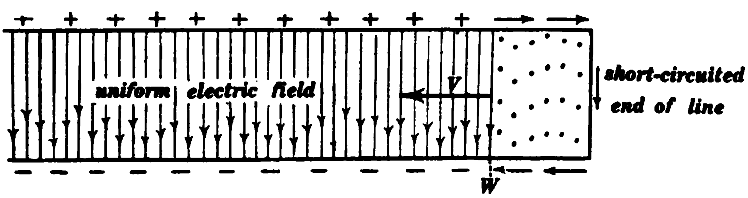


FIG. 1

lishes the initial charge and initial electric field, so that when this wave sheet has reached the short-circuited end of the line, the entire system is in its initial condition and the line has performed one complete oscillation. Line resistance is ignored in this discussion.

Fig. 2A represents a generator delivering current over a line of negligible resistance to a receiver of negligible resistance (the system is short-circuited). The receiver end of the line is suddenly opened, the short-circuited current is suddenly reduced to zero at the opened end, and, in being reduced to zero, builds up an electric field between the lines and charges the lines, as shown in Fig. 2B (voltage assumed not to rise to a sufficiently large value to break across from line to line). The short-circuit current (and the magnetic field which corresponds to it) is wiped out by a wave sheet W'', and the electric field and charges are laid down by this wave sheet which travels towards the generator. A complete cycle of oscillation may easily be traced very much as in the case of Fig. 1.

A battery is suddenly connected to one end of a transmission line as shown in Fig. 3A, the other end of the transmission line being open. A wave sheet W' shoots out from the battery end of the line establishing a current in the line and magnetic field between the lines, and also establishing electric charges on the lines and an electric field between them, as shown. When the wave sheet W' of Fig. 3A reaches the open end of the line, the current in the line is suddenly reduced to zero, and in being reduced to zero builds up a doubled charge and doubled electric field as shown in Fig. 3B, this doubled charge and doubled electric field being established by a wave sheet W'' which travels back towards the battery end of the line. A complete oscillation of the line may easily be traced in this case, as in the case represented in Fig. 1.

A very interesting case is the establishment of current in a long transmission line, short-circuited at the far end, when a

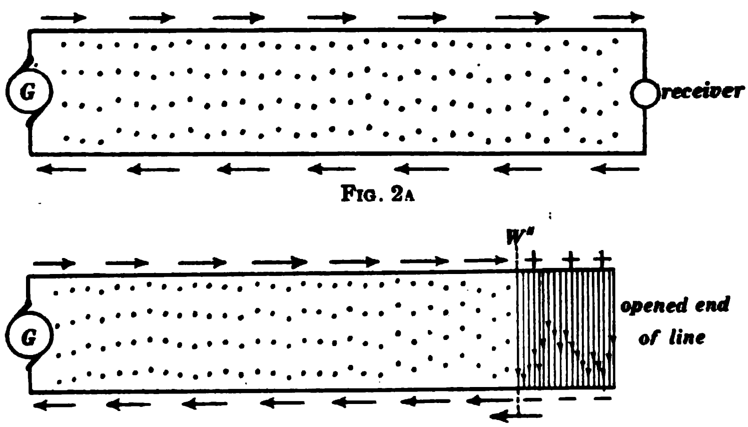


FIG. 2A

FIG. 2B

battery is connected to the near end, resistance being neglected. A wave sheet shoots out from the battery as represented in Fig. 3A ($I=E\sqrt{\frac{C}{L}}$, where L and C are line inductance and line capacity per unit of length, respectively, E is the voltage of the battery, and I is the current in the line behind the wave sheet). When this wave sheet reaches the short-circuited end of the line, the electric field between the wires drops to zero at the extreme end and the current rises to $2I$, and this doubled current and zero voltage is established over the whole line by a wave sheet which travels back towards the battery end. When this wave sheet reaches the battery end, the whole action is repeated on top of the existing short-circuit current $2I$. That is to say, a current $3I$ and battery voltage E are established over the whole line by a wave sheet which shoots out from the battery, and when this wave sheet reaches the short-circuited end of the line, a current 4 I and zero voltage are established over the whole line by a wave sheet which travels back towards the battery end. And so on. The action is precisely analogous to the surging motion of a frictionless railroad train which gets under way under a constant locomotive draw-bar pull. At the beginning, the constant pull establishes a certain velocity I which travels backward until it reaches the caboose; then the stretched coupling springs jerk the caboose forward and a velocity 2 I is established which travels forward until the whole train is moving at velocity 2 I with coupling springs entirely freed from stress; and then this process is repeated on top of the existing velocity 2 I.

A stretched string, A B, Fig. 4A, is pulled to one side at the

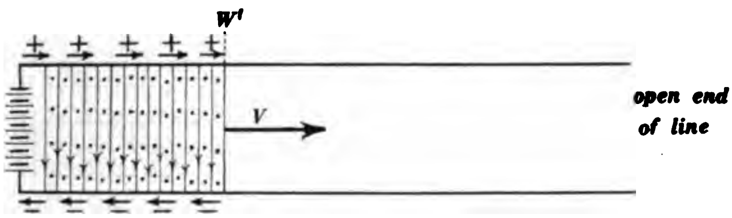


FIG. 3A

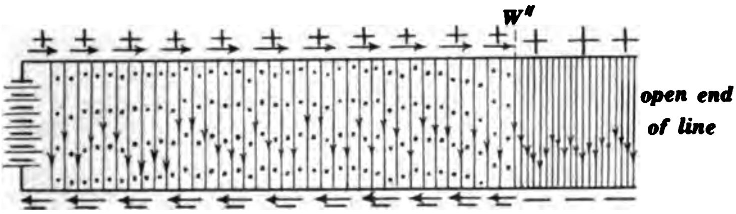


FIG. 3B

point P; the water at one end of a canal C D is held uniformly elevated and the water at the other end uniformly depressed by a gate at the center of the canal; a transmission line, E F, open at both ends and broken at the middle, has two batteries connected as shown, thus producing an upward electric field between the wires at one end of the line and a downward electric field between the wires at the other end of the line. The three diagrams in Fig. 4B represent the state of affairs when the initial disturbances which are represented in Fig. 4A are released. When the point P is released the middle portion of the string gets into motion at a uniform velocity and is entirely relieved from the stretch due to the distortion, which is represented in A B, Fig. 4A. The middle portion of the water in the canal immediately settles to its normal level and is set in uniform motion as represented in the diagram C' D'. When the switches in $E\,F$, Fig. 4A, are closed, the charge near the middle of the line disappears, and a uniform current (and magnetic field) is established as shown in the diagram $E'\,F'$. A complete cycle of oscillation may easily be followed in each case.

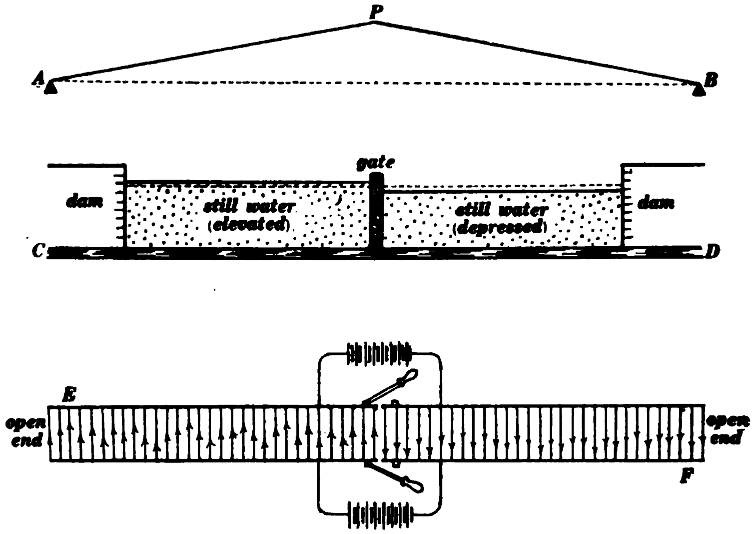


FIG. 4A

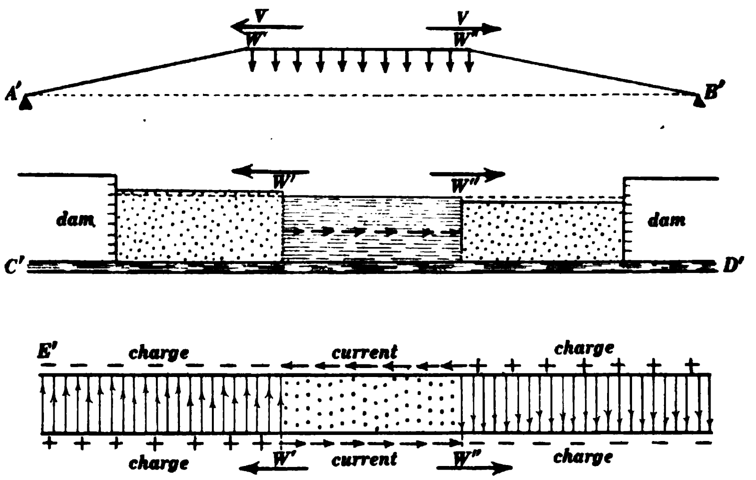


FIG. 4B

The paper which I am preparing on electric-wave motion is devoted (1) to the establishment of clear ideas of the physical action which is involved in the electromagnetic wave; (2) to the discussion of a great variety of special cases; and (3) to the modification of the results so reached by the effects of line resistance and line leakage. I wish to call attention to the extremely simple discussion of electromagnetic waves which is to be found in the first volume of Heaviside's *Electromagnetic Theory*, pages 306 to 455, in which the fundamental ideas of electric-wave motion are developed on the basis of negligible attenuation (due to resistance of line and leakage between wires), and the effects of line resistance and line leakage are then described in extremely simple terms as a diffusion phenomenon modifying the results that are arrived at in the first instance. I know of no discussion of electric waves more edifying than this of Heaviside's. It would be misleading, however, if I did not mention the fact that throughout the second volume of Heaviside's *Electromagnetic Theory* the mathematical formulation is based, as it must be, upon exponential and periodic functions very much as Dr. Steinmetz has developed it, and that the mathematical formulation of the wave pulse is accomplished by the aid of Fourier's integrals. Such methods must be used if one is to arrive at numerical results, but I think that a clear conception of any physical phenomenon is a thing without which no one can make use of numerical results, or even be led to seek for them.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 2, 1908.

PRIMARY STANDARD OF LIGHT

BY CHARLES P. STEINMETZ

Light is not a physical quantity, but a physiological effect, that of certain wave-lengths of radiation, and therefore can not be expressed in absolute physical units; it must be measured by comparison with an arbitrarily chosen standard of physiological effect. As a result thereof, even with the best existing primary standard of light, the amylacetate lamp, the difficulties of reproduction, and maintenance of its constancy, are such as to involve errors very far beyond those considered permissible in physical measurements. A radical increase in the accuracy of reproduction and maintenance of a primary standard of light appears possible only by relating the standard of light in such manner to physical quantities, that it can be determined by energy measurements.

This led to the recommendation of defining the primary standard of light by the energy of radiation. It requires, however, a definition of the quality of radiation, since the physiological effect of radiation has no direct relation to the energy of radiation: one watt of radiation of a wave-length from the center of the visible range gives a far higher physiological effect, that is, more light, than one watt of a wave-length near the ends of the visible spectrum. This is not merely a function of the quality or color of the light, as light of the same intensity and same color, identical physiologically, may be entirely different physically, and therefore represent a different amount of power. For instance, the physiological effect of white light is produced by the combination of all colors of the spectrum, but also by an infinite number of combinations of two, three, or more spectrum colors; and these physically different forms of white light, while indistinguishable on the photometer screen, and thus physiologically identical, represent different amounts of radiation power per unit of physiological effect, or per candle-power. The same reasoning applies to colored light. The power of the visible radiation is, therefore, not a measure of light.

The definition of the absolute unit of light as the physiological effect of one watt of power of visible radiation, requires, therefore, an arbitrary definition of the distribution of power throughout the visible range. As such distribution of power through the visible range, may be chosen that of the black-body radiation at a definite temperature, and the temperature measurement may be eliminated by specifying the energy-ratio between two definite regions of the spectrum. For instance, such a definition would be:

The unit of light is that given by a black body radiating one watt of power between the wave-length of 39 and 72, under the condition (that is, at the temperature) where the power of radiation from wave length 39 to 55 bears to that from 55 to 72 the ratio a.

It does not appear to me that such a standard would be very satisfactory, and for the following reasons:

1. Though the intensity of radiation at the red end of the spectrum is very high, the physiological effect is very small, and an inaccuracy at this limit would therefore seriously affect the result.

2. The absorption constant of glass or any other envelope is different for different wave-lengths.

3. Any deviation of the radiator from the black-body radiation vitiates the result.

4. The color of the light would not be white, but yellow, due to the temperature limitation imposed by the radiator.

A more satisfactory primary standard of light, based on measurement of radiation power can, as I believe, be produced by selecting three primary colors of the visible spectrum. Let these colors be of definite wave-length, about equidistant from each other, and of such character that they can be absolutely reproduced at high intensity, as the spectrum lines of a luminous gas or vapor. Then define the unit of light as that given by one watt of power radiated at these three wave-lengths, in definite proportions. chosen so as to give white, or yellowish-white, light. That is, the primary unit of light is the physiological effect, as observed on the white photometer screen, of one watt of power, radiated in the three definite wave-lengths A, B, C. in the proportion $a \div b \div c$.

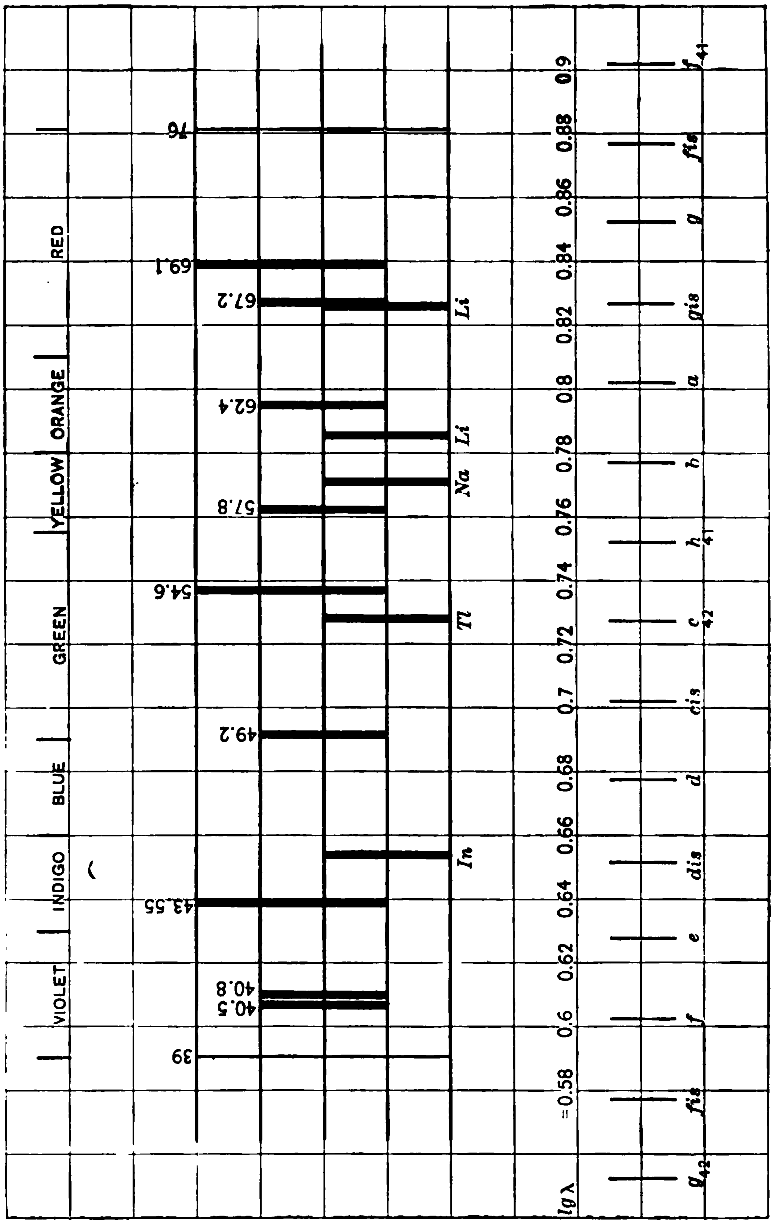
The adoption of this standard of light would eliminate the sources of error which now affect the primary flame-standard. The radiation density or light flux intensity of the three primary colors is measured separately before they are combined on the same photometer screen, and the accuracy of determination would be limited only by that of physical measurement of radiation energy, by bolometer or otherwise.

This standard of light requires three sources of monochromatic radiation, which can be maintained constant with any desired exactness. This requirement seems fulfilled by the mercury lamp. When reasonably protected from air drafts, changes of surrounding temperatures, etc., the intensity of radiation of a mercury lamp remains extremely constant at constant current, and, for minor variations of current, varies directly as the current, while that of the incandescent lamp varies as a high power of the current. It must be considered that a reproduction of the same intensity in the lamp is not required, as is the case with standard lamps, since the mercury lamp is merely the source of monochromatic light, and its intensity is measured and adjusted.

I should therefore recommend mercury lamps as the sources of the three monochromatic radiations, which combined give the primary standard of light.

Approximately, some of the brighter lines of the mercury spectrum are shown in Fig. 1 in geometric scale; that is, with the logarithm of wave-length as abscissas. When investigating the combination of different frequencies to a resultant effect, this rational scale, which is the scale of acoustics, is preferable.*

* The usual way of recording spectra, with the wave-length, or the frequency as abscissas, is irrational; one wave-length in the ultraviolet represents a far greater range than in the ultrared. The infinite number of radiations of shorter wave-length, or higher frequency than the visible, are crowded into a finite space, when using wave-length as abscissas, while the lower frequencies or longer waves cover the whole range from the ultrared to infinity. The reverse is the case with the frequency as abscissas. The intensity curve of radiation, measured and recorded with the wave-length as abscissas, is different and its maximum at a different point, than the intensity of the same radiation, plotted with the frequency as abscissas. The rational scale of any periodic quantity is the geometric scale, where equal intervals represent equal percentual increase or decrease; that is, the logarithmic scale. This was realized long before science existed: it is the scale used in music, with the octave; that is, log 2, as abscissas


RED
ORANGE
YELLOW
GREEN
BLUE
INDIGO
VIOLET
76
69.1
67.2
62.4
57.8
54.6
49.2
43.55
40.8
40.5
39
Li
Li
Na
Tl
In
lgλ
=0.58
0.6
0.62
0.64
0.66
0.68
0.7
0.72
0.74
0.76
0.78
0.8
0.82
0.84
0.86
0.88
0.9
g42
fis
f
e
dis
d
cis
c42
h41
b
a
gis
g
fis
f41

In Fig. 2 the mercury spectrum is given in polar coördinates with the octave, or the ratio of wave-lengths $2 \div 1$, represented by 360°. For illustration, the frequency denotations of acoustics are recorded in Figs. 1 and 2; that is, one half-tone represents 30°.

For comparison the spectrum lines of Li, Na, Tl, and In are also shown.

Of the spectrum lines of mercury of approximate wave lengths:

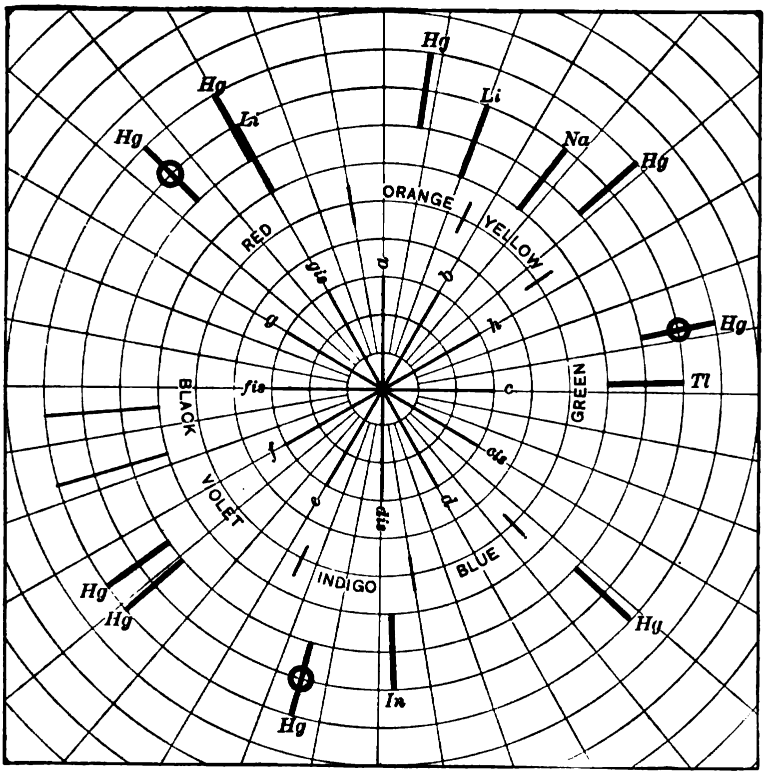


FIG. 2

40.5; 40.8; violet	43.55; blue	49.2; 54.6; green	57.8; yellow	62.4; orange	67.2; 69.1; red*,

the lines:

Blue.............................43.55
Green.............................54.6
Yellow.............................57.8

* The wave-lengths of these red lines require re-determination. Some of the other lines are twins.

are the most prominent. The three red lines, and many more red lines, appear only at higher temperature, as in the Heraeus quartz lamp.

The three spectrum lines of mercury:

Blue	43.55
Green	54.6
Red	69.1

are about equidistant and therefore appear the most suitable as primary colors for a standard of light.

Two low-temperature mercury lamps would be required for the blue and the green, and one high-temperature quartz lamp for the red. These would be maintained at constant radiation by maintaining the current constant, and also the condition of ventilation and surrounding temperature.

Resolved by a prism, the blue line of the first, the green of the second, and the red of the third lamp are thrown on the same white screen, and their radiation energy measured separately. What energy proportion to select for the three colors, to give a color suitable for a primary standard, remains to be investigated. Probably a yellowish-white would industrially be most convenient.

It is interesting to note that such a selection of three primary colors as components of a standard of light would also allow an exact numerical expression of the physiological color of any light, by the ratio of the three intensities, $a \div b \div c$; that is, the color of light could be measured by varying the intensities of the three standard wave-lengths until their combination, on the white screen, becomes identical in color with the observed light.

Discussion on "Primary Standard of Light." Atlantic City, N. J., July 2, 1908

A. E. Kennelly (by letter): The paper presents a certain fundamental physical proposition relating to a fundamental standard of luminous intensity, and also a practical proposal suggesting a means for carrying out the proposition. In so far as concerns the physical proposition, it seems to be unassailable, and to offer a theoretical solution for the problem of realizing a standard luminous intensity of definite physiological value, and definitely measured energy; that is, in ergs per steradian, or ergs per cubic centimeter from point to point of the luminous beam.

As regards the practical suggestion of carrying this method into effect, it seems to me a great deal of experimental work will be necessary to develop the method. It would be necessary not only to develop the bolometric method of measuring the energy in the three component beams with a satisfactory degree of accuracy for standard purposes, but it would also be necessary to determine the physiological stimulus of each of these three colors throughout the range of intensity required in photometric measurements. This is for the reason that in order to measure the light given by any particular lamp, such as a Hefner flame, for example, it would be necessary to arrive at a color balance on the photometer, by altering the ratio of the intensities in the three component beams. This would involve an alteration in the luminous stimulus, or light sensation, of unit intensity of the composite light so produced, and the luminous intensity of the Hefner flame could only be expressed in terms of standard light after taking into account the change in physiological effect of the altered components.

Nevertheless, Dr. Steinmetz's proposal is definite, and offers a definite chance of realizing a true determination of light in terms of energy, as the outcome of experimental research along three defined wave lengths or luminous frequencies.

The suggestion in the paper that the scale of abscissas should be logarithmic, instead of numerical, seems to be excellent for many graphical purposes. According to this suggestion, the visible spectrum would be divided into three larger regions (violet, green, and red) and four smaller regions, or "semi-tones," namely, indigo and blue, on one side of the green, and yellow and orange on the other. This would make an optical scale of approximately three tones and four semi-tones, or ten semi-tones altogether. The division of the spectrum would thus naturally fall into decimal subdivisions instead of into Angström units. Each first decimal place, or tenth root of 2, would correspond approximately to a semi-tone of color, while two such decimal divisions would correspond approximately to a tone of color.

Edwin P. Hyde (by letter): It would be very desirable indeed, both from theoretical and practical considerations, to be able to define our standard of light in terms of the C. G. S.

system of units, rather than have it without any known connection with other physical quantities. The suggestion of Dr. Steinmetz would seem to make possible such a definition. According to the suggestion we would define, not a standard of luminous intensity, but a standard of luminous flux; and we would measure the intensity of an unknown source by comparing the illumination produced by the flux from that source with the illumination produced by the standard flux. Since according to our present methods what we actually do is to compare illuminations and reason by means of known or assumed laws to the luminous intensities of the sources producing the illuminations, it is only necessary that we have a standard luminous flux, and not that we have a source of standard luminous intensity.

The principal difficulty to be overcome in realizing such a standard as Dr. Steinmetz suggests is in accomplishing with the requisite accuracy the absolute measurement of the small amount of energy contained in a single bright line in the visible spectrum. The determination of the proper proportions of energy in the three regions of the spectrum to give the required color, and the reproduction of these three colors in their proper proportions, could probably be accomplished with sufficient accuracy, but the determination of the energy of each region in absolute units would be a much more difficult undertaking. It would seem to me that a suggestive method of procedure might consist of: first, in obtaining as intense radiation as possible for the objective measurements, and then cutting down the intensity, if necessary, by sectored disks or other suitable means for the photometric measurements; and secondly, in measuring objectively the energy both of the component wave-lengths and of the combined flux—the former to give the proper proportions, and the latter to give the total luminous flux. By this method, slight errors in determining the proper proportions of the component parts would only enter as second order effects if the total flux is measured accurately; and the larger the total quantity of energy measured objectively the more accurately can the measurement be made when we are dealing with relatively small quantities of energy.

Although there would be numerous advantages in such a standard as Dr. Steinmetz proposes, if such a standard could be realized, there is one slight disadvantageous feature, which, however, is not serious. It has usually been customary to take the standard of white light as the standard also of spectral distribution in spectrophotometric measurements. This can only be done, of course, when the standard has either a continuous spectrum, or else a large number of bright lines well distributed in the visible region of the spectrum. The use for spectrophotometric measurements of such a standard as that proposed by Dr. Steinmetz would not seem feasible.

With the adoption of a common international unit of luminous intensity maintained arbitrarily by means of incandescent lamp secondary standards, such as has been proposed, the demand for a suitable primary standard of light is not as urgent as it has been heretofore; it is nevertheless just as important as ever, and the proposal of Dr. Steinmetz to define the standard in watts, with a prescribed spectral distribution, is very suggestive.

W. S. Franklin (by letter): The establishment of a reliable and reproducible light standard is of very great importance, and we naturally look to our Bureau of Standards to carry out such researches as may be necessary for its realization. I therefore move that the Bureau of Standards be requested by the Institute of Electrical Engineers to undertake such researches as may be necessary to determine the practicability of the light standard suggested by Dr. Steinmetz,[1] and, to develop such a primary standard of light, if it be found practicable.

In asking the Institute to place itself on record in this matter, I must say that, as it seems to me, Dr. Steinmetz's proposal cannot be a complete solution of the problem. Indeed, I do not believe the problem is capable of complete solution. Our present ideas of precision, and the theory of errors of observation as it stands at present, have been developed almost wholly in connection with extremely precise measurements, such as measurements in astronomy and geodesy, where the thing which is measured approximates very closely an actual "state of permanency," that is, where the thing which is measured actually exists as an unvarying physical condition. In attempting to establish a standard of light, however, it is important, as it seems to me, to understand that the present ideas of precision of measurement are in many cases inapplicable. Measurement in its most general aspects consists in bringing two things or systems into a kind of congruence, and all errors of observation originate in the incompleteness of this congruence test. In view of this general statement it is possible to distinguish two classes of errors of observation, namely, (a) errors which are due to erratic variations in the thing or system which is being measured, and (b) errors which are due to erratic variations in the measuring devices. The former being inherent in the thing which is measured, may be called intrinsic errors; and the latter, being outside of the thing which is measured, may be called extrinsic errors.

The probable error of a result, in so far as it depends upon extrinsic errors (irregularities in the measuring devices), is a measure of precision in the ordinary sense of that term, whereas the probable error of a result, in so far as it depends upon intrinsic errors (irregularities in the thing or system which is being measured), is a measure of the degree of indefiniteness of the thing which is measured.

1. PROCEEDINGS American Institute Electrical Engineers, March, 1908, p. 297.

It is generally possible to distinguish between extrinsic errors and intrinsic errors, and it is desirable to use the term probable error (as calculated from extrinsic errors) as a measure of precision, and to use the term probable departure for that part of the probable error, ordinarily so-called, which depends upon intrinsic errors.

In order to make these ideas perfectly clear, let us consider a particular case of measurement relating to erratically[2] varying conditions. For example, let us consider the repeated measurement of the current which is delivered by an electric railway power station. The successive measured values of the current differ greatly from each other, and one might proceed in the ordinary way in the determination of the "probable error" of the result. Suppose, however, that the ammeter is carefully studied by measuring with it a very carefully controlled current output which is kept as nearly constant as possible. In this way the systematic and erratic errors of the ammeter may be determined, and the probable error of an ammeter reading under these conditions would be a measure of the precision of the instrument. Let a be the probable error of a single observation of the ammeter, so determined. Let A be the probable error of a single reading of the ammeter as determined from the residuals of the observed values of the current output of the station. Then $A^2 - a^2$ is the probable departure of the current output of the station at any given instant from its mean value; that is to say, the difference between the mean value of the current output of the station and its actual value at any instant is as likely to be greater than $A^2 - a^2$ as it is to be less than $A^2 - a^2$.

Suppose the value of a in the above discussion to be one ampere and the value of A to be ten amperes. Then it is evident that the extrinsic errors (due to irregularities in the measuring device) in the above measurement are entirely negligible in their effect on the result as compared with the intrinsic errors (due to irregularities in the system which is being studied).

Ordinary photometry presents a case in which intrinsic errors are very great, because the actual intensity of light sensation depends upon so many varying conditions which are beyond control; in fact, the intensity of the light sensation which is produced by a given beam of light depends upon manifold individual peculiarities of the person, upon the degree of fatigue of the retina, and upon the degree to which the sensation is enhanced by attention, to say nothing of variations in composition of the light (relative intensities of the various wave-lengths which

2. Any set of varying conditions must be thought of as erratic when no attempt is made to analyze the varying conditions into systematic groups. Thus the variation of current output of an electric railway power station is to be thought of as erratic unless an attempt is made to study the mean daily and hourly fluctuations of load. For the present purpose, the current output of such a station is assumed to be erratic.

are present). It would therefore seem to be hopeless, and indeed meaningless, to attempt to establish a unit of light which would be completely definite except in so far as the physical character of the light is concerned. The proposal of Dr. Steinmetz does, in fact, seem to provide for complete definiteness of the light unit from a physical point of view, and the degree of precision with which this light unit might be reproduced would probably be so great as to leave nothing to be desired, considering the very great intrinsic errors in photometric measurements.

Carl Hering: I have been very much interested in this paper by Dr. Steinmetz, and am very glad to see a suggestion in a practical form of a new unit of light, based on the power equivalent. It interests me personally, because I have for a number of years (since 1901 or 1902) recommended that our unit of light should be based on the power unit, that is, that the unit of light should be defined as the light produced by one erg per second converted entirely into light.* But I suggested no specific way in which that could be carried out practically other than to base it on one selected wave-length, or on a certain average wave-length, and I am therefore very much pleased to see that Dr. Steinmetz has offered this very interesting and apparently possible solution.

There is another reason why it is desirable to use the power unit in which to express our unit of light. We have at present three groups of units, the absolute (or electrical) units, the gravitational units, and the heat units. All of our units, except those of light, can be referred to one of these three. They have as connecting links the mechancial equivalent of heat, the acceleration of gravity, and the specific heat of water. Each of these three groups of units was originally started on an entirely independent basis, and the connecting links had to be derived afterwards by experiment. We have at present a unit of light which does not belong to any of these; it is based on an entirely new foundation, and it will be necessary to find by experiment the connecting links with these three groups of units. Dr. Steinmetz's suggestion is not to form such a new group, but to start by making the unit of light coincident with one of the existing groups, namely, the electrical or absolute units.

The term "unit of light", used throughout Dr. Steinmetz's paper, might be confused with the term "candle-power" by those who have not paid particular attention to the subject of light units. Of course, he does not mean candle-power, he means flux of light, as he stated in his abstract. There is evidently no possibility of an equivalent between watts and candle-power. The only equivalent can be between watts and flux of light. Flux of light is equal to candle-power multiplied by a solid angle, or to illumination multiplied by a surface.† We use the term

* See Conversion Tables, Hering, p. 3; also p. 147.

† See Conversion Tables, Hering, p. 147.

" watts per candle-power "—but, of course, that is well known to be an irrational expression, although it has come into our nomenclature. We mean, of course, watts per spherical or hemispherical candle-power.

I call attention to an error in the diagram of the musical scale, in which the " a " to the left should be " e." I suggest that the English equivalents be added to this musical scale, as it will then be more easily understood by English readers; Dr. Steinmetz has apparently adopted the German nomenclature. Why does he reverse the conventional methods in the diagram by having the shorter wave-length to the left and the longer wave-length to the right, instead of the reverse, which is the universal method in musical practice; also in showing the spectrum, in which, I believe, it is conventional to show the red end to the left and the blue end to the right? There is an error in the text a little further on: one tone represents 60 deg., not 30 deg.

At the close of the paper, Dr. Steinmetz says that any color can be made up of these three. I am interested in knowing whether he is really quite sure of that, for the following reason: If we take three colored pigments and mix them together, we will get a certain color; now if we take some of that mixed pigment, and mix with it some white pigment, and then take some more and mix with it some black pigment, we will have, with the original mixture, three distinct colors. Each one of them will have the same relative quantities of the three original colored pigments in it, and yet it will in itself be an entirely different color from the other two. It might be said that they are shades of the same color. Now the question is, whether a parallel to this exists in the mixing of light rays. That is, is there in the mixing of light rays, something which corresponds to the admixture of white pigment or black pigment to a colored pigment?

Some years ago, when I was interested in this subject of the equivalent between power units and light units, I desired to know about what the magnitude of that unit would probably be; that is, what the conversion factor between light and power would be. I found some determinations made quite a number of years ago and after recalculating them to bring them to our present units, I found the figures to be 5.3 spherical candles per watt.* That figure might be termed the electrical equivalent of light. Hence, if the original tests were correct (I took them as I found them and am not responsible for their correctness), the figure will be about of that magnitude, which corresponds to 0.188 watt per spherical candle-power. If this figure be correct, that would be the ultimatum which we will ever be able to reach, as it would mean an efficiency of 100 per cent.

---

* Mechanical Equivalent of Light, Hering, *Elec. World*, April 20, 1901, p. 631.

The unit which Dr. Steinmetz proposes naturally involves the solid angle when reduced to candle-power, and we therefore have the choice now in defining that unit, of selecting either the steradian (or absolute unit solid angle) or the sphere, as the solid angle to be included in that unit. Selecting the sphere eliminates the troublesome 4π from most of our calculations of light flux, which are made in spherical candles, and are not based on steradians.

Clayton H. Sharp: I would like to submit that when any member of this Institute is going to present to us a general solution of the electric circuit and the general solution of the perhaps equally difficult problem of the standard of light, he should at least give us a chance to catch our breath in between.

It looks to me as if this were, in a way, a general solution of this problem. The whole question of the standards of light, the history of the subject, shows how difficult the problem has been. For many years the flame standards were used, and were sufficiently good for the purpose. With the introduction of electric lamps, which were capable of more accurate measurement, a greater precision in photometry was at once demanded, and certain fundamental difficulties which exist in flame standards became known for the first time.

As a solution of the question of a more reliable standard of light, Violle came to the front with his propositions, namely, the use, as a standard, of the light emitted by a square centimeter of platinum at its point of solidification. This proposal, being on the face of it very attractive, was received and adopted at several electrical congresses, but it has never been found feasible to put it into practical operation, because a sufficiently accurate reproduction of this so-called " absolute " unit of light could not be carried out. Other propositions have been made of a similar nature, and the proposal to use energy measurements is not a new one. We find that Lummer made an extensive investigation of a standard of light which was defined in this way: "the standard light shall consist of the luminous radiation from a surface of platinum brought to such a temperature that the ratio of the radiation which it will transmit through a certain cell containing water to its total radiation per unit surface is as 1 to 10." Now, it should be theoretically possible to define a standard in this way. The great difficulty with such a definition, however, is that it involves the measurement of the ratio of the intensities of radiation, one of which, at least, is very low. Another difficulty with it is that it depends upon the radiating properties of a certain substance, namely, platinum. This latter difficulty might be obviated by using, instead of the platinum surface, a surface of a theoretical black body. However, the first difficulty would remain, and it seems to be of sufficient importance to preclude the possibility of establishing an accurate standard in this way.

The proposition which is made to us to-day appears to be the very first one which offers a chance of the establishment of a standard of light based on energy measurements, and independent of the properties of any substance whatever. Not only does it offer us a standard of light, but it gives us also the possibility of a standard for the measurement of colored light, since a variation of the proportions of the different colors used will produce a variation in the resultant tint. The latter feature may very well be one of considerable importance in view of the ever-increasing diversity in tints of the lights that we have actually to measure in the photometer room.

As Dr. Steinmetz has said, we have to deal here with a physiological effect, and not with a purely physical effect. Ordinarily, the physiological effect which we get from an incandescent lamp, or from most of our light sources, is an effect made up of the various wave-lengths of the spectrum; in other words, the spectra of our common light sources are continuous spectra, and have difference in color on account of a continuous difference in the ratio of the intensities of the various wave-lengths.

It is to my mind an open question, one which must be submitted to experimental determination, whether we shall get ultimately the same effect from the admixture of three monochromatic lights that we would get from a polychromatic source; whether it is not possible that minor differences in the sensibility of individual eyes to the different colors will not become much more apparent when we measure a polychromatic source against a source which is made up of three colors, than when we are measuring two polychromatic sources, one against the other. Will the physiological effect of three colors mixed together in all cases be the same as the physiological effect of all the colors mixed together, though in varying proportion?

In regard to the difficulties of the actual energy measurements which are involved in the realization of this solution of the standard of light, it seems to me, while they are very considerable, yet they are not necessarily insuperable, because it is not necessary, as I understand it, to deal with any extremely small quantities of energy. There is no reason why the sources which are used for the production of these three lines should not be sufficiently powerful so that there will be no small concentration of energy in each of the lines, and so that it will become physically possible to measure the radiation-intensity of each of these lines with a sufficient degree of accuracy for the purpose. I think there is no doubt that the method ought to be given a very thorough trial indeed, and I think also that the place where it ought to be tried and investigated is the Bureau of Standards, at Washington, where the answer to the question can be found, if it can be found anywhere.

I think there was some confusion in the statements of the last speaker in regard to the nomenclature of this matter. As

I understand it, Dr. Steinmetz's proposal looks to the production of a standard illumination on a given surface. Now, illumination is not the same thing as candle-power, though it can be referred to the theoretical point-source of light placed at a certain distance from the surface which is illuminated. Neither is illumination simply flux of light, but it is flux of light per unit area, or flux density, and consequently the proposition is to produce, really, a standard illumination on a surface, and if we wish to refer that to an imaginary point-source of light, which really does not exist in the experiment, we can say that illumination is produced by a source of one candle-power. However, the solid angle does not enter into the question directly, neither necessarily does the candle-power enter into the question directly, nor does the 4π come in. It is a question simply of producing a unit or standard illumination on a given surface, or unit flux density, or unit flux per unit area; for when we get right down to the fundamentals, it is flux we have to deal with in the case of light. The candle-power notion or concept is more or less fictitious and superfluous, though convenient for many purposes. For other purposes, the candle-power notion cannot enter in; we must come to the idea of luminous flux and base our computation of illumination on the idea of a luminous flux, as is quite evident when we consider any space illuminated by the so-called indirect method, where the ceiling is brightly illuminated, and, serving as a secondary source of light, illuminates other parts of the room. Evidently, there is no candle-power there, it is all a case of luminous flux from the ceiling falling on other surfaces of the room and illuminating them.

I think we are to be congratulated that this first, and certainly we hope the true, solution for the absolute definition of a standard of light, emanates from this country, and is presented for the first time to this Institute.

C. A. Perkins: It is not necessary for me to add my congratulations to Dr. Steinmetz for his valuable paper, but I wish to emphasize the fact that it is a step toward the solution of a problem which is impossible from its nature.

As Dr. Steinmetz has just explained, illumination is a physiological phenomenon and cannot be measured. It is impossible to say that a red light is as bright as a blue light. It is impossible to say that a Welsbach burner is so many times as bright as a Hefner lamp. The ratio depends on the eye of the observer, and also is a matter of personal estimate or judgment, and is not capable of measurement.

Yet the proposed standard simplifies the problem of comparing lights of different colors very materially, by reducing all the colors to three.

The next step will be to fix upon numbers which may represent the brightness of each of these primary sources (in candle-power per watt) based upon the average judgment of a large number of observers. The number thus determined will be more or less arbitrary but will represent in a fairly satisfactory way the average judgment and the normal eye. When the relative brightness of the primary sources has been fixed upon, the various secondary standards—Hefner, pentane or incandescent lamps—may be expressed definitely in terms of the primary standards, and the comparison of colored sources will be definite, though it cannot be absolute.

John B. Taylor: Dr. Steinmetz has several times told me that he had no proper musical training or appreciation, but in this paper he gives us a rainbow set to music, and leads me to hope that he has taken up a new subject, old as an art, but still having many unsolved physical and mathematical problems.

I wish to point out a few discrepancies as they appeal to me. In the first place, if we are going to replot the spectrum on a rational basis, why do we use wave-length at all? The essence of light, as well as of sound, is frequency, and not wave-length. Sodium light has a certain rate of vibration, while its wave-length is only incidental to the transmitting medium, having one value in air and other values in glass, water, etc. The same is true of sound. Pitch depends on frequency, while the wave-length of a sound of given pitch has different values in air, hydrogen, water, and wood. So it seems we should rightly lay out the spectrum on the basis of frequency instead of wave-length, frequency being really the essence of light and color.

I agree with Mr. Hering, that the English and American names of the musical tones would be more appropriate for this Institute. The German " cis's " and " fis's " are not so familiar to most of us as c-sharp and f-sharp, which in addition to the misprints and backward arrangement, have obscured, the optical and acoustical comparison. The use of frequencies, rather than wave-lengths, will naturally bring red on the familiar left end of the spectrum and the musical scale will read from left to right.

I do not see the arguments for a logarithmic scale in representing the spectrum. The musical system, both melodic and harmonic, is based entirely on the ratios of frequencies, and music as written is essentially a curve or curves with time as abscissas and logarithms of frequencies as ordinates. I have never yet been able to find that there is anything in the optical scale corresponding to this matter of ratios in the acoustical scale. It is true that certain combinations of colors will be harmonious while other combinations will not be pleasing. For example, it is generally agreed that pink and red do not go well together. Is it possible to take two different colors and say that these are not harmonious, on account of the ratio of their frequency? There is a definite period in music. In a well-recognized sense, after the interval of an octave, we go over the same ground again.

I hope Dr. Steinmetz can show more clearly some periodic property of the spectrum or something depending on ratios of frequencies to justify the logarithmic plot.

Carl Hering: Dr. Sharp says there is some confusion of terms in my remarks. I am confident that Dr. Steinmetz will agree with me that his proposed unit is not a unit of illumination, as Dr. Sharp says, but a unit of flux, which is equivalent to illumination multiplied by an area, or candle-power multiplied by a solid angle—either one gives the same result. I am quite familiar with the various units and measures of light and the relations between them, as I gave some attention to this question of equivalency between watts and light some years ago and I am sure there is no confusion in my mind

E. B. Rosa: I am interested in this proposition of Dr. Steinmetz, and should sincerely like to see it realized. It would be important to make a serious attempt to get a primary photometric standard in this way, which should be sufficiently satisfactory to be considered to solve the problem. It seems probable that it might be somewhat difficult to make the energy measurements with the required accuracy, and perhaps this will prove to be an insurmountable difficulty; and yet I have no doubt that if the attempt were made seriously, sufficient information and experience would result from the work to pay for the effort, even if it did not result entirely satisfactorily. It often happens that when a serious effort on a difficult problem is made and continued, a great many new things appear in the work that are of considerable value, by-products, so to speak, of the investigation. I think this field might be a very valuable and promising one to investigate thoroughly, with the hope, of course, that the problem of finding a primary photometric standard would be solved. At any rate, I think it would be proper to consider it very carefully.

I appreciate also the words of confidence in the Bureau of Standards that have been expressed. This investigation is clearly in the field of work of the Bureau of Standards, and the Director of the Bureau would be very glad, I have no doubt, to take up the work without any request from the Institute, as soon as practicable. Nevertheless a request made by the Institute would be very proper, and I would be glad to see that request made formally, because, as you will easily understand, the Bureau has so many fundamental problems on hand, more than it can at present undertake, that it needs support in its work; it needs the expression of confidence of this Institute, and the moral support of the engineering fraternity at large in its work. Such support will enable it to get the assistance and to develop the force that it needs in order to take up such problems, and therefore it can do more of that kind of fundamental work if interest is expressed in such a concrete way as is here proposed. I can easily see that such an investigation might last over a considerable period of time. We now have some problems on hand that have already extended over considerable periods of time, but in most cases they have been very profitable investigations to pursue; and entirely apart from the primary objects of the investigation, the results have been of considerable importance.

I wish again to express my interest in the suggestion of Dr. Steinmetz, and certainly consider it of value.

Carl Hering: I make the motion that the Board of Directors of this Institute be requested to refer to the Bureau of Standards the question of the establishment of a standard of light as proposed in this paper of Dr. Steinmetz. (The motion was unanimously carried.)

H. S. Carhart: I have only a few words to add to what has already been said. I suppose the reason why Dr. Steinmetz has chosen three distinct wave-lengths to enter into this combination for luminous intensity or luminous flux is the fact that there are three distinct primary color sensations. It would be desirable, first, to be assured that we get an average determination of the wave-lengths of these three primary colors, as applied to the eyes of different individuals, because there is individuality in this matter. Secondly, we should measure the relative energy in the three wave-lengths to produce any definite color, such as yellowish white. This task would be one of considerable difficulty. In that way we should be able to get the average energy of the primary colors required to reproduce a light of any given tint and intensity. It can readily be seen that such processes as these would necessarily require a good deal of time in order to realize them.

I think one of the speakers has been confused a little, if I may call attention to the fact—the mixture of three colored lights is a totally different thing from the mixture of pigments. The mixture of pigments is a process of absorption, and the mixture of lights is a matter of superposition and addition. If we have red glass, which cuts off the violet end of the spectrum, and blue glass, which cuts off the red end of the spectrum, and if we put one in front of the other, the only light that comes through is a little of the red. This is like the mixture of pigments. We should be very careful not to confuse mixture of pigments with mixture of colors. What we have in the paper is three colored lights, coming from three sources of definite wave-length. I am greatly interested in the proposition, and think it is really hopeful enough to suggest to the Bureau of Standards, as has already been done, for investigation.

Chas. P. Steinmetz: Three colors I selected as primary colors, since three primary colors are sufficient by their combination to produce all other colors, representing these by the ratio of the energies of the three primary colors. As seen, this is essentially the representation of the points of the color plane by a system of symmetrical coördinates. Combinations of two colors are not sufficient to produce all other colors, but at least three are needed. In regard to the selection of the three primary colors, I tried to get three colors forming as nearly as possible an equilateral triangle in the polar diagram, or in the color plane, as shown, so as to get the maximum accuracy in their combination. There was a second consideration. The three points of the equilateral triangle I tried to select so as to have a maximum distance from the ends of the spectrum, since at the ends of the spectrum the physiological equivalent of power is low and vanishing, and therefore if one of the lines were chosen close to the end, as for instance dark red, it would mean there would always be a large percentage, in power, of red rays, even to produce a color which appeared to the eye deficient in red. For greatest accuracy, it is however desirable to have the three numerical values of power of primary colors as nearly of the same magnitude as possible. The two outside colors, blue and red, are therefore chosen about equidistant from the ends of the spectrum, as two corners of the equilateral triangle, which brings the third corner into the middle of the spectrum, in the green. You can choose any other set of spectrum colors, but I believe the best way would be to choose the three points of an approximately equilateral triangle, with the two outside corners as far away from the ends of the spectrum as possible, as was done by me.

As regards the scale adopted, I must apologize, for my musical education was so badly neglected that I still used the Italian scale, instead of the flats and sharps of the English notation. I believe, however, it will be equally intelligible.

As regards the wave-length; what I give is not really the logarithm of the wave-length, but the logarithm of the frequency. The wave-length of light is not constant, but a function of the medium. However, the logarithm of the wave-length in any medium is the same, and is also the logarithm of the frequency, except with reversed sign and shifted decimal point. It therefore is immaterial, in the logarithmic scale, whether wave-length or frequency is used; either gives the same curve or figure, except turned over, and it obviously is immaterial whether the red end of the spectrum is on the right or on the left side.

In regard to the bearing of the acoustic scale, you will see if you look at the polar diagram, that it is material, and that the wave-length is not the right measure. In the acoustic or polar diagram, two diametrically opposite points are of complementary color, that is, combined give white light, or approximately white light (after all " white " is a rather indefinite expression). This is not the case in the representation by wave-length, nor in the representation by frequency; but in either case you find that one-half of the visible spectrum represents much less variation of the physiological effect, that is, change of color, than the other half.

About color representation: If we represent colors by a combination of the three primary radiations, we still have different values for the same color, representing different color densities. For instance, a certain yellow light may be represented by: red, 50; green, 50; blue, 0. The same yellow light, but of a lighter shade, may be represented by: red, 45; green, 45; blue, 10. Still a lighter shade by: red, 40; green, 40; blue, 20; until ultimately you reach white light.

I believe that even the densest yellow light, produced from the primary colors: red, 50; green, 50; blue, 0, is yet not the spectrum yellow; that is, not as pure yellow as the spectrum yellow, and that the spectrum yellow would be represented by something like: red, 55; green, 55; blue, -10. This ratio: $55 \div 55 \div -10$ represents a point of the plane outside of the equilateral triangle formed by the primary-color points, and such probably is the case with the spectrum yellow and with all the spectrum colors, except the three colors selected as primaries.

Physically, this means that if we desire to compare the spectrum yellow with our three-color combination, no combination of red, green and blue can be found to equal the spectrum yellow, but that a combination of red and green can be found to equal a combination of the spectrum yellow and blue, and in this case the blue becomes subtractive. It is the same as balancing a weight by putting too much weight on the other scale, and then putting some weights on the first scale, *i.e.*, balancing by taking some weights back. We may do the same in color representation. Theoretically, I believe all pure spectrum colors, except the three selected as primary, would be represented by combinations of the three primary colors with one of the primary radiations as negative.

As regards the mechanical equivalent of light, I do not agree with the value mentioned by Mr. Hering, 0.188 watts per candle-power, for the reason that efficiencies of arcs have been measured which give lower watts per mean spherical candle-power than that. In some powerful titanium arcs values considerably lower have been found, and in those cases the efficiency was still far below 100 per cent. Estimating the loss of power by invisible ultra-red and ultra-violet radiation, by heat conduction, heat convection, air currents, etc., the efficiency was undoubtedly below 50 per cent., probably nearer 10 per cent. I believe the approximate equivalent of white light is probably something more nearly fifty candle-power per watt—that is, 0.02 watts per spherical candle-power. I make no claim to this value, but it is merely an estimate based on my experience—it may be more or less, but I would expect the equivalent of white light to be approximately 50 candle-power per watt, or thereabouts. For other colors of light there would be different values. Probably in the case of the green light, the candle-power per watt would be much greater, in red light or blue light much less. It is a function of the color.

A paper presented at a meeting of the Philadelphia Section of the American Institute of Electrical Engineers, Philadelphia, Pa., February, 10, 1908, and at the 25th annual convention, Atlantic City, N. J., July 2, 1908.

AN IMPERFECTION IN THE USUAL STATEMENT OF THE FUNDAMENTAL LAW OF ELECTROMAGNETIC INDUCTION†

BY CARL HERING

The fundamental law of electromagnetic induction, which is the basis of all our present mechanical generators of electric currents, is, in general, stated in two different ways. Faraday's statement is to the effect that if a conductor cuts magnetic lines of force, an electromotive force is generated. This simple statement, however, has been changed by later authorities, who define the same law by saying in effect that when the amount of flux enclosed by an electric circuit is changed, an electromotive force is induced. This is the more usual form taught to-day and recopied in most text-books in preference to Faraday's statement of it.

Maxwell in his classic treatise, Sec. 531, Vol. II, states the law in these words:

The whole of these phenomena may be summed up in one law. When the number of lines of magnetic induction which pass through the secondary circuit* in the positive direction is altered, an electromotive

†The object of this note is to point out that the usual and well-known statement of the fundamental law of the induction of currents by magnetic flux, is not correct as a universal law, and requires to be modified; when, applied as it is usually stated, it sometimes gives entirely erroneous results, although it is correct under the usual conditions. An essential qualification has apparently been overlooked. The proof is given by a simple experiment. The possibility of linking or unlinking an uninterrupted electric circuit with a magnetic circuit, without producing any induction, is shown. It leads to a clearer conception of induction and to a new form of continuous current generator.

* He had just previously shown that a magnet and a primary circuit are here equivalent.

force acts round the circuit, which is measured by the rate of decrease of the magnetic induction through the circuit.

Another high authority, Professor J. J. Thomson, in his excellent and more modern treatise (Elements of Electricity and Magnetism, Sec. 229, p. 388) states the law in these words:

Whenever the number of tubes of magnetic induction passing through a circuit is changing, there is an e.m.f. acting round the circuit equal to the rate of diminution in the number of tubes of magnetic induction which pass through the circuit.

Other writers merely used different expressions to the same effect. One well-known form of expressing this law, is to consider the magnetic circuit and the electric circuit to be like two links of a chain: then, during the process of linking them together, an electromotive force is induced in one direction; when unlinking them, an exactly equal electromotive force is induced in the other direction.

The well known natural consequence of this law, when stated as it usually is, is that the same flux cannot be linked and unlinked repeatedly by the same closed circuit without generating alternating electromotive forces, hence the commutator of direct-current machines, and the inoperativeness of many suggested unipolar machines. Although not stated directly it is presumably understood that the electric circuit is not opened; the magnetic circuit unfortunately cannot be opened as there is no good magnetic insulator known.

Innumerable methods have been suggested, and doubtless many have been actually tried, to obviate, overcome, or neutralize this reverse electromotive force so as to produce direct currents without the necessity of commutation, but they have all failed except the Faraday unipolar dynamo, which has apparently not come into use for the usual voltages. According to this law, as it is usually stated, the former have failed because of this reverse electromotive force. It is now believed to be generally accepted and taught that this law is universal, and the attempts to evade it have lessened.

Recent investigations made by the writer and followed by experimental demonstrations, have, however, shown that the usual statement of this important law is imperfect, and that it requires to be modified in order to become universal. Probably others besides the writer have been misled by accepting this law as usually stated to be correct and universal. It will be shown below that it is quite simple to produce linkages or unlinkages, or to change or reverse the same flux enclosed by the same continuously closed circuit, without producing any electromotive forces whatsoever. It is true that the law as usually stated, seems to apply correctly to all the usual methods of producing electromagnetic induction at the present time; but it will be shown that, taken literally as usually stated, it is quite incorrect under certain other conditions, giving entirely erroneous results and must therefore be modified in order to become universal, or else be limited to those conditions to which it does apply, in which case it cannot of course be called universal.

It is not denied here that Maxwell himself may have elsewhere described these limiting conditions, or may have elsewhere described induction under those unusual conditions. If so, his followers should have made similar explanations in teaching his version of the law, so that students should not be misled.

Some time ago the writer suggested a so-called unipolar machine (that is, one having unipolar instead of bipolar induction) based on an apparently new phenomenon,* which would be operative if this law, as it is usually defined, is correct; but it was thought by him and also by Dr. E. F. Northrup, that it would fail, notwithstanding this law. It was concluded that it was not the change of flux in a circuit, or the linkages of the *circuit* with the flux, which determined the induction, but that it was essential that the *material* itself which composed the conductor must actually cut or move through the flux, (or the flux move across it). It was thought that Faraday's way of stating the law was nearer correct than the supposedly improved and more general form of his successors.

The following simple experiment was devised by the writer to investigate the correctness of these conclusions.

A loop L, Fig. 1, was formed of two flexible strips whose ends pressed together at the joint J, and the circuit of the loop was closed through a galvanometer G. On moving this loop over one leg of a U-shaped permanent magnet $N\ S$ from the dotted position to that shown in full, an electromotive force was induced, as is well understood. The magnetic flux has

* "A Practical Limitation of Resistance Furnaces; the 'Pinch' Phenomenon." By Carl Hering. Trans. Amer. Electrochemical Soc., Vol. XI, 1907, p. 329. Also "Some Newly Observed Manifestations of Forces in the Interior of an Electric Conductor". By Edwin F. Northrup. Physical Rev., Vol. XXIV, No. 6, June 1907, p. 474.

thereby been linked with the electric circuit; the flux enclosed by the circuit has been increased from zero to the maximum; or to use Faraday's terms, the lines of force in the air from one pole to the other, have been cut by the conductor. So far, the law as it is stated is correct.

If this loop be now moved as shown in Fig. 2 from the dotted position to the one shown in full, by passing the leg of the magnet through the joint *J* of the loop, but *without opening the circuit*, the flux and the circuit will be unlinked again; that is, the flux enclosed by the circuit will again be reduced from a maximum to zero, and the circuit will have cut the same lines of force in the opposite direction, as it is well known that all the flux of this magnet passes through its interior.

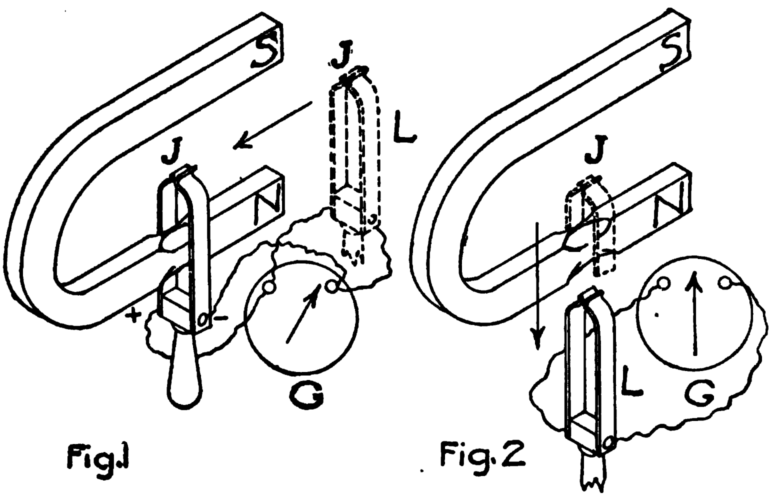


Fig.1 Fig.2

According to the law of induction as quoted above from Maxwell, or J. J. Thomson, and as almost universally accepted, there ought then to be an electromotive force induced which is opposite and exactly equal to that induced by the first movement shown in Fig. 1, which will be seen by reading the law as there stated, on to this experiment. The fact is, however, quite the contrary; *there is absolutely no electromotive force induced by this unlinking*. Moreover, the same circuit is then free from flux and can be linked again repeatedly in the same direction with the same flux as before; a rapid repetition of these two movements increased the deflection more than ten fold.

This is believed to be the first direct and simple experiment which proves conclusively that it is possible to link and unlink a flux with a closed electric circuit without any induction, thereby showing that the usual statement of Maxwell's version of the law should either be restricted to certain conditions, or be modified so as to become universal without the necessity of more or less complicated interpretations.

It is also believed to be the first experiment which shows that in applying Faraday's version of the law it is important to make a distinction, heretofore apparently not recognized, between the conductor itself and the mere current path or circuit, when induction is concerned.

Through the kindness of the Leeds & Northrup Co., the experiments were made in their well equipped laboratory and were confirmed by my friend, Dr. Northrup himself. With his usual care and precision he even amalgamated the contact at the joint *J* and at the surface of the steel magnet where the loop passes over it, so as to be absolutely sure that the total absence of a deflection was not due to a possible open circuit. The magnet was ground down to a knife edge on each side, forming a lozenge shaped cross-section at the place where the passage of the strips over the magnet took place.

To be still further assured that there was no open circuit during this unlinking, the writer modified the test by passing a constant current from a battery, through the circuit during both the linking in Fig. 1 and the unlinking in Fig. 2. The linking was then found to increase (or diminish) the deflection, but during the unlinking the deflection remained constant; had the circuit been opened during the unlinking, the deflection would have indicated it at once by falling as was shown by unlinking the loop over part of the magnet which had been painted. The galvanometer used was one that was suitably adopted for the necessarily low electromotive force and low resistance; its sensibility was 38 megohms, and its resistance was 140 ohms; the deflections were therefore very decided and reliable; the test was repeated numerous times, thus leaving no doubt as to the correctness of the results.

This experiment, therefore, teaches us that it is *not* the linking and unlinking of the magnetic and the closed electric circuits, and it is *not* the changing of the amount of flux enclosed in the closed circuit, which is the prime cause of the induction, as it proves that such unlinking or changing of the flux can take place *without* any induction whatsoever. The cause of the induction must therefore reside elsewhere.

It appears from this experiment that it is the ***material of the conductor of the current***, and not merely the circuit itself, which must actually move across the flux (or the flux move across it) in order to cause induction. In the experiment, the circuit itself did move across the flux during the operation of unlinking because the unlinking of the two circuits was effected completely; both circuits having remained unbroken, it of course follows that they must necessarily have cut each other. The only part of the conductor which did not move relatively to the flux during the unlinking, is that part of the material composing the magnet which is embraced between the two contacts; the circuit here did cut through the flux, but the material of the moving conductor did not; this appears to be the only difference, hence it seems essential that this feature should now be embodied in the statement of a law which is intended to be universal.

As the term " conductor " implies the material thing itself, when distinguished from a theoretical line representing the closed circuit or current path, it might suffice to emphasize the condition that the conductor itself must cut across the flux, in stating the law in Faraday's way.

If that later statement of the law which is in terms of the change of flux included in a circuit, is still to be used as a universal law, some limiting clause will now have to be added to cover the case shown in this experiment. A " change of flux " is not sufficiently definite, the change must occur in a certain way in order to be effective. But in view of the fact that this experiment seems to show that the real or prime cause of the induction does not reside in the "change of flux" at all as is usually believed it would seem to the writer to be preferable not to continue to define induction in those terms at all, but to use other terms which more clearly embody what seems to be the real prime cause. In other words, it seems to the writer to be better now to go back to Faraday's original definition in terms of cutting the magnetic lines of force, than to use the one of Maxwell which is in terms of the change of flux.

Maxwell and his followers, in generalizing this law, seem to have gone too far, because, *as it is quoted above*, it includes in its scope a case in which it gives entirely erroneous results. His predecessor, Faraday, was nearer correct when he described the induction to be due to "cutting the lines of force." Even Faraday's definition as incorrectly interpreted by others who used the word " circuit " instead of " conductor " (Faraday apparently did not do so himself), should now be modified or qualified to the effect that the conductor itself, and not merely the circuit must do the actual cutting.

As to stating the law in terms of the linking and unlinking of the two closed circuits, it seems difficult to so qualify it now as to limit it to the specific and probably the only kind of linking and unlinking which *is* effective, and exclude the kinds which are ineffective. This is unfortunate as this idea of linkages formerly afforded a very convenient and useful conception of the nature of induction.

In Faraday's original experiment with a revolving bar magnet and a loop running from one pole to the middle of the magnet, this same phenomenon occurs. The *circuit* itself cuts the flux in the interior of his bar magnet, as Faraday himself says; but he did not notice that the corresponding part of the conductor itself, does not; if it had, he would not have obtained the results described. He thought the lines of force remained stationary in space while the magnet revolved. The important distinction between the material of the conductor, and the circuit itself, when induction takes place, seems therefore to have been overlooked by him, notwithstanding that in stating the law he used the term conductor and not circuit.

The writer desires to repeat that the usual statements of this law in terms of the change of flux seem to apply correctly to all the ordinary cases occurring in our present practice, and that the limitation now pointed out therefore does not alter in anyway the former applications of that law to those cases. But as those statements of the law are directly contradicted by the present unusual case, the law as usually stated is not universal; and it misleads as it did the author, by making it appear that certain actions will always take place, when as a fact they sometimes do not. These statements of the law are unsatisfactory also in that they intimate that the *prime cause* of the induction resides in the changing of the flux embraced by a circuit, when it seems now as though it resides in the actual passage of the flux across the material conductor itself, and that it is located there and is not a property of the circuit as a whole.

It may be claimed that this experiment does not show an " error " in the usual statement of the law, and that it might equally well be claimed that Ohm's law is in error because counter electromotive forces, inductances, capacities, skin effects, Hall effects, etc., in the circuit, modify the results obtained by it.

But the latter does not seem to be a parallel case; Ohm's law treats of the relations of three quantities, and if there are more than three, the excess must either be eliminated or expressed in terms of one or more of the three, before Ohm's law can apply as such, and it is then correctly stated. The law of induction, however, as usually stated, actually gives totally erroneous results in some cases, as will be seen by reading Maxwell's or J. J. Thomson's statement of the law, on this experiment. This law treats of three things (1) a circuit, (2) an alteration of the magnetic flux enclosed by it, and (3) an electromotive force produced by such an alteration; it treats of nothing else; and if the results specified by this law *as usually stated* are not obtained or are not correct, when no additional quantities are introduced it seems to the writer to be justifiable to say that the statement of the law is faulty and is actually in error when applied to those cases. Aside from this, this law seems to give an erroneous conception of the real cause of the phenomenon.

The present experiment is also thought to be of considerable interest in connection with the long discussion of many years duration, as to which of the two so-called theories was the better one to teach and use, the one of Faraday based on the "cutting of lines of force," or the one of Maxwell based on the "change of flux enclosed by a circuit." It seems to show that after all, Faraday's original statement which was based on experiment, was more universal than Maxwell's, which latter appears to have been a mathematical generalization.

Another feature of interest of this experiment is that it seems to indicate that the induction is localized, and takes place only in that part of the conductor which passes through the flux, and not in any other.*

There seems to be no reason why the present experiment should not give the same results if a piece of copper were introduced in or around that part of the magnet over which the loop slides; and that this piece of copper might be insulated from the magnet, in which case the loop may have more than one turn, if there are a corresponding number of insulated copper pieces. This was not tried.

Fig. 3 was suggested by Dr. Northrup to show how the experiment might be repeated without involving any iron or steel in the phenomenon. Two solenoids are used instead of the permanent

*The so-called "Dead Wire on Gramme Armatures." By Carl Hering. The Electrician and Electrical Engineer. May, 1887, p. 171.

magnet, and one of them is surrounded by a copper ring *C* over which the joint of the loop passes.

A continuous current generator without a commutator may be constructed in accordance with the results taught us by this experiment. Fig. 4 shows diagrammatically a simple way in which this might be done for demonstration purposes, although not for a commercial machine. The diagram explains itself; a number of magnets are placed symmetrically around a center and a single circuit of as many loops as desired may be made to cut all of the fields in such a way as shown, that the electromotive forces induced in the several loops are added in series. Doubtless the machine would operate as a motor also. Unfortunately, however, such a machine seems to have the same

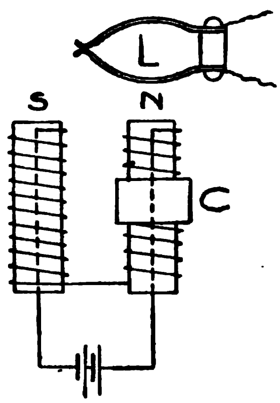


Fig. 3.

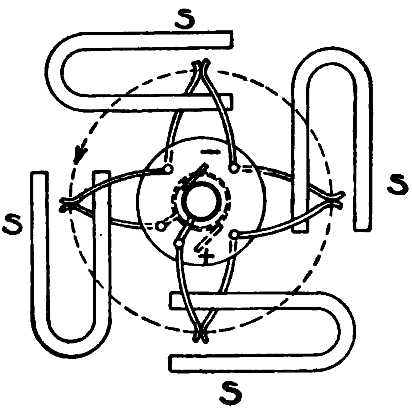


Fig. 4.

limitations as the usual unipolar ones, in that two sliding contacts moving at high speed, are required for each element of the circuit which passes through the flux.

In accordance with general laws, the converse of the phenomenon shown by this experiment should also be true, namely, that when applied to motors it is the material of the conductor itself, and not merely the current or the circuit, which is caused to move through the magnetic field; in other words the electromagnetic forces act on the conductor and not on the circuit or on the current.

In this form it has long been known. For instance in a Faraday disc unipolar machine when used as a motor, it is not the circuit nor the current which is rotated through the disc, but it is the disc itself, on which the forces act. In other words, it is not necessary to make the disc of separate radial wires insulated from each other, in order to confine the currents to radial paths. On the other hand, however, there is the Hall effect in which it is the circuit, that is, the path of the current, which is deflected in a large conductor, when the conductor is not permitted to move.

The phenomenon illustrated by the present experiment may perhaps explain the reasons for the well known action of a magnet in "blowing out" an arc. The magnet appears to exert its forces on the material forming the conductor, namely on the ionized conducting gases which carry the current, and not on the current itself; the current follows the moving conductor, instead of the conductor following the moving current path.

The present experiment naturally gives rise again to the interesting question of what forces exist, if any, which oppose the change of the path or circuit of a current which is flowing through a large moving conductor. If for instance, a current is passed from one side of a rapidly running river to the other through the water, is the path different from what it would be if the water were at rest? A study of this question may not lead to any practical results, but it may nevertheless be of interest and perhaps be instructive in aiding us to obtain a better conception of the true nature of conduction.

I desire to repeat here that the error pointed out by this note does not affect the applications of this law to the usual electromagnetic machinery and apparatus of the present, and is therefore of no importance in that sense; the importance of it lies in showing that the law as usually stated, and when applied directly as stated, gives entirely incorrect results under certain conditions differing from the general ones; also that it misleads; and that it is not universal; to become universal it should be modified instead of having to resort to what is little short of mathematical juggling to make it fit this case. The importance of this experiment is perhaps, that it seems to give us a clearer conception of the real phenomenon of electromagnetic induction, its probable cause and its location; it shows conclusively that the change of flux enclosed in a circuit cannot by itself be the prime cause of induction; unless a further essential condition be fulfilled, such a change of flux fails to induce an electromotive force.

Attention might here be called to a very early experiment of

Faraday published in 1831, which has apparently been overlooked by the followers of Maxwell; it is described in his Researches, Paragraph 101. Maxwell's statement of the law also fails to apply to this, unless modified, as the flux included in the circuit always remains the same even though there is an electromotive force induced in that circuit. It also requires some interpreting of the law to make it apply to the case of the Faraday unipolar machine, a fact which many students and their teachers have experienced.

The writer desires it to be clearly understood that he does not deny that the usual statement of the law involving the change of flux idea, might be made to cover this experiment by adding various conditions or restrictions, or by various more or less complicated interpretations now suggested by the results of the experiment itself, and it might be claimed that there is then no error in it as a universal law. The writer's comments were intended to apply not to such modifications or interpretations of that law, but to the law as it was published and republished, and as it is and has been taught to students. In that form a student would certainly not have predicted the results of this experiment correctly by it, for in that form it directly contradicts the results. Faraday's statement of the law, on the other hand, when stress is laid on the distinction between the conductor and the mere circuit, predicts the correct result directly and without interpretations or other conditions. But aside from the question of whether or not it can be called an actual error, the more important question is now, whether Faraday's version is not simpler, more universal, safer, more direct, and much easier for students to learn and comprehend than Maxwell's.

DISCUSSION ON "AN IMPERFECTION IN THE USUAL STATEMENTS OF THE FUNDAMENTAL LAW OF ELECTROMAGNETIC INDUCTION." ATLANTIC CITY, N. J., JULY 2, 1908

Chas. P. Steinmetz (by letter): Mr. Hering's paper is interesting in that it draws attention to a looseness in the form of expressing this law, which is frequently the cause of serious misunderstanding, and the waste of much energy and time. For instance, in the attempts to invent a coil-wound unipolar machine much useless effort could have been avoided by a clearer distinction between the general law and the special case of its application to a continuous closed conductor. While Mr. Hering's experiment is interesting in showing an instance of a closed electric circuit in which the number of interlinkages with the lines of force changes without inducing an electromotive force, it is not startling to me, as the reverse case, the electromagnetic induction of an electromotive force, in a closed circuit, without any change of the number of interlinkages of the circuit with the magnetic flux, is illustrated by practically every unipolar machine. There are thousands of kilowatts of such machines now in commercial operation.

The general law of electromagnetic induction is:

In a conductor moving relatively to a magnetic field, an electromotive force is induced which is proportional—and in absolute units equal—to the product of the intensity of the magnetic field, and the components of the length, and of the velocity of the conductor at right angles to the magnetic field and to each other.

If an electric conductor moves relatively to a magnetic field, an electromotive force is induced in the conductor, which is proportional to the intensity of the magnetic field, to the length of the conductor, and to the speed of its motion perpendicular to the magnetic field and the direction of the conductor.

Using the pictorial representation of the magnetic field by the lines of magnetic force, as given by Faraday, this means, that the electromotive force induced in the conductor equals the rate of cutting of the conductor through the lines of magnetic force, that is, gives: $e = \frac{d\,\phi}{d\,t}\,10^{-8}$ volts, where $d\,\phi$ are the lines of force cut by the conductor during the time $d\,t$.

Applying this general induction law to the special case—which is the most important, but not the only case met in electrical engineering—of a continuous conductor closed upon itself, or a turn, it follows, as conclusion, that the total electromotive force or resultant electromotive force induced in the turn equals the rate of change of the total number of magnetic interlinkages of lines of magnetic force enclosed by the turn, hence is: $e = \frac{d\,\phi}{d\,t}\,10^{-8}$, where ϕ is the number of lines of magnetic force enclosed by the turn or, leaving Faraday's pictorial representation, ϕ is the magnetic flux enclosed by the turn.

Maxwell and J. J. Thomson's statement, as quoted by Mr. Hering, are not the most general expressions of the law of induction, but its formulation for the special case discussed by these scientists, of a turn moving with regard to the magnetic field.

Mathematically speaking, we may see that Maxwell's law is the integral expression derived from the general or differential law by integration over the whole circuit, under the " terminal " or " limit " conditions of continuity of conductor and continuity of motion, and does not apply to Mr. Hering's experiment, or to the general design of unipolar machines, which do not fulfil the conditions of continuity of motion, but have parts of the conductor sliding over other parts.

Faraday's expression, of cutting of lines of magnetic force by the conductor, is the general law; but in its application to unusual cases it must be kept in mind that the " line of magnetic force " is merely a pictorial representation of the magnetic field in space, as characterized by the two constants, intensity, and direction. This pictorial representation, when carried so far as to apply to its physical existence, may lead to wrong conclusions; for instance, when discussing whether the lines of magnetic force of a revolving magnet move with the magnet or stand still. Assuming, for instance, a bar magnet of circular section $x^2+y^2=r^2$, revolving around its axis z. Then in any point in space, outside of the magnet as well as inside, the intensity as well as the direction of the magnetic field is constant; that is, the magnetic field is constant, or stationary in space, regardless of whether the magnet stands still or revolves. Assuming a second system of coördinates with the same axis, z, as the magnet, and with the other two axes, x_1, and y_1, stationary with regard to the magnet, and revolving in space with the revolutions of the magnet, then with this coordinate system, x_1, y_1, z, the magnetic field at any point, inside of the magnet as well as outside, is also constant in intensity and in direction; that is, is stationary. Or in other words, while the two coördinate systems x, y, z, and x_1, y, z_1, revolve with regard to each other, the magnetic field of the magnet is constant in intensity and in direction, that is, it is stationary, with regard to either. Physically, this is nothing exceptional; it merely means that the condition of stress, which we call magnetic field, is unvarying in its distribution in space as well as with regard to the revolving magnet. Picturing to ourselves the magnetic field as lines of magnetic force, it would mean that the lines of force are at the same time stationary in space and also revolving with the magnet. This suggests that all pictorial representations, no matter how useful, may occasionally become ambiguous. In such cases the only safe way is to go back to the entities proper, in the present case, the magnetic field as a quantity characterized by intensity and d rection.

Unfortunately, in teaching, instead of the general law of

induction, there is frequently given to the student, its specific application to the turn or closed continuous conductor as more convenient to illustrate and to understand. While in the introduction to the elements to electrical engineering this is permissible, to get a complete understanding of the phenomena of induction, it must be supplemented by an exact discussion of the general induction law, that is, the mathematical formulation of Faraday's pictorial representation.

A. E. Kennelly (by letter): The experiment described in the paper is both interesting and instructive. Although the experiment illustrates the proper application of the law of induction when applied to electric circuits, it does not in my opinion controvert the existing law when properly interpreted; that is, when interpreted as intended to be expressed by its founders, Faraday, Maxwell, and others.

When it is stated that the electromotive force round a circuit is equal to the time-rate of change of the flux linked with that circuit, it is inherently assumed that the circuit is not interrupted and then established around a new boundary. It means, as I understand it, that the circuit contains a simply connected region of magnetic flux, through which the boundaries may be flexibly caused to wander at will. Maxwell specifically rules out the case of multiple-connected regions, by a special proposition to that effect. In Mr. Hering's experiment, the boundary of the circuit is cut at one point, and simultaneously a second circuit embracing flux is introduced at the gap, in such a manner that by sliding along the boundaries of the second circuit, the magnetic flux may be caused to disappear from the embrace of the first circuit without any intersection of flux by the edge of that circuit. This, to my mind, is juggling with terms, just as though the circuit were cut and then reclosed through a quiescent loop linked with a magnet. Manifestly, no current would be induced through the galvanometer by reason of that change, although in one sense of the word "circuit", all of the flux in the loop has suddenly been introduced into the circuit.

The instructive value of the experiment described in the paper lies, to my mind, in clearing up the question as to which is the primordial proposition; that electromotive force is induced (1) by the movement of magnetic flux across the boundary, or (2) by the introduction of flux into a loop or circuit. As ordinarily stated, the two propositions are as closely connected as the propositions concerning the priority of the chicken and the egg, because flux cannot cut the boundary of a loop at any point without altering the flux contents of the loop, nor can the flux in a loop alter without cutting the boundary somewhere. The experiment shows, however, that where flux comes into a field from a balloon, as distinct from walking over a fence, no electromotive force is induced, and this indicates hat the cutting is the primordial conception, to which enclosing is secondary. Of course this cutting electromotive force occurs as much in insulators as conductors, but can ordinarily be revealed only through the use of conductors. When, therefore, a closed circular solenoid, or anchor-ring, is wound with a primary and with a secondary coil, we know that in the steady state, if the solenoid is properly wound, there will be no external magnetic flux due to current in either winding; but the change of internal flux due to change of current in one winding induces electromotive force in the other, as in the ordinary transformer. At first sight, this would look as though there were change of enclosing flux without any cutting, but in the light of this experiment it seems clear that this is not the case. There must be cutting of flux passing from outside to inside of the secondary coil, but, incidentally, all the external flux cancels off, or becomes zero in the final steady state.

Summing up, then, I should think that this experiment shows that when the ordinary proposition is enunciated concerning induction of electromotive force with change of flux enclosed in a circuit, it should be borne in mind that the circuit is not to be juggled with by interrupting it and changing it from a simply-connected to a multiple-connected space. It must consist of a single, continuous boundary which, if it moves, moves continuously through simply connected space. In this understanding, I think we shall all agree.

Elihu Thomson (by letter): I agree with Dr. Kennelly in his view of the state of the case. By passing the spring clip over the magnet leg the circuit is virtually opened so far as magnetic induction is concerned, and an immovable section of conductor is substituted in the gap where all the flux to be cut then exists. The conclusion that the real physical substance of the circuit (the matter of the circuit) must cut or be cut by the lines is in accordance with my view of the subject. There are innumerable phenomena which have confirmed that idea of what is really the circuit to be considered. Otherwise in fact, the circuit is more metaphysical than physical, a sort of mental image only. I am glad Mr. Hering has tried the experiment, as it will tend to clear up matters which have troubled students in unipolar induction. I have long regarded Faraday's view of *line cutting* as much preferable and more universal than the theory of linkages simply.

The unipolar dynamo is quite practicable for large units of 220 to 550 volts or more, understanding that greater collector losses offset the commutation difficulties with ordinary types and that magnetic losses will probably exceed the calculated losses considerably.

There seems to be one universal law which might be expressed as follows: "It is not possible without chemical or thermoelectric action to generate a continuous current in a closed circuit without sliding contacts," or "A magnetic induction machine for direct currents must have a commutator or sliding contacts of some sort."

The experiment might be modified by boring holes through the magnet and inserting copper pins, the ends of which would be traversed by the ends of the spring clip. Manifestly since there would be no movement of these pins, there could be no electromotive force generated when they were traversed.

In a magnetic blowout arrester the static stress is exerted at full intensity across the narrower part of the gap in spite of the powerful field surrounding it. As soon, however, as the first slight spark jumps the gap (at the narrower part of course) the gases traversed by the current are deflected. Not, however, until the matter carrying the current has actually bridged the gap does the deflecting action of the field begin.

If the current be assumed to be a flow of electrons (negative) from one molecule to the next contiguous and so on, it is easy to understand why the *matter* of the circuit is the thing concerned and not merely an assumed line or direction.

It may be proper to regard the experiment as involving a form of unipolar induction, inasmuch as it is not possible to use a coil of numerous turns but one of a single turn. If the coil were to be used, it would have to be made up of loops which could open and pass over the magnet by riding upon a series of pins, insulated from each other, projecting through the magnet section, as suggested by me in a modification. Some twenty-five or thirty years ago I leaned to the more generalized view of Maxwell's law referred to in the paper, but gradually grew out of it and adopted Faraday's view of line cutting as the essential thing. The lines must be cut by the moving conductor, or the lines must move and cut the conductor in order to generate an electromotive force, and the potential difference arises in only that section of the conductor which cuts or is cut by the lines.

I think I should have to modify the universal law, which I proposed above, a little in view of Dr. Bruger's direct-current machine. Inasmuch as the resistance of a coil of bismuth wire may be changed by a magnetic field, it is evident that if the magnetism is so used as to increase the resistance of the bismuth when a current in one direction would be induced in it or a part of the circuit in series with it, and if, when the opposite pulse was induced, the resistance of the bismuth were lower, this would amount to a partial commutation but would be accomplished without slipping contacts or any ordinary form of commutator. The result, however, would seem to be more of a pulsating current or a current in which the values of the half waves were unequal, or the wave unsymmetrical with respect to the zero line; a partially rectified current, so to speak. The mercury arc rectifier accomplishes the production of an alternating current from a direct current, and other instances exist, as is well known, of partial rectification by electrolytic cells, vacuum tubes, and the like. It is conceivable that it might be possible to use even a selenium cell in place of the bismuth of Dr. Bruger, putting the selenium in the light at one time and in the dark at another, to affect synchronous alternating pulses differently and give a balance of direct current. A partially rectified current can be sifted so that the direct-current component goes one way, and the alternating-current component another way, as by connecting the terminals by a non-inductive resistance and connecting the direct-current receiving apparatus from the same terminals through a high inductance.

In my paper many years ago, on "Magnetism in its Relation to Induced Electromotive Force and Current" read before the American Institute of Electrical Engineers, I rather emphasized the view that a line of force can only finish by collapsing to a point or infinitesimal closed chain, and can never be broken or opened. By this view, to my mind, we have the only possible explanation for the varied phenomena of induction; that is, we have the only possibility of getting any good physical conception of what takes place.

W. S. Franklin (by letter): What is said in this paper concerning the law of induced electromotive force is, I think, entirely correct. I must say, however, that I have never looked upon Maxwell's statement of this law in a way which would lead me to think of it as not strictly correct. It has always seemed to me that the differential equations of the electromagnetic field in *stationary* and in *moving* media cover the ground completely without any possibility of a misunderstanding.

The point of view of looking upon the circuit as abstracted from definite identifiable material may be illustrated in its extreme form as follows: imagine a sheet of copper placed at right angles to a steady magnetic field; imagine an electric circuit in the form of a circle which starts at a point on the copper sheet and spreads outward like a circular ripple on the surface of a pond. This moving circuit, according to Mr. Hering's interpretation of Maxwell's generalization of Faraday's law, should have electromotive force induced in it, and therefore the stationary sheet of copper in a steady magnetic field should have eddy currents circulating in it. Of course I understand it is precisely this absurd conclusion that is being objected to. My point is this: I do not believe that those who accept Maxwell's generalization have forgotten the idea of *actual cutting of lines of force* by the material of the electric circuit; but it may be said they are likely to be led to forget it because of the form of Maxwell's statement of the generalized Faraday law. Perhaps that is true, and yet the difficulty reduced to its simplest terms is the difficulty of partial differentiation; a physical condition is a function of several independent variables, such, for example, as space and time. One is *obliged* to think of one variable only changing at a time, but one should never for this reason surrender the *knowledge* that all things change together.

Percy H. Thomas (by letter): If I understand this paper correctly, the experiment is not conclusive, and for the following reasons: in drawing off the loop, as long as the ends are rubbing on the sides of the magnet, the lines of force may be said to be still within the loop. When next those ends come in contact, preliminary to their separation from the magnet, the galvanometer becomes short-circuited and protected from any influence of a later change of lines in the loop. In other words, the original loop becomes two loops by being connected across the middle. None of the lines of force is evidently in the lower loop; all are in the upper loop. The upper loop is then *broken* by withdrawing the wires entirely from the magnet, which of course will give no deflection. What has really been done, if I understand the experiment, is that a portion of the original loop containing the galvanometer (but none of the lines) has been *cut off* by short-circuiting, from the whole loop while it still contains the lines, and then the main loop has been opened.

W. P. Graham (by letter): It is perhaps open to question whether the statements of the law of electromagnetic induction by Maxwell and by J. J. Thomson, as quoted by Mr. Hering, are sufficiently precise. Their lack of precision may be simply illustrated as follows: let a wire be bent round the pole of a magnet so as to form a closed circuit linking the magnetic flux. Now let the ends of the wire be separated and the magnet slipped out between the ends so that the flux ceases to link with the wire. Finally, let the wire be again formed into a closed circuit with which the flux does not link. If this re-established circuit be regarded as the original circuit, the flux through the circuit has certainly changed without inducing any electromotive force in the wire.

I think most of us would regard such an interpretation of Maxwell or Thomson as a quibble. But essentially the same interpretation is made by Mr. Hering in discussing the experiment with the galvanometer and copper strips. As soon as the copper strips strike the iron, the original circuit, made up of copper strips and galvanometer is opened, but a new circuit including copper strips, magnet iron and galvanometer is established. And as long as the copper strips make contact with the magnet, there is no change of flux through this new circuit. When the strips leave the magnet, this new circuit is broken, and the old circuit from which the flux has been removed after opening it, is re-established. A more accurate statement would perhaps be that as soon as the strips strike the iron, the old circuit is opened and a new circuit established, to be followed by an infinite succession of other new circuits as the strips slip over the iron, and that the flux is removed from each one of this succession of circuits, by opening each in turn, and establishing its successors; but that there is no change of flux through any individual circuit of the series so long as that circuit is closed.

I agree entirely with Mr. Hering that in presenting the law of induction to a class of students, the cutting of the flux by the conductor is the point to be emphasized and that Faraday's statement of the law is to be preferred.

George T. Hanchett (by letter): It would appear from this that some modification of Maxwell's version of the laws of induction should be made. Let us consider two other methods of trying this experiment. The first of these is illustrated in Fig. 1. The test circuit is slipped over the magnet in the usual manner and the initial deflection is observed. It is then unlinked (?) with the magnetic circuit by the following device: The wires are short-circuited at *A* and the clips at *B* drawn over the magnet limb and removed. There will be absolutely no deflection at the galvanometer as a result of this action, provided, of course, that the short-circuiting is perfect.

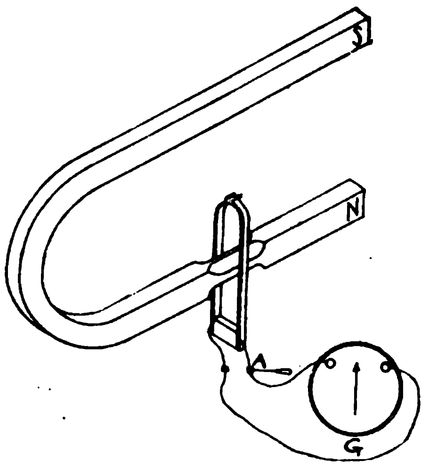


Fig. 1

The second experiment, illustrated in Fig. 2, provides two clips similar to those mentioned by Mr. Hering, but instead of drawing them over the body of the magnet, they are drawn over a tube or annulus surrounding the limb of the magnet and insulated therefrom. It is observed that the detector circuit is first short-circuited outside the flux field before any attempt is made to unlink it, and by this means the circuit is divided into two portions, one containing the galvanometer or detector, which is short-circuited and removed, and the other containing the flux, which is short-circuited and not removed from the magnetic circuit at all. No lines of force are cut by or varied in number within a circuit, and in strict accordance with Maxwell and Faraday no deflection results. In Fig. 2 I have left the loop or ring surrounding the magnet limb in order to point out clearly what Mr. Hering has evidently failed to see—that in performing his experiment he has left an exactly similar flux containing a portion of the circuit behind him in the form of a loop of steel which is a part of the magnet limb itself.

There is no unlinking of the magnetic and electric circuits, but instead a mechanical unlinking which deceives the eye.

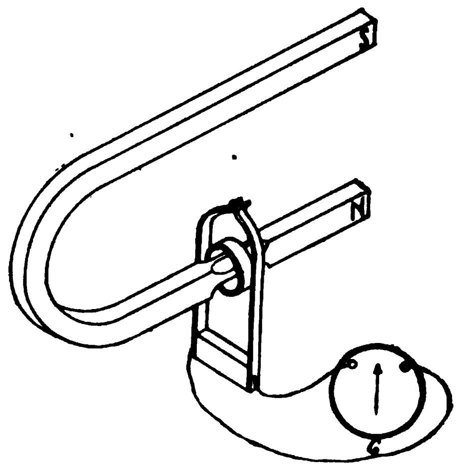


Fig. 2

Electrically, the circuit is linked with the magnet, then a new portion of circuit is added in the form of the magnet limb itself which is ingeniously caused to do all the linking. Next the original circuit now free from flux is removed. This is the exact analogue of the experiment which is the subject of this discussion.

George A. Campbell (by letter): The very simplicity of the facts and principles which Maxwell stated is, apparently, the reason for their being overlooked in the present discussion. The points requiring restatement seem to be the following:

1. Electric currents and magnetic lines of force are both closed. This statement is true irrespective of whether the electric conductors and magnetic cores are open or closed. The current is determined by the integral electromotive force around the circuit, and thus any experimental measurement of the current gives us the total, and not the localized electromotive force. An exact statement of fact must then relate only to the integral values for the entire closed circuit; accordingly, Maxwell's statement (Vol. II, paragraph 541) is in terms of these integral qualities:

" The total electromotive force acting round a circuit at any instant is measured by the rate of decrease of the number of lines of magnetic force which pass through it."

This is the fundamental statement of fact, as distinct from theory, in electromagnetic induction.

2. As the current and the lines of force are invariably closed, it follows as a mathematical necessity that the rate of cutting of lines of force by the circuit is equal to the rate of change in the number of lines of force through the circuit. Maxwell expresses this (Vol. II, paragraph 541) as follows:

"If, therefore, the number of lines which pass through a conducting circuit is made to vary, it can only be by the circuit moving across the lines of force, or else by the lines of force moving across the circuit. In either case, a current is generated in the circuit."

The ordinary "cutting of lines of force" statement is thus a derived law and involves something more than the experimental facts. This addition may be merely the mathematical theorem that for any closed circuit the integral result of the "cutting" statement is the same as that of the "flux" statement; or it may also include, consciously or unconsciously, the hypothesis that the induced electromotive force is physically and locally associated with the cutting of the induction. However natural it may have been for Faraday to localize the induction at the point where there was cutting of the lines of force, it is evident that this was theory and not experiment, for the experimental facts may be accounted for on the hypothesis of action at a distance. If Faraday's theory rather than Faraday's experiments is made the starting point, then, of course, "cutting of lines of force" is fundamental and the flux statement becomes the derived law.

3. The law of induction applies to circuits in non-conductors as well as to circuits in conductors. One of the most important applications of the law is to the free ether, where no materials are present. Mr. Hering's restriction to conductors would therefore be perfectly arbitrary and extremely inconvenient. Where the conductor is not closed the circuit is always completed through the dielectric. The induction in the dielectric must always be borne in mind.

4. It is self-evident that the fundamental statement of the law of induction applies to a linear circuit and that it can be extended to a circuit having a finite cross-section only after the system has been resolved into infinitesimal filaments. This principle is an elementary one in applied theory, but its recognition eliminates all difficulty in the application of the flux statement to Mr. Hering's "crucial" experiment. Thus the sketch shows the springs in the process of sliding over the magnet, which is resolved into a network of conducting filaments. As it is found to be immaterial how this resolution is made in the present case, a simple illustrative network is all that is attempted in the sketch. In the experiment, all of the induction passes through the iron and none of it through circuit 1. Motion of the springs does not change the number of lines

of force threading circuit 1 or circuit 2 or any of the remaining 17 circuits. Therefore, the integral electromotive force around each circuit is zero; and no currents can flow; hence the experimental result.

5. When it is possible to state a law in more than one way it is ordinarily true that the application of the law to different problems will be facilitated by the choice of different forms of the law, and that this is the case with the law of induction is not surprising or significant. As Mr. Hering happened on a unipolar experiment, he found the cutting statement more convenient, as have others before. Had he happened upon an experiment involving toroidal coils or slotted armatures, he would presumably have advocated the flux statement.

6. Mr. Hering's new machine is actually a unipolar dynamo. Faraday in his machines, preferred to make both the cutting of lines of force and the sliding of contacts perfectly uniform; this condition seems to be the ideal one and it is now being found

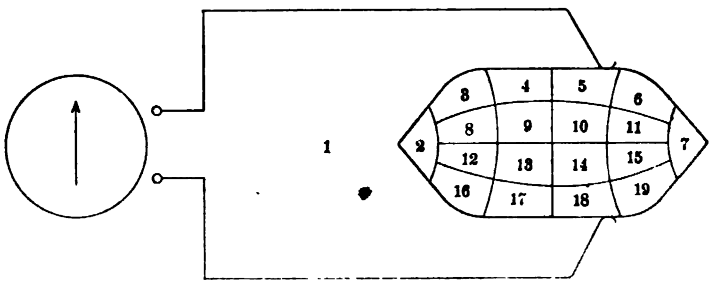


FIG. 1

to be commercial. Obviously this particular arrangement is not necessary. This is Mr. Hering's machine, while the induction cut by each spring is (on the whole) in one direction throughout the motion, the cutting and the sliding of contacts are made intermittent and alternate with each other. The expectation aroused at the beginning of Mr. Hering's paper, that the limitations of the magnetic induction machine were about to be extended, meets with disappointment. It still remains true that for a direct-current machine magnetic induction alone is not sufficient; sliding contacts, a commutator, a variable resistance or some other auxiliary device is necessary.

To summarize: The "flux" law is a literal statement of the observed facts of electromagnetic induction and enables the results to be predicted in any case whatsoever. The unipolar induction experiment described by Mr. Hering presents no exception to this rule and throws no new light upon the subject. The "cutting of lines of force" statement may be regarded either as a mathematical substitute for the "flux" statement, or as a theory localizing the observed action. Both statements of the law are useful in practice.

Tracy D. Waring (by letter): Any great generalization when concisely worded is a likely cause of misapprehension. The novel and ingenious experiment by which Mr. Hering links and unlinks magnetic flux into and out of an electrically closed conducting circuit without producing any inductive effect should go far towards removing a prevalent misunderstanding or misinterpretation of the law which for brevity we may term the Linkage Law of electromagnetic induction.

In considering some of the questions raised by Mr. Hering, it would seem appropriate to call to mind the physical conceptions as to the nature of electromagnetic induction from the point of view of electromagnetic theory.

In any circuit undergoing electromagnetic induction, the electromotive force produced in that circuit is due to, or rather owes its existence to, the electric intensity[1] developed at some or all of the points of the circuit. The seat of the electric intensity (electromagnetically produced) can only be at that place in a medium where the magnetic field is in some way changing relatively to the medium, so we may perhaps describe electromagnetic induction thus: Consider any point of a medium in which a magnetic field is sustained; then electromagnetic induction, if it exists there, may be described as the production of an electric intensity, at the point considered, by a change in the magnetic field at that point.

If this statement be true, what shall we understand by a change in a magnetic field at a point? How is such a change to be specified and by what physical conception may we picture it?

An adequate answer to these questions, if such be possible, would involve a complete physical theory of the ether and matter, a problem so profound and vast that the considerations here presented have by comparison but slight significance. These considerations are ventured, however, as laying stress on a certain aspect of the common conceptions relating to electromagnetic induction.

Taking the case of a magnetic field in an isotropic medium having unit permeability (that is, a medium in which the magnetic intensity and the magnetic induction are numerically identical), how shall we describe a change of the magnetic state at a point in such a medium and the related electric state or electric field accompanying the change of magnetic field? The magnetic field has at every point intensity and direction, either or both of which may change. A change of magnetic intensity at the point will suffice to produce an electric intensity there, the directions of the electric and the magnetic intensities being so related as to be at right angles to each other. But that is not all. Both the magnetic intensity and its direction may remain

1. Also variously termed electromotive intensity, electric force, and intensity of electric field.

constant and yet some change in the magnetic field may take place, at a point in the medium, that will produce an electric intensity there. Relative motion between the magnetic field and the medium will do this. Such relative motion then also represents a change in the magnetic field at a point.

But how shall we picture such motion? Consider any point or points of a medium in which a uniform magnetic field is sustained. Conceive, if you can, a relative translational motion of the uniform field with respect to the point or points fixed in the medium. The magnetic intensity and its direction are everywhere constant, and no instrument fixed in the medium and indicating only intensity and direction would give any indication that the field was moving; yet we can hardly doubt that something does, or may, move relatively to the medium and that the medium responds by having an electric intensity produced in it (unless the line of motion happen to coincide with the direction of the magnetic intensity).

By what physical conception can we make such relative motion have a physical meaning? Perhaps we may put it as follows: Intensity and its direction, as we measure them, do not fully specify a magnetic field at a point. It is not to be expected that so simple an expression could give more than the faintest clue to the physical state we call magnetic, for even a so-called uniform field is not really uniform for very minute dimensions—let us say, for instance, for dimensions of atomic size or perhaps much smaller. If we could only see closely enough, we would see that a so-called uniform field possesses some sort of polarized structure, magnetic filaments of some sort all running the same way and perhaps uniformly spaced.

And so we appear compelled to fall back on a conception of lines of force or tubes of induction having some sort of a real physical existence, if the relative motion between a uniform field and the medium in which it is sustained is to have physical significance.

Take a simple concrete example, say a uniformly magnetized permanent magnet, shaped something like the letter C, with pole faces broad and flat and not very far apart (see Fig. 1). Let the surrounding medium be a dielectric, insulating oil for instance, which we shall suppose remains quiescent around the fixed point, *P*.

The field near the center of the pole faces is practically uniform. Nearer the edges its intensity is weaker and changes in inclination, that is, in direction. Now what happens as the magnet moves forward, relative to the point *P* fixed in the oil, and in the direction indicated by the arrow? It is not difficult to conceive of the magnetic state being created in front of the poles as the magnet advances, and the same kind of state disappearing or being destroyed behind them, while in that part where the field is uniform the magnetic state is constant and apparently does not in any way change as the upper pole face passes over and the lower one under it. But consider the matter more closely. Imagine a minute but intelligent being, stationed at P, equipped with an instrument suitable for indicating to him the intensity and direction of the magnetic field. Watching his instrument as the magnet begins to approach him, he sees the magnetic intensity growing stronger and stronger and ever changing in direction (inclination), but presently it reaches a maximum and remains constant in magnitude and direction. He is then in that part of the field which we call uniform. But is that state, which we call magnetic, really at rest about him? Could he perhaps not see, with a wonderful imaginary eye, the magnetic filaments or magnetic whirls or lines of stress or flow or whatever they are—could he not see them file past him as the pole faces move above and below him, or, as a last resort, could he not devise an instrument that would indicate an electric intensity in the field about him?

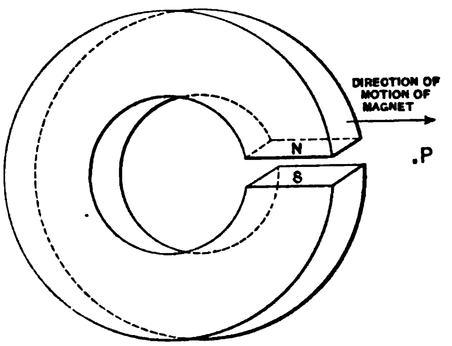


Fig.1

Considerations similar to these arise in considering the rotation of a magnetic field about a line of force as an axis which happens to be also an axis of symmetry for the field. For instance, Mr. Hering raises the question: Does the magnetic field of a round, uniformly magnetized bar magnet rotate with the magnet if the magnet be made to rotate about its axis by mechanical means? Here for any given point in the field the magnetic intensity is constant in direction and magnitude, whether the field rotates or remains stationary?[2]

Sir Oliver Lodge believes that the field does rotate with the magnet, or at least he says: " If a magnet were spun on its axis rapidly by mechanical means, there is very little doubt but that it would act on charged bodies in its neighborhood, tending to

2. We are of course not considering enormous hypothetical peripheral speeds, such as would be comparable with the rate of propagation of magnetic disturbances.

make them move radially either to or from it. This, however, is an experiment that ought to be tried; and the easiest way of trying it would be to suspend a sort of electrometer needle, electrified positive at one end and negative at the other, near the spinning magnet, and to look for a trace of deflection—to be reversed when the spin is reversed. A magnet of varying strength might be easier to try than a spinning one."[3] Mr. Hering suggested the use of a single wire and a condenser for an experiment of a similar character.[4]

To fix our ideas, imagine the arrangement indicated in Fig. 2, which represents a bar magnet capable of rotating around its axis and encircled by two concentric and coaxial cylindrical conducting surfaces. The two surfaces, which we may look upon as the two plates of a cylindrical condenser, are fixed in space, while the magnet may be made to rotate by some mechanical means not indicated in the figure.

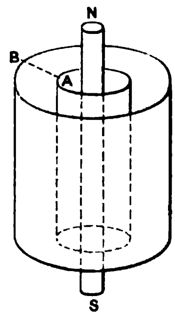


Fig.2

Now assume the magnet to be rotating around its axis at a high speed and then place a conductor, say a piece of wire, across from A to B. If it be true that the field rotates with the magnet, then such of the flux as passes between the cylinders will cut the wire and the cylinders will become charged electrically. Their electrical condition could be investigated after first removing the wire and then allowing the magnet to come to rest.

The experiment might perhaps be modified so as to get cumulative results by means of water-drop collectors as in Fig. 3. With this arrangement the magnetic flux of the rotating field cuts the wire, establishing an electromotive force

3. Modern Views of Electricity, Section 73, 1907 edition, by Oliver Lodge. Also see Lodge, *Phil. Mag.*, June 1889, page 469.

4. A New Factor in Induction; the "Loop" *versus* the "Cutting Lines of Force Law", by Carl Hering, *Electrical World*, March 14, 1908.

between the ends a and b, in consequence of which the water-drops at the extremities of the conductor take on opposite electrical charges. The charges are carried off by the water-drops, and the latter in turn give up their charges to the collecting cylinders which should become more and more heavily charged.

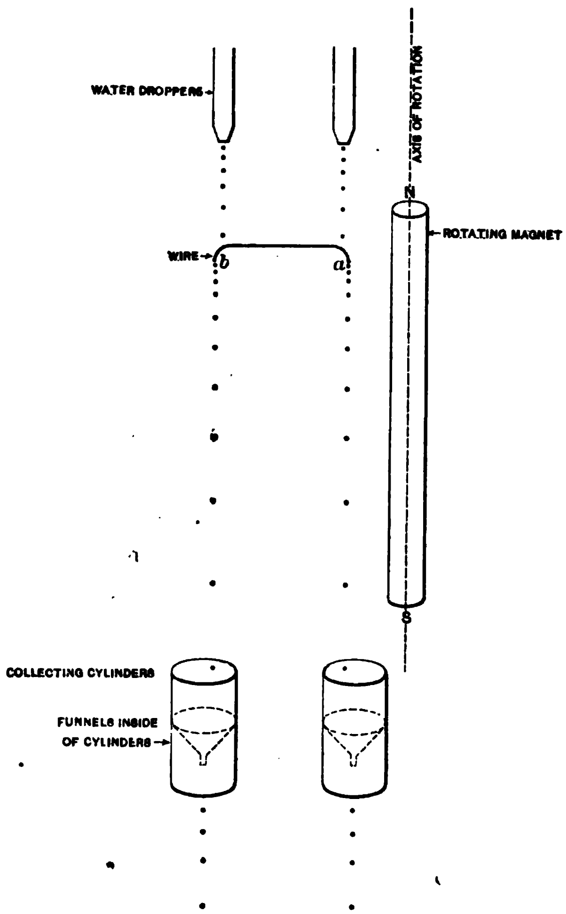


Fig. 3

Of these suggested experiments that proposed by Lodge is perhaps of the greater theoretical interest, as it could be performed in some suitable dielectric without perhaps using any conducting material in the field, and is in line with the experiment so successfully carried out by Dr. H. A. Wilson in which he measured the electric displacement in a dielectric resulting from its motion in a magnetic field.[5]

Carl Hering (by letter): Concerning Dr. Steinmetz's remarks, I am greatly pleased to see that he agrees with me on the main points. The statement of the general law of electromagnetic induction which he gives is probably the first statement which has ever been published of a form of the law which is really universal, so far as we know now, and I hope that in the future it will be copied into text-books freely; I know of no case, no matter how complicated, in which the student would be misled into getting wrong conclusions, or even only doubtful conclusions, when applying this new statement of the law. The only comment I have to make about it is that, to be quite accurate, it should be preceded by a statement that it applies only to elemental conductors, that is, to conductors the cross-section of which is negligibly small; when the cross-section is relatively large, some further limitations would have to be introduced into the law to make it strictly correct and these limitations would complicate it.

What Dr. Steinmetz says concerning the form of induction in unipolar machines, was, of course, well known to me, but the usual unipolar machine does not come directly under Maxwell's law as there is no definite limited loop in which the enclosed flux increases or diminishes. Hence the induction in unipolar machines has, for many years, not been considered a direct contradiction of Maxwell's law, or of the law of linkages. In my experiment, however, the conditions are such that Maxwell's law can be applied directly, there being a very well-defined permanently closed loop and a well-defined increase and decrease of flux enclosed by that loop. It therefore contradicts Maxwell's law directly, while the induction in unipolar machines does not do so, directly. I am pleased that Dr. Steinmetz agrees with me that Maxwell's and Thomson's statements of the law are not universal, but refer only to special cases. If Maxwell's law applies only to " conditions of continuity of conductor

5. See Proc. Roy. Soc., vol. 73, 1904. The experiment consisted in rotating a hollow cylinder or tube of ebonite about its axis, the rotation being performed in a magnetic field the direction of which coincided with the axis of rotation. The inner and the outer cylindrical surfaces were each covered with a metallic film, against each of which sliding brushes made contact. The brushes were connected to a quadrant electrometer, the deflection of the latter thus becoming a measure of the charge displaced through the walls of the cylinder.

The results indicated that an electromotive force was produced between the inner and the outer conducting cylindrical surfaces the same as though a conductor had been used instead of ebonite, but that the value of the electromotive force was less for the ebonite in the ratio of $\frac{K - K_0}{K}$; K being the electric permitivity of the ebonite and K_0 that of free ether, that is numerically $K_0 = 1$ when $K =$ the specific inductive capacity of the moving dielectric (in this case ebonite).

See also Blondlot, *Journal de Physique*, January 1902.

and continuity of motion", as Dr. Steinmetz states it, this limiting condition ought always to accompany the statement of Maxwell's law; but this is not the case in most text-books; and the student should not be left to find out these limiting conditions himself.

Dr. Steinmetz's description of why a magnetic field remains fixed in space while the magnet generating it revolves on its axis is interesting. It seems, however, that high authorities differ on this point,* hence, the question must still be considered an open one.

I am pleased to see that Dr. Kennelly agrees with me that the movement of magnetic flux across the boundary is the primeordial proposition and the increase or diminution of flux in a loop is secondary. This statement is merely another way of saying that Faraday's statement of the law is the fundamental one and explains the seat of the induction, while Maxwell's statement is a deduction from it, applying only to special cases with strict limitations (which latter do not generally accompany the law in text-books).

One of Professor Elihu Thomson's remarks, namely, that he formerly used Maxwell's law, but later adopted Faraday's view of line cutting as the essential thing, bears out the contention that Faraday's law is the more reliable. His proposed universal law, followed afterwards by an admitted exception, illustrates the difficulty of framing a universal law.

Dr. Franklin's remarks show that the teacher who looks at these phenomena. in a broad and general way will be apt to supply in his own mind the omitted limitations of a briefly stated law or rule. Students, however, as also many who are engaged in practical work, accept a law as it is stated and apply it literally; hence they may be very seriously misled unless the limitations clearly accompany such a law.

Mr. Thomas's remarks, in my opinion, illustrate very forcibly my contention that the statement of Maxwell's law is imperfect; for if it requires such a complicated interpretation as he gives in order to make it fit this experiment, and perhaps even more intricate interpretations to fit other possible cases, it certainly is not in correct form to be given to students and practicing engineers to use as it reads. A careful student reading Mr. Thomas's explanation, would naturally feel great uncertainty in applying Maxwell's statement of the law to other unusual cases before he knew the result. If, in this experiment, the continuously closed circuit must be considered as being broken (as Mr. Thomas states in italics), then it must necessarily follow that in all other sliding contacts, such as those on alternators, revolving fields, induction motors, etc., the circuit must be considered to be broken all the time. Hence the

* See Modern Views of Electricity, Lodge, 1907, Section 73, page 142. Also A Treatise on Magnetism and Electricity, Andrew Gray, 1898. page 329, Section 423, in which the author says: " When the magnet moves, its field of force moves with it."

current is flowing through a broken circuit without causing an arc—which, of course, leads to an absurdity.

Replying to Mr. Graham, I need only say that the experiment deals only with permanently closed circuits through which a steady current could be flowing all the time. Hence his analogy to a circuit which is opened and the flux moved out through the opening, is not legitimate discussion of the experiment itself.

Mr. Hanchett's first experiment is essentially different from mine and has no bearing on mine, as it distinctly opens and closes a circuit at a switch. His second experiment is described in my paper, Fig. 3. It seems that Mr. Hanchett has evidently failed to see the very point of the experiment, which was that in teaching students the laws and rules for their practical use, these laws and rules should be so stated that the student will readily understand them, can feel absolute reliance in them, can predict results with certainty and reliance, and will not have to resort to complicated interpretations (like leaving a part of a circuit behind notwithstanding that all of it has been visibly removed) in order to make the laws fit special cases after experimental investigation has shown what the result is.

Mr. Waring's suggestions of methods to determine whether a field moves with its magnet or not, are very ingenious, particularly the one with the water-drop collectors, which it seems to me would not be difficult to carry out. As the result is a simple question of fact concerning which the theories of able authorities give contradictory results, it would be very interesting to have this experiment carried out. I had suggested the use of condensers for such an experiment, but his water-drop method would give cumulative results, which makes the measurement easier. The precaution of course should be taken to stop the rotation of the magnet before connections are made with the electrometer, so that there could not be any question of a possible induction in any leads, which has heretofore been the great stumbling-block in any direct measurements.

Mr. Campbell's statement that "the current is determined by the integral electromotive force around the circuit" is, in my opinion, an evident fallacy, as every student will know that a current is not determined by the electromotive force, but by the quotient of the electromotive force and the resistance; without a knowledge of the resistance the current is a decidedly indeterminate quantity. In his paragraph 2 he virtually says that "as the current" is "invariably closed, it follows as a mathematical necessity that" there cannot be any such a thing as an open circuit, hence Maxwell's loop law is universal!

Of course if one starts out with the arbitrary assumption that an electric circuit cannot ever be opened, then one thereby incapacitates himself to even consider such a thing as induction of electromotive force in an open circuit. Persons who have been left embarrassed in the dark by an opening of the electric light supply circuit, or whose motors have been stalled for a similar reason, will not have much faith in arguments as to the impossibility of opening circuits. If Mr. Campbell will read Faraday he will find that this distinguished experimenter invariably made experiments first and suggested theories to accord with the facts afterwards. I believe he was never guilty of torturing facts to fit premature theories; he never to my knowledge had to interpret a closed circuit to be open or an open one to be closed, in order to fit a pet theory to the facts. To Mr. Campbell's arbitrary assertion that induction "where no materials are present" is "one of the most important applications" of the law under discussion, it is I hope not necessary to reply; the usual industrial use of induction is to get useful currents, which could not flow where there are no materials to conduct and confine them. Why should we be expected to accept his very arbitrary and non-proved statement (4) that "it is self-evident that the fundamental statement of the law of induction applies to linear circuits" (presumably meaning closed circuits or complete loops, as he described before). Authorities, who are recognized both here and abroad, consider the general case to include all kinds of circuits whether open or closed, the closed circuit being therefore a special case; in order to be fundamental, a law must apply to the general case. Many who have been asked have acknowledged frankly, and in a true scientific spirit, that Maxwell's law as usually stated would not have predicted the correct results of this experiment before the results were known: hence Mr. Campbell's arbitrary statement that it "enables the results to be predicted in any case whatsoever" is not borne out by facts. That the experiment "throws no new light on the subject" is a matter about which others have expressed a different opinion to me. One of the chief objects of my paper was to try to show that there is room for improvement in teaching students the fundamental laws, so that they will obtain such a clear and unencumbered conception of those laws that they will have confidence in applying them for predicting results, without involving complicated "interpretations"—such as having to consider an open circuit to be a closed one, or the reverse.—which require an experimental determination before a correct prediction can be made. Mr. Campbell's discussion is a good illustration of this.

Mr. Thomas has claimed above that what is universally considered by electrical engineers to be a closed circuit must now be considered to be an open circuit in order to make his theory fit this case, and now Mr. Campbell claims that what would generally be considered to be an open circuit must now be considered to be a closed one in order to make his theory fit this case. I think this is a good illustration of the point raised in the paper, that Maxwell's law is not in a satisfactory form for teaching students, or for engineers to use.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 2, 1908.

GRAPHICAL TREATMENT OF THE ROTATING FIELD

BY R. E. HELLMUND

The object of this paper is to evolve diagrams by means of which nearly all the phenomena of the rotating field may be easily studied, and the various factors necessary for the calculation of fluxes, exciting current, etc., exactly determined. In order to avoid too frequent repetition of certain expressions, the author has adopted the expedient of using the words " addition " and " subtraction " to mean geometrical addition and subtraction of vectors according to the well-known conventional method.

The derivation of a full pitch two-phase winding with four slots per pole per phase as represented in Fig. 1 may be chosen. The two vectors a and b in Fig. 2 represent the total ampere-turns of the two phases a and b. Under this assumption the ampere-turns of one of the 8 coils of each phase may be represented by one-eighth of each of the two vectors. In Fig. 1 the coils of phase a are numbered from $1a$ to $8a$, those of phase b from $1b$ to $8b$. As in any similar winding the current flows in four coils of each phase in one direction and in the other four coils of the same phase in the opposite direction; the difference of flow is indicated in Fig. 1 by + and − signs in front of the number of the coil.

1. *Diagram of the fluxes in the individual teeth.* Consider, first, tooth number 2. It is seen from Fig. 1, that the coils $-3a$, $-4a$, $+5a$, $+6a$, $+1b$, $+2b$, $+3b$, and $+4b$ exert a magnetizing effect upon this tooth. The effects of the coils $-3a$, $-4a$, $+5a$, and $+6c$ neutralize each other, and therefore the resultant magnetizing effect is in phase with the ampere-turns b and equal to four-eighths of b. The resultant mag-

netizing effect may therefore be represented by the vector $\overline{0-2}$ Fig. 3, which is equal to four-eighths of b and parallel thereto. Passing from tooth 2 to tooth 3, it will be seen that the magnetizing effect of coil $-3a$ must be subtracted and that of coil $+7a$ be added in order to obtain the resultant magnetizing effect exerted upon tooth 3; the subtraction and addition may be accomplished by adding the line $\overline{2-3}$, which is in phase with the vector a of Fig. 2 and = two-eighths of a, to the line $\overline{0-2}$. The resultant line $\overline{0-3}$ represents the magnetizing effect exerted upon tooth 3. In the same way the magnetizing effects exerted upon all 16 teeth may be easily determined. The square 16-4-8-12 is obtained in this way. If a uniform reluctance over all the poles is assumed, the vectors $\overline{0-1}$, $\overline{0-2}$, $\overline{0-3}$, etc., may at the same time be considered to represent the fluxes in the teeth 1 to 16,* if the scale is changed under proper consideration of the air-gap reluctance, etc.

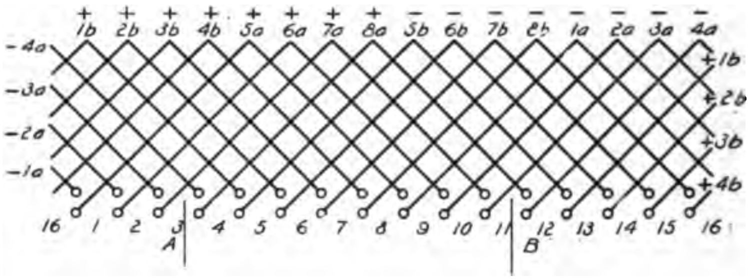


FIG. 1.—Space values of the rotating field.

2. *Diagram of the space values of the total field.* It is obvious that the vector representing the total field over a certain arc of the pole face may be easily obtained by simply adding the flux vectors of those teeth which form part of the arc under consideration. Thus in order to obtain the vector representing the total flux of an arc equal to a full pole pitch, the fluxes of 8 adjacent teeth have to be added. If, for instance, the fluxes of teeth 2 to 9 are added as shown in Fig. 3, we obtain the vector $6a$, which represents the total flux over an arc comprising the teeth 2 to 9; that is, over an arc equal to the pole pitch. In order to find the total flux over the teeth 3 to 10, the complete addition does not need to be repeated; it is only necessary

* This diagram for the tooth fluxes was evolved by Professor H. Görges, Dresden, and is published in *Electrotechnische Zeitschrift*, January 3, 1907.

to subtract from 6*a* the vector 2 and to add the vector 10; thus the desired vector, 7*a*, which represents the total flux over the teeth 3 to 10, is quickly obtained. In the same way we find:

Vector 8*a* representing the total flux over the teeth 4 to 11
" 5*b* " " " " " " " 5 " 12
" 6*b* " " " " " " " 6 " 13
etc.

The vector for the flux between any two points *A* and *B* on the pole face, which are a full pole pitch apart and which do not coincide with the primary current centers, may be just as easily determined. Suppose, for instance, that the points *A* and *B* are respectively one-third and two-thirds of a tooth pitch away from the next current point. The vector for the flux between the

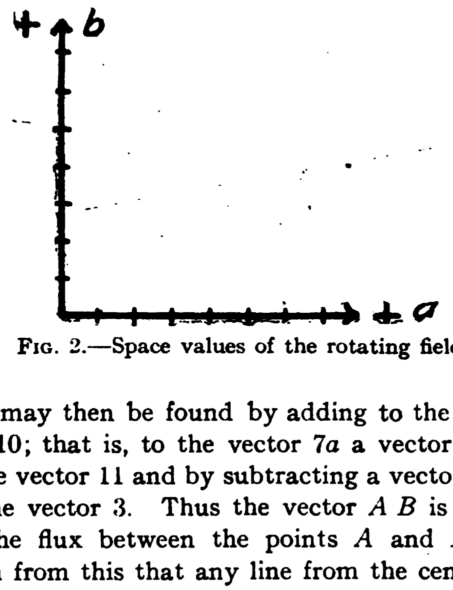


FIG. 2.—Space values of the rotating field.

two points may then be found by adding to the vectors of the teeth 3 to 10; that is, to the vector 7*a* a vector equal to two-thirds of the vector 11 and by subtracting a vector equal to two-thirds of the vector 3. Thus the vector *A B* is obtained, representing the flux between the points *A* and *B*. It will be readily seen from this that any line from the center of the diagram to one of the sides of the polygon 6*a*, 7*a*, 8*a*, 5*b*, ètc., represents the flux of some arc equal to the pole pitch.

3. *Diagram of potentials.* After the total flux over any full pitch area of the pole face is determined, the potential induced in a full pitch coil placed around this area may be represented by a vector proportional to and lagging 90 degrees behind the flux vector. If, for instance, the two sides of a secondary coil were placed on the points *A* and *B* the potentials induced in

the coil would be represented by a vector proportional to the vector *A B* lagging 90 degrees behind it. The potentials induced in the primary coils are represented by vectors proportional to and lagging 90 degrees behind the vectors 7*a*, 8*a*, 5*b*, 6*b*, etc. The potential vectors corresponding to the field vectors 7*a*, 8*a*, 5*b*, 6*b*, etc., are shown in Fig. 4, and correspond to the potentials induced in the coils 7*a*, 8*a*, 5*b*, 6*b*, etc., respectively. The total potential induced in the four coils of each

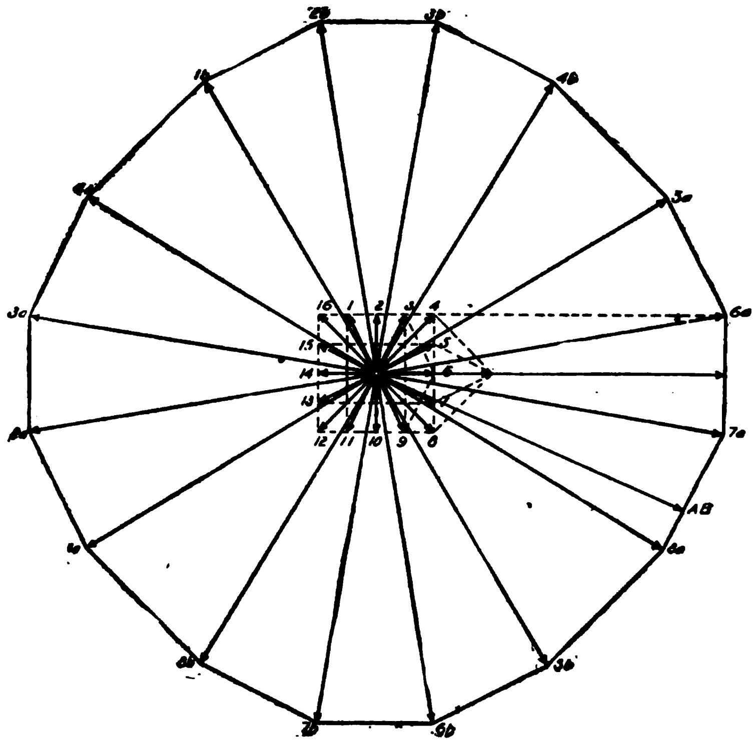


Fig. 3.—Space values of the rotating field.

group may be obtained by simply adding the four corresponding vectors, as shown in Fig. 4.

4. *Diagram of the time values of the total field.* As has been stated, from Fig. 3 can be found the vectors, which represent the maximum values as well as the time-angle of the total fields over any full pitch area of the pole face. It was found, for example, that the vector *A B* represents the maximum flux value between the points *A* and *B* of Fig. 1. This flux does

not, however, necessarily represent the maximum flux which exists at the time at which the flux between A and B reaches its maximum value. It is obvious that there may be fluxes that find some path which does not go through the pole face area between A and B; or there may be fluxes that pass the pole face between A and B twice, and which therefore have no effect upon a coil located with its two sides at the points A and B. The question arises, therefore, what are the maximum

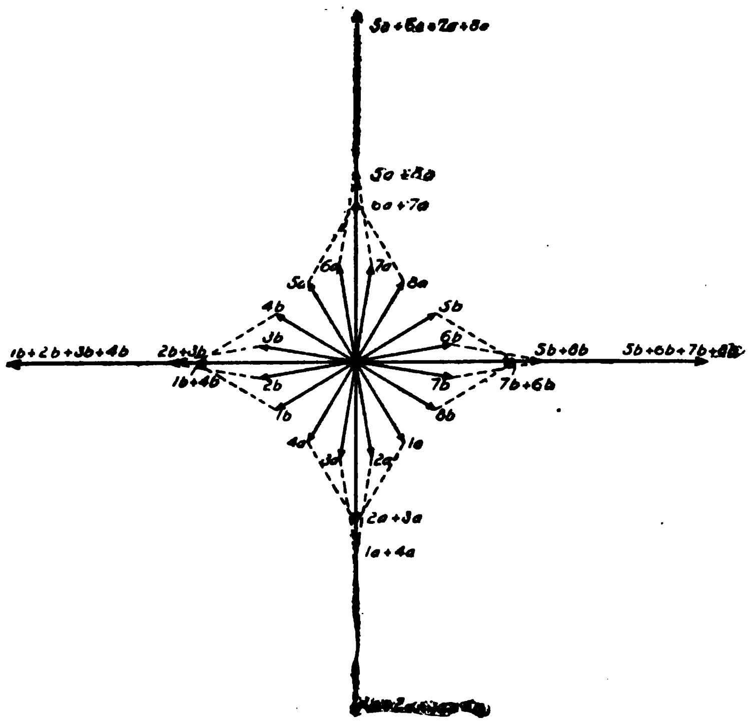


FIG. 4.—Potential diagram.

values of the total field existing at any point of time? (In considering this question the leakage fluxes across the slots and the end connections of the coils may be neglected, since they can easily be considered separately.)

It was found that all the individual fluxes that go through each particular tooth are represented by the vectors 1 to 12 in Fig. 3. The maximum value of the flux at any time must therefore be represented by the sum of all individual fluxes that have the same direction at the time under consideration. If, therefore, in Fig. 5, which contains some of the vectors of Fig. 3, a time-line, $\overline{o\,m}$, is assumed, the maximum field value for this time may be represented by the addition of all the projections of the individual teeth fluxes, which have the same direction, upon the time-line; or, what is equivalent, by the projection of the graphical sum of all the teeth flux vectors that are at one side of the perpendicular, $\overline{x\,y}$, on the time-line. For the time-

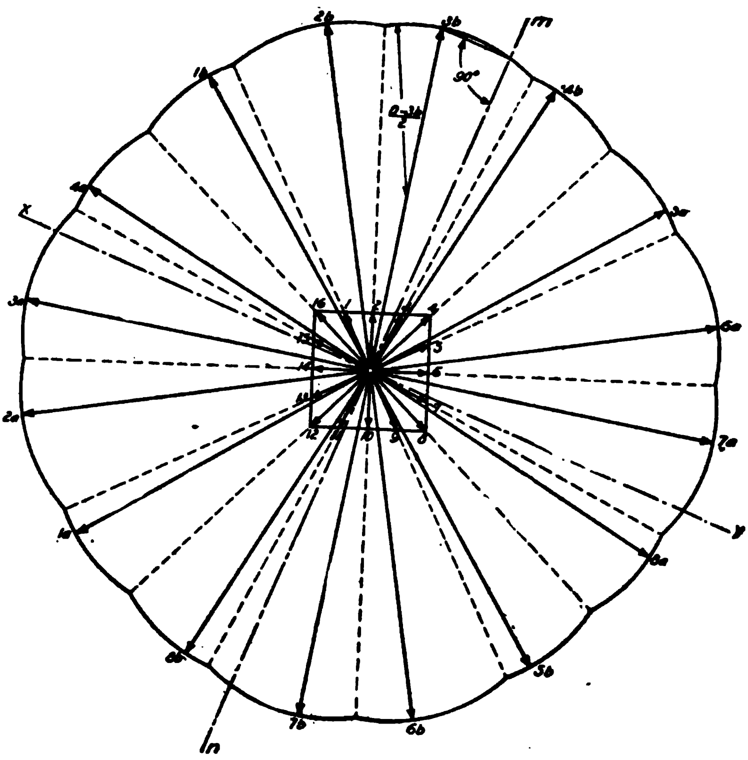


FIG. 5.—Time values of the rotating field.

line under consideration the maximum field value is the graphical sum of the vectors 15, 16, and 1 to 6: The graphical sum of these vectors is represented by the vector 3*b*, as before derived; consequently the value of the total field may be found by projecting the vector 3*b* on the time-line. This holds for any time-line the perpendicular of which falls between the vectors 14 and 15, or between 6 and 7. In the same way the value of the total field for any time-line the perpendicular of which falls between the vectors 15 and 16 or between 7 and 8 may be found by projecting the vector $4b$ on the time-line, etc. It is also obvious that the locus of the projection points upon the time-line is a circle around the center of each vector. A very simple polar representation of the total field value in terms of the time-angle may therefore be obtained by simply drawing these circles for all the vectors, as shown in Fig. 5. The intersection of the various circles occurs when the perpendicular to

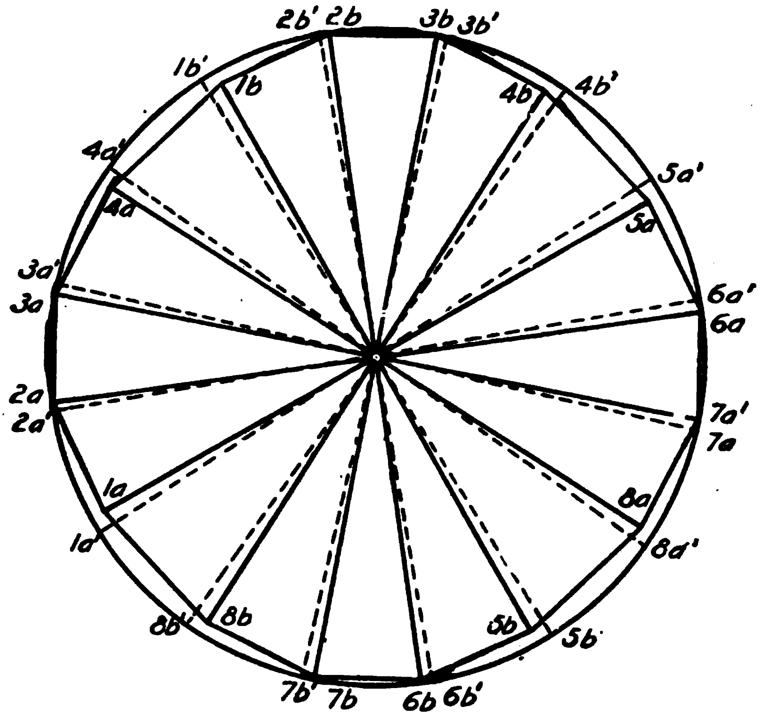


FIG. 6.—Comparison between the actual field and the equivalent sinusoidal field.

the time-line passes the vectors of the teeth fluxes; therefore the time-angles of the intersection points follow directly from the time-angles of the teeth fluxes.

5. *Equivalent sinusoidal field.* After the vector representing the total potential which is induced in one group of coils has been found from Fig. 4 to equal E, it is possible to find the amplitude of the equivalent sinusoidal field; that is, the size of a field that has a sinusoidal space distribution rotates with

uniform speed and has the same effect as the actually existing field.

If n is the number of coils per pole, the angles between the potentials being induced in the coils is

$$\alpha = \frac{180}{n}$$

Moreover, if the potential induced in each coil is x, it follows from Fig. 7, for the case under consideration:

$$x = \frac{E}{2 \sin \frac{\alpha}{2} + 2 \sin \frac{3\,\alpha}{2}}$$

where $\alpha = 22° \, 30'$.

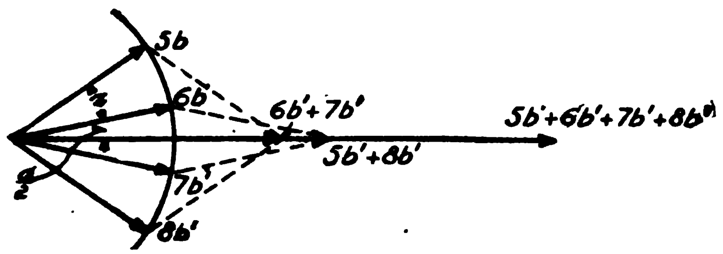


FIG. 7.—Diagram of the equivalent sinusoidal field.

In Fig. 6 a circle with a radius, x, is shown in combination with the diagram of Fig. 3. The dotted line vectors represent the potentials induced in the individual coils by the equivalent sinusoidal field, while the full-line vectors show the potentials induced by the actual field. In Fig. 8 the circle representing the equivalent sinusoidal field is shown in combination with the diagram on Fig. 5.

6. *Characteristics of the rotating field.* From the diagrams evolved so far, the main characteristics of the rotating field may be discerned merely by looking at the illustrations. It appears at once from Figs. 3 and 6, that the space values of the field vary in size; from Figs. 5 and 8 it appears that the total time values also vary in size. From the tooth diagram in Fig. 3, it follows that the maximum values of the individual teeth fluxes are different in size, and their time-angles differ

from the angles giving the space distances between the. teeth; this means that the individual parts of the field rotate at non-uniform speed. That the total field rotates at non-uniform speed appears from Fig. 6, because the full-line vectors, that is, the vectors of the actual field values, would coincide with the dotted vectors, which indicate the time-angles of a uniformly rotating field, if the total field were to rotate at uniform speed. From Figs. 6 and 8 it also appears that the average of the space values, as well as the average of the time values, are smaller than the equivalent sinusoidal field values. Even the average of the flux vectors of the primary coils $5a$, $6a$, etc., is smaller than the equivalent sinusoidal flux value. It seems strange that the average of the primary field values induces the

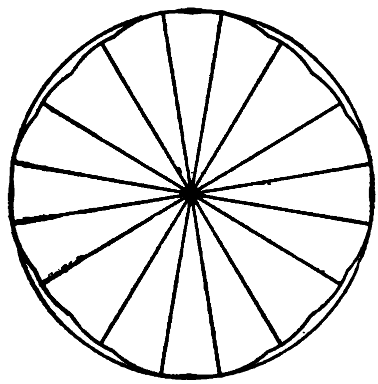


FIG. 8.—Diagram showing the time values of the primary and secondary fields and the leakage.

same resultant electromotive force as the equivalent but larger sinusoidal field. The reason for this follows however at once from Fig. 6, which shows that the actual field vectors $5b$, $6b$, $7b$, and $8b$, for instance, which belong to one group of coils, are less out of phase than are the equivalent vectors of the sinusoidal field $5b'$, $6b'$, $7b'$, and $8b'$. For this reason it happens that for some coil arrangements the actual field reaches at no time the value of the equivalent sinusoidal field.

7. *Reactive effect of the secondary.* So far the diagrams have been evolved and the field treated without considering the reactive effects of the secondary. Assume now that a single coil, which may first be considered to be open-circuited, is rotated at synchronous speed round the pole face. Assume further

that the dotted lines $1a'$, $2a'$, $3a'$, etc., of Fig. 6 represent some time-positions of the rotating coil. They then give the positions of the secondary coil, in which its sides coincide with those of the primary coils $1a$, $2a$, $3a$, etc., respectively. For these positions the time vectors of the fields are represented by the full lines $1a$, $2a$, $3a$, etc., in Fig. 6, and the field inside of the moving coil, while the latter is in position $1a'$ may be found, for instance, by projecting line $1a'$ upon $1a'$, assuming that the sides of the rotating coil happen to coincide with those of the primary coil at the time represented by the vector $1a'$. For any other position of the coil the field inside the coil may be similarly found. It will be seen that the field as set up by the primary in the synchronously rotating secondary coil will fluctuate in size; therefore there will be potentials induced in the secondary coil, and if this coil is short-circuited reactive currents occur therein. It is also obvious that the more the polygon differs from a circle the larger these currents will be. The detailed treatment of the reactive currents takes considerable space and will be given later. It may be sufficient to state here that the currents in the secondary tend to keep up a field of sinusoidal space distribution and uniform size rotating with uniform speed. This field will naturally assume a value equal to an average of the field values originally induced by the primary. From Fig. 8 it follows that this average value will be smaller than the previously found value of the equivalent sinusoidal field. It follows, therefore, that the counter electromotive force induced by the field which is under the influence of the secondary reactance will be smaller than that induced by the original field. This, in turn, will cause a larger primary magnetization current in order to make up for the decrease caused by the secondary reaction. For this reason the magnetizing current of a motor with short-circuited and synchronously rotating secondary will be larger than the current of the motor with open secondary. The ratio of the two currents is the ratio of the average time values of the actual field as originally set up by the primary (open secondary assumed) and the value of the equivalent sinusoidal field. This ratio may be called the reactance factor.

8. *Derivation of coefficients.* The above diagrams lend themselves readily to the derivation of such factors as are required for the practical calculation of the rotating field and its magnetizing current. By drawing a segment of the diagram on

cross-section paper, as shown in Fig. 9, the problem of exactly determining the various factors is reduced to simple geometrical calculations, which may be quickly made with the aid of a table giving the squares and the square roots of figures and the angle functions. Various methods of proceeding and various factors may be chosen for the calculation of the magnetizing current and the field. It seems to the writer, however, that the relative merits of each coil arrangement may be best recognized if a coil arrangement with one slot per pole per phase is chosen as a base.

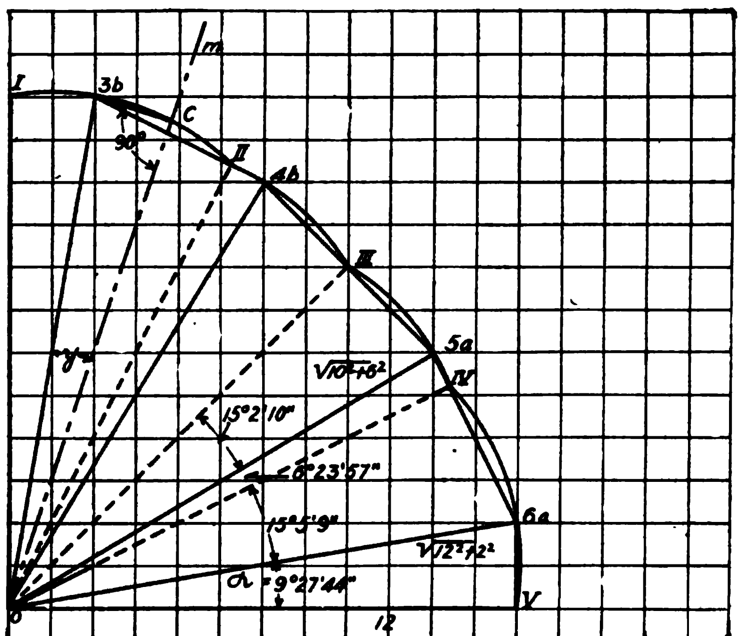


FIG. 9.

a. Potential factor. If, for instance, the potential being induced by a given sinusoidal field rotating with a given uniform speed is E_1, under the assumption of a given full-pitch winding located in one slot per pole per phase, the potential E_n induced by the same field in the same winding distributed over n slots per pole per phase may be

$$E_n = k_p E_1,$$

where k_p may be called the potential factor. The factor k_p is found from diagrams as shown in Fig. 7, and from the formulas given in connection therewith. The curves of Fig. 11 give the factor k_p for two-phase and three-phase full-pitch windings in terms of the total number of slots per pole.

b. Current factor. We assume further that a certain number of ampere-turns in a full-pitch winding wound in one slot per pole per phase induces, in case of an open-circuited secondary, a field, equivalent to the sinusoidal field F_1; and that the same number of ampere-turns in case of the same winding wound in n slots per pole per phase induces with an open-circuited secondary a field which is equivalent to a sinusoidal field F_n. The ratio of the two equivalent sinusoidal fields,

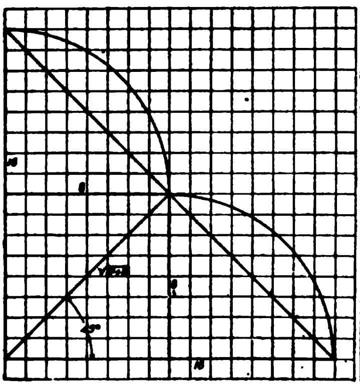


FIG. 10.—Diagrams for the determination of the various factors.

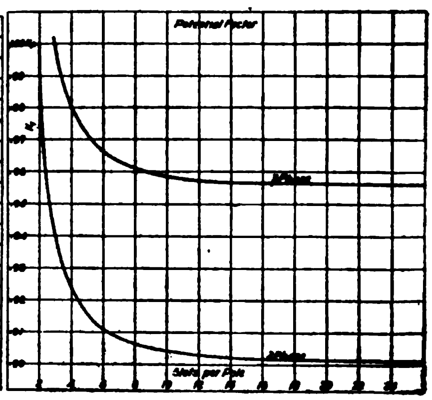


FIG. 11.—Potential factor.

$$k_c = \frac{F_n}{F_1}$$

may then be called the current factor.

The latter may be found by drawing, in addition to the diagram of Fig. 9, the diagram for one slot per pole per phase as shown in Fig. 10, the same number of total ampere-turns being assumed for both cases. The ratio of the equivalent sinusoidal fields found from the two diagrams is then the current factor k_c. The curves in Fig. 12 give the current factors for two-phase and three-phase full-pitch windings in terms of the total number of slots per pole.

c. Reactance factor. The potential and current factors are sufficient for calculating the magnetizing current with an open-circuited secondary. In order to find the exact magnetizing current for the case of a short-circuited synchronously rotating secondary the previously mentioned reactance factor has also to be determined. For this purpose we have to find the average value of the time values of the total field. From Fig. 9 it is seen that for any time-line, $\overline{O\text{-}m}$, during the time period I–II the time value of the field is

$$O\,C = \overline{O\text{-}3\,b}\cos y.$$

The average value over one period may therefore be found by a very simple integration. Since the diagram in our case

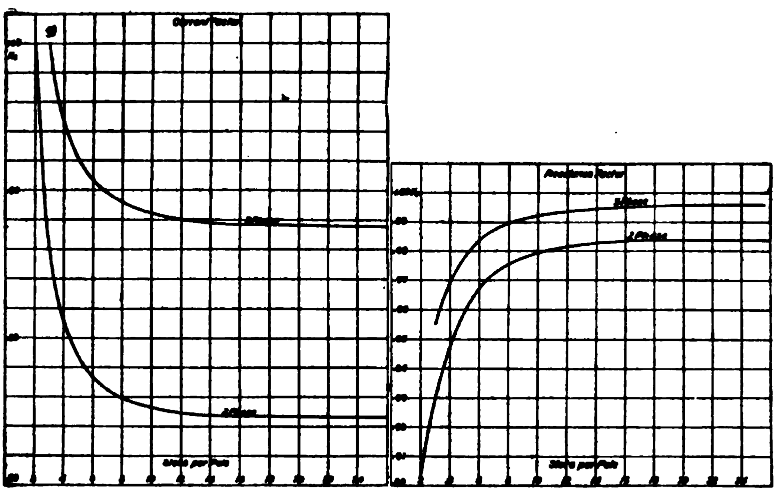


FIG. 12.—Current factor. FIG. 13.—Reactance factor.

has, however, eight symmetrical segments, it is sufficient to find the average value for one of the eight segments. Thus we find the average time value of the field from segment III–IV–V to be:

$$F_a = \frac{4}{\pi}\left[\sqrt{12^2+2^2}\int_{-9^\circ\,27'\,44''}^{15^\circ\,5'\,9''}\cos y \;+\sqrt{10^2+6^2}\int_{-6^\circ\,23'\,57''}^{15^\circ\,2'\,10''}\cos y\right]$$

or

$$F_a = \frac{4}{\pi}\Big[\sqrt{12^2+2^2}\,(\sin 15^\circ\,5'\,9''+\sin 9^\circ\,27'\,44'') + \sqrt{10^2+6^2}\,(\sin 15^\circ\,2'\,10''+\sin 6^\circ\,23'\,57'')\Big]$$

The various angles may be best found from their tangent, which follows directly from the figure. For the first angle α we have for instance

$$\tan \alpha = \frac{2}{12} \text{ therefore}$$

$$\alpha = 9° \, 27' \, 44''$$

The reactance factor is found from

$$k_r = \frac{F_a}{F_n}$$

The curves in Fig. 13 give the reactance factor for two-phase and three-phase full-pitch windings in terms of the total number of slots per pole.

d. Magnetization factor. In practice the calculation of the magnetizing current for the running motor is of most interest. therefore the current factor and the reactance factor may be combined to one factor

$$k_m = k_r \, k_c,$$

which may be called the magnetization factor. The latter is given for two-phase and three-phase full-pitch windings by the curves of Fig. 14.

e. Resultant factor. In cases where the magnetizing current is found directly from the impressed voltage, all three of the factors, k_p, k_c, and k_r, may be combined as a resultant factor

$$k = k_p \, k_c \, k_r$$

The latter is given for two-phase and three-phase full-pitch windings by the curves of Fig. 15.

9. *Calculation of the field and its magnetizing current.* In practise the impressed potential is known as a rule, and the field and its magnetizing current are to be determined. If F_1 is the required sinusoidal field for the one slot per pole per phase arrangement, we find for other arrangements:

a. For open-circuited secondary:

The equivalent sinusoidal field $F_n = \dfrac{F_1}{k_p}$

The average of the time values of the actual total field

$$F_a = \frac{F_1}{k_p} \times k_r$$

b. For short-circuited and synchronously rotating secondary: The equivalent sinusoidal as well as the actual field

$$F_n = \frac{F_1}{k_p}$$

If I_1 is the required magnetizing current for open-circuited

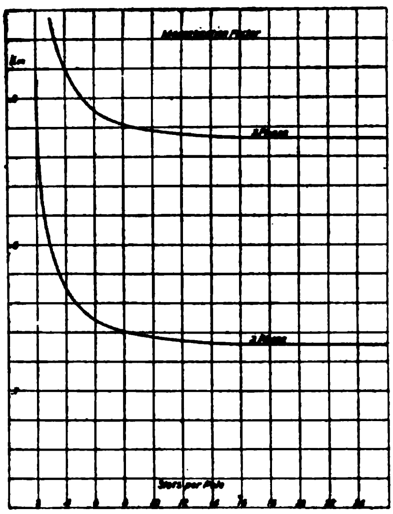


FIG. 14.—Magnetization factor.

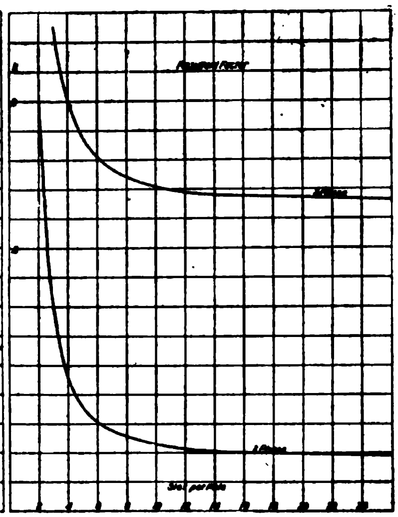


FIG. 15.—Resultant factor.

secondary with the one slot per pole per phase arrangement, we find for other arrangements:

a. For open circuited secondary:

$$I_n = \frac{I_1}{k_p \times k_c}$$

b. For short-circuited synchronously rotating secondary

$$I_n = \frac{I_1}{k_p \times k_c \times k_r} = \frac{I_1}{k}$$

10. *Magnetic leakage.* The diagrams also give a means for deriving the magnetic leakage. The time values of the total fluxes in the secondary may be determined similarly to the time values of the primary flux, by finding the vectors for the fluxes in the secondary coils and drawing circles round their centres.

Let us assume, first, that the secondary has the same number of slots as the primary, and that the secondary is in a position

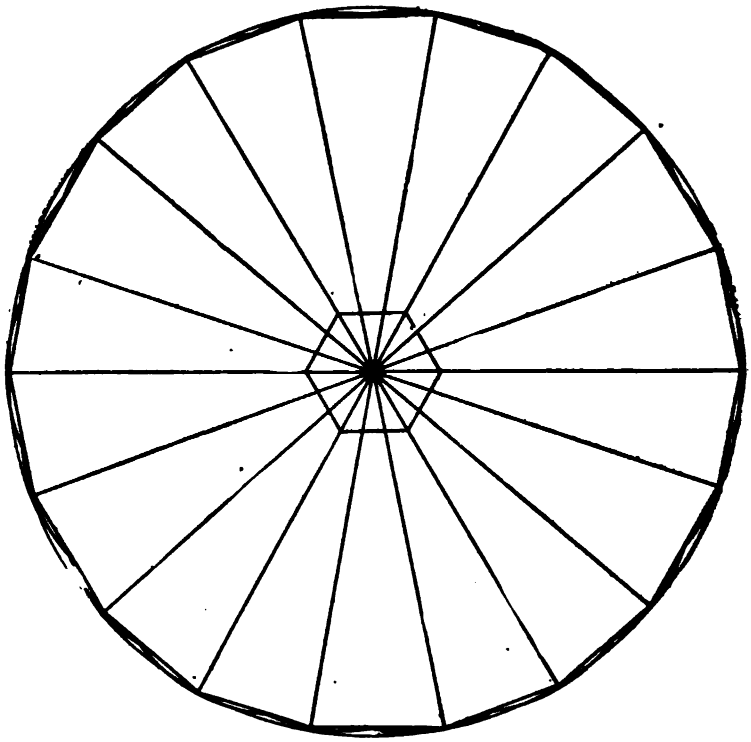


FIG. 16.—Field diagrams for a three-phase motor with three slots per pole per phase.

in which the sides of the coil coincide with those of the primary. Then of course the secondary fluxes are always the same as the primary fiuxes, and the potentials induced in the secondary are the same as in the primary, slot and end-connection leakage being neglected. Therefore the curve of Fig. 8 represents for this case the primary flux as well as the secondary flux; that is, the leakage flux is zero.

The rotor may now be shifted so that the sides of its coil are located in the middle between the coil sides of the primary. The field vectors of the secondary are then represented by the dotted lines of Fig. 17, which go from the center of the diagram to the middle points of sides of the polygon. By drawing the circle segments around the centers of these dotted lines, we obtain the inner curve shown, Fig. 17, which represents the time values of the total secondary flux. The difference between the outer and the inner curve represents the leakage flux for the rotor position under consideration.

In Fig. 18 the same curves are shown for a secondary position between that of Figs. 8 and 17. The three figures show clearly the change of the leakage field with the secondary position.

In Fig. 19 the curves for the fluxes are shown for a rotor in

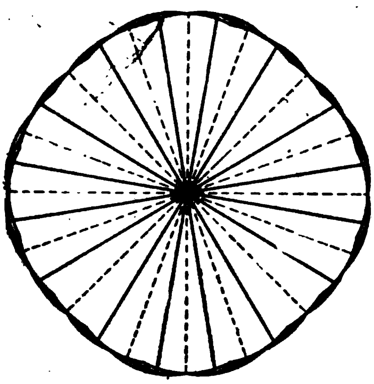


FIG. 17.—Diagram showing the time values of the primary and secondary fields and the leakage.

a position like that of Fig. 17, the number of secondary slots being reduced to half their former number. A comparison with Fig. 17 shows at once how the leakage has increased by the reduction of the secondary slots.

Instead of determining the leakage fluxes as the difference between the primary and secondary fluxes, the zig-zag and belt leakage fluxes may be also determined directly from the diagram of the teeth fluxes. The slot leakage may also be determined by the latter.

11. *Comparison between two-phase and three-phase fields.* It may be well to give for the purpose of comparison, a diagram for a three-phase winding. Fig. 16 shows the diagram for the teeth fluxes, the space values, and the time values of the total

fluxes of a motor with three slots per pole per phase; that is, approximately the same number of slots per pole as for the two-phase case considered before—nine slots per pole instead of eight. It appears at once from the diagram that the three-phase field approaches the ideal condition much more than does the equivalent two-phase field. All three diagrams given in Fig. 16 give

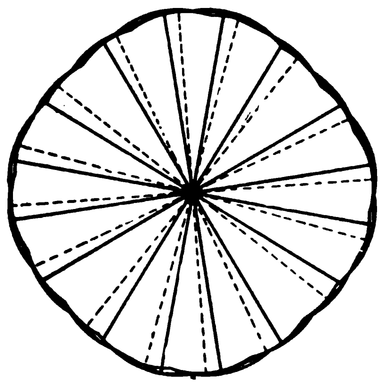


Fig. 18—Diagram showing the time values of the primary and secondary fields and the leakage.

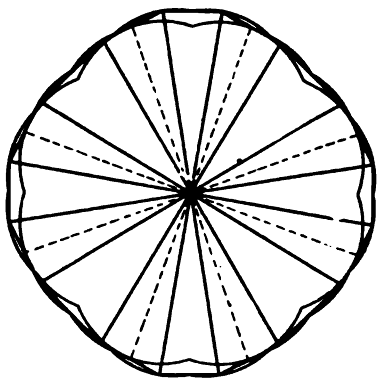


Fig. 19—Diagram showing the time values of the primary and secondary fields and the leakage.

curves that much more approximate a circle than the curves for the two-phase field. The superiority of the three-phase field also appears from the curves of Figs. 11 to 15.

It is well known that the two-phase field requires more magnetizing volt-amperes than the equivalent three-phase field. It may also be shown that the leakage coefficient of the two-phase

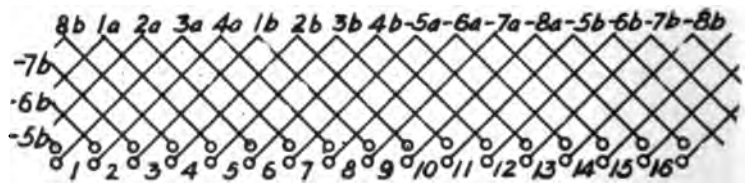


Fig. 20.

field is larger than that of the three-phase field. These facts are considered by every designer, and result in the well known differences of 0.5 to 3 per cent. in the calculated values of the efficiency and power-factor, or possibly in some difference in the starting torque and pull-out torque of the two motor types, if they are designed for equal power-factor.

The comparison of the above diagrams shows, moreover, that a two-phase motor will have larger reactive currents in the secondary. The comparison of the reactance factors shows that the amplitude of the higher harmonics in the primary is larger in the two-phase motor than in the three-phase motor. Consequently both of the latter phenomena tend to reduce the efficiency of the two-phase motor more than that of the three-phase motor;

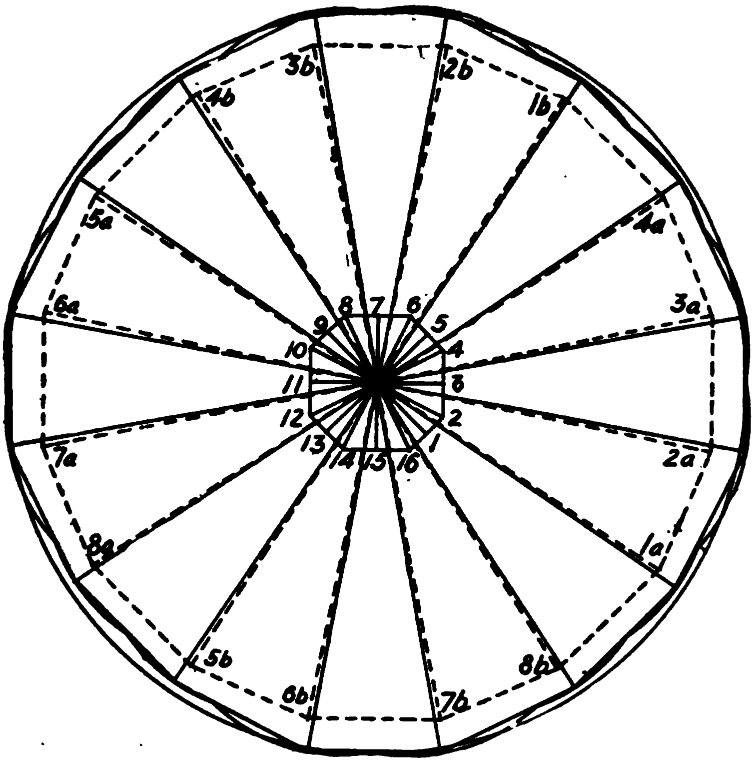


FIG. 21.—Field diagram for a two-phase motor with four slots per pole per phase and a coil-throw 1 to 7.

12. *Fractional pitch.* The diagrams may also be applied to fractional pitch windings. Fig. 20 shows a two-phase winding with four slots per pole per phase and 75 per cent. pitch. Fig. 21 gives the diagram for this winding. The inner small polygon gives the individual teeth fluxes; the outer polygon gives the space values of the field; the curve consisting of segments of circles gives the time values of the total field. These three

figures are derived in the same manner as that previously shown for full-pitch windings. Since in the case of fractional pitch windings the fluxes interlinking with the primary coils are smaller than the total primary fluxes, it is necessary to find the time

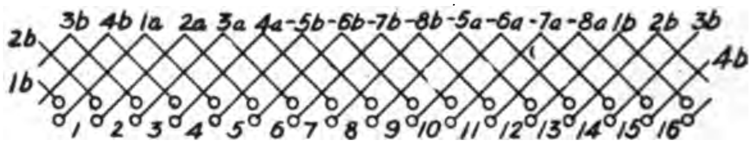


FIG. 22.—Field diagram for a two-phase motor with four slots per pole per phase and a coil-throw 1 to 5.

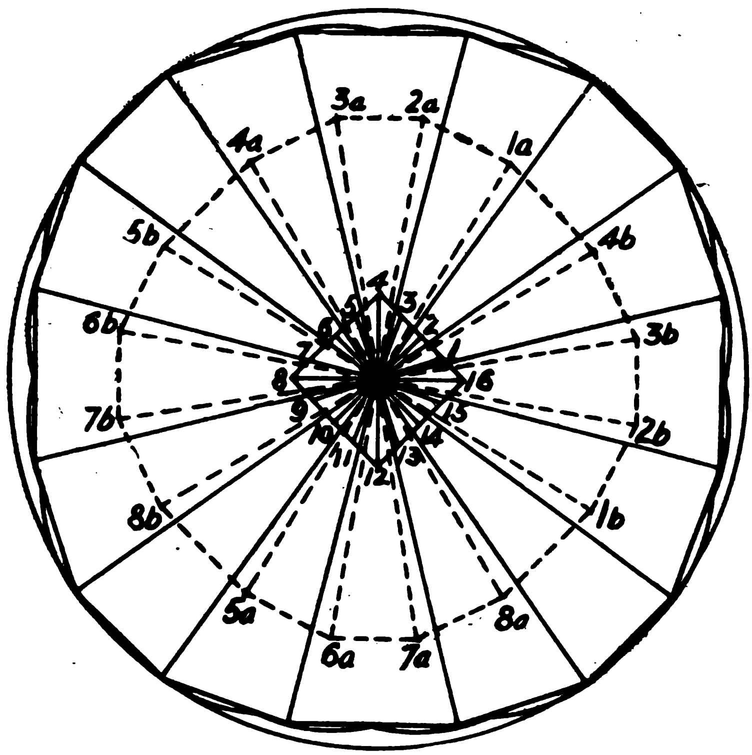


FIG. 23.

values of the fluxes interlinking with the primary coils in order to find the potentials induced in the primary coils. These fluxes are found for each coil by adding the fluxes of the teeth, surrounded by the coil under consideration. The primary coil fluxes thus obtained are shown in Fig. 21 by the vectors 1*a*, 2*a*, etc., which corresponds to the coils 1*a*, 2*a*, etc., of Fig. 20 respectively. The various factors may be derived in the same way as before for the full-pitch windings. The equivalent sine field is represented in Fig. 21 by the circle.

Figs. 22 and 23 give the same slot arrangement as before with a two-phase 50 per cent. pitch winding.

In Fig. 24 the diagrams for the space values of the field for all pitches for two phase 8 slots per pole between 0 per cent.

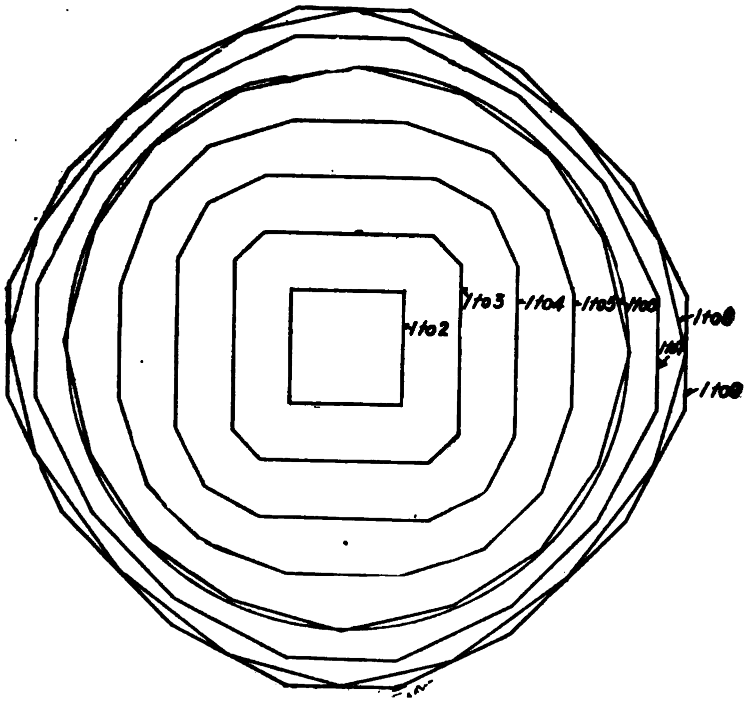


FIG. 24.—Diagram showing the space values of the rotating field for various pitches.

and 100 per cent. are shown. The numbers given indicate the throw of the coils, for which the various polygons apply. It appears that the full-pitch polygon throw 1 to 9 is flattened in a direction 45 degrees to the horizontal and vertical. The same applies to a smaller extent for the 87.5 per cent. pitch —throw 1 to 8— and the 75 per cent. pitch—throw 1 to 7— windings. For the 62.5 per cent. pitch—throw 1 to 6— winding the polygon is closest to a circle. The circle is shown here only for comparison. If the pitch is reduced further, the polygons become flattened in the vertical and horizontal direction and the flattening becomes the larger the more the pitch is reduced. This means that by reducing the coil pitch from full pitch, the field may be improved to a certain point; it will, however, become worse again if the pitch is reduced below a certain amount. Without delving further, it may be safely concluded that for the 75 and 62.5 per cent. pitches the secondary reactive currents and the higher harmonics in the primary are the smallest. It may be also shown that the leakage coefficient for this pitch is reduced. The 75 and 62.5 per cent. pitches are therefore advantageous in various respects for a two-phase winding and 8 slots per pole. For most windings it will be found that the best field is obtained when the phases overlap one-half to one-third. If the phases overlap entirely, the field conditions are not any better than for full pitch.

13. *Dead points.* It is well known that the starting torque of induction motors varies with the rotor position. This variation is due to the variation of the leakage with the rotor position. Since the diagrams shown in Figs. 8, 17, 18, and 19 give a means for studying the variation of the leakage with the rotor position, etc., they also give a means for studying the question of the varying starting torque.

14. *Noise in induction motors.* Although little can be said with any degree of certainty about motors running noisily, it is almost certain that the reactive currents in the secondary and the higher harmonics in the primary are two of the many fundamental causes of noise. Therefore the chances for noise can be reduced by keeping the amplitude of these currents as low as possible. In this respect the above diagrams show that three-phase motors are less likely to be noisy than two-phase motors with a corresponding pitch and number of slots. From Fig. 24 it follows that the chances for noisy operation may be greatly reduced by choosing the proper pitch; in the case under consideration the pitches are 75 and 62.5 per cent.

While it has not been possible to treat in this paper the various topics much in detail it will be readily seen that the graphical method before outlined will answer a great many questions. Moreover, the graphical treatment always has the advantage of giving a clearer conception of the physical facts than analytical formulas. The analytical treatment is rather complicated and requires in many cases too much time.

DISCUSSION ON "GRAPHICAL TREATMENT OF THE ROTATING FIELD." ATLANTIC CITY, N. J., JULY 2, 1908.

Comfort A. Adams: This is not an easy paper to discuss, for although it presents many familiar phenomena, it presents them in a new language with which it takes time to become familiar. It is a very good kind of language, however, as it assists greatly in visualizing the problem in hand. This is of value, not only to the beginner in getting a picture of what goes on, but to the man who has to make the calculations, because when he can see the problem clearly in his mind's eye he is far more apt to get accurate and reliable results from his calculations.

There are, of course, numerous methods of attacking any problem of this kind, with similar or identical results, but some methods are far more valuable because of their intimate connection with the physical side of the subject and this is true of the one presented by Mr. Hellmund. As to the comparative value of this method for the calculation of leakage, etc., I am not yet able to give an opinion, but it certainly is very interesting and throws much new light on several rather troublesome problems in induction motor analysis.

R. E. Hellmund: In regard to the belt leakage phenomena, the diagrams lead me to the conclusion that matters are more complicated than I had previously supposed. It occurred to me lately that the latter, unlike the other leakages, depend not only upon the magnetic dimensions, number of turns, etc., of the motor, but also varies largely with the secondary resistance. The belt leakage phenomena depend upon the current distribution of the secondary, and the secondary current distribution in turn changes with the secondary resistance. It may be shown, for instance, that in a squirrel-cage secondary with a resistance of zero, the current distribution is not even approximately sinusoidal but approaches closely to that of the primary, the secondary being assumed stationary. In this case the counter magnetomotive forces of the secondary are, at all parts of the circumference, equal and opposite to those of the primary. Therefoie the belt leakage fluxes cannot exist. Assuming the other extreme; that is, a secondary of very high resistance, the current distribution in the secondary will be approximately sinusoidal, and the secondary magnetomotive forces are no longer distributed like those of the primary. A certain differential action; that is, belt leakage fluxes, seems therefore to be possible in this case at certain parts of the circumference. It will be of little value to discuss these questions more in detail at this time since I have not yet arrived at definite conclusions as to their practical importance.

Discussion on "A Trigonometric Method for the Solution of Alternating-Current Problems." Atlantic City, N. J., July 2, 1908

Comfort A. Adams: Eight or ten years ago I worked out, for the purpose of transmission line calculations, a method which, with tables and curves, was an almost exact equivalent of Mr. Pender's method. If one had frequent occasion to make transmission line calculations these tables and curves would be a great assistance, but for infrequent use it takes longer to become familiar with the method than to solve the problem by the long way.

Moreover, I cannot help viewing a matter of this sort from the standpoint of the teacher, and from that standpoint it seems to me that these short-cut methods are undesirable since they do not bring out step by step the physical meaning of the problem; they do not keep one reminded of exactly what one is doing, and an error is less likely to be detected. The natural method has proved more serviceable in my own work, where a given type of problem is considered occasionally, and I think it far more valuable for students. However, I do not wish to minimize the value of any such labor-saving device, as it has a legitimate field where it serves a most useful purpose.

W. A. Del Mar (by letter): The treatment of alternating-current problems has been subject to rapid evolution. When Dr. Steinmetz's method of imaginary quantities appeared it seemed that the summit of development in that line had been reached, but Mr. Pender's method is as much in advance of the Steinmetz method as the latter was of the plain vector method which preceded it. It is a more important advance than that due to Steinmetz's method, because it places the solution of alternating-current problems within the reach of those of very limited mathematical training.

To the teachers Mr. Pender's method should particularly commend itself, as it can be quickly acquired and students can be taught the solution of alternating-current problems before they reach the fairly advanced mathematics. One effect of this would be the possibility of inculcating elementary ideas about alternating currents before the professor of mathematics had propounded the theory of vectors and imaginary quantities.

A preliminary reading of the new method gave the impression that for didactic purposes it would suffer from the disadvantage of not offering a direct physical understanding of the various operations. This impression soon vanished, however, when the ease of working became apparent, for the observations of the writer have led him to believe that only a small portion of technical graduates understand the theory of alternating-current machinery so that they can apply it to the solution of practical problems. This condition arises either from indifference to mathematics, or from the ease with which the subject is forgotten in after years. The simplicity of Dr. Pender's method overcomes these two difficulties in an admirable way.

The writer has had occasion to try the new method to solve a number of problems, including the eddy-current brake and transmission problems, and has found the application extremely simple in every case.

L. W. Rosenthal (by letter): I should like to point out the character of the error resulting from the approximation for E_0

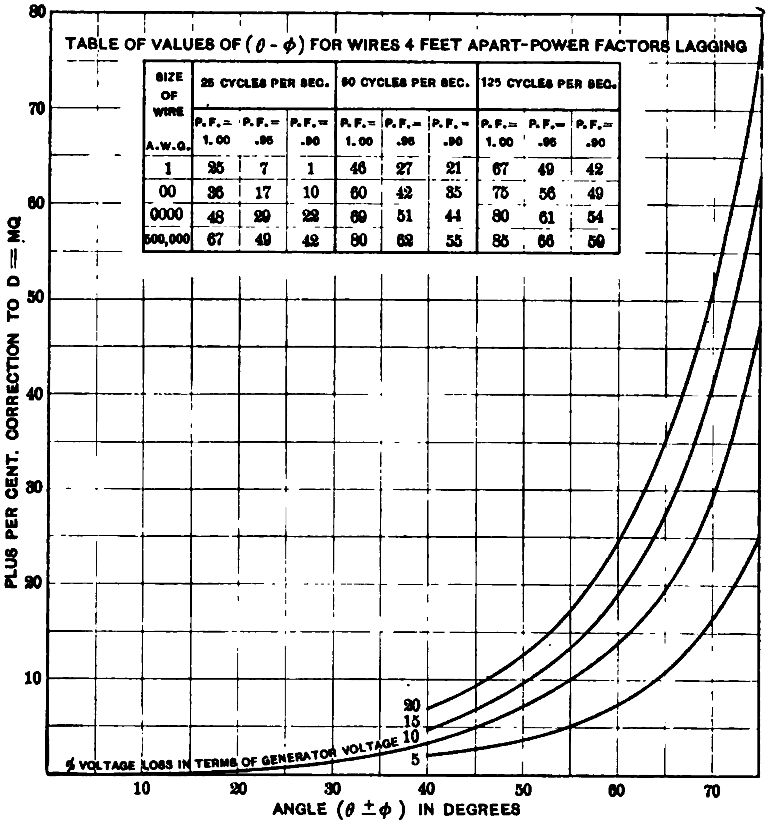


TABLE OF VALUES OF $(\theta - \phi)$ FOR WIRES 4 FEET APART-POWER FACTORS LAGGING

SIZE OF WIRE	25 CYCLES PER SEC.			60 CYCLES PER SEC.			125 CYCLES PER SEC.		
A.W.G.	P.F.= 1.00	P.F.= .95	P.F.= .90	P.F.= 1.00	P.F.= .95	P.F.= .90	P.F.= 1.00	P.F.= .95	P.F.= .90
1	25	7	1	46	27	21	67	49	42
00	36	17	10	60	42	35	75	56	49
0000	48	29	22	69	51	44	80	61	54
500,000	67	49	42	80	62	55	85	66	59

FIG. 1

between formulas (17) and (18). Denoting the power-factor angle of the load by ϕ and that of the line by θ, the error will depend on $(\theta + \phi)$ for leading currents and on $(\theta - \phi)$ for lagging currents. The error also depends on the per cent. voltage loss. The attached curves show the correction which must be applied to the approximate values of D, from which it is evident that for the range of 0 to 20 per cent. loss the approximation is sufficiently close only to about 45°. The values of $(\theta - \phi)$ for various wires, frequencies, and lagging power-factors are shown in the table on Fig. 1.

As an extreme case of the possible error, suppose in the first numerical example that the frequency be taken at 125 cycles per second, the power-factor at 100 per cent., and the load reduced to 3500 kw. in order to obtain about the same voltage loss. The exact formula (17) will then give a voltage loss of 21.8 per cent., while the approximate formula (18) shows a loss of 12.8 per cent. In other words, the "approximate value" requires a correction of 70 per cent., as is also shown by the curves at 75° and a loss of 18 per cent. of the generated volts, corresponding to 21.8 of the received volts. In the example chosen by the author $(\theta - \phi)$ equals 17°, and hence the error is small.

Although the author has drawn attention to the increasing error of the approximation by the letters *a*, *b*, and *c* in Table III, yet it would have been better to omit the values below *c*, which approximately correspond to $(\theta \pm \phi)$ equals 40°. It seems, therefore, that for high frequencies or power-factors, and for large wires or per cent. voltage losses, the approximate formulas (18) may give erroneous results. They should not be used in any case where the drop-factor M is below the *c* of its column.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 2, 1908.

THE RELATIVE PROPORTIONS OF COPPER AND IRON IN ALTERNATORS

BY CARL J. FECHHEIMER

Many radical changes have accompanied the rapid development of alternating-current generators. In the early days, when alternators were designed by rule-of-thumb methods, the relative proportion of copper to iron was high. These machines had very poor inherent regulation, and in many cases failed to give the voltage for which they were designed. These "copper machines" were relatively light, and were considerably cheaper than the "iron machines" which succeeded them. The iron machines, though much heavier and more expensive than their predecessors, had much better regulation. Following this, the characteristics of alternators were more carefully studied so that the designer was able to reduce the weight and again approach the copper machine. Competition between the various manufacturing companies made it necessary for their engineers to design cheaper machines, resulting in a compromise between the "copper" and "iron" machines. We are now at this period, and the designer is to-day confronted with the question: what shall be the relative proportions of copper and iron in this machine? Doubtless the answer is, so to proportion the iron and copper as to obtain the cheapest possible machine.

The usual method of carrying this out consists in designing a number of machines with different fluxes, and possibly different diameters, and, after calculating the weights and costs of the various parts, to select that design which is believed to be the most desirable. This method requires a great amount of time, and is very laborious. In this paper the writer has endeavored to bring out a method which will enable the designer to determine, after a few substitutions, what value of flux to employ to give the cheapest machine. He has made no attempt to determine mathematically the best diameter to use; in fact, he has assumed certain simple relations involving the diameter, such as the ratio of pole axial length to pole pitch. However, the relations are such that the designer, having studied and designed a number of machines in the past, can readily decide upon the best values to use for these constants.

So many variables enter into the design of alternators that it has been essential to establish definite relations between some of them in order to reduce their number. Some of these relations are empirical, in which case the evaluation of the constants involved in the empirical equations is left to the designer. Other relations, such as the fundamental equation used for determining the temperature rise of the armature coils, and the relation between field and armature ampere-turns per pole, are combinations of fundamental principles with empirical constants.

Briefly, the method which the writer has pursued in this paper is as follows: Equations are derived for the weights of the principal parts of the alternator, the weight in each case being expressed as some factor which is easily determined, multiplied by some power of the flux per pole. The weight of these parts is then multiplied by the price per pound of material used and by some other factor to allow for the unavoidable scrap material, and also for some of the parts not included in the equations, as for example, the stator teeth and the spider arms. The sum of these costs will give the cost of the material in the principal parts of the machine in terms of the flux. We can find what value of flux to employ to give the minimum cost of material in the parts considered by differentiating the equation obtained with respect to the flux, equating to zero, and solving the equation for the flux. Having determined the flux, the number of conductors necessary to give the desired electromotive force at once follows. The arrangement of these conductors, the diameter and length of the machine, the dimensions of the field conductor, etc., are left to the judgment of the designer, as has been the custom in the past.

The method is intended to apply to revolving-field alternators having rectangular poles. It is quite possible, however, to apply the method to machines of the revolving-armature type, although, judging from present indications, this type of machine will be built very little in the future. The round-pole machine necessitates so large a diameter that the cost of this type of machine would generally be much greater than the rectangular pole machine, even though there would be a considerable saving in field copper. There are special cases in which the round-pole machine is desirable, and in such cases it would be possible to modify the equations as derived by the writer.

The cost of the bases, shafts, bearings, and other " dead " material increases somewhat with the flux, but it is impossible to consider every detail in determining the most suitable value of flux to use. The labor also increases somewhat with the flux. As this, omitting the burden, is usually much smaller than the material, it has been neglected in the method which has been employed. The freight also increases with the quantity of flux employed, as a large flux machine means a machine that will be great in weight. All things considered, it will usually be best, from the standpoint of cost, to select a value for the flux which will be slightly less than that given by means of the equations.

If we were to employ a value of flux as given by the equation for the cheapest machine, it would be impossible in some cases, with our present methods of cooling, to prevent an excessive temperature rise, as, for example, in some of the small, high-speed, 25-cycle machines. In these, the fields become so crowded near the bases of the poles that it is very difficult to put sufficient copper in the fields to keep their temperature rise within limits, especially when the machine is to operate at low power-factor. In such cases, the best we can do is to approach as near to the copper machine as possible. In a few other cases there are limits which prevent the direct application of the writer's method.

In some cases it is possible to reduce the weight below the weight of the cheapest machine as determined by the writer's method, without sacrificing the electrical qualities of the machine. In such cases it may be advisable to build the lightest machine rather than the machine which will be the cheapest for the factory to build, especially when the machine is to go to some district to which the freight rates are very great.

There are many cases in which it is desirable to employ some other means of procedure than that brought out by the writer in this paper. The writer believes his method to be especially applicable to alternators which are to be entirely new or nearly so. For machines in which a large number of old parts can be used, it is usually best to make use of them rather than to build an entirely new machine in which the material is used most economically.

The method is only an approximation, but is believed by the writer to be sufficiently accurate for commercial purposes; for when we consider the variation in weight of yoke castings made from the same pattern, the not inconsiderable difference in temperature rise of duplicate machines, and the wide limits between which the market value of the raw material fluctuates (copper in particular), a rough approximation is doubtless sufficiently accurate.

As previously stated, no attempt has been made to determine mathematically the best diameter and axial length to employ. Within reasonable limits, a decrease in diameter (and increase in length) usually necessitates an increase in copper and a reduction in iron and total weight. Conversely, an increase in diameter requires an increase in total iron and a reduction in copper. This follows from the well known facts that the ventilation in a long machine is poorer than in a short machine; that the field mean-turn is greater in the former than in the latter; that the weight of the yoke and spider increase more rapidly with an increase in diameter than with an equivalent increase in length, etc. The labor is usually slightly less on the machine having the small diameter. The freight is generally less on the small diameter machine. There are some cases, as in engine-type, 60-cycle generators of small output, in which the machines must be made large in diameter to prevent so small a pole pitch as to cause the field to become too crowded. On the other hand, the diameter is often limited by the peripheral speed. The choice of diameter and length of machine depend upon the judgment of the designer.

The writer appreciates the fact that a mathematical treatment of this kind has a tendency to cause the designer to drift away from the physical conception of the problem in hand. The designer can, however, check up his calculations roughly as he proceeds, by comparing the weight of each part with the weight of corresponding parts of machines previously built, he having assumed an approximate value for the flux. He can further simplify his calculations by plotting curves. For example, he can plot a series of curves to aid him in evaluating K^x in equation (42). He may plot K^{x} against the number of poles, p, fixing the value of $K^{II} K^{IV}$, and plotting different curves for different values of B_a, the armature core density.

NOTATION

a = number of active conductors per slot = number of conductors per slot divided by number of parallel circuits in armature.

A = ampere conductors per inch of armature periphery, full load.

B_a = flux density in armature core in kilolines per sq. in.

$B_{f.r.}$ = flux density in field ring in kilolines per sq. in.

B_p = flux density in poles in kilolines per sq. in.

d_1 = outside diameter of field ring in inches.

d_2 = inside diameter of field ring in inches.

D_1 = outside diameter of armature laminations, in inches.

D_2 = inside diameter of armature laminations, in inches.

E = alternating-current electromotive force per phase or per leg if star connected.

E' = direct-current exciting electromotive force per pole.

F_a = equivalent section in square inches of one active armature conductor = section of one conductor multiplied by the number of parallel paths in armature.

F_f = section in square inches of field conductor.

H = radial dimension of pole including head.

I_a = alternating current amperes per leg in armature.

I_f = direct current amperes excitation on full field.

k = distribution factor = ratio of vector to algebraic sum of electromotive forces induced in armature conductors in one leg.

K = ratio of net to gross iron in poles.

K' = ratio of field ampere-turns per pole required for full field to the armature ampere conductors per pole with full load.

K^{II} = ratio of W to λ (ratio of tangential dimension of pole to axial length of pole).

K^{III} = resistivity in micro-ohms of an inch cube of copper at the temperature of the fields when hot = .85 approx.

K^{IV} = ratio of pole length λ to pole pitch τ

K^{V} $= \dfrac{3130\ K^{III}\ \sigma}{K\ B_p\ P_f}\ \dfrac{(K^{II}+1)^2}{K^{II}}\left[\dfrac{P_a\ K'}{k\ \sim}\right]^2$

K^{VI} = factor by which the product of amperes per square inch and ampere conductors per inch should be divided to give the temperature rise in degrees centigrade of armature coils.

K^{VII} = ratio of length of armature end connections per coil to pole pitch.

$$K^{VIII} = \frac{323 \times 10^6 \, (2 \, K^{IV} + K^{VII})}{p \, T \, K^{VI}} \left[\frac{P_a}{\sim k} \right]^2$$

K^{IX} = ratio of net iron in armature to axial length of pole λ.

$$K^{X} = \frac{22{,}000}{B_a} \sqrt{\frac{K \, K^{II} \, B_P}{10 \, \sigma}} \left(\frac{1}{B_a \, K^{IX}} + \frac{0.636 \, p \, \sigma}{K \, K^{II} \, K^{IV} \, B_P} \right)$$

K^{XI} = coefficient used for determining weight of yoke =

$$\frac{W_y}{D_1^{\frac{3}{2}} \, \lambda^{\frac{1}{2}}}$$

$$K^{XII} = 565 \, K^{XI} \sqrt{\frac{p}{K^{IV}}} \left[\frac{.318 \, \sigma \, p}{K \, K^{II} \, K^{IV} \, B_P} + \frac{1.5}{K^{IX} \, B_a} \right]$$

K^{XIII} = ratio of pole radial heighth H, to square root of pole waist W.

K^{XIV} = $1580 \, p \, K^{XIII} \, (K^{II}/K)^{\frac{1}{4}} \, (\sigma/B_P)^{\frac{5}{4}}$

$$K^{XV} = \frac{4450 \, \sigma'}{B_{f.r.}} \sqrt{\frac{K \, K^{II} \, B_P}{\sigma}} \left(\frac{p \, \sigma}{K \, K^{II} \, K^{IV} \, B_P} - \frac{1.57 \, \sigma'}{B_{f.r.}} \right)$$

$$K^{XVI} = \frac{4900 \, \sigma'}{B_{f.r.}} K^{XIII} \left(\frac{\sigma \, K^{II}}{K \, B_P} \right)^{\frac{1}{4}}$$

$(M.T.)_a$ = mean length of turn, in inches, of armature coil.

$(M.T.)_f$ = mean length of turn, in inches, of field coil.

N = number of field turns per pole.

n = number of phases.

p = number of poles.

p_a = price, in dollars per pound, of armature copper, including transportation, etc.

p_{ai} = price, in dollars per pound, of armature active iron, including transportation, etc.

p_f = price, in dollars per pound, of field copper, including transportation, etc.

$p_{f.r.}$ = price, in dollars per pound, of field ring material, including transportation, etc.

p_p = price, in dollars per pound, of pole iron, including transportation, etc.

p_y = price, in dollars per pound, of yoke iron, including transportation, etc.

P_a = rated kilovolt-ampere output of alternator.

P_f = kilowatts excitation required for full field on alternator.

S = number of stator slots per phase per pole.

S_a = cost of armature copper in dollars.

S_{ai} = cost of armature active iron in dollars.

S_f = cost of field copper in dollars.

S_m = cost of material for principal parts of alternator in dollars.

$S_{f.r.}$ = cost of field ring in dollars.

S_p = cost of poles in dollars.

S_y = cost of yoke in dollars.

T = temperature rise in degrees centigrade, of armature coils.

W = pole waist, or tangential dimension of pole.

W_a = weight in pounds of armature copper.

W_{ai} = weight in pounds of armature active iron.

W_f = weight in pounds of field copper.

$W_{f.r.}$ = weight in pounds of field ring.

W_p = weight in pounds of poles.

W_y = weight in pounds of yoke.

X = full field ampere-turns per pole.

Greek Letters

α_1 $= p_f K^{\mathrm{v}}$

α_2 $= p_a K^{\mathrm{viii}}$

α_3 $= p_{ai} K^{\mathrm{x}} + p_{f.r.} K^{\mathrm{xv}}$

α_4 $= p_y K^{\mathrm{xii}}$

α_5 $= p_p K^{\mathrm{xiv}} - p_{f.r.} K^{\mathrm{xvi}}$

Δ = current density in armature copper, amperes per square inch

λ = length of pole parallel to shaft.

σ = pole leakage factor $= \dfrac{\text{flux in one pole}}{\text{useful flux per pole.}}$

σ' = field ring leakage factor.

τ = pole pitch in inches.

ϕ = flux in megalines per pole.

$\sim$ = frequency, cycles per second.

WEIGHT AND COST OF MATERIAL REQUIRED FOR ALTERNATORS.

FIELD COPPER

We have the following well-known fundamental equations:

$$\phi = \frac{100\,E}{2.22\,a\,S \sim p\,k} \tag{1}$$

$$A = \frac{a\,S\,I_a\,n}{\tau} \tag{2}$$

$$B_P = \frac{10^8\,\phi\,\sigma}{W\,\lambda\,K} \tag{3}$$

Combining the above equations, we have:

$$\phi = \frac{100\,E\,I_a\,n}{2.22\,p\,k \sim A\,\tau} = \frac{W\,\lambda\,B_P\,K}{10^8\,\sigma} \tag{4}$$

The ratio of the field ampere-turns per pole required for full field to the armature ampere conductors per pole is fixed, within reasonable limits, by the regulation required of the alternator, the power-factor of the circuit, the amount of overload required of the generator, and the manner in which the magnetic circuit is proportioned; it being understood that the alternator is to give its normal voltage when operating at the proper frequency and subjected to the specified overload at the power-factor of the circuit. This ratio varies from 1 to 2.2 in polyphase machines, depending upon the factors given above, and in accordance with guarantees required by consulting engineers at the present time. Calling this ratio K', we have:

$$X = K'\,A\,\tau \tag{5}$$

Combining (4) and (5),

$$X = \frac{10^8\,E\,I_a\,n\,K'\,\sigma}{2.22\,p\,k \sim W\,\lambda\,B_P\,K}$$

But $E\,I_a\,n = 10^3\,P_a$ (6)

Therefore,

$$X = \frac{10^8\,P_a\,K'\,\sigma}{2.22\,p\,k \sim W\,\lambda\,B_P\,K} \tag{7}$$

The following equations are well known:

$$F_f = \frac{K^{III}\,X\,(M.T.)_f}{10^6\,E'} \tag{8}$$

$$W_f = 0.318\,(M.T.)_f\,N.\,p\,F_f \tag{9}$$

And the following is sufficiently accurate for the present estimate, when the poles are rectangular in section:

$$(M.T.)_f = 2.2\,(W + \lambda) \qquad (10)$$

Combining (8), (9) and (10),

$$W_f = \frac{1.54\,K^{III}\,X\,p\,N\,(W + \lambda)^2}{10^6\,E'} \qquad (11)$$

The designer will know approximately at the start what the ratio of the tangential dimension, W, of the pole, to the axial length of the pole, will be. This ratio varies from 1 in high speed 25 cycle machines to 4.0 in slow speed 60 cycle generators. Calling this ratio K^{II}, we have

$$W = K^{II}\,\lambda \qquad (12)$$

The amperes excitation on full field is:

$$I_f = \frac{X}{N} \qquad (13)$$

Combining (11), (12) and (13),

$$W_f\,I_f = \frac{1.54\,(K^{II} + 1)^2\,\lambda^2\,p\,K^{III}\,X^2}{10^6\,E'} \qquad (14)$$

Combining (7) and (14),

$$W_f\,(I_f\,E'\,p) = 31.3\,K^{III}\,(K^{II} + 1)^2\,\lambda^2\,10^8 \left(\frac{P_a\,K'\,\sigma}{k \sim W\,\lambda\,B_p\,K}\right)^2 \qquad (15)$$

But

$$I_f\,E'\,p = 10^3\,P_f \qquad (16)$$

and from (3),

$$W\,\lambda\,B_p\,K = 10^8\,\sigma\,\phi \qquad (17)$$

The designer will recognize the fact that the ratio of pole length to pole pitch is very nearly fixed, varying from .5 in high speed 25 cycle machines to 2 in slow speed 60 cycle machines. Calling this ratio K^{IV}, we have

$$\lambda = K^{IV}\,\tau \qquad (18)$$

Combining (12) and (18),

$$W = K^{II} K^{IV} \tau \tag{19}$$

Combining (3), (18) and (19),

$$\lambda = 10 \sqrt{\frac{10\, \phi\, \sigma}{K^{II} K B_p}} \tag{20}$$

Combining (15), (16), (17) and (20),

$$W_f\, P_f = \left\{ \frac{3130\, K^{III}\, \sigma}{K\, B_p} \; \frac{(K^{II}+1)^2}{K^{II}} \left[\frac{P_a\, K'}{\sim k} \right]^2 \right\} \frac{1}{\phi} \tag{21}$$

Equation (21) illustrates a condition with which designers are familiar; that is, without sacrificing the good qualities of the generator, the excitation can be reduced only at the expense of field copper; and if we approach a copper machine by decreasing the flux, it results in a proportional increase in weight of field copper for a given excitation.

It is usually known at the start what full field excitation to allow, this being limited in some cases by heating; while in other cases it is necessary to reduce the excitation to prevent its becoming too large a percentage of the output of the alternator. As typical examples of the first class, we have the moderate and high speed 25 cycle alternators up to about 500 kw. output, in which the output of the machine must be reduced to prevent excessive field heating when operating at low power factors. As examples of the second class we have fairly large (1000 kw. or over) engine-type, 60-cycle generators, the field heating of which seldom exceeds 40 degrees rise when operating on full field, the single layer edge-wound field being used in both cases.* Therefore, the designer, knowing the full field excitation in kilowatts at the start (having made a study of the limiting conditions), will know the value of all of the quantities in equation (21), except the flux ϕ and the weight of field copper W_f. Hence,

$$W_f = \left\{ \frac{3130\, K^{III}\, \sigma}{K\, B_p\, P_f} \; \frac{(K^{II}+1)^2}{K^{II}} \left[\frac{P_a\, K'}{\sim k} \right]^2 \right\} \frac{1}{\phi} \tag{22}$$

* A means of determining that value to assign to the excitation, which will make the cost of the entire equipment a minimum, is given in the appendix.

And $$W_f = K^v \frac{1}{\phi} \tag{23}$$

Where K^v is the expression in the brace in equation (22).

If p_f represent the price in dollars of one pound of field copper, the total cost of field copper for the alternator is:

$$S_f = p_f K^v \frac{1}{\phi} \tag{24}$$

ARMATURE COPPER.

The weight of copper required for the armature of an alternator depends primarily upon the temperature rise per watt loss per square inch of radiating surface. The loss is not only affected by the core loss and copper loss due to the resistance of the copper, but also by the eddy-current loss in the copper. The core loss differs considerably in duplicate machines, due chiefly to variation in material. All revolving-field alternators depend for the cooling of their armatures upon the air thrown into the stator by the rotating element. It frequently happens that this air is not uniformly distributed over the surface of the armature, causing a greater cooling at one side than at the other side of the armature, resulting in a greater elevation in temperature in one part of the armature than in another. The designer must allow for the maximum elevation in temperature. With the inconsistent data obtained from heat runs on a large number of machines, it is next to impossible to obtain any reliable means of predetermining the temperature rise within a few per cent. The best that can be done is to obtain a rough approximation. Machines having the same quantity of air thrown upon each square inch of stator surface per unit of time, and when worked at the same magnetic densities in the stator, will, when operated on open circuit, have approximately the same elevation on the stator iron. It is therefore reasonable that the incremental heating due to the copper loss be alone considered.

It may be easily shown that the watts copper loss (due to resistance only) per square inch of surface is proportional to the product of ampere-conductors per inch and the current density in the armature conductor (in amperes per square inch). The surface is taken as the cylindrical surface of the armature iron plus the surface of the end-connections. Hence, we may write:

$$T = \frac{A \Lambda}{K^{vi}} \tag{25}$$

T is the temperature rise in degrees centigrade, A the ampere conductors per inch, and Δ the density in amperes per square inch. K^{vi} varies in values from 1.8×10^4 in poorly ventilated, or slow speed, high voltage machines to 9×10^4 in low-voltage, well ventilated machines, not having appreciable eddy current loss in the conductors. These values of K^{vi} are consistent with modern practice in methods employed in cooling the stators of revolving field alternators. If at some future time the methods of cooling are improved, K^{vi} should be modified accordingly.

Since $$\Delta = \frac{I_a}{F_a}, \tag{26}$$

where I_a = alternating current amperes per leg, and F_a = section of one active armature conductor, we have, by combining with (2) and (25),

$$a\,S = \frac{T\,F_a\,K^{vi}\,\tau}{I_a^2\,n} \tag{27}$$

The weight of armature copper in pounds is:

$$W_a = .318\,(M.T.)_a\,F_a\,a\,\frac{S}{2}\,n\,p \tag{28}$$

The mean length of turn is given by:

$$(M.T.)_a = 2\,\lambda + K^{vii}\,\tau \tag{29}$$

K^{vii} has a value of about 4.2 for 2 or 3 phase chain windings and about 3.6 for full pitch double layer windings. It is somewhat greater than 4.2 for chain windings intended for high voltage machines; and for short pitch double layer windings, it should be correspondingly reduced.

Combining (27), (28), (29) and (18),

$$W_a = \left[\frac{.159\,(2\,K^{iv} + K^{vii})\,T\,K^{vi}\,p}{I_a^2}\right](\tau\,F_a)^2 \tag{30}$$

Eliminating $a\,S$ between (1) and (27),

$$F_a\,\tau = \frac{100\,E\,I_a^2\,n}{2.22\,p \sim k\,T\,K^{vi}}\;\frac{1}{\phi} \tag{31}$$

Combining (30), (31) and (6),

$$W_a = \left\{ \frac{323 \times 10^6 \ (2\,K^{\text{IV}} + K^{\text{VII}})}{p\,T\,K^{\text{VI}}} \left[\frac{P_a}{\sim k}\right]^2 \right\} \frac{1}{\phi^2} \qquad (32)$$

And

$$W_a = K^{\text{VIII}} \frac{1}{\phi^2} \qquad (33)$$

where K^{VIII} is the expression in the brace in equation (32).

This equation illustrates how rapidly the cost of copper increases as the flux is reduced and we approach a "copper" machine.

If p_a represent, in dollars per pound, the price of copper used in the armature, then the cost of armature copper is:

$$S_a = p_a\,K^{\text{VIII}} \frac{1}{\phi^2} \qquad (34)$$

The value of K^{VIII} should be increased by about 5 to 15 per cent. to allow for scrap.

Armature Active Iron.

In punching the laminations for the armature there is always an unavoidable amount of scrap, varying from 25 to 60 per cent. This scrap, or at least a portion of it, should be charged up to the cost of the material required. It complicates the equations considerably, to bring in the iron in the teeth, while it is not very difficult to form an equation considering only the iron behind the teeth. As it is necessary to add a percentage for the scrap material, it is easy to increase this percentage so as to include the iron in the teeth. This is the method pursued, no attempt being made in the equations to find to what extent the weight of the teeth is affected by the flux.

Neglecting the fact that the diameter of the armature is not exactly the same at the bore as at the tooth roots, we have

$$W_{ai} = 0.28\ (D_1^2 - D_2^2)\ \frac{\pi}{4}\ K^{\text{IX}}\ \lambda = 0.22\ (D_1 - D_2)\ (D_1 + D_2)\ K^{\text{IX}}\ \lambda \qquad (35)$$

Since we are considering only that portion of the armature active iron through which the flux passes after emerging from the teeth, the depth of the armature punching is $\frac{1}{2}(D_1 - D_2)$. If B_a represent the flux density in the armature behind the teeth, in kilolines per square inch, we have:

$$B_a = \frac{10^3 \phi}{\lambda (D_1 - D_2) K^{IX}} \tag{36}$$

Whence,

$$(D_1 - D_2) = \frac{10^3 \phi}{\lambda B_a K^{IX}} \tag{37}$$

Also

$$D_1 + D_2) = (D_1 - D_2) + 2 D_2 = \frac{10^3 \phi}{\lambda B_a K^{IX}} + 2 D_2 \tag{38}$$

But

$$D_2 = \frac{\tau p}{\pi} \tag{39}$$

Combining (18) and (39),

$$D_2 = \frac{\lambda p}{K^{IV} \pi} \tag{40}$$

Combining (35), (37), (38) and (40),

$$W_{ai} = 220 \frac{\phi}{B_a} \left(\frac{10^3 \phi}{B_a \lambda K^{IX}} + \frac{2 \lambda p}{K^{IV} \pi} \right) \tag{41}$$

Substituting for λ from equation (20),

$$W_{ai} = \left\{ \frac{22000}{B_a} \sqrt{\frac{K^{II} K B_P}{10 \sigma}} \left(\frac{1}{B_a K^{IX}} + \frac{.636 p \sigma}{K^{IV} K^{II} K B_P} \right) \right\} \phi^{\frac{3}{2}} \tag{42}$$

And

$$W_{ai} = K^{X} \phi^{\frac{3}{2}} \tag{43}$$

where K^{X} is the expression in the brace in equation (42).

The weight of armature active iron as expressed in (42), should, of course, be multiplied by a factor to allow for the weight of teeth and unavoidable scrap previously referred to. If p_{ai} represent the price, in dollars per pound, of the iron used in the armature, then the cost of armature iron S_{ai} will be:

$$S_{ai} = p_{ai} K^{X} \phi^{\frac{3}{2}} \tag{44}$$

Even in the so-called " copper machine." the cost of armature active iron is usually as great as that of the armature copper, and in most cases is slightly greater. As we depart from the " copper machine " and approach the " iron machine," the armature active iron increases very rapidly, as shown by the above equation; and this partially accounts for the fact that the " iron machine " compares unfavorably in cost with the " copper machine ".

The Yoke.

Although the mechanical design of alternator yokes has undergone many changes, the writer has been able, by comparing the weights of consistently designed yokes, to obtain an equation which will give the weight within about 15 per cent. The yokes considered for obtaining the equation belonged to a line of alternators, the bores of which were from 25 to 250 inches, and whose pole lengths were from 6 to 20 inches. They were all of modern, open type, and the material used in them was cast iron. The equation is:

$$W_y = K^{XI} D_1^{\frac{3}{2}} \lambda^{\frac{1}{2}} \qquad (45)$$

W_y is the weight of the yoke in pounds, D_1 the outside diameter of the stator laminations in inches, λ the length of pole parallel to the shaft, and K^{XI} a coefficient whose value lies between 1.2 and 3, the exact value depending upon the design adopted. K^{XI} had nearly the same value for the yokes considered for obtaining the equation. The extreme values are for yokes which were very light or very heavy.

From equations (37), (39) and (18) we have

$$D_1 = D_2 + (D_1 - D_2) = \frac{\lambda p}{K^{IV} \pi} + \frac{10^8 \phi}{\lambda B_a K^{IX}} \qquad (46)$$

Hence

$$W_y = K^{XI} \lambda^{\frac{1}{2}} \left[\frac{\lambda p}{K^{IV} \pi} + \frac{10^8 \phi}{\lambda B_a K^{IX}} \right]^{\frac{3}{2}} \qquad (47)$$

In expanding the above binomial, it is unnecessary for our purposes to consider more than two terms in the series; since, except for very small diameter machines, the first term in the binomial is considerably greater than the second; and secondly, the fourth term of the series is negative and nearly cancels the positive third, the negative sixth term nearly cancels the positive fifth, etc. We have then, after expanding,

$$W_y = K^{XI}\,\lambda^{\frac{1}{2}}\left[\left(\frac{\lambda\,p}{K^{IV}\,\pi}\right)^{\frac{3}{2}}+\frac{3}{2}\left(\frac{\lambda\,p}{K^{IV}\,\pi}\right)^{\frac{1}{2}}\frac{10^3\,\phi}{B_a\,K^{IX}\,\lambda}\right] \tag{48}$$

Substituting for λ from (20) and simplifying,

$$W_y = \left\{565\,K^{XI}\sqrt{\frac{p}{K^{IV}}}\left[\frac{.318\,\sigma\,p}{K\,K^{II}\,K^{IV}\,B_p}+\frac{1.5}{K^{IX}\,B_a}\right]\right\}\phi \tag{49}$$

And $$W_y = K^{XII}\,\phi \tag{50}$$

where K^{XII} is the expression in the brace in (49).

If p_y represent the price of material used in the yoke in dollars per pound, and S_y the cost of material required for the yoke, then

$$S_y = p_y\,K^{XII}\,\phi \tag{51}$$

In order to allow for the errors introduced by neglecting the depth of the stator teeth, and by discarding all but the first two terms in the series after expanding the binominal, the value of K^{XII} should be increased by about 15 per cent.

Poles.

The radial height, H, of the pole evidently must depend upon the number of ampere turns required on full field. As previously pointed out, the full field ampere turns are proportional to the armature ampere-conductors per pole, and these latter are equal to the product of the pole pitch and ampere conductors per inch (see equation 5). Since the pole waist, W, must also be nearly proportional to the pole pitch, H should increase in some definite relation with W. Comparing the dimensions of a large number of poles for consistently designed alternators having single layer edge-wound fields, the writer has found the following relation to hold:

$$H = K^{XIII}\sqrt{W} \tag{52}$$

K^{XIII} lies between 2.5 and 4.5, depending upon the number of ampere conductors per inch; and the designer who has the data of a large number of alternators at his command, can readily determine the best value for this coefficient. H includes the radial dimension of the pole head, the projections of which are used for holding the field copper in place when the alternator is in operation; it does not include the radial dimension of the dove-tail, if dove-tails be used for fastening the poles to the field ring. If the projections of the pole head be neglected in comparison with the remainder of the pole, we may write as the weight W_P of the poles in pounds:

$$W_P = 0.28\, p\, W\, K\, \lambda\, H \tag{53}$$

Substituting for W from (12), for H from (52) and for λ from (20), we have:

$$W_P = \left\{ 1580\, p\, K^{XIII} \left(\frac{K^{II}}{K}\right)^{\frac{1}{4}} \left(\frac{\sigma}{B_P}\right)^{\frac{5}{4}} \right\} \phi^{\frac{5}{4}} \tag{54}$$

And
$$W_P = K^{XIV}\, \phi^{\frac{5}{4}} \tag{55}$$

where K^{XIV} is the expression in the brace in (54).

Representing by p_P the price, in dollars per pound, of the iron used in the poles, and by S_P the cost of material for the poles, we have:

$$S_P = p_P\, K^{XIV}\, \phi^{\frac{5}{4}} \tag{56}$$

K^{XIV} should be increased by some percentage to allow for the pole tips (which were neglected in the derivation of the equation), and for the scrap material.

Field Ring.

In the following derivation, it is assumed that if the field ring be made sufficiently large to carry the flux (allowing for poor material, blow holes, etc.), it will be large enough for mechanical purposes. This is usually the case, although it occasionally happens that it is necessary to make the field ring heavier for mechanical reasons than would be demanded for electrical purposes.

Considering the field ring to be solid and of the same axial length as the pole, we have as its weight:

$$W_{f.r.} = .28 \times \frac{\pi}{4}\, \lambda\, (d_1^2 - d_2^2) = .22\, \lambda\, (d_1 - d_2)\, (d_1 + d_2) \tag{57}$$

But

$$(d_1 - d_2) = \frac{10^8\, \phi\, \sigma}{B_{f.r.}\, \lambda} \tag{58}$$

And
$$(d_1 + d_2) = 2\, d_1 - (d_1 - d_2) \tag{59}$$

From (39) and (52)

$$d_1 = D_2 - 2\, H = \frac{\tau\, p}{\pi} - 2\, K^{\mathrm{XIII}} \sqrt{W} \tag{60}$$

Combining (12), (18), (59) and (60),

$$(d_1 + d_2) = \frac{2\, \lambda\, p}{\pi\, K^{\mathrm{IV}}} - 4\, K^{\mathrm{XIII}} \sqrt{K^{\mathrm{II}}\, \lambda} - \frac{10^8\, \phi\, \sigma'}{B_{f.r.}\, \lambda} \tag{61}$$

Substituting (58) and (61) in (57)

$$W_{f.r.} = .22 \left(\frac{10^8 \phi\, \sigma'}{B_{f.r.}}\right) \left(\frac{2\, \lambda\, p}{\pi\, K^{\mathrm{IV}}} - 4\, K^{\mathrm{XIII}} \sqrt{K^{\mathrm{II}}\, \lambda} - \frac{10^8 \phi\, \sigma'}{B_{f.r.}\, \lambda}\right) \tag{62}$$

Substituting for λ from (20),

$$W_{f.r.} = \left\{ \frac{4450\, \sigma'}{B_{f.r.}} \sqrt{\frac{K\, K^{\mathrm{II}}\, B_{\mathrm{P}}}{\sigma}} \left(\frac{p\, \sigma}{K\, K^{\mathrm{II}}\; K^{\mathrm{IV}}\, B_{\mathrm{P}}} - \frac{1.57\, \sigma'}{B_{f.r.}} \right) \right\} \phi^{\frac{3}{2}} -$$

$$\left\{ \frac{4900\, \sigma'}{B_{f.r.}}\, K^{\mathrm{XIII}} \left(\frac{\sigma\, K^{\mathrm{II}}}{K\, B_{\mathrm{P}}} \right)^{\frac{1}{4}} \right\} \phi^{\frac{5}{4}} \tag{63}$$

And
$$W_{f.r.} = K^{\mathrm{XV}}\, \phi^{\frac{3}{2}} - K^{\mathrm{XVI}}\, \phi^{\frac{5}{4}} \tag{64}$$

where K^{XV} and K^{XVI} are respectively the first and second expressions in braces in (63).

Representing by $p_{f.r.}$ the price in dollars per pound of the field ring material, and by $S_{f.r.}$ the cost of the field ring, we have:

$$S_{f.r.} = p_{f.r.}\, K^{\mathrm{XV}}\, \phi^{\frac{3}{2}} - p_{f.r.}\, K^{\mathrm{XVI}}\, \phi^{\frac{5}{4}} \tag{65}$$

K^{XV} and K^{XVI} should be increased by some percentage to allow for the spider arms, the weight of which will depend somewhat upon the flux.

Collection of Terms for the Final Equation.

Summing up the costs of material required for the principal parts of the alternator, we have

$$S_m = S_f + S_a + S_{ai} + S_y + S_p + S_{f.r.} \tag{66}$$

$$= p_f\, K^{\mathrm{v}}\, \frac{1}{\phi} + p_a\, K^{\mathrm{viii}}\, \frac{1}{\phi^2} + p_{ai}\, K^{\mathrm{x}}\, \phi^{\frac{3}{2}} + p_y\, K^{\mathrm{xii}}\, \phi + p_p\, K^{\mathrm{xiv}}\, \phi^{\frac{5}{4}}$$

$$+ p_{f.r.}\, K^{\mathrm{xv}}\, \phi^{\frac{3}{2}} - p_{f.r.}\, K^{\mathrm{xvi}}\, \phi^{\frac{5}{4}} \tag{67}$$

Each of the terms in the right-hand member of equation (67) should be multiplied by some factor to allow for the unavoidable scrap and for parts which are not allowed for in the equation, such as the stator teeth, the pole tips, the spider arms, etc.

Rewriting equation (67), we have:

$$S_m = p_f\, K^{\mathrm{v}}\, \frac{1}{\phi} + p_a\, K^{\mathrm{viii}}\, \frac{1}{\phi^2} + (p_{ai}\, K^{\mathrm{x}} + p_{f.r.}\, K^{\mathrm{xv}})\, \phi^{\frac{3}{2}} + p_y K^{\mathrm{xii}}\, \phi$$

$$+ (p_p\, K^{\mathrm{xiv}} - p_{f.r.}\, K^{\mathrm{xvi}})\, \phi^{\frac{5}{4}} \tag{68}$$

Or

$$S_m = \alpha_1 \frac{1}{\phi} + \alpha\ \frac{1}{\phi^2} + \alpha_3\, \phi^{\frac{3}{2}} + \alpha_4\, \phi + \alpha_5\, \phi^{\frac{5}{4}} \tag{69}$$

where α_1, α_2, etc., are respectively p_f, K^{v}, $p_a\, K^{\mathrm{viii}}$, etc.

Differentiating equation (69) with respect to ϕ, we have:

$$\frac{d\, S_m}{d\, \phi} = -\frac{\alpha_1}{\phi^2} - \frac{2\, \alpha_2}{\phi^3} + \frac{3}{2}\, \alpha_3\, \phi^{\frac{1}{2}} + \alpha_4 + \frac{5}{4}\, \alpha_5\, \phi^{\frac{1}{4}} \tag{70}$$

S_m will be a minimum when $\frac{d\, S_m}{d\, \phi} = 0$; hence, after clearing we have:

$$\frac{3}{2} \alpha_3\, \phi^{\frac{7}{2}} + \alpha_4\, \phi^3 + \frac{5}{4}\, \alpha_5\, \phi^{\frac{13}{4}} - \alpha_1\, \phi - 2\, \alpha_2 = 0 \tag{71}$$

This equation is of such high degree that, as well as the writer has been able to determine, it can be solved only by trial. However, after having determined the values of the various constants, the writer has been able to find that value of

ϕ which will satisfy the equation in two or three trials, requiring from three to five minutes. Equation (71) is used in determining that value of flux to employ to obtain the most economical use of material. The algebraic values of the constants occurring in equation (71) will be found in the notation.

Example of the Application of the Writer's Method.

Design of an 800-kw. at 85% power-factor, 480 volts, three-phase, 60-cycle, 360 rev. per min. alternator to meet the following specifications:

Regulation. Rise in voltage with 800-kw. load, 100% power-factor shall not exceed 8% above normal, speed and excitation remaining constant. Rise in voltage with 800-kw. load, at 85% power-factor, shall not exceed 22% above normal, speed and excitation remaining constant.

Heating. After operating for 24 hours with 800 kw. load at 85% power-factor, the temperature rise shall not exceed 35° cent. in any part. After operating for two hours following the full-load run, with a load 25% above full load at 85% power-factor, the temperature rise shall not exceed 45° cent. in any part.

Efficiency. With 85% power-factor, the generator will have an efficiency not less than 93.5% at full load; 92.2% at 0.75 load; 89.5% at 0.50 load. Friction and windage to be included in the losses.

Excitation. The excitation will be at 120 volts. The excitation will not exceed 125 amperes when the generator yields its full load at 100% power-factor; the excitation will not exceed 180 amperes when the generator yields its full load in kilowatts at 85% power-factor.

The generator is to be capable of delivering its normal voltage of 480 when operated at 25% overload in kilowatts at 85% power-factor.

The generator must be able to stand an overspeed of 50%.

In accordance with the above specifications, the generator will have an output of 942 kilovolt-amperes, and will have 20 poles. We shall assume that if the field ring be made of cast steel, and large enough to carry the flux, it will be sufficiently strong to stand the 50% overspeed. This will be checked after the machine is designed.

As the voltage is low, the armature conductor will be large, and there will be an eddy-current loss in the copper. We shall therefore take $K^{vi} = 3.5 \times 10^4$, and $T = 30°$, allowing a margin of 5 degrees.

The alternator must have sufficient field margin to give its voltage with 25% overload in kilowatts at 85% power-factor. We shall, therefore, take $K'=1.8$. We shall assume 23.5 kw. excitation on full field, this being 2.5% of the output of the alternator. We shall then be able to meet the excitation guarantees.

We shall assume the following values for the quantities in the equations:

$B_a=33$; $B_{f.r.}=63$; $B_p=97$; $k=.96$; $K=.92$; $K'=1.8$; $K^{II}=.45$; $K^{III}=.85$; $K^{IV}=1$; $K^{VI}=3.5\times 10^4$; $K^{VII}=3.7$; $K^{IX}=.81$; $K^{XI}=1.9$; $K^{XIII}=2.5$; $p=20$; $p_a=.23$; $p_{ai}=.037$; $p_f=.2$; $p_{f.r.}=.036$; $p_p=.028$; $p_y=.025$; $P_a=942$; $P_f=23.5$; $T=30$; $\sigma=1.2$; $\sigma'=1.28$; $\sim=60$.

Substituting the above values, we obtain the following:

$K^{V}=6300$; $K^{VIII}=23{,}000$; $K^{X}=510$; $K^{XII}=1180$; $K^{XIV}=380$; $K^{XV}=354$; $K^{XVI}=99$.

We shall increase the value of these factors by the following percentages to allow for scrap and parts not included:

Field copper (K^{V}) 0%; armature copper (K^{VIII}) 15%; armature laminations (K^{X}) 70%; poles (K^{XIV}) 25%; yoke (K^{XII}) 15%; field ring (K^{XV} and K^{XVI}) 15%.

Substituting in equation (71), we obtain

$$69\,\phi^{\frac{7}{2}}+34\,\phi^{3}+11.5\,\phi^{\frac{13}{4}}-1260\,\phi-12{,}200=0$$

from which we find by trial that $\phi=4.5$ megalines per pole.

We wish to build a machine that can be wound for either two or three phase, and for voltages at least as high as 2300, in order that the alternator "carcass" may be useful in the future. The first of these requirements necessitates a number of slots per pole which will be a multiple of 6. If we use 12 slots per pole, they will be rather narrow, and we must have twice as many coils as by using 6 slots, resulting in a great deal more labor. Moreover, the die work for the 12-slot punching would be considerably greater. On the other hand, for the same weight of material, the heating with the 12-slot punching would be slightly less and the wave-form slightly better. We shall, however, decide in favor of 6 slots per pole. With $\phi=4.5$, we shall require 1.2 conductors per slot with Y connection, or 2.07 with delta connection. Using 2 conductors per slot, and connecting delta, we find that $\phi=4.66$, which we shall adopt.

In order to find the diameter and length, we shall use equation (20) as a trial, and find $\lambda = 11.8$ inches. From (18) and (39) we find that $D_2 = 75$ in., giving a pole pitch of 11.8 inches, and a peripheral speed at the bore of 7100 ft. per minute. With this diameter we shall have 670 ampere conductors per inch.

We shall take the length of the pole λ to be 12 in. Taking the axial length of the armature to be 13.5 in., including two 0.5-in. outside, ventilating ducts, and allowing for three other 0.5-in. vents, the gross iron will be 11 in. and the net iron 9.7 in. Allowing for four teeth through which the flux will pass, and a

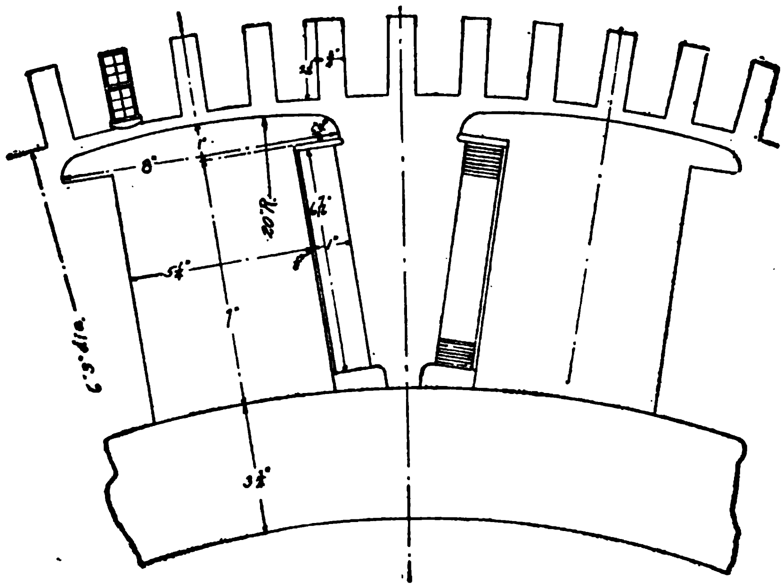


FIG. 1.

tooth density of 98 kilolines per square inch, the tooth width will be 1.22 in. This will give a slot width of 0.75 in.

In order to reduce the eddy-current loss in the armature conductors to a minimum, we will use a double layer wave winding and laminate the conductor by using square double cotton covered wire. Two number 2 square B. & S. fit nicely in width, and allowing for three in depth in the half slot, we obtain a current density of 1670 amperes per square inch. Allowing for the stick and for insulation, we shall make the slot depth 2.25 in. Conforming with our assumed density of 33 kilolines per square inch in the armature core, we shall make the depth behind the teeth 7.25 in., corresponding to a density of 33.2.

As the pole length λ is 12 in., we find from equation (3) that W, the pole "waist", is 5.25 in. With a 1 in. width of strip, the mean turn will be 37 in. Taking K' to be 1.8, we find from equation (5) that $X = 14{,}200$ ampere-turns. From equation (8) we find that 0.074 square inches are required. 0.07 in.x1 in. has slightly less section, but we can find from the saturation curves which will be plotted, whether this gives sufficient margin. By winding 75.5 turns on each pole, we shall meet both the temperature and excitation guarantees. This will require a radial length of pole of 7 in. excluding the head, and allowing 1 in. for the head, gives $H = 8$ in.

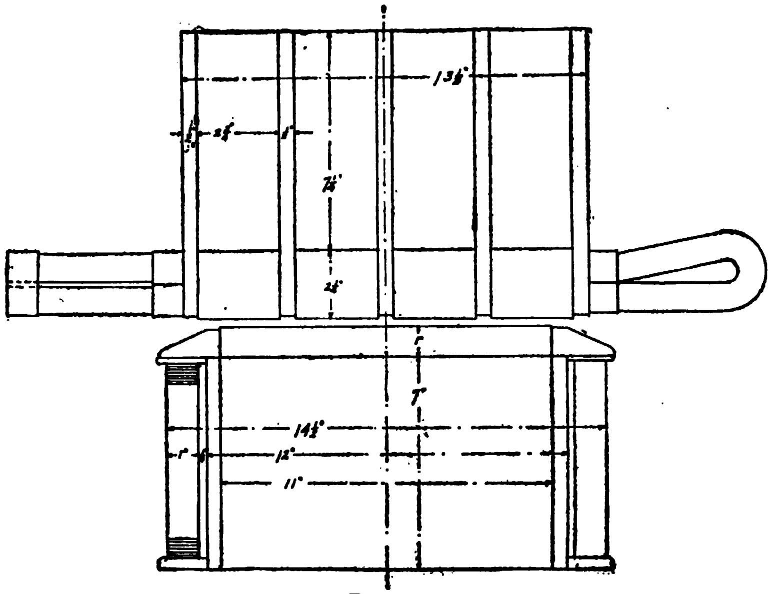


FIG. 2.

Employing cast steel for the field ring, and allowing a density of 63 kilolines per square inch, the radial depth will be 3.75 in. for an axial length of 12.5 in. Allowing 1.5 in. for the radial depth of the pole dovetail, we obtain a stress of 5200 lb. per square inch due to centrifugal force at normal speed. At 50% overspeed this will be 11,700, which is safe, as it allows a factor of safety of about 5.

In order to secure the regulation guaranteed, we shall use an air gap of 0.3 in. at the middle of the pole, making the pole eccentric with a 20 in. radius. This will make the gap ampere-turns per pole approximately 5500. The magnetic circuit of the machine is shown in Figs. 1 and 2.

The calculated saturation curves are not given, but in Fig. 3 will be found the test curves. The air-gap came a little smaller than was specified, 0.28 in. instead of 0.3 in.; but notwithstanding

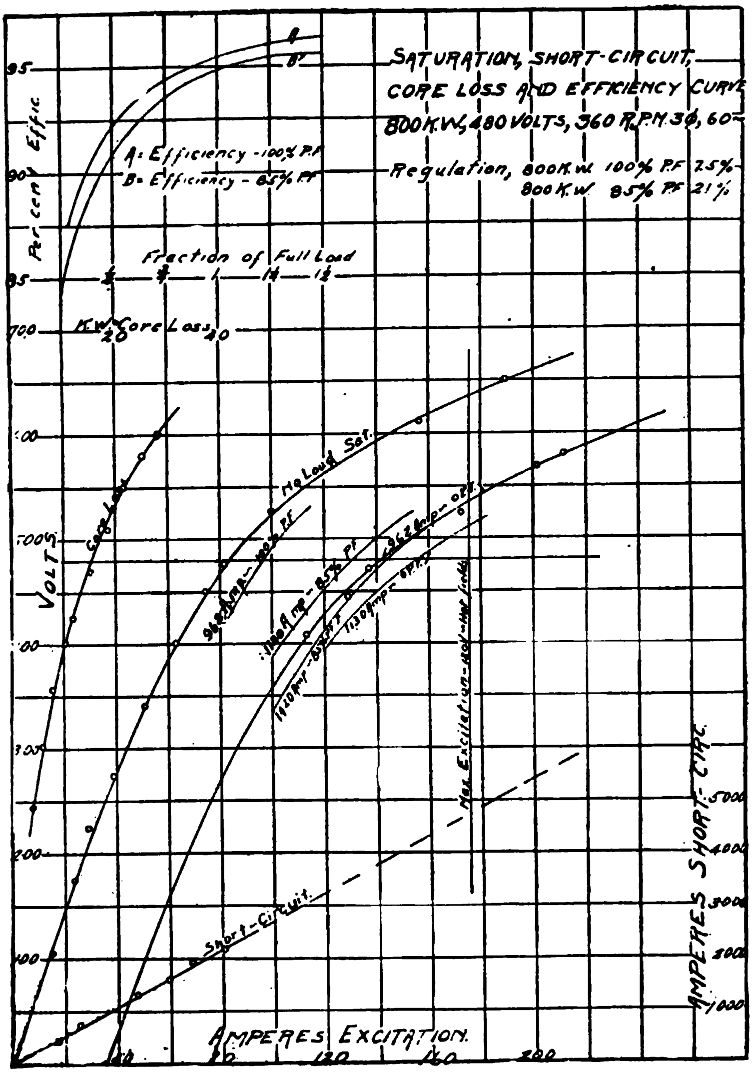


FIG. 3.

this discrepancy the regulation guarantees are easily met, being 7.5% and 21% with 800 kw. load and at respectively 100% and 85% power-factor. The efficiencies obtained were: full load in kilowatts, 85% power-factor, 94.72 per cent.; 0.75 load 93.54 per cent.; 0.50 load 91 per cent. The heat run showed the following results: 12 hours, 480 volts, 1005 amperes armature, 144 amperes field. Temperature rise stator iron, 22° cent.; stator coils 20° cent.; field coils 34° cent. (taken at a power-factor less than 20%). From this heat run it will be seen that the temperature rise with 800 kw. load at 85% power-factor (1130 amperes) will not exceed 25.5° cent. on the stator coils; nor will the rise exceed 40° cent. with 25% overload. As in neither case will the excitation be greater than 145 amperes, the field heating is also within the guaranteed temperature rise. It will also be seen, from an inspection of the saturation curves, that the alternator will give 535 volts when operated full field, normal speed, with 25% overload in kilowatts (1420 amperes) at 85% power-factor. The clause in the specifications calls for the alternator to give its normal voltage of 480 under these conditions.

Comparing the weights of the principal parts of the alternator as computed by means of equations (23), (33), (43), (50), (55), and (64), [employing the same value for K^{v}, K^{viii}, etc., as originally assumed ($K^{v} = 6300$, $K^{viii} = 23{,}000$, etc.), and $\phi = 4.66$], with the calculated weights determined from the dimensions of the machine, we obtain the following:

	Calculated by equation		Calculated from dimensions	
Armature copper	1060		1020	
Field copper	1360		1260	
Summation copper		2,420		2,280
Armature laminations	6400*		6240	
Pole laminations	2860†		3350	
Yoke	6300‡		6300§	
Field ring	2700		2300	
Summation iron		18,260		18,190
Sum of iron and copper		20,680		20,470

*25% added to allow for teeth.

†10% " " " " pole tips

‡15% " " " " inaccuracy of equation.

§Actual weight of rough casting.

In the above calculations involved in the design of the alternator, but one diameter and one length were considered. This was done in this case to prevent the paper becoming too lengthy.

The alternator as above designed well illustrates the application of the writer's method to an actual example. In an alternator for this rating it is possible to secure the same electrical characteristics by approaching either the copper or iron machine, increasing the air-gap and possibly the flux density in the poles in the former, to secure regulation. Had the circulation of air been increased, thereby reducing the heating for the same weight of copper, and had the densities and other constants in the equations remained the same, the equation would have indicated a nearer approach to the copper machine. However, in a case of this kind it might be advantageous to increase the flux densities in the armature. It will therefore be seen that the evaluation of many quantities entering into the equations must depend upon the judgment of the designer.

Appendix.

As was noted in the paragraph following equation (21), other things being equal, the product of the weight of the field copper and the kilowatts excitation is constant. If, as is often the case, there is no limit placed by the field becoming crowded near the bases of the poles, or by field heating, the designer may have some difficulty in deciding upon the best value to assign to the excitation. He may overcome this difficulty if his chief aim is to reduce to a minimum the cost of the entire equipment, including alternator and exciter, by bringing the cost of the exciter into the equations, as is carried out in the following derivation:

Rewriting equation (21),

$$W_f P_f = \gamma \frac{1}{\phi} \tag{72}$$

where γ = expression in the brace in equation (21); that is,

$$\gamma = \frac{3130\, K^{\text{III}}\, \sigma}{K\, B_p} \; \frac{(K^{\text{II}} + 1)^2}{K^{\text{II}}} \left[\frac{P_a\, K^{\text{I}}}{\sim k}\right]^2$$

If S_e represent the cost, in dollars, of the field copper plus the cost of the exciter, and p_e the cost in dollars per kilowatt of the exciter, we have:

$$S_e = p_e P_f + p_f W_f \tag{73}$$

$$= p_e P_f + p_f \frac{r}{\phi P_f} \tag{74}$$

In order that S_e be a minimum, $\dfrac{d S_e}{d P_f} = 0$, hence:

$$\frac{d S_e}{d P_f} = p_e - \frac{p_f r}{\phi P_f^2} = 0$$

Whence

$$P_f = \sqrt{\frac{p_f r}{p_e \phi}} \tag{75}$$

Substituting this value of P_f in (72),

$$W_f = \sqrt{\frac{p_e r}{p_f \phi}} \tag{76}$$

The cost of the field copper will be:

$$S_f = p_f W_f = (\sqrt{p_f \, p_e \, r}) \frac{1}{\phi^{\frac{1}{2}}} \tag{77}$$

The cost of material in the principal parts of the alternator will then be:

$$S_m = (\sqrt{p_f \, p_e \, r}) \frac{1}{\phi^{\frac{1}{2}}} + p_a K^{\text{VIII}} \frac{1}{\phi^2} + (p_{ai} K^{\text{X}} + p_{f.r.} K^{\text{XV}}) \phi^{\frac{3}{2}} +$$

$$p_y K^{\text{XII}} \phi + (p_p K^{\text{XIV}} - p_{f.r.} K^{\text{XVI}}) \phi^{\frac{5}{4}} \tag{78}$$

Letting $\alpha_6 = \sqrt{p_f \, p_e \, r}$, we have, after differentiating with respect to ϕ, equating to zero and clearing:

$$\tfrac{3}{2} \alpha_3 \phi^{\frac{7}{2}} + \alpha_4 \phi^3 + \tfrac{5}{4} \alpha_5 \phi^{\frac{13}{4}} - \tfrac{1}{2} \alpha_6 \phi^{\frac{3}{2}} - 2 \alpha_2 = 0 \tag{79}$$

Applying the above to the 800 kw. alternator, the design of which was outlined in the conclusion of the paper, and allowing p_e = \$20.00, the evaluation of the other quantities being the same as previously,

$$\alpha_6 = \sqrt{.2 \times 20 \times 148{,}000} = 770.$$

Equation (79) becomes:

$$69\,\phi^{\frac{7}{3}} + 34\,\phi^{3} + 11.5\,\phi^{\frac{13}{4}} - 385\,\phi^{\frac{3}{2}} - 12{,}200 = 0$$

Whence ϕ = 4.34 megalines per pole. This value is slightly less than that given by equation (71), this latter being 4.5 megalines. The exciter kilowatts will also be somewhat less than assumed for carrying out the design of the alternator, being 18.5, as obtained from equation (75), as compared with 23.5 kilowatts assumed.

Discussion on " The Relative Proportions of Copper and Iron in Alternators." Atlantic City, N. J. July 2, 1908

W. L. Waters: Mr. Fechheimer is to be congratulated on the success obtained in his attempts to reduce the design of alternators to a series of mathematical formulas. A good many attempts have been made in the past to reduce electrical machine design to an exact science, but they have met with little success. The first attempt was made in regard to transformer design, when rules for the most economical dimensions were formulated by various engineers, but even in this extremely simple case such rules met with very little success. I think that such attempts only serve to emphasize the well-known statement that electrical machine design is an art and not an exact science.

In the case of an alternator, there are many variables, such as voltage, speed, and frequency, which make radically different designs necessary, and these different designs may again introduce further complications in regard to cooling and windage of the machine. This being the case, it is not surprising that Mr. Fechheimer finds such extreme variations in his constants.

I think this paper is more useful in an academic way than as an assistance in practical designing. The probability is that for practical commercial designing we shall find it necessary to keep to our old method of trial and error, basing our designs and changes in design on past experience. Mr. Fechheimer will no doubt find that so much latitude must be allowed in fixing his constants that his method will in the end practically amount to a trial and error method, similar to that which has been in use for the last fifteen years.

Comfort A. Adams: While there is undoubtedly much truth in what Mr. Waters says, there is, on the other hand, no doubt as to the very great usefulness of a general analysis of design problems carried out as indicated in Mr. Fechheimer's paper. This is particularly true in the case of a new line of machines where the limitations introduced by the necessity of employing old frames or stampings are absent; but even in many other cases the general analysis reduces the amount of the cut-and-try process and renders the work of design more rational. Designing engineers are too much inclined to assume, because their machines operate satisfactorily and are reasonably successful commercially, that they cannot be improved. That this is far from true in many cases would, I feel sure, be made clear by the application of just such methods of analysis to machines which are the results of years of the cut-and-try process. Of course, as Mr. Waters intimates, a vast amount of practical experience and judgment is necessary to produce a good commercial piece of apparatus; but can there be any doubt that the more rational, scientific, theoretical, general analysis of this sort, when illumined by the same experience and judgment, will give better results than the experience and judgment alone? The claim often made by the practical designing engineer that he has no time for such careful theoretical analyses can hardly be upheld by experience. To me, the obvious fact is that the designing engineer cannot afford to omit the consideration of any point of view which tends to make nearer perfect the product of his work. Who has not had the experience of pushing through a hurry-up job, only to find that the saving of a few hours or even of a few days has cost the customer weeks or months of delay and consequent loss, and the manufacturer heavy financial loss, due to changes made in the finished product which might have been made in the design.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 2, 1908.

THE NEW METHOD OF TRAINING ENGINEERS

BY MAGNUS W. ALEXANDER

A century ago higher education in American colleges and universities aimed primarily to develop a man for the professional life of a preacher, a doctor, a lawyer, a teacher, a writer, or a philosopher. Colleges and universities responded to the demands of the life of that time. A college bred man occasionally, from choice or through circumstances, entered business activities, but the direction of the commerce and industry of the country rested chiefly with those who had worked their way up to important positions through all the steps of practical commercial and industrial life. The advantages of the broad culture and of the power of observing and reasoning, secured and developed in institutions of higher learning, were very little recognized in commercial and industrial work; and the mathematical and physical knowledge with which the colleges equipped their graduates found comparatively little call in the business activities.

The invention of the steam engine ushered in the wonderful industrial development of the world, strikingly manifested in the extensive building of steam railroad systems all over the country. This activity, requiring the construction of locomotives and rolling stock, called also for the surveying of land, the building of roadbeds, the spanning of rivers and valleys by bridges and viaducts, the leveling of hills, and the tunneling of mountains. A fair knowledge of the mathematical and physical sciences became a prerequisite of the equipment of the men who were to take charge of this kind of work. Colleges and universities, wisely then began to pay particular attention to the teaching of mathematics and physics, which were expanded until they became part of special courses devoted to instruction in civil engineering. The further development of the industries through added inventions and the extended use of machinery reacted on the colleges in demanding of some of the graduates an enlarged engineering knowledge that became an important factor in the various industrial activities. This call of the industries led to the establishment of mechanical engineering courses in many of the colleges, finally resulting in distinctive technological schools for the teaching of the various branches of the then known engineering sciences. At first, most of the graduates of the scientific courses and technological schools devoted themselves to the teaching of the sciences; or entered practical industrial life, in positions connected with railroad systems where a knowledge of civil engineering was required. Those who went into the workshops, however, found it necessary to acquire a knowledge of industrial processes and the art of the application of the sciences to the practical problems of the factory before they could assume responsible positions; and they even found that their college training worked rather against than for securing an opportunity of studying the practical side of industrial life.

A further marked change in the relation of the colleges to the industries became apparent some twenty-five years ago when electricity began to play an active part in daily life. The call for college men who had received a thorough training in the mathematical and physical sciences underlying electrical engineering now grew imperative, as such men were needed in the designing of electrical apparatus and in its manufacture, as well as in the technical management of electric street railways which were replacing the old transportation methods. Many colleges responded to this new demand by the establishment of electrical engineering courses. The rapidly changing conditions of the electrical industry, on the one hand, required the colleges to watch closely the practical applications of electrical engineering theories in the factories, and the remarkable development of new theories in college laboratories, on the other hand, obliged the manufacturers to keep in close touch with the work in the colleges. These conditions brought about a greater coöperation between engineering colleges and technical industries than had ever been the case before; and with this, grew the interdependence of the two institutions.

Graduates of electrical engineering courses more than any other engineering graduates felt the necessity of securing a good deal of the practical side of their profession in the workshops before they could become effective economic units in an industrial organization. The electrical industry had called into play many new processes of manufacture with which the student was not made familiar at college, nor had he even that general knowledge in regard to them which most young men acquire as boys in regard to ordinary mechanical processes, either by observation or by actual contact with the work. Realizing the gap between the theory of the college and the practice of the industry, some colleges added courses in practical shop and field work to their curricula. They established and maintained college work shops for that purpose.

The machine equipment and manufacturing methods of to-day soon become those of yesterday. This applies especially to colleges which neither have the resources to keep their equipment up-to-date, except at comparatively long intervals, nor for obvious reasons, can adjust their processes and methods quickly to the rapidly changing industrial conditions. Many of the men in charge of the practical shop courses, to be sure, are taken from practical life, but even they find it difficult to keep abreast of the development in the industries.

While this process of adjustment on the part of the colleges to the new industrial life was going on, manufacturers of steam engines, locomotives and similar devices, and especially manufacturers of electrical apparatus, evolved their own methods through which the college education of the young man was rounded out by practical experience in the industry before he could be placed even in a minor position of responsibility. These "student courses" proved very valuable from the beginning, and became so important that in most cases they were placed under the supervision of men especially selected for the training of students. The general aim was to give the student an opportunity during a two years' course, to apply the theories learned at college to the practical problems of daily industrial life, to acquaint him with factory conditions, and, as far as time would permit, with some of the factory processes; he was usually required to work regular factory hours and to submit in general to the rules governing the conduct of men in the shop.

With all due recognition of the excellent systems for training college graduates that have been developed in many establishments throughout the country, it may not be amiss to outline here briefly the system that was evolved a few years ago by the General Electric Company in the Works at Lynn, Mass., where the effort for a well conceived and well conducted student course has resulted in a system of recognized efficiency. The student course at Lynn is planned to meet the requirements of the General Electric Company for designing and estimating, construction and commercial engineers, and technical salesmen. The company takes graduates of technical schools and trains them during a period of two years, giving them during this time practical experience in the handling and testing of apparatus, in order to fix in the students' minds the practical application of engineering theories, to enlarge their engineering knowledge in general, to acquaint them with the competitive value of the product of the factory, and to develop them along lines of their future usefulness to the company.

In order that each student might receive careful individual attention thereby directing him into the field of his greatest capacity, a supervisory committee was organized a few years ago, consisting of four members of the engineering corps of the Lynn Works, one being the superintendent of the testing department. The committee meets either weekly or fortnightly. Each new student is called before the committee sometime during his first three months of service with the company in order that the committee may get acquainted with him and he with the committee. He is questioned as to his future plans and is advised as to the best way of carrying them out. Proper record of the student and his aims and of the impression that he has made on the committee is kept in the minutes of the meeting, to be referred to again when the student is called the second time within the next six months. He is then examined quite fully with regard to his theoretical and applied technical knowledge, his alertness in following up the applications of engineering theories and in taking advantage of the educational facilities offered by the course. The examination is of a more or less informal character, but of eminently practical value; it does not aim to find fault with the student but rather to assist him in his work and point out to him the way to success.

As a rule, the young man is not asked to recite theories and formulas, but is rather tested as to his power of comprehension in explaining the reasons for certain actions in engineering work. A question as to whether a constant current transformer should be short-circuited or open-circuited so as to protect the station equipment, if in case of an emergency quick action in the power house is imperative, will bring out more truly the student's real knowledge of transformer design than any question as to the theory of transformers; it will also give a clue to the student's mental alertness and his ability to think for himself. Similarly, a question as to the proper apparatus for lighting a city under stated conditions or as to the reasons for the superiority of certain motors over those of competitors will open an almost unlimited field for testing the student's engineering knowledge, his understanding of electrical apparatus, and his native ability as a technical salesman. These examinations are repeated two or three times during the student course. Those who do not come up to the standard, after they have been warned by the committee, are dropped from the course. Those who give a satisfactory account of themselves are usually placed in suitable positions as soon as they graduate from the course.

The efforts of the supervisory committee have resulted in encouraging the students in their work and in giving the company a fair idea of the value of each candidate for an engineering position. This system has also stimulated in the students a desire for increased engineering knowledge which has manifested itself in the formation of students' clubs to compare notes and to discuss the various problems. It has even led to the publication by two students of a very interesting booklet on " Questions and Answers about Electrical Apparatus ", a valuable aid to the young men in the course. The members of the committee, on the other hand, have been kept informed, through close contact with the students, of many matters with which they would not have otherwise become familiar, on account of the complexity of the company's business, and some of the ideas advanced by students have led the committee to adopt methods which have proved of advantage to the students and to the company.

As a member of this supervisory committee, I have had an unusual opportunity to study at close range several hundred of the graduates of technical schools, and to follow their early careers as junior engineers in various positions with the company. Although the present system of engineering training possesses many points of merit and has turned out engineers of high rank, nevertheless I have become convinced that this combination of four years of mental activity in college with two subsequent years of practical shop work in the student course, calling for physical exercise to a large extent, is not the most effective method of training designing, commercial, and construction engineers. It fails to give that insight into the practical side of electrical engineering and into the proper relation of the economic forces of an industrial organization that is more and more demanded of those who wish to take leading positions in the industrial field. Moreover, the atmosphere at college is charged with little of the seriousness of business. The correlation of theory and practice is not sufficiently close to facilitate the proper appreciation of the s iences in their concrete applications. While shop practice courses at college endeavor to approximate the desired condition, it must be admitted that they can give at best only a faint idea of the real industrial situation. The truth of this statement will be readily admitted by those who have employed many college graduates in engineering positions and have followed rather closely their early careers as engineers. The college has initiated the young men only in a very general way into the practical processes, and has given them but a speaking acquaintance with machines and materials; of course, hardly more can be expected of the colleges in view of their limited equipment and the brief time available for this purpose. Moreover, consideration of the elements of time and money in carrying out practical work is entirely neglected at college, although the proper appreciation of economic values is the important factor that makes for success in industrial life. Only extended experience in practical work in which time and money play leading parts can instill a proper conception of these values. It is natural, therefore, that colleges leave to subsequent practical life the young men's development along these lines. Similarly, no amount of the study of political economy at college alone can give the student a true perspective of the relation of employer to employee and of the many economic and sociological phases that prove more and more perplexing in our complicated industrial system; a thorough knowledge and appreciation of these forces, however, are to-day considered essential for those who are called to positions of responsibility in the executive and administrative departments of industrial life.

A serious study of this situation has led me to believe that the best engineering education can be obtained under a plan which provides that the teaching of the theory and practice should go hand in hand, and, so far as practicable, successive steps in the one should be based on similar advances made in the other, at such intervals as to permit of the most advantageous interplay of the two; and further, that the colleges devote their whole time to the teaching of the theory for which they are so eminently adapted, leaving it to real workshops to initiate the student into practical work, for which they in turn are best equipped. From an educational standpoint, this plan should prove efficacious, for mental conception of any activity is facilitated by the physical perception of a given process, as illustrated in the whole development of the human being; and on the other hand, mental visions are more firmly clinched by concrete application. Moreover, under such a plan, the freedom enjoyed by students during the college career is happily interrupted by the stern discipline that must prevail in a business organization; the advantage of this college freedom in the development of the young man's character, in the spreading of his wings, so to speak, is not lost, but his freedom is regulated by frequently recurring intervals of discipline in the factory, so that he may be prevented from soaring to the skies in his fanciful ideas engendered by his personal irresponsibility and after four years find himself all too rudely pulled back to earth by the stern call of practical life with its demand for coöperation of all forces. This plan also trains and develops the young man in the very life to which he will devote his future efforts and gives him the love for it, which, after all, is necessary for his success. Economically this coöperative education is sound in principle, in that it will give to the industries engineers who are known to be capable of assuming responsibility and can therefore be placed in positions of leadership. An arrangement of this kind, carrying with it financial remuneration during a part of the time, will attract to engineering work young men who are mentally and physically adapted to it but who at the present time do not enter college for financial reasons or for lack of appreciation of the value of higher education. The poor boy, if otherwise fitted for it, will be given a chance to acquire a college education by "working his way" through college in activities that have a direct beneficial bearing on his future career. The influence of the college will therefore, be extended and in no way restricted by the coöperative plan as contemplated.

It is obvious that only those should be permitted to enjoy the benefits of coöperative education who can prove both their mental fitness for a college training and their adaptability to the requirements of practical work. The test, therefore, should be a successful passing of entrance examinations into an engineering school and satisfactory service during a trial period in the factory. The former should precede the latter, so that the workshop may not be put to any expense for the preliminary training of young men who could not enter upon the course on account of educational deficiency. The trial period should be of at least three months' duration, which I consider necessary for a fair judgment in regard to a young man's ability for, and right attitude toward, practical work. If properly conducted, the trial period will afford a splendid opportunity to differentiate between those who give fair promise for a successful career as engineers and those who show a lack of the essential qualifications for "making good". The coöperative course, involving as it does the expenditure of money on the part of a business organization, should of course be open only to promising young men, especially since the demand for admission to the course will undoubtedly many times exceed the available opportunities.

After passing a satisfactory educational and practical test, the young man begins a coöperative course of six years, corresponding to the six years at present occupied by the engineering college education and the factory student course. Under either plan, therefore, the junior engineer will start his life's work after the same length of preparation. The plan which I have in mind provides that the first five years be spent in alternating periods at the college and factory, leaving the sixth year to be devoted entirely to college work. Under this arrangement, coöperative students will be taught in separate classes for the first five years at college, but in their senior year, they may be merged with the regular seniors. The advantage of the latter provision lies in the opportunity which it gives to the engineering apprentice for uninterrupted attention to his thesis and original research work and for forming wider college associations by coming in contact with the larger body of regular college men. This plan has an economic value to the college and places both classes of college students on the same plane with regard to their final examinations and the attainment of their college degrees. The fact that the last year is spent away from the factory will strongly appeal to manufacturers in that those engineering apprentices who upon completion of the coöperative course enter the engineering staff of competing firms will not possess the data relating to the latest developments and experiments. For myself, I rather believe that the last year at college will be interpreted by most students as a leave of absence at the expiration of which they will gladly return to the establishment which made it possible for them to receive an engineering training, and which in turn will willingly offer adequate inducements to secure the services of those with whose special aptitudes it is familiar.

As to the length of the alternating periods during the first five years of the course, extended experience alone will be able to determine the most efficacious arrangement. We may in the meantime, however, consider various proposals, and, by the process of elimination, narrow our consideration down to the few which in the light of logical reasoning might appeal as efficient. The length of the alternating period is a very important element in this plan, for too long or too short a time may defeat the very objects which the coöperative course seeks to accomplish. In the light of the aims of the course previously advanced, short periods seem to recommend themselves to us. A year or even six months spent alternatingly at college and at the factory would, I think, fail to give that close coördination of theory and practice that is an essential feature of the plan; nor would it establish that balance between the college freedom and the factory discipline which has already been referred to as very desirable. Short periods, on the other hand, will develop that facility in the young man's physical and mental make-up that will enable him to adjust himself quickly to the interacting influence of the college and the factory. The engineering apprentice should enter upon his factory work as a college boy with all the mental alertness and the inquiring mind fostered by the college; and he should return to the college as an industrial worker with the physical energy and the determined spirit of achievement that will be developed in a hustling factory organization. The attainment of these characteristics will to a large degree determine the success of the plan. Alternating periods of a day or even a week, to take the other extreme, might keep the young man's mind in a rather chaotic state, might not give the seed sown in the class room and factory, respectively, a fair chance to take root. Such time arrangement, moreover, would seriously interfere with the best economy at the factory, and largely forfeit that sympathetic interest of the shop foremen and workmen which seems to me not only desirable but decidedly necessary. Periods of such short duration would prevent also the complete carrying out by the same men of many pieces of work which should form part of their practical education. The finishing of such work by another set of men might often entail loss of time and even spoil the work itself.

As already stated, any estimate arrived at now in regard to the length of alternating periods must be looked upon as experimental; different factory conditions might lead to different conclusions. Personally, I believe in the efficacy of an arrangement under which the periods increase in duration from the first toward the last year of the course, beginning perhaps with alternations of four or five weeks' duration and ending with time elements of college semesters. In that way, all the advantages of the coöperative course would be emphasized strongly at the beginning when they are of determining influence, and the economic consideration of the college and the factory would receive growing attention in latter years as justified by the increasing importance of the work. An important advantage under a system of alternating periods lies in the fact that one set of students can work in the shop while the other set is engaged at the college, and vice versa, thereby keeping the educational and physical equipment employed practically at all times. This plan, of course, presupposes that during the college summer vacations, all engineering apprentices will be utilized at the factory, one-half of them during the first part of the summer with a vacation following during the latter part, and the other half enjoying a rest at the close of the college year and entering upon practical work again during the middle of the summer.

It goes without saying that engineering apprentices should receive a fair compensation for services in the factory, while they would, of course, pay a tuition fee for their college training. In the latter case, they receive education from the college without giving anything in return; quite differently, however, at the factory where they receive a practical education, they make immediate returns in the commercial work which they perform. I believe, and in fact from similar experience, I know, that if the factory end of the coöperative course is well organized and efficiently conducted, an astonishing amount of good commercial work can be turned out by engineering apprentices, which will compensate fully for the wages of the apprentices and for the expenditures for special supervision, materials spoiled work, and other incidentals. Considering that the young men admitted to the coöperative course will be at least eighteen years of age, possessing a high school education and that, furthermore, they will be selected according to a rather high standard, I should advocate an initial wage which would make them entirely self-supporting while they are at the factory. Their remuneration should increase at yearly or semi-yearly intervals both to give incentive and stimulation to the young men and to keep pace with the increasing value of their work; in that way, they would also be enabled to lay up a sufficient amount for the payment of their college tuition and for other necessaries. The exact amount of the wage will, of course, depend somewhat on local conditions as to the prevailing average compensation for work and the educational and living expenditures of the particular community. As a general proposition, I might suggest a starting wage of $6 a week with a yearly increase of $2 a week, amounting therefore to $14 a week during the fifth year of the course, when the shop work ceases. On this basis, engineering apprentices would earn in the neighborhood of $1300 during the entire course as against expenditures of $600 to $1000 for the college training, according to the tuition fee prevailing at the different engineering schools.

The college as well as the industrial establishment must, of course, have the right, fully understood beforehand, to terminate the course of any engineering apprentice at any time for good reason, and discharge from one institution for unsatisfactory work or conduct must be followed by discharge from the coöperative course by the other institution. Either institution, however, may make any special arrangements with such student, and if he is transferred to the regular course at college, he should receive credit for studies already pursued.

The administration of the coöperative course would involve the appointment of a supervisory board with representation from each institution. This board would work out and supervise the details of the course of study both at the college and at the factory, and have general charge of the social and cultural needs of engineering apprentices. It would be an interesting matter to lay out a course of study which would cover the present four years' college course and at the same time lay particular stress on those subjects that may be classed under the general head of applied economics. In view of the elimination from the college course of most of the time now devoted to shop practice, mechanical drawing, and electrical testing, to which subjects the factory will give particular attention, and also on the assumption that the coördinated practical work in the shop will make it possible to cover more ground at college in a given time, it would seem that the present four years' college program could be very nearly covered in the first five years of the coöperative course. The sixth year, which is entirely spent at college, might, therefore, be devoted to thesis and research work and to special lectures both by the college instructors and by men of affairs in practical life on such subjects as business law and business organization, cost-keeping and factory accounting, the economics of production and methods of equipping and laying out of manufacturing establishments. Seminars for coöperative students in these latter subjects and in many more of a similiar character which will take the place of some of the regular fourth year subjects already covered, will prove most interesting and instructive, because practical, and the presentation and discussion of such matters by men of affairs will bring the latter into contact with the student body to the advantage of both. The supervisory board will also from time to time confer and advise with the officers of the college and the industrial establishment as to their respective work, but they shall have no authority or responsibility in regard to the work of either as long as the course is carried out in conformity with the general plan approved by both institutions.

This coöperation between the college and the industry will undoubtedly produce that kind of engineering education which will adequately meet the demands of changing industrial conditions, for engineers capable of filling executive and administrative positions. It will result in giving college instruction just as complete, thorough, broad, and cultural as in regular college courses, and a practical training in the factory infinitely more thorough and practical than can be given in any engineering college or can be obtained as advantageously in present student courses. Above all, it will produce engineers, technical in their specific knowledge, cultured in their usefulness in life's activities, sympathetic in their understanding of the aspirations and needs of men, and broad and enlightened in their conception of their own obligations as engineers and as citizens. This may sound like a very idealistic forecast, yet I fully believe in its realization under proper conditions. Furthermore, it is well that we set up for ourselves an ideal of achievement in all our work and then endeavor to approximate it as far as our own strength and circumstances will permit.

About a year and a half ago the University of Cincinnati and the manufacturers of Cincinnati arranged for a coöperative course somewhat similar to the one outlined in this paper, the principal difference being that the students at the university are spending every other week at the factory or college, respectively, and that it is contemplated to maintain this arrangement throughout the six years of the coöperative course. For reasons which I have already given, I do not agree with these features and believe that especially the short periods will give way to longer ones before the experiment has gone much further.

Manufacturers are agreed as to the advantage of interweaving theory and practice in the training of engineers, and many educators are looking in the same direction for an advance in engineering education. It is significant that a technological college of the very first rank has recently expressed willingness to establish a coöperative electrical engineering course along the lines which I have set forth in this paper, and there is good reason to believe that a very prominent concern manufacturing electrical apparatus will soon join in such an undertaking.

I trust that I have explained the advantages of the new method of training engineers in such a way as to induce action by colleges and manufacturers in various parts of the country. Members of the American Institute of Electrical Engineers, most of whom have received a college education, and are, therefore, in a position to judge the case broadly, can do a great service to the rising generation of engineers and to the industries of the country by deliberating over this problem of engineering education, so that a safe and sane policy may be adopted regarding this matter.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers. Atlantic City, N. J., July 2, 1908

RELATION OF THE MANUFACTURING COMPANY TO THE TECHNICAL GRADUATE

BY DAVID B. RUSHMORE

Conditions in the commercial world necessitate an increasing consideration being given to the perfecting of organization and to raising the standard of individuals composing the same. The present may be said to be distinctly an age of organization, and in the competition which naturally exists, the relative efficiency attained by different organizations has much to do with their survival in the competitive struggle of modern times. Much thought is now being expended on this question and a great deal of good is resulting from the study being made of it. A whole can reach its best condition only when all of the parts which compose it are of the first class. This means that the individuals composing an organization must be of the best quality both as regards natural and acquired ability. There is an increasing demand for highly trained men, and there is considerable discussion as to what that training should be. A man who has concentrated his energies for four years on a special line of work is naturally more efficient in that line than would be possible otherwise, and consequently the properly trained college man is more efficient than the man without a college education. The ranks of industrial manufacturing concerns are, without question, becoming more largely filled with college graduates and therefore the importance of the technical graduate is increasing. There will always be men of unusual natural ability who will succeed without a college education, but such individuals undoubtedly would have been able to accomplish more if they could have had the advantages of concentrated training correctly applied for a number of consecutive years.

Manufacturing companies must now to a large extent train their own men. These men must have a thorough acquaintance with the organization, must know the apparatus manufactured by it, and must be more or less familiar with the general business conditions and with the individuals handling that business. In a large electrical manufacturing company the ability to handle business in almost any department is to some degree dependent upon the familiarity of the individual with the apparatus concerned, as commercial, legal, and engineering questions are almost always involved together.

A manufacturing company naturally desires a considerable variety of men. There is a large amount of routine business to be transacted, and there is also a large amount of work calling for originality, initiative, and invention, which demands a different type. Men may remain in the shop organization, enter the general commercial department, enter the drafting room or engineering force, work out into the district offices in their engineering or commercial lines, may go with operating concerns to take charge of power plants and transmission systems, or they may branch out into the very wide variety of allied industries which use electrical apparatus and where trained electrical men are required. A considerable number find a useful field in consulting engineering, and a few get into positions where technical engineering ability is utilized in financial lines.

Some manufacturer has said that he could lose his whole factory equipment and still win out if allowed to keep his organization intact. Certainly organization is one of the most important factors in present industrial conditions. An organization means an aggregation of individuals each of whom is performing some more or less specialized function, but all working harmoniously together to a common end. Many men never learn that they are part of an organization, and persist in working as individuals and therefore lose much of their value to a large concern. To get the advantage of a large organization everybody's mind must be used for the whole, and the old military system of passing down all the orders from the top has largely disappeared. Coöperation is the watchword of modern industrial development. Conferences have become a standard way of arriving at decisions, but executive work must, of course, be single-headed. Concerns which make use of the idea of conferences and of using the minds of all of their members, while they may be a little slow in action, will always win in the end. The best organizations allow a free development of individuals, while restricting them to harmonious action. Initiative must be rewarded, suggestions freely accepted, and a healthful rivalry carefully watched and, if necessary, restricted. An individual entering a large concern becomes at once a member of an organization and must work as such. The entire line of thought and action is very different from a condition where he performs all functions himself. The ability to work as part of an organization can be largely acquired, and should be learned in a college course. It is usually one of the things which a man has to learn when he first gets out of college, and is sometimes rather a difficult lesson. Great importance is placed by manufacturing organizations also on personal habits, characteristics, etc., as well as mere technical knowledge.

The condition of a man after graduation depends to a considerable extent on the individual and somewhat on the college from which he comes. The recent graduate is necessarily somewhat theoretical in his point of view, and, while having a good foundation, has not the practical experience or the acquaintance with actual conditions which makes him of immediate value. There is a tendency at present to connect educational institutions with manufacturing establishments so closely that there shall be no sharp line of transition from one to the other. This looks somewhat ideal, but undoubtedly is a desirable tendency. A great deal in the present system is to be commended, and splendid men are being turned out. They are all willing to start in at the bottom, but very few are willing to stay there long. The ability to succeed in an industrial organization depends on a great many other things than mere technical knowledge, and the incidental developments at college bring out many faculties which are useful in later life. The participation in college athletics, debating societies, class functions, fraternities, publications, etc., develops the individual and assists him to follow such work after graduation. Cramming for examinations has even been commended on the score that this is what one has to do continually afterwards. Among the most importtant things which a man learns to do at college, especially in athletic contests, is to work with others; to do team work and not act as an individual, because a large part of the effectiveness of an organization depends on just such work. A college man naturally becomes something of an organizer, promoter, manager, and contestant, and gets a start in taking a broad view of conditions. It is dependent upon himself to develop this afterwards.

One of the most promising signs of the times is the close study and coöperation between manufacturers and colleges. College students frequently work in the shops during vacations, as do also college professors. In some colleges it is the intention to have the professors engage for a year or so in active practice, and then return to teaching for a few years. The whole trend is along a line which brings the college man in close touch with actual conditions. Much that the colleges are doing is excellent, and much can be said in favor of the ideas now being given consideration for future modifications. A few suggestions only are offered here.

The college graduate during his first year or two after graduation, is placed under new conditions, and requires some time to find himself. It has been suggested that he keep some relationship with his college during this time and that some systematic study in connection with this work be followed. Another suggestion is that college work should be from the beginning associated directly with the idea of costs. No real engineering is ever dis-associated from this feature. It usually takes a college man some time to learn that the best engineering is that which will serve the purpose, and cost the least. At least one college runs its shop in this way, and to instill this point of view into men while getting their education would be of much use afterwards. It is very easy to say what should be added to or taken from a college course, and it is only too easy for a man to make suggestions in this line, as is evidenced by the amount of material produced in the way of articles and discussions. Many men after graduation find a use for commercial law, for accounting, for a certain knowledge of business methods, and financial factors. It is suggested that some attention be paid to these in connection with engineering work. Professional men are very largely working on salaries, and a salaried position is always more or less of an insurance against the acquisition of wealth, so that for professional men no subject is of greater importance than the study of investments, and from a personal point of view this cannot start too soon.

A paper presented at the 25th annual convention of the American Institute of Electrical Engineers, Atlantic City, N. J., July 2, 1908.

THE RELATION OF THE MANUFACTURING COMPANY TO THE TECHNICAL GRADUATE

BY B. A. BEHREND

Even though in times of business depression it may seem to us as if there were more applicants than positions, yet no thoughtful man denies that, while the number of positions in such times may have greatly decreased, the importance of filling them by thoroughly capable and competent people has correspondingly increased. For example, a position which, in times of exceptional prosperity may be filled fairly well by an average man, will be sorely in need of a man of exceptional ability when business prosperity is on the wane. If there is any one fact more patent than another it is the fact that there is always room at the top of the ladder for men of integrity, of moral courage, and of intellect. There are plenty of men who possess any one of these qualities; there are few who possess all three, and these the manufacturing companies, or the world at large, most require. In their anxiety to secure the raw recruits for officers thus endowed, the manufacturers turn of necessity to the universities and their graduates.

It must be granted that the colleges possess an almost unlimited potentiality for improving the human material turned over to them to shape and polish, and, it seems to me, considering the results obtained, they are doing very well in this shaping and polishing process. But they are not always supported in the right direction by the manufacturing companies. To allude to one instance only, I refer to the manner in which the manufacturing company seeks the young graduate, instead of letting him do the seeking. He thus gains, at the outset, an exaggerated idea of his importance, and an independence which is not conducive to the development of those qualities which make thoughtful and painstaking men. Our graduates are chiefly deficient in these qualities, and this is due, not so much to an innate deficiency in this direction, as to the fact that the incipient faculty has never been properly awakened and cultivated. There is abroad amongst our colleges and their graduates a most ominous disdain for painstaking accuracy and devotion to laborious detail, so essential to all really great work in engineering. The wish to take someone-else's thought and work and make it a commercial success, which is so prominent a feature of our business life, is easily explainable, though not so easily excusable. Men of ability realize that the same effort turned into the channels of commercial work will be productive of better returns than they would obtain by painstaking working out, for instance, the design and construction of electrical machines. In this case, even though their work may have met with eminent commercial success, our business methods are not much concerned with a debt of gratitude or obligation to the men that did the building up. Here lies a menace to the stability and continued prosperity of our manufacturing industries which must, in time, be remedied, lest it produce a far-reaching result in discouraging graduates of our colleges from the pursuit of new and important creative engineering work, with the result that we will have to draw on other countries for a supply of well-trained engineering brains.

The managers of our manufacturing plants can do as much toward the development of right views and proper education, as can the teachers and organizers of the colleges. The former need to study more sympathetically the condition of the latter, and *vice versa*. There are very poor pretenses of both managers and teachers in this world, and it is obvious that " the highest gifts are not always brought to the highest place." Education is a very good thing, but it cannot give the qualities which it should develop. Those who, like the writer, have been instrumental in the building up of large manufacturing organizations, recognize that the absence of moral qualities frequently mars a successful career, as frequently, perhaps, as the absence of purely intellectual qualities. Success often accompanies the work of men possessing ability, untempered by scrupulous restraint. Examples of this kind, so plentiful, have left a detrimental impression on the minds of aspiring young men. The thoughtful words of Mr. James Bryce, that this country " . . .

has the glorious privilege of youth, the privilege of committing errors without suffering from their consequences", remind us that many of our faults are not visited upon us with the unerring justice they deserve, because of the actual and potential wealth of this country in its present state of youthful and vigorous development. But let us not be deceived permanently into believing that, with our population increasing in geometric progression and thickening in our cities and manufacturing centers, many of the crude and lavish methods, despite which our industry is the most flourishing in the world, can be permanently retained without doing infinite harm. Space does not permit me to do more than to indicate the line of thought I wish to suggest. We need both character and intellect in our graduates, which should be cultivated by close cooperation between the manufacturing companies and the colleges, but for creating which no panacea has as yet been devised.

DISCUSSION ON "A NEW METHOD OF TRAINING ENGINEERS," AND "THE RELATION OF THE MANUFACTURING COMPANY TO THE TECHNICAL GRADUATE." ATLANTIC CITY, N. J., JULY 2, 1908.

B. A. Behrend: Permit me to discuss the paper by Mr. Alexander, suggesting a new method for training engineers by alternating shop work with college work. Let us see how difficult it would be to have such courses at institutions situated far away from manufacturing centers like Cornell, Leland Stanford, University of Wisconsin, or Pennsylvania State College. How could the students in these universities attend the lectures for one week and go into the shops for the next week, or for one month and go into the shop for the next month? There are no shops or manufacturing plants near enough in which the students could pursue practical work, and thus the proposed "Coöperative System" is altogether impossible in a large number of our universities. Furthermore, be it remembered that there are only half a dozen manufacturing companies in this country in which the variety of work is great enough to give the student an idea of the engineering and manufacturing problems arising in the building of electric motors and generators, incandescent lamps, steam turbines, gas engines, and other machinery. In small shops, confined to the making of specialties, the student is not apt to learn more than the mere handicraft. Trades have ceased to be crafts, as discoveries and inventions are constantly changing the industries so that apprenticeship to a certain trade is gradually losing its importance. Knowledge of fundamental principles alone can successfully take the place of the old apprenticeship course. The coöperative system of education, as I interpret it, in accordance with Mr. Alexander's suggestions, would effectively prevent the student from concentrating his mind on any subject whatsoever. I have never seen a complex question which could be completely solved or brought to a conclusion within a week's time or a month's time. Take a boy off the work and put him into the shop, and what is to become of the labor he has expended in familiarizing himself with the problem? Vice versa, put a boy in a machine shop or foundry and teach him for a week the kinks of the trade, and then send him back to school for another week or a month: what is to become of the labor which he has expended in familiarizing himself with his shop work? In either case the work and time spent in preparation will be entirely wasted. It is like starting and stopping, accelerating and retarding the boy's mind continually instead of allowing him to obtain a given velocity and momentum.

The term "engineer" is made to cover so wide a field that it is necessary to define it to prevent confusion. The engineer who is bred at a university should have full command of theoretical knowledge. He should learn at the university principally what he cannot learn in after life. After graduation he may obtain some practical shop experience and, if he happens to be an electrical engineer, he may pass through the two years students' course of any one of the large manufacturing companies. With some shop work during his vacations, and probably a year's shop work after graduation from high school, his training should not be held responsible if he does not become a successful engineer.

There is, however, a much wider field for men in possession of some moderate theoretical training and of good practical knowledge. A large number of electrical and mechanical engineers are needed in the manufacturing industries as foremen, superintendents, and master mechanics, to whom a long scientific training would be of little or no advantage. For the training of such men there are provided abroad technical schools, professional schools, or technical middle schools as they are called, in which such a coöperative course might be feasible. You need to give these men chiefly a practical education and enough light, as far as theory is concerned, to obtain an insight into the principles and the working of the machinery which they handle and take care of.

If we set aside certain colleges of ours for the purpose of giving a good practical education to a class of men who are not likely to be concerned in later life with the difficult and complex questions of engineering, I can see no objection to adopting a coöperative course in these technical schools. When, however, we are told that this alternating course should be introduced into our universities, permit me to say with much emphasis that I think this is the most vicious educational innovation that has been proposed in recent years.

J. P. Jackson: I have heard many times the statement that the college graduate is not prepared for his place in the world when he leaves college. In reply to this criticism, I would like to say to the practising engineers, that if such is the case, they alone are responsible. The engineering college is their servant, and they can make it what they wish. It is their duty, as practising engineers, to take a positive and actual part in the college work.

Mr. Rushmore said that in teaching, we must consider the costs and economic questions. In our technical courses we have been weak in political science, in social and economic science, and in history. I consider that general history should deal with scientific development, as much as with political and economic movements, and I want to refer the weakness in this subject to your educational committee as one worthy of consideration. Along with suitable history, extending to the end of the course, should be studies in political science.

Elihu Thomson (by letter): Mr. Alexander has so fully described the proposed plan of coöperative education for the training of engineers that it is not necessary to refer to any of its details which would naturally be the subject of modification, according to the results of experience. The plan itself or some modification of it has seemed to me to present the possibility of results of such value that it is worthy of an earnest trial. Experience only can determine its true relation to other methods and courses of education. Involving as it does the feature of allowing the student to earn reasonable pay for his shop work, it would serve as a material assistance to those upon whom the expenses of the ordinary technical school would bear very heavily, or become a preventive of the attendance of those otherwise well fitted therefor. It would act to broaden opportunity, and this is of course very desirable in these days. Naturally such an education must be limited to those who manifest a capacity for it, and it would assist the student in an early determination of his fitness or natural aptitude for various kinds of work.

The earlier this discovery or determination is reached the better it will be, serving as a guide in future development. Naturally a combined course such as that outlined in Mr. Alexander's paper cannot be completed except by an extension of time of the school period. It is assumed, however, that upon the completion of the period of six years, the student will have the very real advantage of having had contact with actual practical work and its conditions instead of requiring much time to adapt himself to them. The attitude of the engineering or manufacturing company towards the plan will depend on possible benefits, such as securing for its force a selection of men well-trained practically and theoretically.

Percy H. Thomas: Broadly speaking, the theoretical training of an engineer, in my judgment, should come properly before his familiarity with commercial work. On the other hand, he cannot get the full benefit of his theoretical training without some knowledge of commercial conditions. Since the latter part of his theoretical training is the period in which a knowledge of practical work is of the greatest importance, it would seem that the gain which can be obtained by interlinking the school training with the practical work may be obtained by overlapping the last part of the school course with the beginning of the practical work. To carry this overlapping further at the present time would appear to me, however, to be a great risk.

I notice that Mr. Alexander suggested the cutting out of the shop work at the college. I do not believe that this would be wise. The character of the training from the college shop work should be a very different thing from the practical experience in the factory. There is much about the nature of materials and the characteristics of machine tools which is gotten very imperfectly, if at all, in the factory, and is gotten very quickly in the college workshop.

The distracting of attention from the theoretical ground-work of a man's education before its substantial completion, by awakening a keen interest in the commercial work, is a great danger. I do not believe that the man before graduation should be permitted to get deep enough into the commercial work of the manufacturing company really to feel its inspiration.

If there be any advantage to a poor student, by his being able to utilize his labor in the factory, this fact is to be considered, and commended; but personally I do not believe that this consideration is a controlling factor. The dropping of the work in the factory to return to the college is an incidental disadvantage to the man who wishes to get ahead, in that whatever connection he may have gained in a certain definite line of work is thereby broken.

Morgan Brooks: Mr. Alexander has taken up this question from the manufacturer's standpoint. I desire to say a few words from the point of view of the technical school. Many of the boys entering college directly from the high school are entirely too young to do satisfactory work in a manufacturing establishment. College training for the youth is not merely mechanical drawing or mathematics, but it is the developing of character. I assume that Mr. Alexander has in mind men of twenty-one or more who lack a college training. We sometimes get boys of sixteen who need college restraint and discipline as well as physical training before they are ready for heavy work.

The difficulty of arranging alternating periods for a group of students would be very great, unless every individual were required to take the same work. Election of subjects such as German, French, English, or history, and the selection of different technical subjects as well, would be impracticable. We would find only one or two students out of twenty who would wish to take certain courses, and it would not be feasible to assign separate instructors to each of these small classes. The student would be obliged to adopt a course selected by others, and thus lose one of the opportunities of a university—the selection of subjects according to his own taste or need. By making the period of alternation the half year or college semester this objection would be greatly reduced.

College training should not necessarily be expected to turn out a productive engineer upon graduation. At first, graduates are often at a disadvantage compared with regular apprentices, but the fact that they rapidly attain success is proof of the value of a technical training. The failure of some technically trained men ought not to create surprise. Training can only increase opportunity or accelerate progress; it cannot produce an engineer from nothing. The men who are successful without college training, and we have many such, would no doubt have gained success more quickly with it. They are the kind of men to make their mark in spite of the lack of technical education.

Henry H. Norris: Mr. Alexander outlines a plan for improving technical education which differs from the standard method in that practical and theoretical instruction are separated and alternated. While it possesses many obvious advantages, from the teacher's standpoint this coöperative plan appears to ignore certain difficulties with which the educator has to contend. Although the technical student is in many ways an ideal pupil, he has certain characteristics which must be kept in mind in devising plans for improving present methods, which are the result of evolution and which are, presumably, as satisfactory as can be expected under existing conditions. In technical education the teachers must work and plan for the average student. The brilliant student does not concern them particularly, nor does the very poor student; the former looks out for himself, and the latter is automatically taken care of by a natural process of elimination. The average student is the one whose characteristics must be kept in mind in discussing any particular plan.

In the first place, the average man prefers to work with his hands rather than with his head; for example, a student will usually take more interest and pleasure in building and testing a motor than he will in calculating accurately what the torque in that motor will be when the machine has certain physical characteristics. Again, students lose interest in theory when they become very much absorbed in practical work; that is, they seem to like the practice better than they do the theory. Therefore, in technical training there should be included just enough practice to produce facts to think with, and no more; just enough facts to furnish the material for organization.

A second and important feature is that there is always a loss of time in starting up after a change in mental occupation This is illustrated particularly in the case of vacations. There is a lag of a week or ten days after a vacation before the mental machinery gets into smooth working order. The student in taking up his work at the end of a vacation acts as if he had a large inductance, and toward the end of the term he acts as if he had a large capacity; he starts slowly, he stops quickly. During the past year at Sibley College, the problem of reducing the frequency of these oscillations or alternations of work has received careful attention. Many professors advocated terms of four months, without interruption, alternated with vacations of substantial length. The general faculty of the university rejected the plan because there was difficulty in arranging a schedule to meet the needs of all the colleges of the university, but Sibley College stood for a plan involving two complete cycles per year.

A third fact which has a bearing on the coöperative plan is that when an individual knows that a task is to be dropped, whether completed or not, at a certain time, he loses a sense o responsibility with reference to that task. This sense of responsibility is an important element in successful teaching and it should be retained to the maximum possible extent.

There has been some discussion as to the place of shop work in the college. In my opinion shop work should be kept in the college curriculum, because the shop is the laboratory for teaching the principles of manufacturing. The principles of manufacturing can be taught better in college, by competent persons, than in the manufacturing establishment itself.

Mr. Rushmore suggests another plan for improving technical education. This involves no radical departure from existing practice, but rather a more efficient use of the means already at command. In most of the colleges there exist arrangements whereby advanced degrees may be secured *in absentia*, by proper registration and examination. In many cases the present methods of administering and supervising this work need considerable improvement.

Charles P. Steinmetz: Speaking from the point of view of the college professor, I appreciate that there are many things which it would be very desirable and important to teach in college, such as factory management, shop work and languages. We could teach them if we had a college course of eight or ten years. The question is not what it would be desirable to teach in these four years, because we cannot possibly teach in four years everything that the electrical engineer should know.

It appears to me that the guiding principle, in regard to what should be taught and what should not be taught, is that the college should teach all that which the young man can never learn after leaving college, and drop all those things which the young man can learn after leaving college. The humanities, the scientific foundations of electrical engineering—such as mathematics, principles of mechanical and civil engineering, engineering mechanics, physics, chemistry—these cannot well be learned afterwards, but subjects like practical work, factory management, business administration, etc., while very essential and very important, can be learned afterwards, and usually better than in college.

Colleges have been criticised because the young men are not fit immediately to fill important industrial positions, but have to learn in practice what the college can not give them. Now from my other point of view, that of a user of young men, I do not consider this state of affairs a criticism of the college. On the contrary, if a college were to turn out young men who could immediately drop into positions in our industrial organizations, I would very strongly suspect that that college had wasted the young men's time by teaching them things which they could learn just as well, if not better, afterwards; and that, therefore, many things which they should know but can not learn afterwards had not been taught. I do not believe the college graduate should be expected to be ready to enter immediately into full engineering practice; I believe he should have all the requirements to develop in due time into a first-class engineer.

We must not forget, however, that the college is not the only educational institution. There are other important educational institutions for electrical engineers. There are trade schools and correspondence schools. They do work the importance of which many of you do not appreciate, work of the highest value, and I want to speak here for them and draw attention to them so that we may give them proper recognition.

Dugald C. Jackson: We must remember that the problem of education is physiological and psychological. Education is not one of the exact sciences. The American Institute of Electrical Engineers has had placed before it in more than one paper the very distinct difficulty which engineers find in applying mathematical equations to the phenomena of what we call the exact sciences, such as physics, and we may as well start upon the proposition that we have no basis upon which to lay even the foundation for an equation of education. We are consequently compelled to rely entirely on the experience of the past in drawing our conclusions in respect to education. This is a very unsatisfactory basis upon which to found an advance, but it is the best we have. In passing, I may say that it is as good a foundation as Watt had for his physical science at the time that he made the modern steam engine; it is as good a foundation as George Stephenson had for his physical science when he succeeded in demonstrating that steam locomotives could cope with horses and displace them for transportation; and it is as good a foundation as was possessed for their physical science by most of the great inventors who produced the fundamental processes of our present industries and brought us the kind of civilization we know today. We may therefore believe that there is no reason for pessimism because we do not yet possess a firmer foundation for our reasoning about the methods of education.

As far as engineers' education is concerned, I think we will agree that we are here looking at it as *professional* education. We will probably all of us agree that we want to see the greater engineering schools aim at producing men of tremendous worth who will go far as engineers. The lesser engineering schools, trades schools, and others, will take care of the lower grades in the engineering arts; but so far as the greater engineering schools are concerned, their educational efforts should rest on a foundation of fine, thoughtfully taught literature, history and the other subjects of "humanistic" interest, including the languages; but whatever these efforts are founded on, it is distinctly professional education that the engineering schools must carry on.

The history of engineering training has much in common with the experiences of other professional branches. The early engineers were trained by becoming articled apprentices to principals who were supposed to direct their reading in natural philosophy and the like, and at the same time give them opportunities for actual engineering work. The earlier engineering schools dealt with civil engineering. They afforded advantages over the older way because the students were required to study with greater fidelity, and practice of the old type of civil engineering could be actually mixed with the studies. The situation has been quite different with the provisions for mechanical-engineering and electrical-engineering students. School shops, however well operated, have never afforded an equivalent of the inspiration or the practice that may be derived from association with a large, diversified, and well-run manufacturing establishment. In the latter respect, the engineering schools now labor under the disabilities analogous to those pertaining to the law schools before the general study of cases and practice in the moot courts were introduced in association with the study of the philosophy of the law, and to those pertaining to the medical schools before the association with hospital and general practice was thoroughly worked out.

It seems to me that Mr. Alexander's proposition has a strong foundation of precedent resting on professional education in other branches, and that the plan is one to which it is desirable to give a full and complete trial; and it is to be hoped that arrangements can be made for such a trial to be given. I have had an opportunity to study carefully Mr. Alexander's particular proposition. I have become convinced that this plan properly utilized may do much for engineering education, as the analogous plan properly utilized has already done much for law and medical education. Such a plan would not convert the young men into engineers who would be prepared to do the work that can only be done by men of twenty-five years' professional experience, but it would afford the young men a scientific and humanistic training equal to that which they now get in the best engineering courses plus much more acquaintance with the qualities of men and affairs than they now get in two years of what are called "student apprentice courses" in the factories. In addition to all that, the young men who followed a well-administered plan of the type proposed by Mr. Alexander would have pursued their scientific and humanistic studies in the school with an appreciative understanding of the tremendous results that can be brought about through coöperative effort, and of the necessity of dependability and thoroughness in all important work, and also of the tremendous possibilities which exist for the use of the highest type of talent in the engineering industries. In these three important factors, which should lend an inspiration and a quickening of ambitions of the highest importance, our young men now graduating from engineering schools are commonly defective. It seems plain to me that the engineering schools must in some manner improve their processes if they are to make the men who will *go far* in the engineering industries of the future; and if they are to try an experiment, there is nothing which looks more likely to bring good results than this plan which is an analogue of the plan that has worked advantageously for the medical schools and the law schools.

I do not mean to suggest that it is necessary to have every engineering school close to a great manufacturing establishment in order that the improvements which are here in mind may be substantially brought about. An engineering school of fine grade close to a great manufacturing establishment which carries on a highly diversified business, and which is well organized, and which will deal with this sort of experiment in a sympathetic manner, will possess an advantage due to its position. But the schools that are isolated can work out their own shop and laboratory methods so as to bring very much of the same plan into effect; though it is my belief that no engineering school has yet approached the requirements, and also that an inspiration may be secured by the students' association with proper manufacturing establishments which cannot be obtained in the school shops.

C. A. Adams: Whatever prejudice or preconceived opinions one may have against this plan of coöperative engineering education, there are certain of the advantages claimed for it which are so desirable as to make one wish to see the experiment have a fair trial. Personally I am not yet ready to take a decided stand for or against the plan, although I incline to the opinion that its success will be much more limited than its promoters seem to anticipate.

The principal advantages which would accrue under the best conditions, in the order of their importance in my mind, are: first; social—the value of a fairly intimate association with large numbers of the so-called laboring class, the new point of view usually involved in such association; a consequent better preparation for the duties of citizenship; a better appreciation of the advantages of a college education, and more sympathy for those less fortunate: secondly; educational—the advantages under this head have been fully described by Mr. Alexander.

The principal disadvantages are: first, the numerous practical obstacles to the carrying out of any such plan, such as the difficulty in coördinating the college and shop work where there are several groups of men taking quite different college courses; the frequent change of residence; and the difficulty in finding suitable shops where the influences are such as to make possible the attainment of the advantages above described; secondly, the danger of over-emphasizing the commercial side of engineering. This I consider a very serious danger, although I realize that there are different opinions on this point.

A system adopted in Harvard University recently, follows pretty closely the suggestions made by Dr. Steinmetz. Each undergraduate registers in Harvard College, where his studies are almost wholly elective. If his preference be engineering, he is advised to lay a good broad foundation in science, mathematics and languages, and, where time permits, to take some general or cultural courses. Meantime he associates more or less with men studying in widely different fields and has every opportunity to determine whether or not his taste and ability lie in the direction of engineering, and if not, in what direction they do lie. After the completion of a certain number of courses, he receives the appropriate degree, A.B. or S.B. If the student still prefers engineering after graduation, he registers in the Graduate School of Applied Science where he completes his engineering studies in two or three years more, and receives the degree of master in electrical engineering or in the appropriate field. This school is thus a professional school similar to the law and medical schools. Its advantages over the old arrangement are: first, it insures a more mature choice of profession and eliminates many students wholly unfitted for engineering, but started in that direction by some one of the numerous chance elements; secondly, it encourages a more substantial foundation and a broader education: thirdly, it does not force each man through the same machine, but adapts the work to the wants of the individual. In short, it makes for a higher efficiency of education.

A. F. Ganz: If I correctly understand Mr. Alexander's coöperative plan, it is based on the principle that the student should learn some theory, then go into the factory and get some practice on that theory; on that basis he has planned a six year course, which is equivalent substantially to three years of factory work and three years of college work. Now all of us who have been teaching in technical schools know that our present four-year' course is already crowded, and this proposition would cut down our time for teaching theory from four years to three years. Mr. Alexander suggests that the time can be gained by cutting out shop work, a large part of laboratory work, and mechanical drawing. I do not believe that is possible. If the shop work, the laboratory work, and the mechanical-drawing work have been properly planned, —that is to give the student that information which he cannot get in practice—then I do not believe that the practical work in the factory can be made to take the place of the laboratory work and of the mechanical drawing. I believe that it is the main function of the laboratory to run parallel with the theoretical course in the class-room, and to give to the student practice, with his hands, of the theory which he has learned in the classroom.

I thoroughly believe in allowing the graduate from the technical school go into the shop and get practical experience; but three years of practical work mixed in with three years of theory is too large a proportion of practice. When we look through the usual curricula of our technical courses, we find in the first two years, largely, subjects like mathematics, physics, chemistry, and general cultural subjects. Now, surely, in the factory the

The *average* college student is not a man of much directive force. His tastes are not strong, and he is not very particular which way he goes. He needs polarization. If he can get it before he enters college or in the early part of his college career it will enormously increase the efficiency of his education. I attribute the greater effectiveness of the college graduate who has had experience to his early polarization in the field of life and his early submission to the discipline of the world.

From the manufacturer's point of view there are many obstacles to Mr. Alexander's scheme, even if there are none from the point of view of the educator. In surveying the field and taking note of the number and size of the electrical factories and the number and output of the electrical educational institutions, I think each manufacturer would be unable to take his quota of the electrical students. One very large manufacturing company might absorb all the students of one educational institution; a rough calculation indicates, however, that a proportional distribution of electrical students among electrical manufacturers would be more than the manufacturers could digest. The manufacturer holds on to even relatively low-grade employees, carrying them over dull times, remembering their loyalty and faithfulness in performing various operations. This asset, of greater value than most people realize, would be lost if workmen were changed once a week, once a month, or even once a quarter, whether these workmen were college students or not.

Without wishing to be taken too literally, it is a good thing for a student, somewhere in his college career, to "drill brass name-plates for six months" as in the case cited by Mr. Rushmore. It is a mental training. It corresponds to what a man gets in other countries in the army; and if while he is drilling brass name-plates the man is of the kind of selected material that Mr. Steinmetz would have the colleges accept, he will absorb, through having his eyes and ears open, a vast amount of knowledge about the organization of business, the democracy of relations between hands and foremen, the necessity of cooperation, and many other things that sooner or later he must surely learn.

I would add a word in most earnest support of the idea that we train only selected material at colleges. This is the true keynote to higher efficiency in education. It carries along with it, however, the obligation to employ only selected instructors. In many of our educational institutions we have as instructors men who, however learned, do not possess the art of imparting their knowledge, who do not know how to make a lasting impression on the minds of their students, who do not appreciate what to emphasize and what to suppress; men who in the delivery of their lectures, for instance, do not have that touch of dramatic power that causes students in later life to look back to those lectures as the very foundations of their technical training. A high standard for instructor material must go hand in hand with a high standard for student material.

M. W. Alexander (by letter): The discussion of my paper indicates an impression on the part of some that my plan intends to supplant existing college engineering courses. A careful reading of the paper, however, would show that this is not the case. The underlying thought of the proposed plan is to provide an education especially adapted to the needs of prospective engineers who are to specialize in life along administrative and executive rather than purely engineering lines. These engineers must have an adequate practical as well as a thorough theoretical knowledge of engineering; and the best method to accomplish this, I believe, lies in an arrangement whereby the engineering education at college is aided and supplemented by a familiarity with the industrial methods of progressive manufacturing enterprises, which may best be gained through simultaneous participation in the practical work. The plan, moreover, aims to enlarge the college sphere through a course which, for reasons already outlined, would attract to it capable young men, who, through one cause or another, do not or cannot avail themselves of a college education under present conditions. Obviously, the proposed arrangement, if carried into effect, will in time react upon present engineering education by suggesting changes which will bring the regular engineering courses more fully into synchronism with the demands of modern industrial life.

Mr. Behrend agrees with me regarding the general proposition when he says that " a large number of electrical and mechanical engineers are needed in the manufacturing industries as foremen, superintendents, and master mechanics "; he does not, however, fully appreciate the value of a broad education for these men who, after all, occupy positions as foremen, master mechanics, and superintendents only as stepping stones to something higher. Inasmuch as they are to become eventually leaders of men and managers of enterprises, they should receive an adequate cultural development in addition to their scientific and practical training. Again Mr. Behrend is pleading for my proposition—although unintentionally perhaps—when he claims that the proposed plan is " like starting and stopping, accelerating and retarding the boy's mind continually instead of allowing him to obtain a given velocity and momentum." That is the very thing I am aiming at, namely, to make and keep the young man mentally so alert that he will be able to adjust himself quickly to varying conditions as they arise; he should not need a headway, so to speak, in order to be effective. The mind of the future manager must be trained to work like a polyphase motor, starting under load and quickly acquiring full speed, and not like the ordinary single-phase motor which must run under full speed before it can carry load.

Professor Norris falls into a similar error when he criticises the proposed plan on the score that there is loss of time in starting up after a change in work. It is one of the very objects of the plan to minimize this loss by offering frequent opportunities for practice in changing quickly from one line of activity to another; and this inability to make quick adjustments is a weakness which manufacturers see in recent college graduates. A manager has to change his work many times a day; one hour he is occupied with engineering problems, another hour with financial questions and perhaps the next hour with matters pertaining to the labor problem or the economics of production. Certainly, we should aim to train for such managerial capacity those students who will take up administrative work. This change from one class of work to another may of course be carried too far. It is for this reason that I do not advocate weekly changes such as are provided under the coöperative plan at the University of Cincinnati. I am advocating alternative periods of from six to ten weeks' duration, fully ample for the concentration of thought and work to which Mr. Behrend properly refers. The courses of study in technical colleges will bear me out in this respect, for many of their subjects are completely covered during a similar period of time.

Again, I note with pleasure Mr. Behrend's support of my contention that a correlated practical and theoretical engineering education during the so-called college career will prove very beneficial for engineers who wish to engage in manufacturing business or to become executive officers of industrial establishments and public service corporations, for he says: " If we set aside certain colleges of ours for the purpose of giving a practical education to the class of men who are not likely to be concerned in later life with the difficult and complex questions of engineering, I can see no objection to adopting a coöperative course in these technical schools."

Assuming all the time that we are dealing with recognized engineering colleges which confer the usual degrees upon their graduates, it must be admitted that whether such courses are to be given in separate colleges or side by side with regular courses in the same college is, after all, a matter of detail. Local conditions will have to be taken into account, and it is by no means necessary to have these coöperative courses introduced in all engineering colleges. Naturally they would be established, in such colleges as are located near the large manufacturing industries, and they could prove their value in these particular instances, even though other colleges may not be able to offer the same opportunity. This argument against the proposed plan, put forth by Mr. Behrend, would hold just as good in advising Harvard University not to start its graduate school of business and commerce because other colleges are not in a position to do the same. Furthermore, inasmuch as I am advocating alternating periods of from six to ten weeks, and not of one week's duration, as Mr. Behrend erroneously seems to have in mind, it would be feasible for colleges and manufacturing companies to establish a coöperative course even though they should be separated by a distance of fifty or one hundred miles.

The coöperative course can only be open to a comparatively few, at the most, taking the factory side of the question into account; and it is therefore intended for those especially fitted for the work for which the course aims to train. While I, therefore, fully agree with Professor Norris that elementary education must be laid out for the average boy, I differ from him with regard to higher education in believing that the latter and especially the one of a technological character should differentiate and concern itself with the various capacities of the students so as to allow each, within reasonable limits, a full and free development. A young man should not go into an engineering college unless he is well fitted for an engineering career. Dr. Steinmetz is right in advocating measures by which the engineering college will eliminate the mediocre and develop only the well-qualified student. The latter should be taught the humanities as well as the sciences, and I am ready to go with Professor Norris as far in this respect as he wishes, for I plead for a broad educational training at all engineering colleges, so that the man as well as the engineer will be developed. Professor Norris' fear that students may lose the sense of responsibility for a task when they know that the task is to be dropped at a certain time whether completed or not, would disappear under proper training. To me this danger appeals with much less force than that concerning the lack of serious responsibility of our present college students, who, for four years, enjoy a delightful college freedom without any restraining influence such as intermittent discipline of practical life would bring. I should not want to deprive young men of this college freedom with all its great value, but I should wish to give it greater significance by giving it a background such as the factory discipline would furnish. The ideal of the college linked to the real of the factory is bound to be wholesome in its effect upon the character and stamina of the young men, who, under such conditions, will be developed, rather than injured by the awakening of a keen interest in commercial work, such as Mr. Thomas seems to fear. Experience, furthermore, refutes his contention, for those who during vacations engage in some gainful occupation, even though not at all related to the science of engineering, usually turn out to be good engineers. These students feel the inspiration of practical work, and through it, see clearer the value of a theoretical education.

The statement of Professor Brooks that high school graduates as a rule are entirely too young to do satisfactory work in a manufacturing establishment is surprising to me; if it were true it would be a serious indictment of our trade apprenticeship systems, under which grammar school graduates are required to do commercial work in factories, which they have been entirely able to perform without any inujry to their physical development. It goes without saying, of course, that the work in the factory must be suited to the physical maturity of the young man just as much as the theoretical work in college must be suited to his mental maturity. It must also be borne in mind that thousands of young men with a complete or partial high school education enter our factories and workshops and grow up into healthy and efficient men. Would not a periodical rest from the physical work with a corresponding period of mental activity develop these young men better physically as well as mentally? The elective system under the proposed plan would have to be practically eliminated, as Professor Brooks points out, but this is no objection to the plan. The elective system, however, even though our foremost educational thinker, President Eliot, fathers and favors it, is by no means regarded as the only method of securing an effective college training. The coöperative engineering course, aiming to train young men for a definite career, is an application of the elective system in the broadest sense of the term, for the student elects specific subjects which he must pursue in order to complete the requirements for a degree. The makers of such a course must lay it out on the broad basis of cultural as well as scientific instruction.

Some of the speakers have questioned the advisability of cutting the shop work out of the college curriculum. I think, we will all agree that so far as the term "shop work" at college refers to the teaching of the mechanical principles involved in machine work, the college should still attend to this part of education. The familiarity with machine processes, however, inasmuch as it is based on a more or less extended practice, could be left to the factory under the proposed plan, where it can be taught more effectively, inasmuch as it is taught by practical men and under real industrial conditions. In the proposed plan provision is made for the college to go much further in the educational phase of shop work by giving a more thorough and extended course in shapes of tools, cutting speeds and feeds, and similar important questions, than is done at the present time. Whether the college professor gives a course in these latter subjects at college or at the factory, or whether a practical man is selected as special instructor in these subjects, is, after all, only a detail in the whole plan. Similar arguments would apply to the part which the college and the manufacturing company ought to play in the teaching of mechanical drawing.

Considering the whole matter in this light, and studying carefully a detailed course of both the theoretical and practical studies under the proposed plan, Professor Ganz's criticism that a four-years' theoretical training is to be crowded into a three-years' period falls to the ground; for it will be seen that under the proposed plan as many, and even more weeks, are devoted to the college studies than are provided for at present in most leading technological colleges for the same subjects. As a matter of fact, the concrete plan which I have worked out provides for one hundred and thirty-two weeks at college, one hundred and forty-one weeks at the factory and thirty-five weeks for vacation, or a total of three hundred and eight weeks for the period beginning with the middle of July after high school graduation, and ending with the middle of June six years later. The particular college which I have in mind, devotes only one hundred and twenty weeks to corresponding studies during a four-years course. The extra time in the proposed plan is gained by a trial period of two months of practical work previous to the college work, and by a shortening of the present, I believe all-too-long summer vacation periods from sixteen to five weeks each. Furthermore, I firmly believe that coördinate theoretical and practical work as proposed will make it possible to cover more ground at college in a given time than can be done under the present system. A knowledge of practical work will materially help the student in a quick understanding of most of the engineering subjects taught at college.

In closing, I wish to emphasize again, that the proposed plan aims to train high grade manufacturing and executive engineers who may receive their college degree on the basis of a thorough engineering education, and besides have obtained an adequate practical training with special reference to their future usefulness.

Application of any theory, after all, will prove whether or not the theory is correct. Thus a trial of the plan which I have proposed will prove if it is efficacious; and I am glad to find so many members of the American Institute of Electrical Engineers ready to see a sufficient value in the proposed coöperative course to hope for a fair trial where conditions permit.

A paper presented at the 230th meeting of the American Institute of Electrical Engineers, New York, October 9, 1908.

HIGH-POTENTIAL UNDERGROUND TRANSMISSION

BY P. JUNKERSFELD AND E. O. SCHWEITZER

Among the many electrical developments of the day the most striking one, the most far reaching in its direct and indirect results, is the successful transmission of energy at greatly increased pressures. From present indications systems operating at potentials of 100,000 volts and over will not be uncommon in a few years, and none dare prophesy what further increase the future has in store. What this means to many communities from an economic and sociologic standpoint can readily be appreciated. Cheap power will enable present industrial communities to improve and new ones to build up in pleasant localities many miles from its source, whether coal or waterfall, and cheap traction will quickly carry their wares to market.

Within the confines of the largest cities the conditions are sometimes such that it is not advisable to carry these high potentials overhead. In such cases it may be necessary to place the conductors underground. The limitations of insulation and size have kept the transmission voltage for underground cables at about one-third that of overhead transmission, and while there has been a considerable advance in the art the approximate ratio of 1 to 3 seems to be still maintained. However, in large systems of underground transmission, insulation and size are not the only cable limitations. The line constants may become such that the cable is frequently subjected to dangerously high potentials. It therefore becomes advisable carefully to study the system so as intelligently to provide conditions for reliable service.

This brings us to the subject of our paper in which we shall discuss high-potential underground transmission, with a view of pointing out some lessons taught by experience and tests and of suggesting further investigation and discussion. It is not the intention to discuss cable manufacture or in any detail the installation or protection of underground cables, as these matters have already received careful and thorough attention in other Institute papers. Nor is this paper in any wise to be an argument for underground transmission as against overhead transmission. Conditions are frequently such that overhead transmission is the only practicable method. The increasing use of very high potentials is tending to raise the boundary between high and low tension, but at the present time the dividing line is still generally accepted as 5000 volts.

Typical large cable system. This paper will, therefore, not deal with experience at pressures lower than 5000 volts. A typical system using high-potential underground transmission extensively is that of the Commonwealth Edison Company of Chicago and its experience will be cited principally. The system is fairly well known, having been described in more or less detail in Institute papers and discussions and in the electrical press. Suffice it to say that at present (September, 1908) the system consists of two generating stations (a third is in process of erection) with a maximum capacity of about 18,000 kw. and 100,000 kw. respectively, sixty-six 9000-volt, three-phase, transmission lines aggregating 272 miles, supplying forty-four sub-stations containing synchronous converting equipment almost exclusively. Three additional sub-stations and 68 miles of additional 9000-volt cable are now under construction.

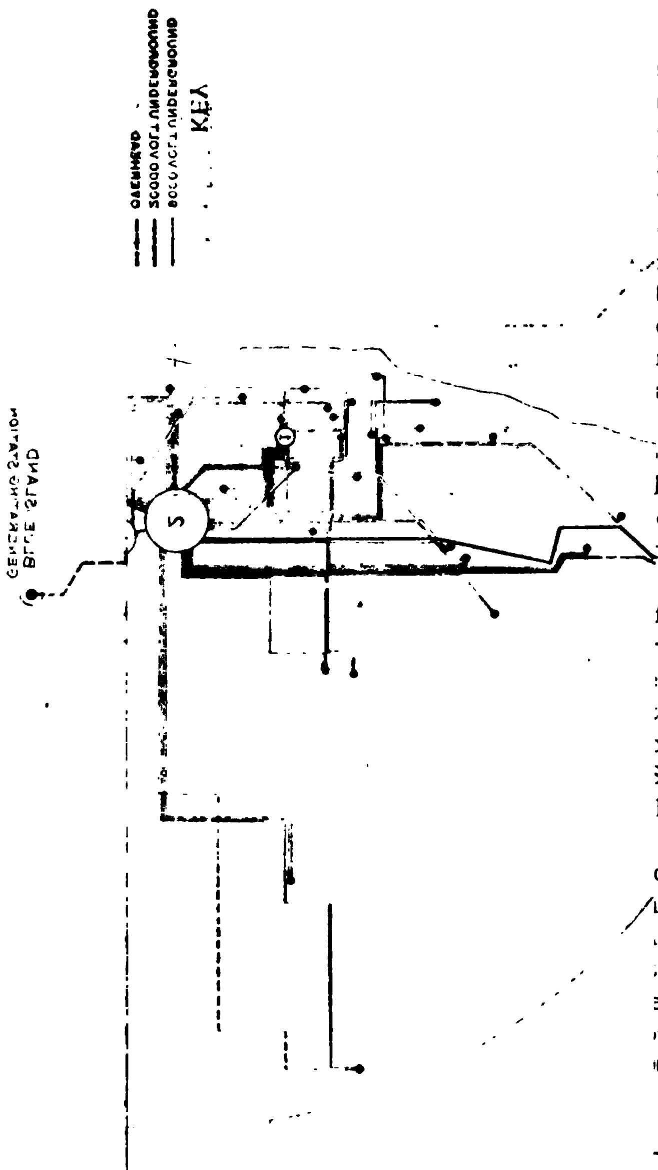
There is also a 20,000-volt line eleven miles long connected to the 9000-volt system through step-up transformers, and feeding railway and frequency changer sub-stations located in an outer or suburban zone. Forty-four miles of additional 20,000-volt underground lines are under construction, which will bring the total up to fifty-five miles. The entire cable system is shown in Fig. 1. This 20,000-volt system will be permanently supplied from 5000 kw., three-phase, step-up transformers connected delta to star with the neutral grounded. The standard size of 20,000-volt cable is 00, and of 9000-volt cable 0000 and 250,000 cir. mils. The 340 miles of 9000-volt cables supplying the densely populated inner zone form a so-called "radial system", while the 55 miles of 20,000-volt cable when completed will form two so-called "ring systems", one supplying the more sparsely settled outer zone northward and the other southward.

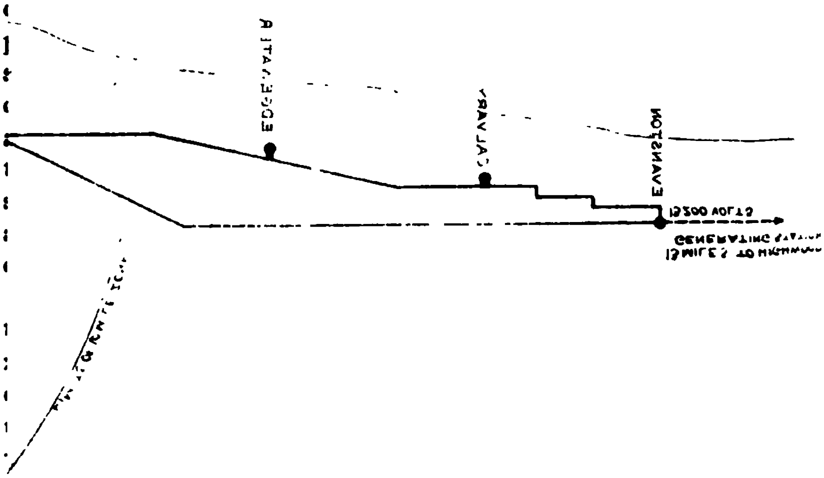
)1

in
m
w.
r-

17
er
nt
is
7.
)0
u-
er
n.
th
er
n.
a-

le
he
ce
on
y,
ff-
o-
c-
er
e.
St.
ht
of
on
ig;
u-
h-
et-
ut
00
of




EDGEWATER
CALVARY
EVANSTON

The radial and ring systems are shown diagrammatically in Fig. 2. The principal station (Fisk St.) is equipped with steam turbines exclusively, four with a maximum capacity of 7500 kw. each and six of 12,000 kw. each. The generators are all star-wound and the neutral is grounded.

In February 1902 the 25-cycle, three-phase system of about 17 miles of three-conductor cable and seven synchronous converter sub-stations was raised from 4500 to 9000 volts, the equipment and cable having been installed with this change in view. This transmission system was first put into service in August, 1897. The greater part of the cable at the time of the change to 9000 volts was 00, some 0 and about one-fifth of 0000 size. The insulation on the original 4500-volt cables consisted of 5/32 in. inner wall of paper around each conductor and an outer wall 3/32 in. thick surrounding all three. The thickness of the lead sheath was ⅛ in. In the 9000-volt cable the thickness of the paper insulation was finally standardized at 6/32 in. and 4/32 in. respectively. The increase per year in mileage of cables operating at 9000 and at 20,000 volts is shown in Fig. 3.

Other early cable systems. Some of the high-tension cable systems in operation or under construction in 1897, when the transmission system in Chicago was first put into service at a lower voltage, were the following: Brooklyn Edison Company, 6600 volts (1897); New York Edison Company, 6600 volts (1898); Niagara Falls Power Company in Buffalo, 11,000 volts at end of overhead line (1897); the Metropolitan Street Railway, New York, 6600 volts; Twin City Electric Company of Minneapolis, 12,000 volts (1898). Soon after came Manhattan Railway, New York, 11,000 volts; Providence. Rhode Island, 12,500 volts; Milwaukee, 13,200 volts; and St. Paul, 25,000 volts (1901). The cables of the St. Paul Gas Light Company were two in number and formed the receiving ends of two parallel 25,000-volt, three-phase, overhead transmission lines 30 miles long. Each cable was about three miles long; one had rubber insulation and the other paper. Cable manufacturers at that time generally offered to make cable to withstand 30,000 volts pressure for individual lines, but for a network about 15,000 volts was considered the safe limit. About a year later this limit had apparently advanced to about 22,000 volts.* The insulation used on these various cables might be of interest and is given in Table 1.

* Fisher, A. I. E. E. Trans. 1903, page 440.

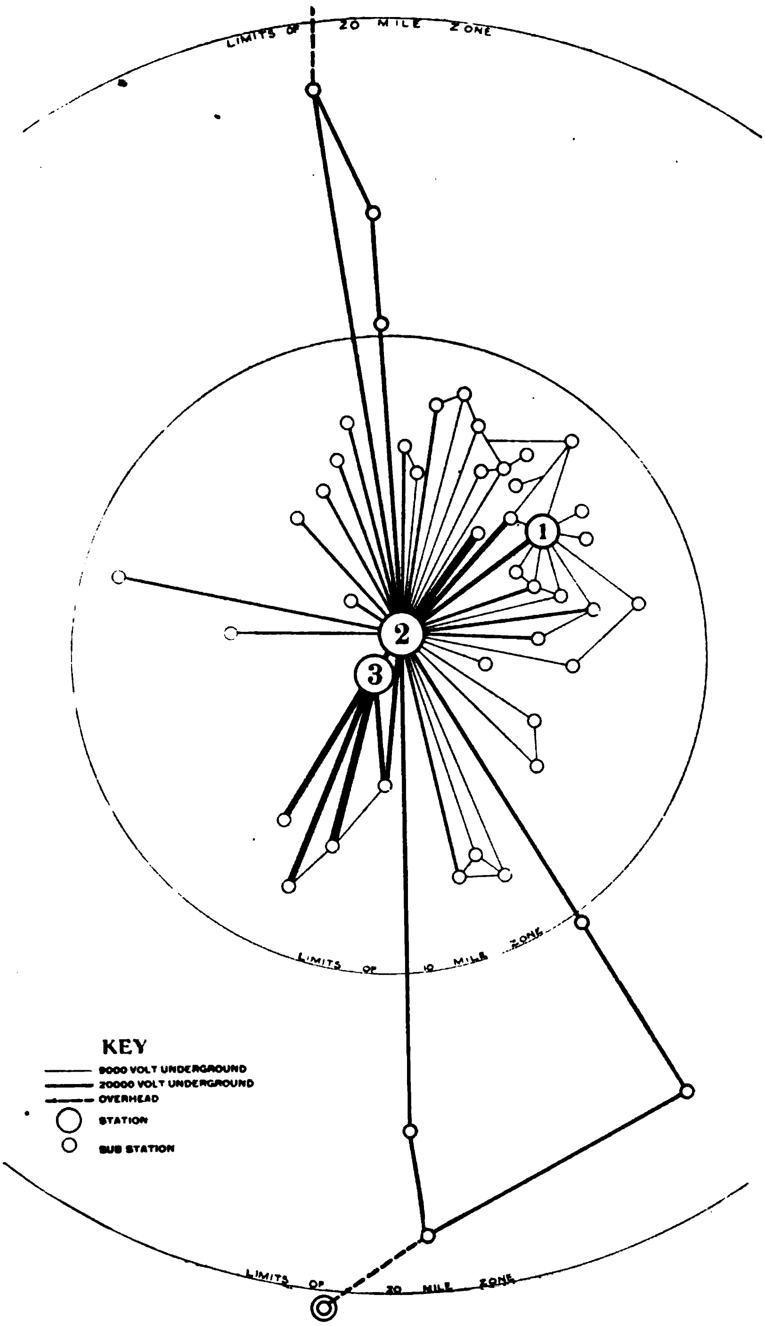


FIG. 2.—Diagram of 9,000-volt radial system and 20,000-volt ring system, showing number of lines and sub-stations in each zone.

Experience. The early experience in Chicago and elsewhere showed the necessity for avoiding sharp bends and for using extreme care in making the splices. A small number of burn-

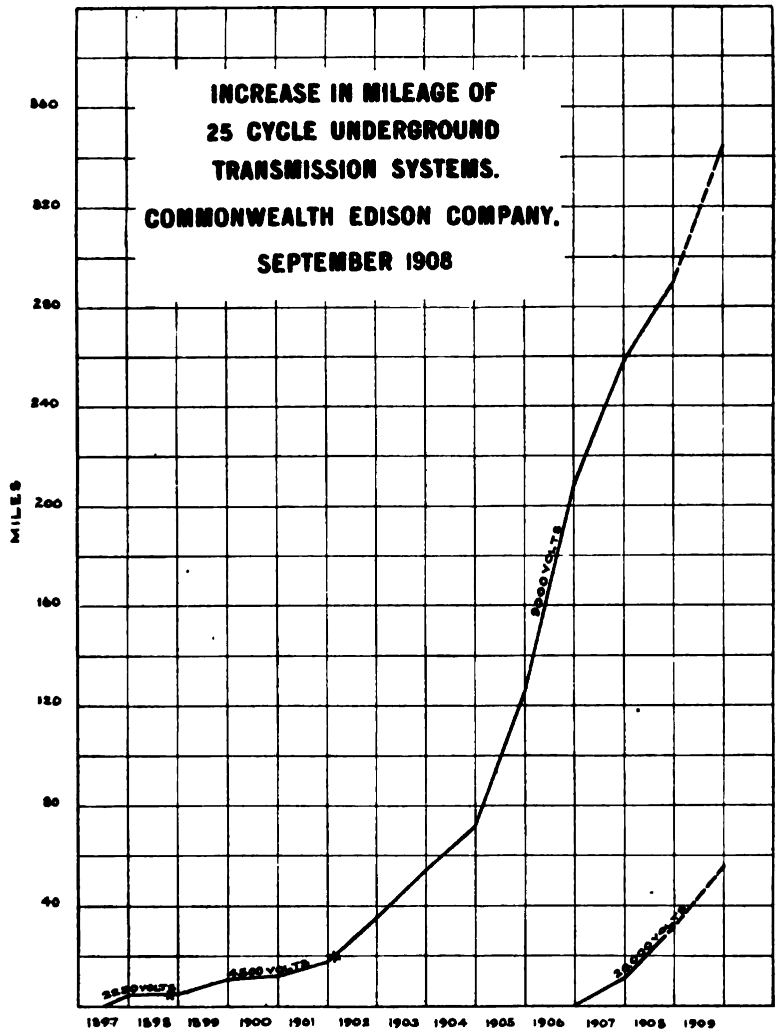


FIG. 3.

outs occurred during the first year's operation at 9000 volts, and nearly all were due to sharp bends or to moisture at the joints. In order to determine how quickly moisture in a cable causes a breakdown, after it has entered through an opening in the sheath, in 1903 Mr. Burch* of Minneapolis made some interesting experiments. He found that it usually requires from a day to a week for the moisture to work down sufficiently to cause a burn-out. Paraffine had up to this time been used at the

TABLE I

TABLE SHOWING INSULATION USED BY DIFFERENT COMPANIES ON THREE CONDUCTOR HIGH TENSION CABLES.

FROM PAPER BY W.G. CARLTON IN 1902

Company	Line Voltage	Kind of Insulation	Thickness of Insulation in Thousandths of an Inch: Over 1 Conductor	Over 3 Conductors	Between Conductors	Between Conductors and Ground	Per 1000 Volts: Between Conductors	Per 1000 Volts: Between Conductors and Ground
New York-Edison	6 600	Paper	156	156	312	312	47	47
New York-Metropolitan	6 600	Paper	218	125	436	343	67	52
New York-Subway Co	11 000	Paper	218	250	436	468	40	43
Buffalo-Niagara Lines	11 000	Paper	203	203	406	406	37	37
Buffalo-Niagara Lines	11 000	Rubber	281	—	562	281	51	25
Chicago-Edison	9 000	Paper	203	140	406	343	44	38
Milwaukee	15 000	Paper	250	187	500	437	33	29
Saint Paul	25 000	Paper	281	203	562	484	22	19
Saint Paul	25 000	Rubber	218	156	436	374	17	15
Providence	12 500	Rubber	281	—	562	281	45	22

joints and in the terminal bells, but being hygroscopic in effect, due to voids left after cooling, it was not satisfactory at the higher voltage. About this time some of the engineers of the company, after considerable study and experimentation, de-

* Burch, A. I. E. E. Trans. 1903, page 433.

veloped a high-grade insulating compound which since then has been used exclusively on all high-potential work.

The splices are made up with great care and the compound is poured into the sleeve at a temperature of 150° cent. in a manner to exclude all moisture. The success of this work is evidenced by the fact that there have been but two failures in splices during the last five and one-half years and there are now some 3400 splices in the high-tension system. The compound-filled terminal bells were formerly made of brass cylinders with a cast-brass base, a substantial ground connection being made at the base. On some recent work a few bells made of a special impregnated concrete have been used with very satisfactory results. The proper protection of cables against damage due to burn-outs of adjacent cable has received considerable attention, and all cables in manholes are covered with split vitrified-clay tile cemented in. Entirely separate duct lines, either on different streets or on opposite sides of the same street, are provided, so as to divide the energy along different routes when possible, so that all important sub-stations have at least two lines, each of which follows a different conduit route.

Semi-annual potential tests were made at the beginning, but were discontinued when the system became more extensive. The danger and liability of accident from these frequent high-potential tests were thought to more than offset the doubtful advantages derived. Since then testing has been limited to new, altered, or repaired cable, which before being put into service is subjected to a test of double working voltage for one minute.

The record of cable trouble (Table II) for the last five and one-half years' operation shows that failure of cables thus far has really not been a very serious matter. The line protective devices are now so perfected that a cable burn-out causes little disturbance. Of the total of 44 cases recorded in the table, only four seriously disturbed the system, and each one of these was aggravated by faults in the protective relays. We believe that these faults have now been eliminated. The records of other companies have also been reported as good. Thus the New York Edison Company[1] in the nine years (1898 to 1907) during which its cable system grew from three to 200 miles had a total of 66 burn-outs, of which 38 were due to external causes, 18 at or in splices, 4 in bends, and 6 in the cable proper. The Cataract Power & Conduit Company of Buffalo[2] had a total

1. Torchio, A. I. E. E. Trans. Dec. 1907, page 1711.
2. W. S. Clark, A. I. E. E. Trans. 1906, page 209.

of four burn-outs in seven years (1899 to 1906) on 12 miles of three-conductor rubber-covered cable operating at 11,000 volts; two were due to mechanical injury, one in a splice, and one in the cable. The rubber surrounding each conductor has a thick-

TABLE II
SUMMARY OF CABLE BURNOUTS
JAN. 1903* TO JULY 1-1908
9000 VOLTS

YEAR	MILES OF CABLE DEC. 31.	BREAKDOWNS				
		IN JOINT	IN BEND	IN CABLE	EXTERNAL CAUSE	TOTAL
1903	54	1		3	4	8
1904	71			2	1	5
1905	128		1	1	1	3
1906	209	1		2	2	5
1907	254			4	3	17
1908	Jan. 1 264		1	3	4	8
TOTAL		2	2	18	25	44

20000 VOLTS

YEAR	MILES OF CABLE DEC. 31.	IN JOINT	IN BEND	IN CABLE	EXTERNAL CAUSE	TOTAL
1907	11				1	1
1908	11	1		2	—	3
TOTAL		1		2	1	4

* DURING THE FIRST YEAR'S OPERATION AT 9000 VOLTS IN 1902 THERE WERE SEVERAL BREAKDOWNS IN JOINTS AND IN BENDS BUT THE TROUBLE RECORDS, UNFORTUNATELY, ARE NOT ENTIRELY COMPLETE.

ness of 9/32 in. and there is no overall jacket. The rubber shows no deterioration. Mr. Mershon[3] mentions four three-conductor cables, each about two-thirds of a mile long, operating at a pressure of 22,500 volts, on which there had been but one

3. Mershon, A. I. E. E. Trans. 1904, page 610.

case of trouble in three years and that at a poorly made splice. The terminal bells on these cables are made of hard rubber.

On the 20,000-volt line in Chicago, which has now been in service about 15 months and which was an initial installation at this higher pressure, there have thus far been a total of four burn-outs, one of which was due to a mechanical injury, one to moisture in a poorly made joint, and two were in the cable itself.

Investigations. It will be seen, therefore, that the experience with high-potential cable has not been such as to cause alarm. The most frequent sources of trouble are from without, and of these electrolysis is apparently the most persistent. However, with rapidly growing systems the destructive effect and danger of breakdowns tend constantly to increase, and it becomes advisable to forego complacency for a while and to look into the system with inquisitive eyes to see just what is going on.

It was this desire to know more about possible dangerous potential rises, so that proper safeguards against them could be provided, if found necessary, that led the Commonwealth Edison Company to undertake a series of spark-gap and oscillograph investigations, the principal results of which thus far obtained are described in this paper. A further purpose of the tests was to obtain information regarding these high-voltage systems for use in their further development. The investment required for such 20,000-volt and higher transmission cable system is very large; it therefore becomes all the more imperative to know that such investment is a safe one commercially.

Tests on 9000-volt system. About a year ago a series of oscillograph tests was started on the 9000-volt system to determine whether or not resonance existed in any part of it, or if dangerous potential rises occurred from any other cause. The oscillograms obtained were taken at the Fisk Street station and included current and pressure waves for all principal switching operations, as well as several for a special cable connection in which resonance was sought. None of the records showed any appreciable excess voltage.

These investigations were later continued and expanded, but no dangerous rises were found. Spark-gaps were installed at four different points of the system and their record carefully kept. The gaps consisted of needle points in series with which were resistance and fuse. They were connected between each phase and ground. These spark-gaps showed that there are frequent rises of about 70%, and occasional ones of about 100%, which apparently have no connection whatever with any switching. These occurred at the station and also in remote sub-stations. At times of cable breakdown, one or more of the spark-gaps invariably discharged at double potential[1]. With grounded systems cables break down first between a phase conductor and ground. When the protective devices fail to act properly, as happened occasionally before recent improvements were made,[2] the arc continues until it is communicated to one or both of the other phases. The effect on the system is then far more severe. Such occurrences were, however, the exception rather than the rule. When, during a breakdown, one or two of the spark-gaps discharge it is on the phase or phases other than the one on which the burn-out occurs. Sometimes all three of the gaps discharge during cable trouble, but not simultaneously. It is quite probable that the potential rises evidenced by these discharges are due to magnetic effect in the generators as much as to surges in the cables. There was only one case of cable breakdown which seemed clearly the result of a surge. During some arrangements which were being made at Fisk Street station to put a potential test on a bus-bar, one of the test leads accidentally came in contact with a live switch stud, the door over which had been opened. This grounded one phase of the system through 50 ft. of heavily insulated No. 12 wire in series with the high-tension coil of a dead 150-kw. testing transformer. The heavy insulation was punctured in a number of places and the entire 50 ft. of test conductor was vaporized. No other damage was done in the station. A few seconds later one of the cables burned out at a point about one mile from the station.

The net result of this investigation indicates that at the present time the 9000-volt system is reasonably free from dangerous conditions, as far as destructive potential rises are concerned. It also indicates the advisability of continuing to make tests on newly installed or repaired cable at double potential for one minute.

Tests on 20,000-*volt line.* Attention was then turned to the new 20,000-volt line in the attempt to learn what difficulties are to be met with there. On the diagram of this line in Fig. 4 are

---

1. Double potential when units with no neutral resistance were grounded, and about 130% over potential when neutral was grounded through resistance ($R=2.6$ ohms).

2. June paper A. I. E. E. Chicago, R. F. Schuchardt.

shown the locations at which current and pressure connections were made for the oscillograph. The numerals on the oscillograms refer to the corresponding ones of Fig. 4. Spark-gaps were connected to ground at points marked s.g. These spark-

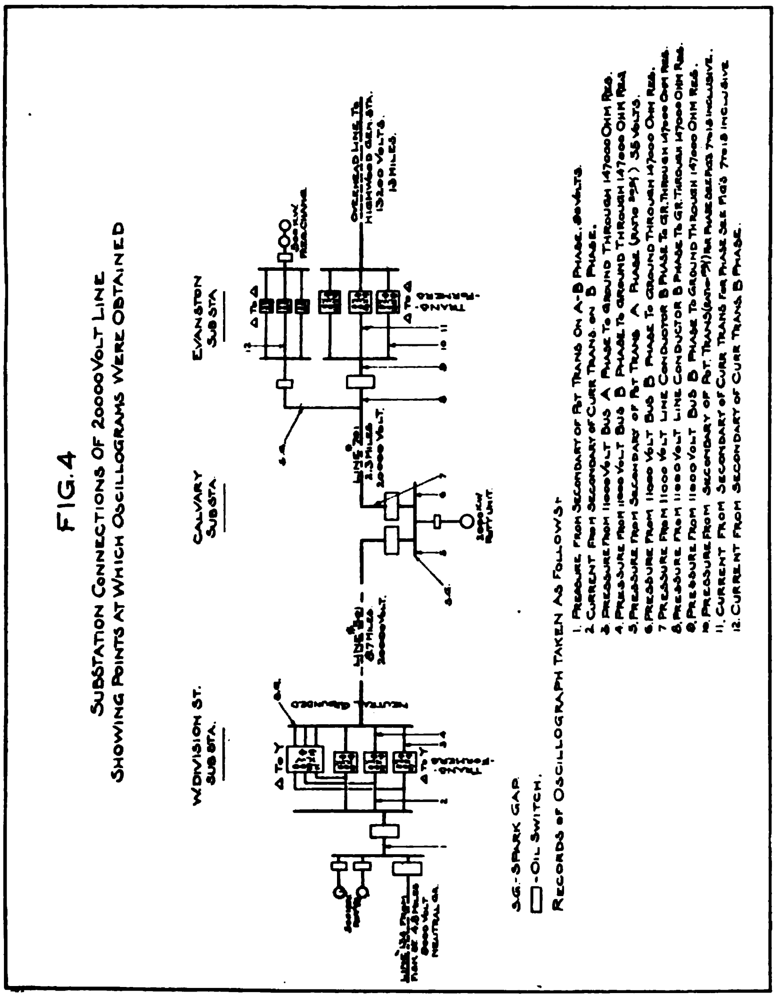


FIG. 4

SUBSTATION CONNECTIONS OF 20000 VOLT LINE SHOWING POINTS AT WHICH OSCILLOGRAMS WERE OBTAINED

gaps were first installed in the Evanston sub-station at the end of the line and originally set at 1.1 in., corresponding to 21,700 volts. During fifteen days *A* phase discharged eight times, *B* phase five times, and *C* phase six times. Five times during this period two gaps discharged at the same time (four on *B* and *C* and one on A and B). All other discharges were on one phase at a time. On only one day were there more than one discharge per phase, and on that day B and C phase gaps discharged together twice, the interval between the discharges being 18 min. With the exception of a single discharge on A phase, none of the discharges could be connected with any switching or other apparent source of disturbance. In this single case a heavy lightning storm passed over the overhead transmission line, which is fed through transformers from the 20,000-volt line. It seems probable that a static charge passed from the line into the transformer and caused the rise which discharged on the A phase gap. Fourteen days' experience with a gap of 1.15 in. (22,500 volts) showed seven discharges on A phase, six on B, and five on C. The only coincident disturbances were as follows: one discharge on A phase when a steam-driven generator at the Highwood station was "synchronized"; B and C phases discharged together when a steam-driven unit in parallel with a generator or frequency changer in Evanston governed poorly, causing heavy cross-currents; one discharge on A phase when a steam-driven unit was disconnected at Highwood, and one discharge on B phase when a steam-driven unit was shut down in Evanston. With the setting increased to 1.2 in. (23,300 volts) there were only five discharges recorded in four months, one on A phase, two on B phase alone, and one on B and C phases together. The simultaneous discharge on B and C phases occurred during a burn-out of the line, probably a breakdown of A phase to ground. One of the B-phase discharges took place when a connection between the 9000-volt coils of the C phase transformer at Division Street burned out. A further increase in gap setting was made to 1.25 in. and left for six weeks, during which time A phase discharged when the line broke down. B phase also discharged, but the connection with a disturbance could not be learned. A final increase to 1.3 in. was made. This corresponds to 25,000 volts. During three months there was one discharge on A phase and two on C phase, but here again no cause for the potential rises could be ascertained. Spark-gaps were also installed at the step-up end of the 20,000-volt line in the West Division street sub-station during a test at that point. They were set at 1.1 in. and during the few hours that they were connected each of the phases discharged heavily at different times. Recently spark-gaps were installed at the Calvary sub-station. They are set at 1.1 in. and each phase has discharged heavily at times.

An attempt was made with the oscillograph to determine the relation between the potential rises and operating conditions. The oscillograph was connected at various times at the three switching points of the line and some records were obtained for switching operations. Unfortunately the time during which the line could be kept out of service for these investigations was so limited that only a few records could be obtained for each operation. It is recognized that in such a limited number (and especially where, as in most cases, only one phase pressure could be recorded) many disturbances may have escaped detection. However, many of the curves are of considerable interest and may serve to bring out more discussion. A number of them are therefore included in this paper. It will be noted that none of the oscillograms shows potential rises as high as those recorded by the spark-gaps. Had more oscillograms been taken, or had all phase pressures been recorded simultaneously, higher rises might possibly have been observed. However, the function of the spark-gap is clearly to detect high potentials, while that of the oscillograph is to show the character of the disturbance and its duration. The spark-gap watches the line continuously, while the oscillograph records occurrences for but very brief intervals.

In the following Figures the source of energy is always from the left, the opening or closing operation being performed on the switch marked *S*. There being but one oscillograph and no pressure wires available, the records at the different sub-stations were not taken simultaneously. The apparatus was set up at one place and curves were obtained for the various operations. The apparatus was then moved to the next sub-station and the operations were repeated.

Fig. 5 is for closing and opening line No. 501. The current wave in the upper curve is the characteristic exciting current wave of a transformer when first energized. The logarithmic curve due to hysteresis is clearly shown. The dying of the pressures in comparatively slow oscillations when the line and transformers are disconnected is shown in the lower curves.

Fig. 6 gives curves for closing and opening line No. 701. A record obtained back at Division street for this operation at Calvary sub-station is given in the upper curve.

The next curves, Fig. 7, are for closing and opening the entire 11 miles of 20,000-volt line from Division street to Evanston. Curves of pressure were also obtained at the Calvary sub-station.

Fig. 8 shows curves for closing dead transformers on to a live bus-bar. The first rush of current in this case was nearly six times full-load current. An interesting curve is the one for West Division street showing the duration and amount of the

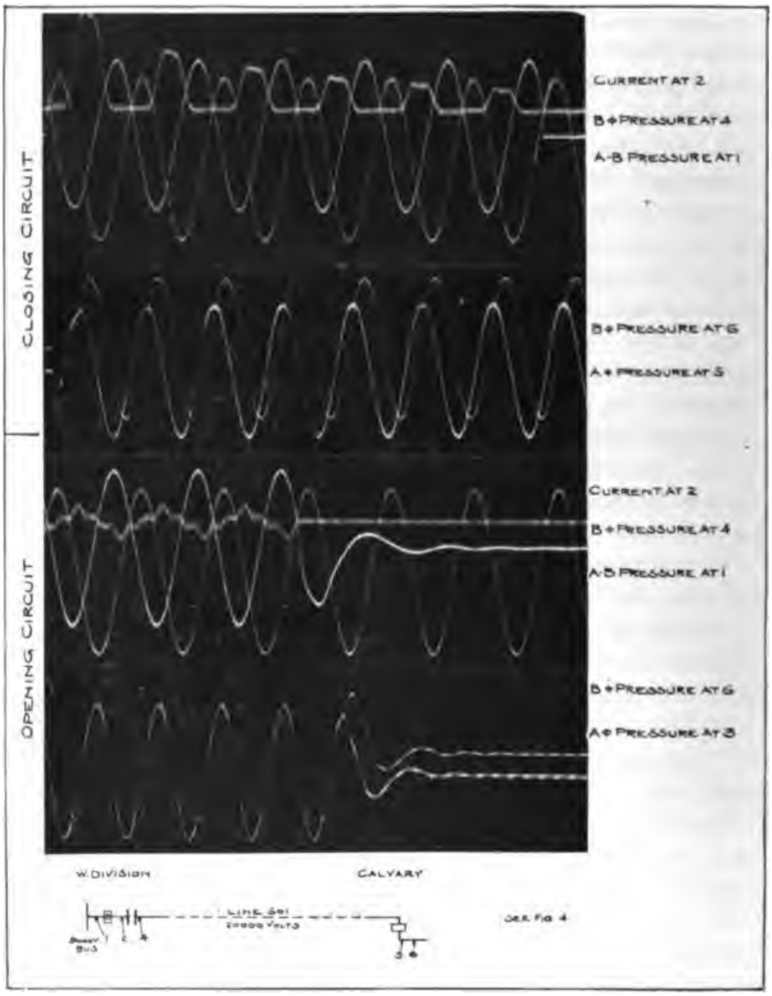


Fig. 5.

current rush. The curve obtained on opening the switch to the transformers is also given.

Line No. 701 was closed and opened with the transformers connected at Evanston, and the curves for these operations are given in Fig. 9. Records were also obtained at Division street for the closing operation and are shown in the upper curve.

The same operations were performed with the entire 11 miles of line, the records being given in Fig. 10.

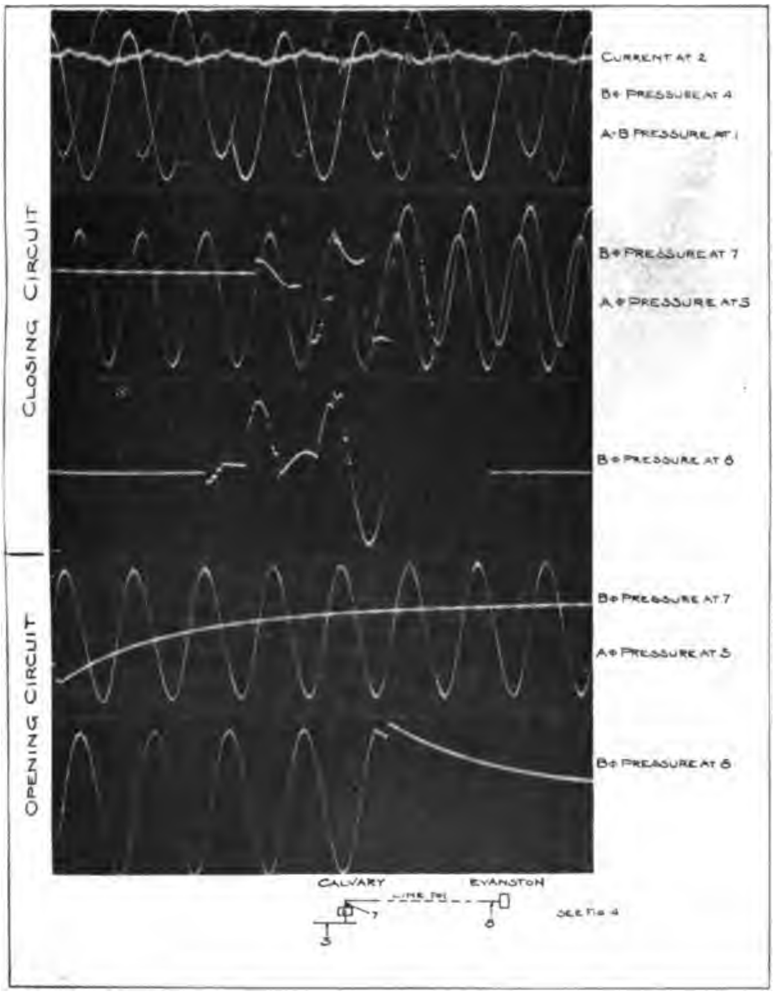


FIG. 6.

The motor of the frequency-changer set at Evanston is wound for 9000 volts, the line pressure being stepped down in three 200-kw. transformers. In Fig. 11 are shown curves obtained when closing these transformers to a live bus-bar. In the

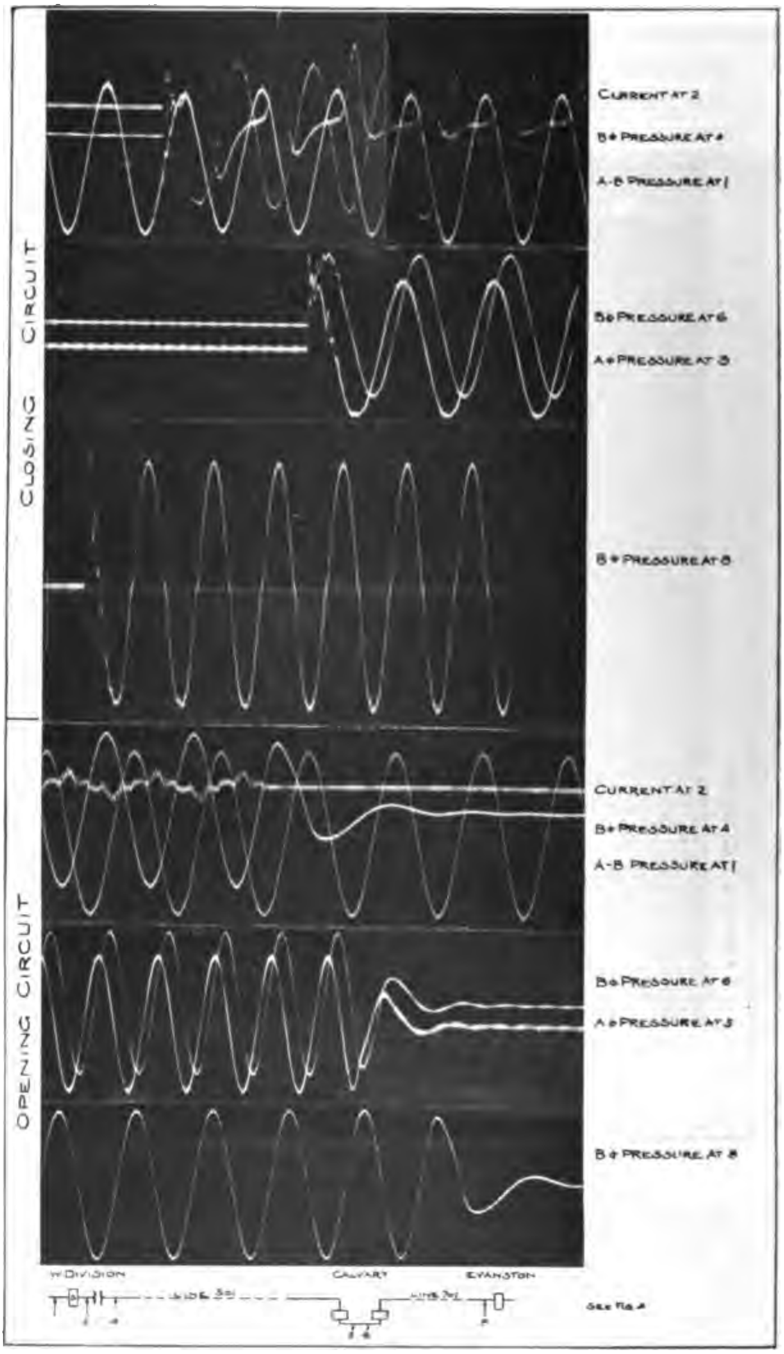

CLOSING CIRCUIT
CURRENT AT 2
B φ PRESSURE AT 4
A-B PRESSURE AT 1
B φ PRESSURE AT 6
A φ PRESSURE AT 5
B φ PRESSURE AT 8
OPENING CIRCUIT
CURRENT AT 2
B φ PRESSURE AT 4
A-B PRESSURE AT 1
B φ PRESSURE AT 6
A φ PRESSURE AT 5
B φ PRESSURE AT 8
W DIVISION
CALVARY
EVANSTON

FIG. 7.

upper curve the transformers had previously been demagnetized by keeping them connected to the motor while being shut down until it had come to rest, the fields being kept excited. In the lower curve they had been disconnected at full pressure. The

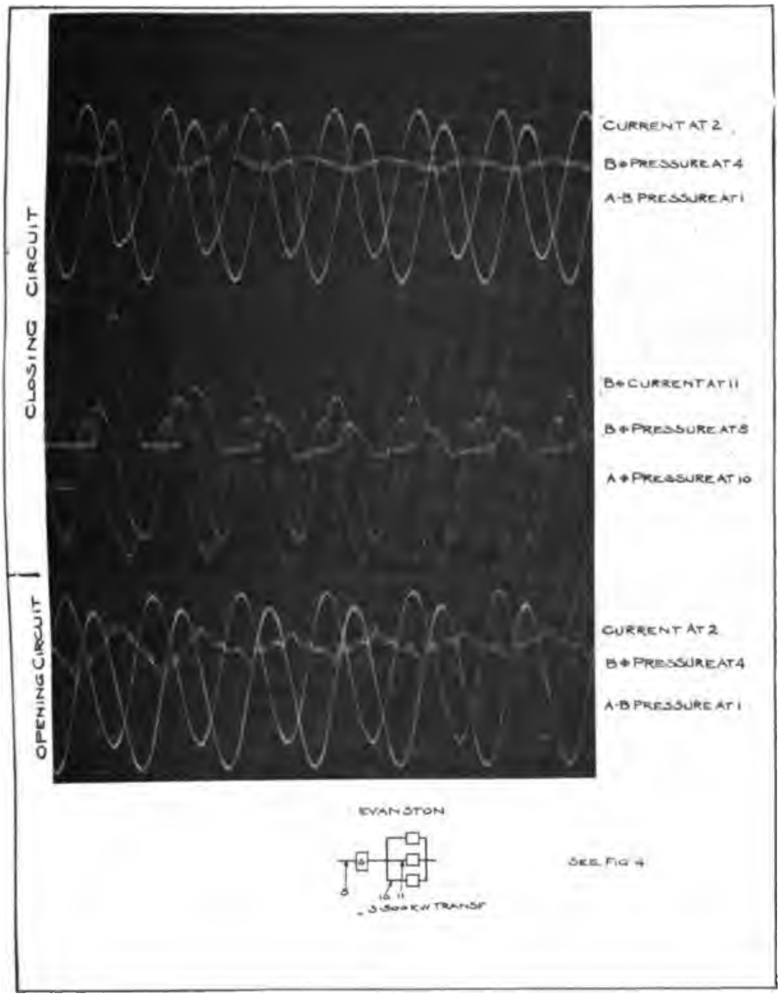


FIG. 8.

difference in current rush emphasizes the advisability of keeping the transformers connected to a dying machine when shutting down. This applies also to transformers of synchronous converter sets.

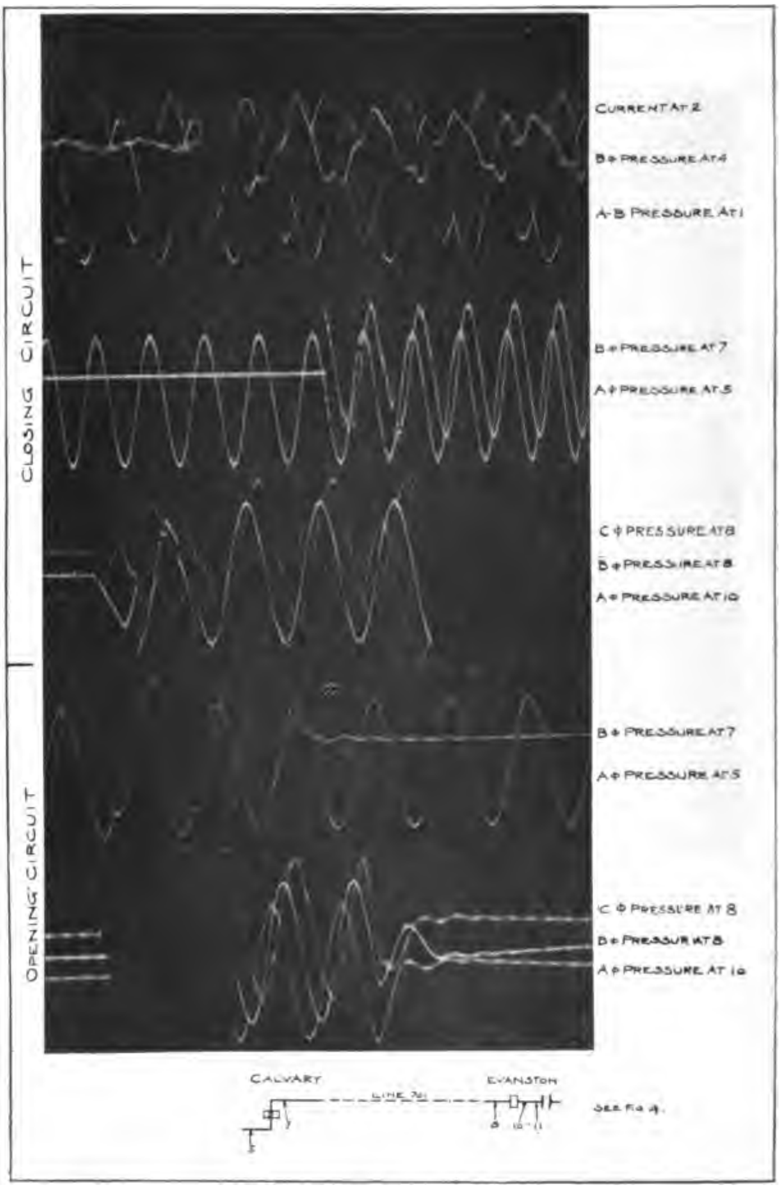

CLOSING CIRCUIT
CURRENT AT 2
B Φ PRESSURE AT 4
A-B PRESSURE AT 1
B Φ PRESSURE AT 7
A Φ PRESSURE AT 5
C Φ PRESSURE AT 8
B Φ PRESSURE AT 8
A Φ PRESSURE AT 10
OPENING CIRCUIT
B Φ PRESSURE AT 7
A Φ PRESSURE AT 5
C Φ PRESSURE AT 8
B Φ PRESSUR AT 8
A Φ PRESSURE AT 10
CALVARY
EVANSTON
SEE FIG 4.

Fig. 9.

The curves for starting the frequency-changer set from these transformers, Fig. 12, show no potential disturbance.

Two load-curves are given in Fig. 13, taken at Division street. The conditions were as follows: upper curve, volts 9000, amperes 85, kilowatts 600, power-factor 45%; lower curve, volts 9000, amperes 105, kilowatts 1600, power-factor 97.5%.

Fig. 14 shows curves obtained at Evanston when a steam-driven generator at Highwood was " cut in " with the line.

At the Evanston sub-station there are three 200-watt potential transformers connected in star with their neutral grounded on both primary and secondary. It will be remembered that the 20,000-volt line is grounded solid at the Division street transformers, but at no other point. The oscillograph was placed in the neutral connection to ground of the Evanston potential transformers' primary and a current of 76 milliamperes was obtained, this amount remaining unchanged even when a resistance of 10,000 ohms was placed in series with the ground connection. The impedances of the three transformers have not been measured, but their load is not balanced. In Fig. 15 this current wave is shown taken simultaneously with pressure. The wave for phase-to-phase pressure shows an unstable condition. A summary of the results shown by these oscillograms is given in Table 3.

Conclusions from investigations. From the spark-gaps on the 20,000-volt line we learn then that pressure-rises in excess of 100% occur occasionally. The oscillograms thus far obtained give no clear evidence of the cause, but the cable broke down three times in 15 months, two of these times in the cable itself, either as a result of these rises, or of weakness developed by mechanical stresses imposed upon the cable at the time of installation. This record is reasonably satisfactory, as during the first year's operation we must expect to eliminate the weakest points. The second 20,000-volt line which will complete this northern ring is being installed at the present time. After it is put in service the investigation will be repeated. The spark-gaps are kept in service continuously. If dangerous potential rises still exist, the remedy will be applied, which, in the form of the aluminum arrester, is fortunately at hand. It is therefore quite prudent to state that 20,000 volts is a safe pressure for an underground system. Whether a higher voltage will be possible, is a matter which has received and is receiving considerable attention.

Possible future developments. The awakened interest in and

TABLE III

SUMMARY OF DISTURBANCES SHOWN ON OSCILLOGRAMS

FIGURES 5 TO 15 INCLUSIVE

Circuit Tested			Stepup Transf. and Line #501	Stepup Transf. and Lines No. 501 and 701	Line #701 Only	Line #701 and 3-500 K.W. Transf	Stepup Transf. Lines No. 501 & 701 and 3-500 K.W. Transf.	3-200 K.W Transf. Demagnetized	3-200 K.W Transf. Not Demagnetized	3-500 K.W. Transf.
Closing Switch	At Switching Point	Bus Pressure	Slight Oscillation	12 % Drop	Slight Rise	Slight Oscillation	Drop	No Rise	No Rise	15 % Drop
		Line Pressure	Slight Oscillation	50 % Rise	Oscillation for Two ~	55 % Rise	30 % Rise	No Rise	No Rise	15 % Drop
		Current	127 %	140 %	——	300 %	200 %	200 %	500 %	600 %
	At Intermediate Point	Pressure	——	40 % Rise	——	——	50 % Rise	——	——	——
	At Receiver End	Pressure	40 % Rise	50 % Rise	Oscillation for Three ~	10 % Rise	Slight Oscillation	——	——	——
		Current	——	——	——	——	400 %.	——	——	——
Opening Switch	At Switching Point	Bus Pressure	No Effect	No Effect	Slight Oscillation	Slight Oscillation	——	——	——	15 % Rise
		Line Pressure	Died down in comparatively slow oscillations where transformers were connected to line. Where line only was opened, a slight pressure rise occurred and the line then discharged through the high resistance of the oscillograph circuit.							
	At Receiver End	Pressure								
Curves Shown In			Fig. 5.	Fig. 7.	Fig. 6.	Fig. 9.	Fig. 10.	Fig. 11	Fig. 11.	Fig. 8.

growing demand for a "City Beautiful" mean the ultimate removal of smoky chimneys, dirt producing factories, and in some cases, of overhead lines from within the heart of the city. This will increase the demand for a means of economically transmitting energy underground in large quantities, and in many cases for comparatively long distances. When that time comes, and the herald has already appeared above the horizon, the operating companies will ask of the manufacturer a cable successfully to withstand the higher pressures required for the service.

Another condition which brings about a demand for high-voltage transmission is the better efficiency at present obtainable from large generating units in large stations, compared with small units in small stations. Recent improvements in turbine design are resulting in greatly decreased steam consumption, especially for smaller units. With a sufficiently high transmission voltage, however, it will in many cases be found cheaper to generate large quantities in a large station and transmit to the distant center where the load is comparatively small, than to generate in a smaller station at that center.

Regarding the maximum cable voltage, Mr. E. J. Berg in his new book on "Electrical Energy" (page 50) says:

> With the present state of the art it would not be conservative to use a cable at more than say 25,000 volts between conductors although it is to be expected that at least with rubber covered cables, or cables with insulation of varying specific inductive capacity a much higher voltage can finally be used.

Mr. H. G. Stott stated in 1906 in a paper on "Underground Cables",* that in his opinion cable voltages of 44,000 could be used, provided the neutral of the system were grounded, thus limiting the potential to ground to 25,000 volts. Cables have been made which in the factory have successfully withstood pressures as high as 80,000 volts or more for an hour, but they have not yet been made for commercial use at such pressures. In the *Elektrotechnische Zeitschrift* of July, 1907 (page 676) appeared an abstract of a paper by de Marchena discussing pressure limitations for cables in which he made the interesting statement:

> As at very high pressures, potential rises are expected to occur less frequently than at low standard pressures, the factor of safety could be

* Read before the American Street & Interurban Railway Engineering Association, Oct., 1906.

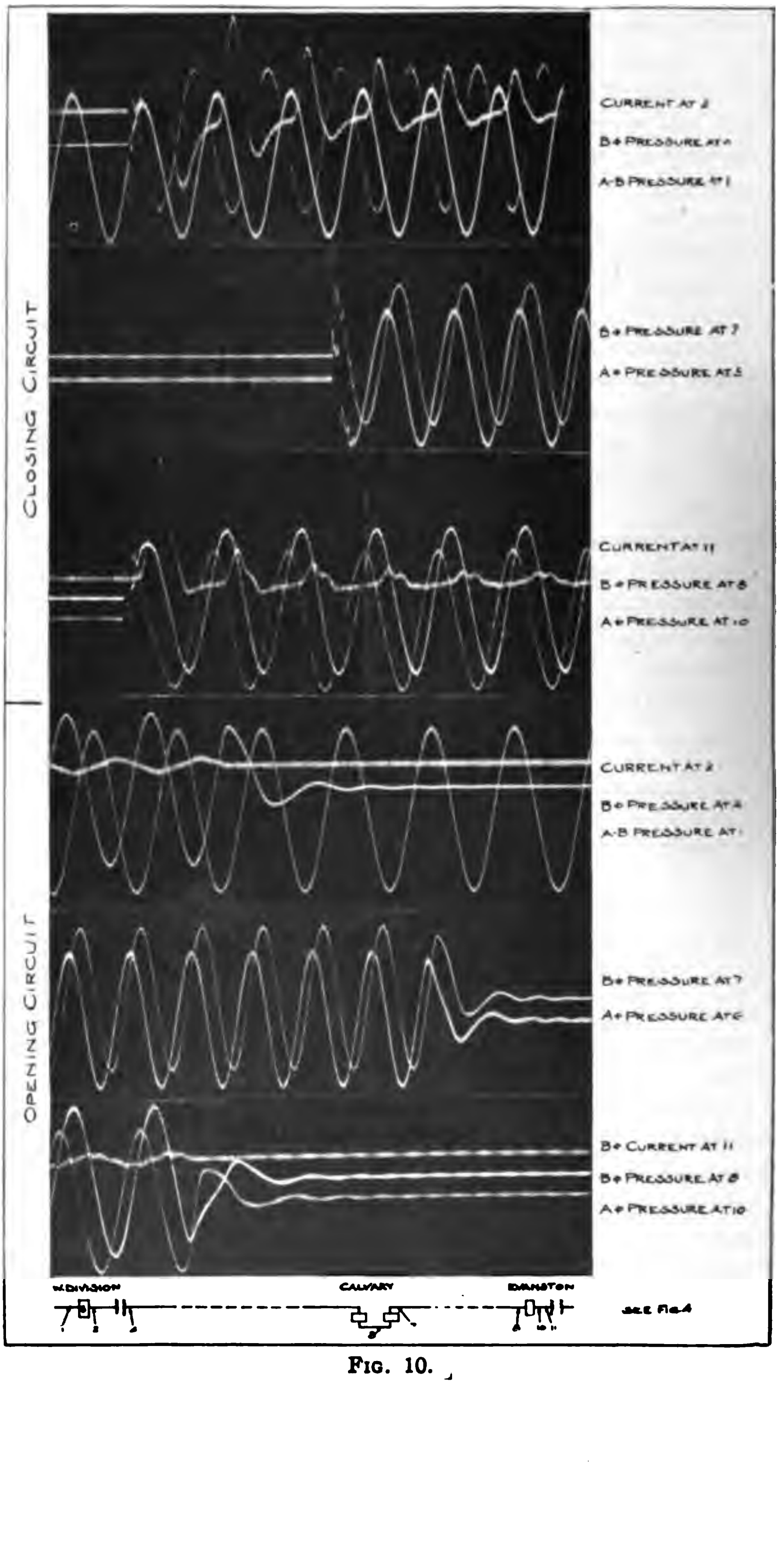

CLOSING CIRCUIT
CURRENT AT 2
B+ PRESSURE AT 7
A+ PRESSURE AT 5
CURRENT AT 11
B+ PRESSURE AT 8
A+ PRESSURE AT 10
OPENING CIRCUIT
CURRENT AT 2
A-B PRESSURE AT 1
B+ PRESSURE AT 7
B+ CURRENT AT 11
B+ PRESSURE AT 8
A+ PRESSURE AT 10
W. DIVISION
CALVARY
EVANSTON
SEE FIG. 4

Fig. 10.

decreased so that cables for a standard pressure of 40,000 volts may be manufactured especially for shorter distances.

It would be of interest to know de Marchena's reason for this statement. Messrs. Feldman and Herzog in the *Elektrotechnische Zeitschrift* for December, 1907 (page 1163) cite many cases of cable installations for voltages over 10,000 and some over 20,000, and also mention the very severe tests to which these cables were subjected. The general subject of high-tension transmission by underground cable is ably discussed from the manufacturer's standpoint by Mr. Richard Apt, Technical Director of the Berlin Cable Works (A. E. G.) in the *Elektrotechnische Zeitschrift* of Feb. 1908 (page 159). Single-conductor, concentric, and three-core cables are discussed, as well as the relative merits of different insulating materials and of aluminum for single-conductor cables. Mr. Apt prophesies that the single-conductor cable for very high pressures is the cable of the future. The installation for the County of Durham Electrical Power Distribution Co. Ltd., England, where about 40 miles of three-conductor cable was put into service for the first time for 20,000 volts, is described. The cable is imbedded in troughs filled with an asphaltum compound. Telephone and pilot wires are placed under the trough. It is stated that this is the method generally employed in England for laying cables.

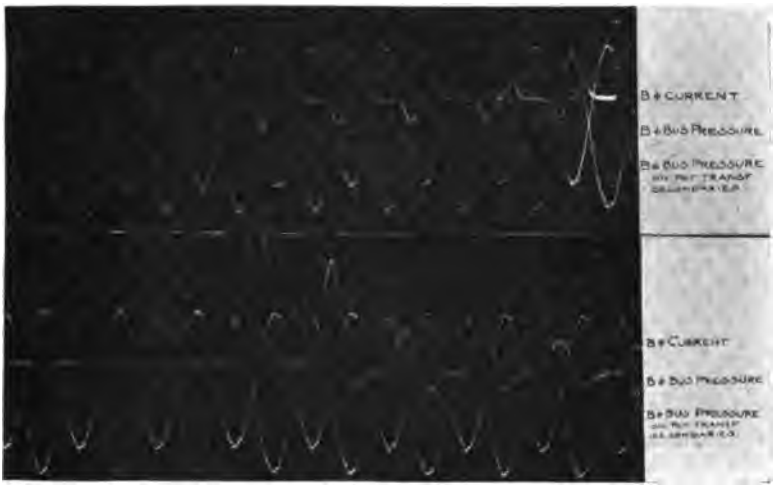


FIG. 11.

If we interpret the discussions of American cable engineers aright, they concur with Mr. Apt in his prophecy regarding single-conductor cable. The heating of the dielectric due to electrostatic stress, with consequent weakening of insulation, the critical voltage which no practical thickness of insulation will withstand—these and other conditions limit the stress to which insulation can safely be subjected. With single-conductor cable in a three-phase system the line pressure can thus be $\sqrt{3}$ times the maximum safe cable pressure. A thin wall of high dielectric strength is desirable to lessen heating and to keep the cable from being too large and too stiff, but then the corona effect due to highly concentrated electrostatic stress must be considered and guarded against. Asphaltum troughs certainly guard against electrolysis and also insulate the lead sheaths of

FIG. 12.

adjacent cables from each other. These sheaths should of course be grounded at one end. It is possible that the commercial demand for underground cables at voltages from 30,000 to 40,000 will result in the use of single-conductor cable, possibly embedded in asphaltum.

With the increasing popularity of all-the-year country life and with the spread of communities as mentioned in the beginning of this paper, it is not hard to imagine a time when it may in exceptional cases be desirable to put some cross-country lines or parts thereof under cover. The general dependance on electrical energy and the immense investments at each end of these transmissions may in special cases justify a fairly large expenditure, provided a very reasonable assurance of reliability is obtained. As many of these lines will be operating at pressures beyond the reach of underground cables, it is possible that a recently suggested protected bus-bar construction may be used. Such a construction could follow the general surface of the land and be built on a right-of-way in conjunction with a railway. The art of insulator manufacture will undoubtedly advance so that pressures of 200,000 volts or more could be employed, if conditions should ever demand such pressures. This construction appears to obviate many of the shortcomings of present high-tension cable systems.

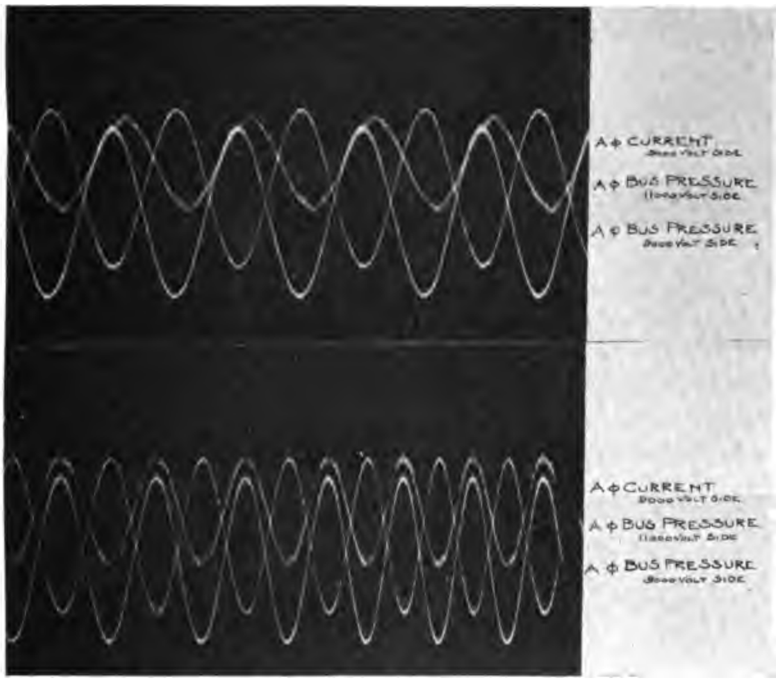


Fig. 13.

Summary

From the viewpoint of the investor or operator the state of the art regarding high-tension underground transmission might be summarized as follows:

1. Underground cable systems of 11,000 volts and under, if properly made, installed, and operated, will give at least equally and probably more reliable service than most of the other elements in the electric power system of which the cable system is a part.

2. Where local and commercial conditions justify, pressures as high as 25,000 volts can be satisfactorily used even for systems aggregating as much as a hundred miles of cable. No single line of such a system would be much longer than twenty miles. If higher voltages are needed to meet operating require-

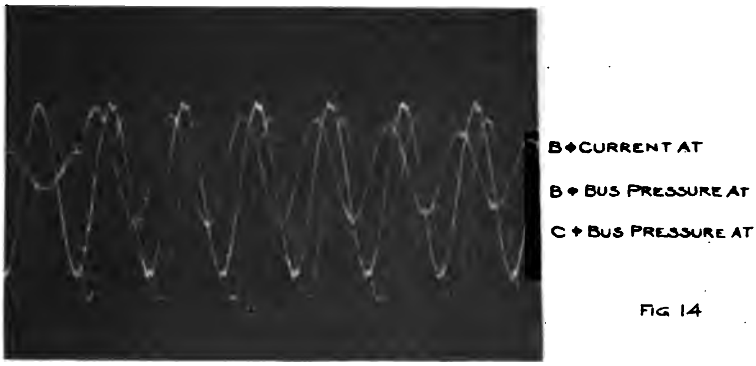


FIG. 14.

ments, and can be justified commercially, special construction will be necessary to overcome limitations in paper, rubber, or varnished cambric insulation, and also in the standard forms of underground conduit or subways used in this country.

3. On comparatively short lengths, underground or under

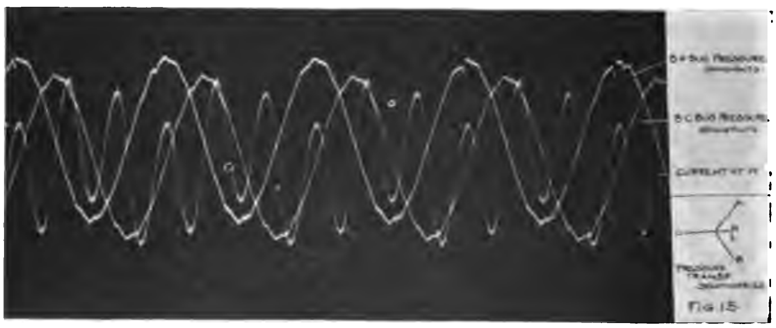


FIG. 15.

water, as a part of a long overhead transmission line, cables operating at 40,000 volts can be used.

4. Potential rises of 50% and 100% are not uncommon in large underground cable systems, although this fact may not always be manifest, due to high factor-of-safety.

5. Definite knowledge of what actually occurs in large high-potential underground systems is still meager, especially regarding intensity and frequency of surges, heating effects, and critical temperature for various kinds and thicknesses of insulation, corona effects, and similar matters.

In conclusion we would urge all companies having high-potential cable systems to keep a complete and systematic record of all troubles. To quote Dr. Steinmetz:

> To determine the origin and cause of high voltage disturbances, so as to be able to guard against their recurrence, the most important thing seems to be to very carefully observe and record all the details of the phenomena, even those which appear unessential. The existence of static on switchboards, lines, etc., and the existence of voltages and currents different from those which may be expected, require special attention. Either of these is sufficient to raise the suspicion of some dangerous fault in the system or some dangerous arrangement of apparatus, which requires consideration . . . The severity of the phenomena depends almost entirely upon the power momentarily available in the system and very rapidly increases with the size of the generating stations. With the increasing power and extension of electric systems we must expect to see these disturbances increase in destructiveness. The prospect which this holds out for the future is not very agreeable; it shows, however, the increasing importance for the operating engineer to study and become familiar with the subject, so as to be able to cope with the problems which are before him, which perhaps are the most serious in the field of electrical engineering . . . It would therefore be a great help if a far closer interchange of experience existed between different operating engineers.

May we not urge on all engineers to remember these words and to appreciate how, by following Dr. Steinmetz's bidding, we all can help to advance the art.

Addendum

Spark-gaps. In using spark-gaps as a means for measuring potentials, certain precautions must be taken to insure reliable results. This fact is, of course, well known. The spark-gaps mentioned in the paper were made up on the usual insulated standard and provided with ordinary No. 5 sewing needles. The series resistance consisted of carbon rods, of an amount to limit the current to about one-half ampere at normal pressure. In order to get the correct interpretation for the various settings, the conditions of humidity and temperature of the places where the spark-gaps were installed were duplicated in the laboratory and the gaps were repeatedly calibrated from 0.4 to 1.3 in. gap.

Within this range they were found to agree to within about 2%. of the Institute Standardization Rules. It is safe to assume then that the indications recorded in the paper are correct within 5%, which is well within the accuracy required for this work.

Where the spark-gaps are in continuous service, to detect occasional high rises it is necessary either to renew the needles daily, or to brush the points frequently to avoid an accumulation of dust which would alter the effective gap-distances. Any tight enclosure around the gap to prevent this accumulation would introduce errors due to the ionization of the enclosed air. During a discharge the needle points burn to a bead, the size of this bead being an index of the severity of the discharge, and thus roughly of the intensity of the disturbance at that point. A recording meter is placed in series with the gaps to record the time of the discharge.

Oscillograph. The development of a successful portable oscillograph marked an important step in the development of cable systems. It afforded a means for studying the system analytically, so that causes for possible unstable conditions could be determined and guarded against. In the particular application of detecting voltage rises of short duration its usefulness, while still very great, is more limited. The time-interval, already mentioned in the paper, introduces an external limitation. The limitations of the apparatus itself are principally as follows:

1. The free periodicity of the vibrators is less than that of some occasional waves of very short length. In the oscillograph used in the test the vibrators were adjusted for their highest free periodicity, about 6,000 cycles per second. Any peak having a periodicity higher than this would not be correctly recorded. However, a peak of such extremely short duration would have too little energy to cause a harmful disturbance and would thus be of little interest.

2. The vibrator cannot be made absolutely dead-beat, but fair results are obtained with five parts of castor oil and one part of turpentine as the damping medium.

3. The self-induction of the vibrator circuit is not entirely nil, but is sufficiently low to be practically negligible.

4. The sensitiveness of apparatus cannot be made perfect, and its degree of perfection in this regard determines the degree of accuracy of its indications. The amperes per millimeter of deflection of the three vibrators used is from 0.00518 to 0.00578,

While in these four important qualities the oscillograph does not attain the ideal, yet for all practical purposes the apparatus is quite satisfactory. The limited time-interval of application is the most serious shortcoming, and probably the cause for the absence of double-potential peaks on the curves. A great number of oscillograms could be taken and yet none of them happen to coincide in time with a high peak. This was actually the case in the tests made, during which there was never a spark-gap discharge simultaneous with the taking of an oscillogram.

Discussion on "High-Potential Underground Transmission." New York October 9, 1908.

President Ferguson: The subject for discussion this evening, gentlemen, is not only interesting from a scientific standpoint, but is of great importance commercially, because of its possible influence upon the investment and methods of operation of our large lighting and power systems.

With the concentration of the production of electrical energy, with its attendant advantages of lower initial investment and of increased economy in operation, which seems to be the order of the day, there naturally follows the necessity for higher voltages. This demand is coming now not only from the country districts, where water power exists, but also from the more congested districts, such as our great cities where immense power houses have been established to supply not only the lighting, power, and railways of the cities themselves, but also of the suburban area for many miles around.

With the increase in values of property in the centers of our large cities, there comes a gradual expansion of the congested areas and with it, in the interest of public safety, a limitation in the use of overhead construction, especially for high-tension transmission lines, and we find ourselves facing the necessity of placing them underground.

Although a great deal of thought has been given to the subject of overhead high-tension transmission, there appears to be much less definitely known regarding high-tension underground transmission, possibly because the necessity for such knowledge has not previously existed. The possibilities of the future are so great and the trend of the times so marked that it appears the subject before us is a timely one, and it would seem very important that all operating companies supplying energy in bulk should investigate the matter thoroughly and give it much serious thought, since a knowledge of the conditions existing may result in great saving in investment and greater reliability in operation.

It is with the idea in view of bringing out such knowledge as may be available and impressing upon those interested the importance to them of assisting in the development of the work that the paper of the evening is presented to you.

Charles H. Merz: Those of us who, by periodical visits to the states and by meeting American engineers when in England, have been able to keep in touch with American progress, have always been much impressed by the development and distribution of electrical energy in large cities in the States.

High-potential underground transmission has been used in such cases, but in the outlying districts it is perhaps not incorrect to say that underground transmission has only recently commenced to take the place of overhead transmission. The fact that a large proportion of the work hitherto has been in crowded and congested districts probably explains why a " draw-in " or " conduit " system has been, so far as the writer is aware, generally adopted for underground transmission in the United States in preference to the so-called " solid " system of laying cables, largely in vogue in England, or armored cables which are used almost exclusively on the Continent of Europe and also largely in England. The extension of underground transmission to outlying and less congested districts will probably result in the merits of these other two systems of laying transmission cables being considered. It may be of interest to refer briefly to these different systems of laying cables and to some considerations affecting the layout of the cable network in such districts.

For congested districts, such as are met with in the centre of all large cities, the " duct " system is often the only practicable system because it is impossible to take up the streets frequently to lay further cables or to carry out repairs. Accessibility to the cables is, however, some would say, the only advantage of this system for, when laid in this way, the lead of the cables is probably more exposed to deterioration by electrolysis and chemical action than in either of the other two systems referred to above. Further, the system is expensive, especially if the manholes are really substantially constructed and if proper precautions are taken to prevent a fault on any one cable interfering with the other cables. It is also probable that the carrying capacity of the cables for a given section is somewhat less than with either of the other two systems, due to the bunching together of a large number of cables in a confined space and the presence of air between the cable and the duct, which must interfere with the transfer of heat from the cable. This latter point is influenced, of course, to a great extent by the number of ducts laid together and the arrangement of the ducts relatively to one another.

In the so-called " solid " system the cable is laid in an earthenware or wooden trough sufficiently large to allow, say 0.75 in. clearance all round the cable, which space is filled with bitumen, pitch or some similar substance. The trough is covered by 3 in. brick or tile.

According to the writer's experience, the solid system in either earthenware or wooden troughs is the cheapest of the three systems mentioned. In saying this it is assumed that armored cables are only permissible if the lead and armoring are effectively protected from moisture, etc. This necessitates not only a sufficient number of wrappings between the lead and the armor and outside the armor, but also effective impregnation of these coverings with some waterproof and insulating composition that will not crack when the cable is bent and which will retain its qualities. An efficiently armored cable costs, laid in the ground, in England from 5 per cent to 10 per cent more than a lead-covered cable laid on the solid system, but it would seem fair to say that an armored cable which is made and finished in the factory, where proper supervision is possible, should have a longer life than cables laid either on the solid system or drawn into ducts. The solid system has, however, proved an extremely efficient system in practice if care is taken in the composition of the material used for filling in the troughs, so that in this country, at any rate in outlying districts, it works out somewhat cheaper than a thoroughly well designed and manufactured armored cable, and it will no doubt continue to be largely used for such purposes.

Apart from the question of life of the lead, which, as far as present experience goes, is the same thing as life of the cable itself, the accessibility of the conduit system is of little or no advantage in outlying districts. In the writer's opinion it is of little advantage anywhere except in the most congested districts, for the following reasons:

1. It is more expensive, even if there are several ducts laid together and all ducts contain cables.
2. It is much more expensive in initial capital expenditure if spare ducts are laid.
3. Apart from the question of the greater security of supply, if mains are laid by different routes it does not pay in most cases to lay more than one cable by the same route; for it is obvious that if it is a question of supplying any given large area with power, the more the cables can be distributed along different routes the greater will be the number of power ,users that can be supplied without large expenditure on branch cables.

The question of the most economical arrangement of network is one that grows in importance as the area to be supplied increases. In congested districts where very large powers are dealt with from one sub-station situated within a relatively short distance of the generating station, it is frequently only possible to deal with the load by means of a radial system; that is, a system in which several mains, one or more of which are spare, run from the generating station to each sub-station. When, however, it comes to a question of distributing power over wide areas and in outlying districts, a radial system in which two or more cables, one of which is a spare, are laid by the same route to each sub-station, becomes very expensive and it is necessary to consider higher voltages and the use of so-called "ring mains." The use of high voltage also enables the most economical section of copper to be used. This economical section, according to the writer's experience, is in general somewhere between a 0.1 sq. in. and a 0.15 sq. in. cable (say 125,000 circular mils and 190,000 circular mils) for three-core cables; that is to say, it is more economical in first cost per kilowatt transmitted to transmit a certain amount of power by means of a cable of this section working at a sufficiently high pressure to enable it to carry the required quantity of power than by any other section or voltage. This, of course, arises solely on account of the fact that the smaller cable can be worked at a considerably higher current density for the same temperature rise. In congested districts where it is impossible to avoid the use of several cables on the same route to a sub-station fed direct from the generating station, it would be more economical, if armored cable or cables laid on the solid system were used, to lay a relatively large number of cables of the economical section rather than a smaller number of cables of the larger section working at the same voltage. If a conduit line be used, however, the number and size of cables are decided more by the total quantity of heat that a block of cables so laid can radiate than by any other consideration; and for the sake of simplicity and to reduce losses and heat generated, the tendency will be to keep down the number of ducts and increase the section of the cable to 0.25 sq. in., or even more, with a corresponding increase in the total cost per kilowatt capacity. This is a further argument in favor of armored cable or cables laid solid.

If the network be made up of ring mains, the power may flow in either direction along any individual cable and it is not possible automatically to isolate a length of faulty cable by means of automatic cutouts of the ordinary overload and reverse-current type. According to the writer's experience, however, any difficulty in this direction can be got over by the use of some form of balanced relay system.* With this device faulty lengths of cable can be instantaneously cut out without any interference with the system or with synchronous plant running thereon, and without interrupting the supply to any consumer, however many sub-stations may be connected to the ring main, and, what is perhaps more important, there is no danger of a section of the system that is not faulty being also cut out.

This system has acted in such a satisfactory manner in actual practice that it is now being adopted largely both in this country and abroad for protecting all kinds of cable systems, whether laid out as ring mains or on the radial system.

Experience in the north of England shows that a 20,000-volt underground cable system may be installed with assurance that it will work well. In the counties of Northumberland and Durham there is now nearly 100 miles of such cable in use. The cables are three-core paper cables lead-covered and laid on the solid system in earthenware troughs; they are made to stand a test pressure of double the working pressure after laying and jointing, and short samples of the cable are specified to stand in the factory pressures of four times the working pressure after bending tests have been applied. There is also a very

*For further details of this system see *Electrical Review* (London,) August 28th, 1908, and the *Elektrotechnische Zeitschrift*, dated 19th and 26th March, and 2nd April, 1908. (Paper by Herr Kuhlmann).

large mileage of 12,000-volt and 6,000-volt cable working in conjunction with the 20,000-volt system. In cables for extra-high-tension work the joints have generally been found to possess the lowest factor of safety, and it is well worth while devoting special attention to their design so as to provide ample margin in insulation to cover indifferent workmanship on the part of the jointer, and to make sure that after laying, the cable and not the joints would break down if a sufficiently high pressure were applied.

Spark-gaps are installed at each end of each cable length. At special points, such as where overhead lines join the cable system, water resistances are installed, thus providing a path for any surges that may occur under working conditions.

Experience has proved that there is no more trouble with such a system than with a 5,000-volt system. On several occasions consecutive lengths of 20,000-volt cable connected in series to form a single length of over 20 miles have been switched on, but in no case has there been any evidence at the spark-gaps or elsewhere of any excessive pressure. Cables directly connected to a large step-down transformer at the farther end have also been switched on without trouble. 11,000-volt overhead lines and cables have been working in series on the same network for some time without any dangerous surging. In some cases when faults have occurred in 20,000-volt joints, the automatic cut-outs have been found to isolate the fault before the compound in the joint has become thoroughly carbonized, and it has been necessary to close the switches and allow them to operate again several times before the localization tests could be applied. This treatment must subject the system to heavy rushes of current, but there has been no evidence of dangerous surging in the pressure which could not be taken care of by the spark-gaps, etc.

H. W. Fisher: Though the results of the tests given by the authors indicate what voltages cables may be subjected to under working conditions, yet I think it must be admitted that under other working conditions similar tests might have shown quite different results. It is, therefore, highly desirable that other operating companies should make similar tests covering a long period of time and present the results before the Institute. When this is done a better idea of the conditions producing maximum voltage stress can be obtained.

The authors have not specified clearly what they mean by double working voltage, for with a grounded neutral the voltage to ground on the 20,000-volt lines would be 11,550 volts. This doubt impels me to ask if the voltage applied between conductors and ground is $2 \times 11550 = 23100$ volts, and that between conductors, 40,000 volts? If these are the voltages applied, the test to ground is not sufficient, because in some cases the spark-gap method gave over 25,000 volts. I would suggest two and one-half times the working voltage as a figure that is more in keeping with tests made on other large installations of cables in New York City and elsewhere, as it is to the interest of both manufacturer and operator to have the assurance that the cables in question can withstand for five minutes two and one-half times the working voltage without failure or injury to the dielectric.

I presume that the 11,000 volts mentioned several times at the bottom of Fig. 4 should really be 11,550 volts, the electromotive force to ground on a 20,000-volt circuit with the neutral grounded.

With cables operating at extremely high voltages, limitations in duct size, test of cable, and the possibility that impulsive rises of voltage are proportionally less than with lower voltages, may make a double working voltage test sufficient.

Statements made in the paper convey the impression that a good insulating compound is the all-important feature of cable joints. I believe that the method of joint construction is the first and most important consideration; after this work is done well, the use of any good insulating compound of proved merit with fairly high melting point, a compound which does not in compounding with the insulation of the cables themselves form an inferior mixture, will insure perfect joints. It should be remembered that the insulating of the individual conductors is a matter which almost necessarily and unavoidably varies with the workmen. I have found that by standardizing joint-making by the use of properly prepared paper tubes of suitable size and composition, I am able to insure at least the necessary minimum thickness of insulation at all points. In my experience this plan works out better than where the thickness and tension under which insulating tapes are applied by individual workmen are left to individual judgment. I understand that such tubes were used on the cables referred to by the authors.

In one of the foot-notes I see that the voltage rise was 30 per cent more when the neutral was grounded through 2.6 ohms. Was this increase due to a distortion of the electromotive force wave curve by increasing the ratio of maximum to mean effective volts? I have obtained this effect, but only when the series resistance was high.

It is interesting to note that the voltage did not rise when the circuit was opened. This I suppose was due to the fact that oil circuit-breakers open at or near the zero part of the electromotive force wave. I have frequently obtained rises of voltage by opening a switch which made an air arc when operated.

The authors say that "asphaltum troughs certainly guard against electrolysis." I must take exception to this statement, because the worst cases of electrolysis I have known were on cables laid in troughs filled with asphaltum. If a cable could be perfectly protected from moisture by asphaltum, I admit that electrolysis would not occur, but this is scarcely possible, and at the moist spots the electrolysis is greatly accentuated causing pit holes in the lead. I know of cases where thousands of feet of cable laid in asphalt troughs were completely destroyed by electrolysis. While it may seem desirable to insulate the lead covers of cables, it must be remembered that when these cables are of the single-conductor type and carry alternating currents these induced voltages between the lead covers may be considerable. I have obtained from 15 to 30 volts per 1000 ft. with an ordinary lead-covered cable, and in the case of a steel-wire armored cable the lead volts per 1000 ft. were 100. The lead volts per 1000 ft. on the same kind of cable armored with two wraps of steel tape were 350. From this it is evident that at least in the case of armored cables, the armor and lead should be connected frequently, especially as the lead current may be as high as 80 per cent of the conductor current, and this is of sufficient magnitude to cause destructive alternating-current electrolytic action if the current be concentrated at isolated points. Fortunately, in practice the sheath of armored cables is generally continuously grounded.

The thickness of insulation on the 9000-volt cables is given but it is not given in the case of the 20,000-volt cable.

H. G. Stott: I recently passed through Buffalo, and I was interested to inquire about the history of certain cables which I put down there in 1891. They were rubber-covered cables, $\frac{4}{32}$ in. rubber, with lead jacket, and put down to carry 60-light arc circuits, having approximately 3000 volts pressure. I found that they are using these same cables to carry 100-light enclosed arc lamp circuits of 7500 volts. As these cables are now 17 years old, the permanency of the investment seems to be very gratifying. Other cables, I think 0000 copper size, which were purchased and put down at the same time, 17 years ago, for use on 1100 volts, having also $\frac{4}{32}$ in. rubber insulation with lead jacket, are to-day carrying 2500 volts, and are said to be the best cables they have.

Referring to Table I, the first column shows the year, and the second the miles of cable. It will be noticed that after the first 2 years the miles of cable are a little more than doubled. The first 2 years apply to the cables of the Manhattan Elevated Railway, and subsequently they apply, from 1904 on, to the subway cables also. For the last 3 years there have been over 350 miles of 11000-volt cables in service. Following along the first year there will be found a classification of the troubles, which shows a total of 7 burn-outs, due to various causes, one of which was due to a surge. The last column, excluding external causes and damages due to crowbars, etc., gives the number of burn-outs per 100 miles of cable per annum. In the first year there was considerable trouble with the joints, but there was only one fault in 122 miles of cable, and there were only 4.91 burn-outs per 100 miles, due to all internal causes.

In the second year, 1903, there were four breakdowns due to surges. Dr. Steinmetz has expressed an opinion on what probably took place,* and concluded that these surges probably caused a rise of potential of 120,000 volts, so that an 11,000-volt cable can not be blamed for breaking down under these conditions.

TABLE I.

HIGH-TENSION CABLE BREAKDOWN SUMMARY.
JAN. 1902 TO OCT. 1908.
11,000 VOLTS.

Year	Miles of cable	In joint	In bend	In cable		External causes	Total	Excluding external causes and surges per year per hundred miles
				Faults	Surges			
1902	122	3	2	1	1		7	4.C1
1903	144	2		1	4	1	8	2.09
1904	330	1		1		1	3	0.61
1905	351		1	2			3	0.85
1906	353	3		3	1	4	11	1.70
1907	353	1				5	6	0.28
1908	368	1				4	5	0.28
Total		11	3	8	6	15	43	

NOTE.—

External Causes { Mechanical injury. Steam leaks. Armor burns from other cables.

The number of burn-outs per 100 miles of cable per annum has fallen during the last 2 years to 0.28, or practically one fault per 400 miles of cable per year. That is a reassuring record; when our overhead transmission lines can show anything like it, we can look forward to reliable long-distance transmission.

Fig. 1 is an oscillograph record taken, which shows the turbine generator running on the circuit. No. I is the voltage on phases 1 to 2, 10,000 volts; No. II is No. I phase to neutral, 5780 volts; No. III is neutral to ground.

Fig. 2 shows the same machine simply switched on to 38.3 miles of cable. This is rather an interesting record. No. 1 is the potential wave, No. II is charging current to cable phase 1, and No. III charging current to cables through neutral There is a very pronounced ninth harmonic it amounts probably to

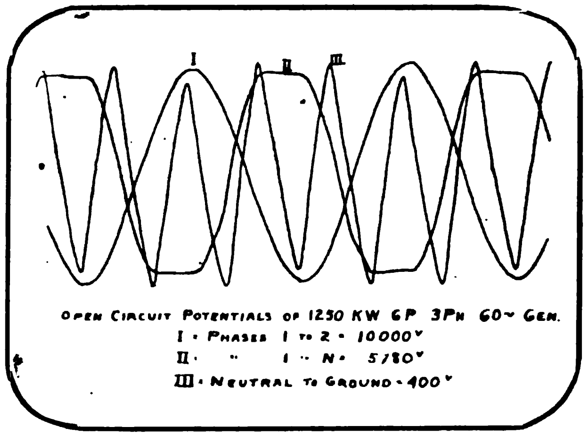


FIG. 1

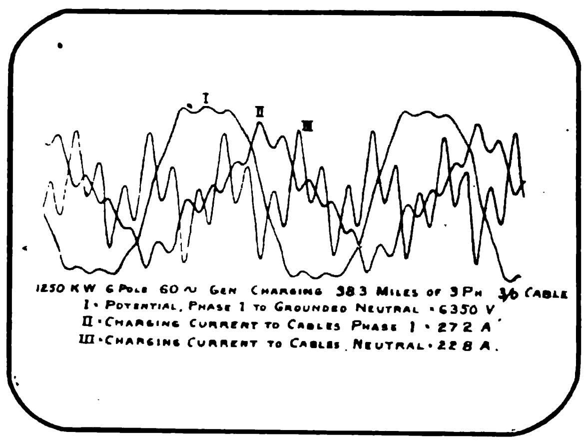


FIG 2

seven per cent of the fundamental. That was a revelation, as we had no idea that any such harmonic as that existed, and we are going to make further investigations. I think it also illustrates the point that further investigations on underground cables will probably yield great rewards for our trouble.

Mr. Merz is a distinguished engineer of very wide experience, but I think that we should apply his conclusions with great caution. Two systems, absolutely similar as to cables, capacities, inductance, etc., but having widely different amounts of power connected, may act in entirely different ways. A system absolutely harmless with perhaps a couple of thousand kilowatts connected to it, may become subject to the most destructive surges and phenomena of all kinds when 100,000 kilowatts are connected to it. In the particular case of the Interborough system, it is possible to develop more than 300,000 kw. on short-circuits between stations. In Mr. Merz's system there are comparatively small and scattered stations, as I understand, so in our American plants there is an entirely different set of conditions.

An apparatus which is going to help out materially in the development of large power plants is the induction type of generator. We cannot, I believe, go on adding indefinitely to the capacity of synchronous generators in one plant. We will soon reach a point where the short-circuit current will be so great that it will be absolutely impossible to take care of it by any automatic apparatus. We all know that in the case of a bad short-circuit on a large system the time-element relays and reverse-current relays fail to act selectively, and everything goes out. With the induction generator, however, we can add on to our present plants indefinitely without increasing the short-circuit current, as the induction generator has the peculiar characteristic of practically ceasing to generate under short-circuit conditions.

E. J. Berg: This paper gives a clue to the probable maximum voltage for high-potential cables. This clue is more discernable when the results obtained by the authors are compared with those obtained in similar tests recently made by Mr. W. F. Wells and myself on the Brooklyn Edison Company's system. In these tests the voltage was 6,000 instead of 9,000, and the neutral was not grounded. We also found from spark-gap indications that under apparently normal conditions double voltage existed at times. These abnormal voltages occurred relatively seldom, perhaps once in a week, or once in two weeks, whereas with the 9,000-volt cables in the Chicago system these abnormal voltages took place every day; and in the case of the 20,000-volt system they found them much more frequently. From this it seems reasonable to conclude that the difficulties due to abnormal voltages will increase with the cable voltage, as should indeed be expected, since a large part of the available energy is stored electrostatically and is proportional to the square of the voltage. Beyond question, commercially a 20,000-volt cable system seems perfectly practicable, the question is how much higher can the pressure be raised.

At the St. Louis International Congress in 1904 Mr. Jona read a paper on cables, discussing the dielectric strength of various insulating compounds and insulators, in general. If I remember correctly, he stated that the maximum permissible potential gradient at the surface of the conductor, using rubber as insulator, was 300,000 volts per inch. Assuming that to be correct, it is possible to calculate the limiting voltage for a given size conductor. Calculations show that it would not be practicable to use small conductors at 20,000 volts and above. At 40,000 volts quite large insulators would have to be used even if the insulation were of the very best.

With large conductors, it would be hardly practicable to make three-conductor cables, they would be unwieldy; consequently single-conductor cables would be used. With single-conductor cables, difficulties may be anticipated, since there is considerable losses in the lead if the lead is grounded, and if the lead is insulated it is subjected to considerable voltages. However, these voltages are proportional to the current, and the losses to the square of the current, and the current will be relatively small at 40,000 volts, so that these difficulties may not be serious.

The authors mention the large current taken by transformers in switching. The oscillograms show six times normal full-load current; that is, perhaps sixty to one hundred times the normal exciting current or the current which would flow after a second or two. This abnormal current depends upon the magnetic condition of the transformer and the phase of the impressed electromotive force at the time the switch is closed. In general, the lower the saturation under normal operation the less instantaneous current. If the density of the iron in the transformer were 65,000 lines per square inch, double voltage could be applied without saturating the iron. Thus such transformers would not be saturated when closed to a circuit of normal voltage even if the phase relation of the electromotive force were most unfavorable, assuming that the residual magnetism were small. With such transformers the maximum current would no doubt be less than full-load current. They would, however, require from 30 to 50 per cent more iron, and would therefore cost more. This construction would not be warranted unless great advantages were gained. These advantages would be: first, small current, which often is of little value; secondly, small mechanical stresses on winding which may be of advantage. So far I have been unable to locate any large stresses, but they undoubtedly exist.

Wallace S. Clark: The authors say that in a grounded system—I presume they mean star-connected—with the neutral grounded, the burn-outs take place to ground. That is not necessarily so. It happened in this case because: first, the insulation to ground is less than that between conductors; secondly, the cable is paper-insulated, and the usual cause of failure of paper-insulated cable is some injury to the lead. It is probable that the burn-out would go to earth, but if the cable had the same thickness of insulation to lead that it had between conductors, it is possible that surges would also have caused some rise of potential between phases, and might have caused a burn-out between conductors.

I do not think that the putting of high voltage periodically on cables for the purposes of tests does the cable any good; it does not show anything, and it is no assurance that the cable tested that way will not burn out the next day or the next hour. Insulation tests are perhaps a little better made periodically. Furthermore, with the present tendency to make paper cables with very low insulation resistance, it is rather difficult to get good results from such tests. There is however, one form of inspection which is greatly needed, the inspection for electrolysis.

If the same amount of money were invested in watching for electrolysis, as in periodic high-tension insulation testing, just as many burn-outs would probably be avoided.

That with a star-connected circuit neutral grounded about seven-tenths of the insulation between conductors is all that is needed to ground, seems to be more of a theory than a fact, because the potential rise noted in this paper seemed to be to ground, to be apparently double the working potential between conductors, and a good deal more than double the working potential to ground. The overcoming of this condition would really be of tremendous economical importance, because it would mean a very much cheaper cable for a given service.

There is one feature in practically all types of cable installations that has not been referred to; that is, the cable that is tested for five minutes at two and one-half times the working pressure will easily stand for five seconds four times the working pressure. That the puncture is a factor, not only of the pressure, but of the length of time the pressure is applied, has been a boon to every cable installation in this country. Several years ago I made some tests to verify this. A few days ago, after reading Messrs. Junkersfeld and Schweitzer's paper I made similar tests, and these gave almost exactly the same results as those formerly obtained.

From the cable manufacturers' standpoint there is no hesitation about using 25,000 volts. But I do not think that it is practicable with any of the forms of insulation in use at present to make very small conductors for 25,000 volts satisfactorily. Furthermore, 25,000- and the 20,000-volt cables that are in use in this country are really working at a less factor of safety than most of the 11,000-volt cables.

I understand that the thickness of insulation in thousandths of an inch between conductors on the Chicago 9,000-volt cables is about 44, and on the 20,000 volt cables about 28 per 1000 volts. It is undoubtedly true that the 20,000-volt cables are made with more care, and probably with a higher dielectric strength in the insulating material, but I doubt if that is enough to make up for so great a difference in thickness of insulation per 1000 volts, so it must be acknowledged that these cables are worked at a less factor of safety. This condition may be a justifiable one, because these cables are less important to the system than the 9,000-volt cables; if this is not so then the 9,000-volt cables have altogether too much insulation on them.

In the matter of cable economics, assume that there are two cables, a 25,000-volt cable and an 11,000-volt cable and that they will last almost exactly the same length of time. Then it is necessary to write off more money per year depreciation on the 25,000-volt cable than on the 11,000-volt cable because the copper for which scrap value can be obtained is the same. Assuming the scrap value of the copper at 15c. per pound for a 4/0, 11,000-volt paper cable, then there will be written off for whatever may seem to be the life of the cable, 66c. a foot on the 11,000-volt cables, and on the 25,000-volt cable about $1.19 a foot.

There are some other cables besides the ones mentioned by Mr. Stott which are running along peacefully under 11,000 volts pressure, and are just about as good as when they were installed in 1898. I think they are the oldest 11,000-volt cables in the country. Unfortunately these cables, as well as the cables Mr. Stott mentioned, are rubber-insulated and do not depend entirely on the lead covers. Just how long a lead-covered paper cable will survive is still undecided.

Alex Dow: In Detroit there are two cables operating regularly at 23,000 volts. Each cable is 39,075 ft. long. The cable conductor is 19-strand, equal to No. 2 B. & S. The insulation is $\frac{8}{32}$ in. rubber and $\frac{6}{32}$ in. of varnished cambric on each conductor, and $\frac{8}{32}$ in. varnished cambric over all, and $\frac{3}{32}$ lead sheathing. The lead sheath is sectionalized at every second manhole, between 700 and 800 ft., in order to prevent the travel along the lead sheath of stray railway return current. Each of these 700- and 800-ft. lengths of lead sheath is solidly connected to earth. One of these cables has been in service for 13 months. It is not the older of the two cables, but it is that which has the most continuous and useful history. It went into service September 4, 1907. Since that time it has broken down six times. One of these breakdowns was obviously due to high temperature from a nearby steam heating pipe. The other five are not attributable to any known cause; four of the breakdowns were in the cable itself, including the one due to the known cause; two of them were in joints. There were four to earth, and two between phases. Adding to this the experience had with the second cable, after it was rejoined, I may say that the total breakdowns were twelve, seven of which were in the joints and five in the cable. Seven were short circuits and five to earth. So that nothing seems to be proved.

The most notable observation is that the faults occur in the middle third of the cable, and are not accompanied by any great disturbances at the terminals. This seems to confirm the observation made by Messrs. Junkersfeld and Schweitzer, that such disturbances do not travel the entire length of the cable, but may spend their energy in one part thereof. Our conception which we will attempt to confirm or disprove by experiment, is that in a cable of that length, working at that frequency, connected in the way in which that is connected, the tendency is for such disturbances to attain their maximum in the middle third of the cable. A desirable precaution would be to cut the cable in the middle, looping it into a sub-station in that neighborhood which is available, and there connect it to electrolytic static dischargers. This will be done, and the action following thereon may be of interest to us, or it may not.

The joints of the first cable were originally insulated according to instructions received from the cable manufacturers by applying a straight wrapping of the varnished cambric tape. The three-conductor joints were wrapped over all and the sleeve filled with a compound. It was obvious to us that the compound used could not possibly penetrate between the layers of tape, and that it was not at all likely to penetrate thoroughly all the larger interstices between conductors. It did not, in fact. The joints broke down continuously from the time the cable was put in service. We stopped using it during the day, but operated it at 4600 volts during evenings as a reserve cable. During the day we made over again the one hundred and twenty-five joints. In the new making up of the joints we restored the rubber as nearly as might be, with the usual vulcanizing tape. We also restored the varnished cambric insulation, thoroughly painting each lap of tape with a non-drying varnish of the same kind as that used in making the cable. The result was satisfactory. The joints, placed on test in comparison with the original joints, showed that they could remain intact under a prolonged test of 30,000 volts. The original joints always broke down within a week or two. The cable put into service with these new joints proved as satisfactory as the other cable which had such joints to begin with.

Our conclusion, in addition to the one already given, is as follows. It is essential that the insulation of joints shall be solid and not include air, as air in the lapping sooner or later causes the joint to break down.

On emergency work and repairs made in a hurry we have used rubber and paper and paraffin. We admit that paraffin is not entirely satisfactory, but we have never had a breakdown which could be attributed to moisture in a paraffin joint, and there is no doubt that thoroughly boiled out paraffin has no air left in it. Whatever of shrinkage there is in the paraffin, is to all intents and purposes followed up and the joint is homogeneous.

Another conclusion is for a higher factor of safety to earth. It will be noticed that the over-all insulation is $\frac{3}{32}$ in. which is theoretically correct, with the grounded neutral. We think that there should be a higher margin of safety, and have adopted it on a third cable which is now going in.

Mr. Merz wants to put his cables down solid in asphaltum so that they will not be attacked by electrolysis. That is quite a common and doubtless a proper practice in England, but it is questionable whether it is worth while in America. Of late years we have adopted the practice of laying pavements on water-proof foundations. In almost all American cities, with a few exceptions, like New Orleans, the soil is under drained, so that the soil in which the conduits are laid is reasonably dry. Any one who has had experience with cables has noticed that in a street in which electrolysis was frequent, and failure of lead coverings common, the trouble has ceased as soon as the street was paved with a water-proof pavement. It is obvious that there is no electrolysis in dry soil, but perhaps there is no dry soil in England.

The system referred to by Mr. Merz is along the line of a distribution system, as distinguished from a transmission system. In none of our large systems, except perhaps the central districts of New York and Chicago, would the comments of Mr. Merz be quite applicable. The cables in Detroit and in most American cities go into sub-stations, from which the distribution is by overhead lines. Where cable distribution is required it is under conditions which call for radial distribution. Circular distribution, while occasionally called for, is not frequent. Therefore the comparison of English and American cable methods is not direct, but can only be indirect.

Warren Partridge: The experience of the Public Service Corporation of New Jersey with high-potential underground transmission when reduced to burn-outs per mile of cable per year has been considerably better than the 20,000-volt experience in Chicago, as reported by Messrs. Junkersfeld and Schweitzer, but when compared with the Chicago 9000-volt system, the results have been very much worse. The exact facts will be given, however, with the hope that they may help engineers of cable systems to tend toward the straight and narrow path where cables never puncture nor picks break through.

The essential facts of the Public Service system are as follows:

Marion Plant with	10,000 kw.	in 25-cycle	turbine units
and	6,000 kw.	in 60-cycle	" "
Coal Street plant with	9,000 kw.	in 25-cycle	engine units
City Dock " "	5,950 kw.	in 60-cycle	" "
and	3,000 kw.	in 60-cycle	turbine unit

The City Dock plant generates 2400 volts two phase and connects to the transmission system through 3000 kw. in transformers connected three-phase—two-phase Scott system. With this exception all generators are three-phase, 13,200 volts star wound. The neutral is not grounded.

Connected to the 13,200-volt bus-bars at Marion, Coal Street, and City Dock are twenty-one underground feeders, aggregating 81 miles of cable and two overhead feeders aggregating 65 miles of line. These cables and overhead lines feed nine 60-cycle sub-stations in which the greater part of the apparatus is composed of step-down transformers, and twelve 25-cycle sub-stations in which there are synchronous converters. The 60- and 25-cycle systems are independent except that some cables can be used on either service.

One cable is No. 4, four cables No. 1, while the remaining sixteen cables are 00. All are three-conductor, paper-insulated $\frac{7}{32}$ in. over each conductor and $\frac{7}{32}$ in. over all, giving 0.437 in. between conductors and between each conductor and ground. The cable sheath is $\frac{1}{8}$ in. lead and the majority of joint sleeves are paraffin filled, compound having been used on a few of the cables. The conduits are part fibre duct and part tile duct.

The record of cable troubles from January 1, 1905 to October 1, 1908, a period of 3.75 years, is as follows: breakdowns, in joints, 11; from external causes, 16; in the cables, 25; a total of 52. The total number of cable-mile-years is 255. Therefore the actual cable breakdowns not accounted for by external causes or by defective installation is approximately 0.1 per mile of cable per year.

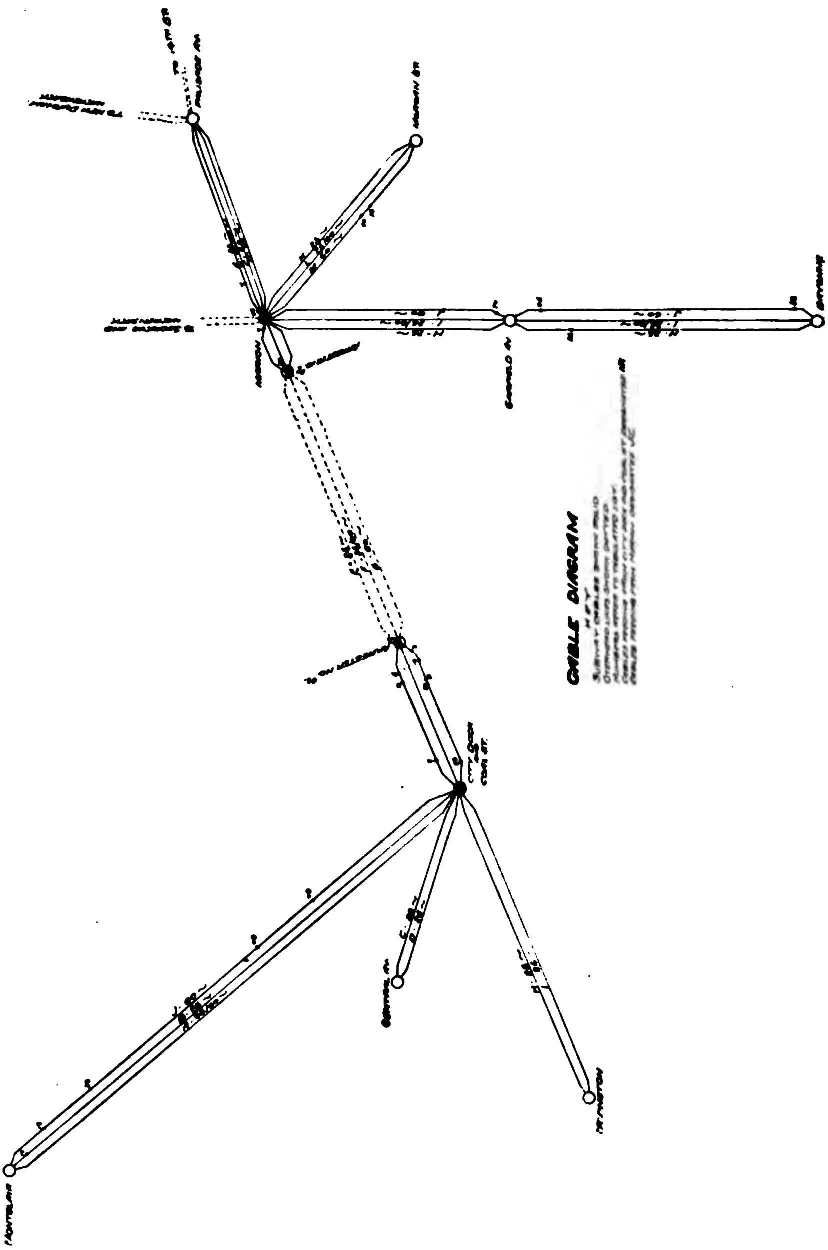
Estimating from the data given by Messrs. Junkersfeld and Schweitzer, the results in Chicago on the 11,000-volt system are approximately 0.02 and on the 20,000-volt system 0.14 breakdowns per mile of cable per year.

The data given by Mr. Rhodes last year for the Interborough Rapid Transit Company 11,000-volt system showed about the same results as the Chicago 9000-volt system, while Mr. Torchio's figures for the New York Edison were, I think, even better.

Analyzing the breakdowns by cables, shows that eleven of the twenty-one cables have had no trouble whatever, that five cables have had one each, two cables two each, and that the remaining sixteen breaks have occurred on four cables.

These four cables stand out by themselves as follows: two of them are tie lines between Marion and the Coal Street and City Dock plants. One is used exclusively for 25-cycle and one exclusively for 60-cycle service. Both cables run from Marion in conduits for 2500 ft., then as armored submarines for 1000 ft., then as overhead lines for 16,000 ft., then in conduits 11,000 ft. and again as armored submarines 1000 ft. into the Coal Street and City Dock plants. The tie lines have, therefore, peculiar physical characteristics. Their use in actual operation differs also from the other straight feeder lines, for as ties between generating plants a considerable amount of synchronizing is done over them.

It is not the practice to operate the plants in parallel over these lines; they are, however, often synchronized in order to transfer load from one station to the other without service interruptions. We know that some of the breakdowns over


CABLE DIAGRAM

these lines have been due to synchronizing or rather to non-synchronizing and believe that others have been due to this same cause even when it was impossible to trace the sequence of events.

The other two lines that have a large number of burn-outs are both long lines, seven miles each, are both operated exclusively on 60-cycle service, and are both smaller than the standard 00 cable, being No. 4 and No. 1 respectively. One feeds from the Marion bus-bar and one from City Dock bus-bar. What the relation is between the above characteristics and the resulting burn-outs is hard to determine. Spark-gap and oscillograph records during different switching operations would undoubtedly throw some light on the subject and will be made later in connection with proposed installation of electrolytic dischargers.

Voltage surges of more or less severity have taken place a number of times on the system, as indicated not only by the cable punctures but by the wrecking of many sets of static dischargers and lightning-arresters.

In spite of all difficulties experienced we agree with the authors of this paper, that cable systems are fully as reliable as other elements in the electric power system. Our records for a period of three years show that cable breakdowns caused but 7 per cent of all the interruptions to service and that the duration of time of cable interruptions was no longer than the average for interruptions from other causes.

A quick summary of the essential differences between the Chicago system and the Public Service system might be made as follows:

1. Chicago, 9000 volts. Public Service 13,200 volts.

This condition is at once much more severe, being a voltage increase of nearly 50 per cent, and assuming a possible surge voltage of double potential would give a possible maximum of 26,400 against 18,000.

2. Chicago, essentially all underground feeders.

Public Service, a considerable amount of overhead lines operated in conjunction with the cable system.

3. Chicago, thickness of insulation per 1000 volts 0.44 inch between conductors and 0.38 in. between conductors and ground.

Public Service, thickness of insulation per 1000 volts 0.33 in. between conductors and between conductors and ground.

However, as thickness of insulation is not a direct function of voltage it is probable that the factor of safety of the two insulations is not radically different.

I would like to inquire what thickness of insulation is used on the 20,000-volt cables at Chicago, and whether ducts larger than standard 3 in. have been employed to carry these cables?

4. Chicago, neutral grounded.

Public Service, neutral not grounded.

There may be other differences between the two systems which would be evident to one more familiar with that of the Chicago system.

The above comparison is made, in order to give, if possible, some clue to the reasons behind the greater percentage of breakdowns that have occurred on the Public Service system.

Breakdowns
13,200-volt cable system

No.	Date	Cable	External injury	Joints	Break-downs
1	5- 7-05	Nk. B	..	1	..
2	7-16-05	" E	..	1	..
3	9-16-05	" G	..	..	1
4	10-22-05	" E	..	..	1
5	11-17-05	" E	..	..	1
6	12- 2-05	J. C. A	1	..	..
7	5- 4-06	" E	..	..	1
8	5- 7-06	Nk. B	1	..	..
9	7- 6-06	" E	..	..	1
10	7- 6-06	J. C. A	1	..	..
11	7-15-06	" A	..	..	1
12	7-17-06	Nk. J	..	..	1
13	7-19-06	" A	..	1	..
14	9-25-06	" E	..	..	1
15	9-29-06	" F	..	..	1
16	10-10-06	" G	..	..	1
17	10-20-06	" B	..	.	1
18	11- 4-06	" E	1	..	..
19	11- 4-06	" F	1	..	..
20	1- 5-07	" G	..	..	1
21	1- 7-07	" G	..	..	1
22	3-15-07	" B	..	1	..
23	3-27-07	J. C. M	..	..	1
24	3-27-07	" J	..	..	1
25	4-24-07	" H	1	..	..
26	5- 4-07	Nk. G	..	..	1
27	6-17-07	" J	..	1	..
28	6-17-07	" G	..	1	..
29	6-17-07	" G	..	1	..
30	8-24-07	" E	1	..	..
31	8-25-07	J. C. M	1	..	..
32	8-29-07	" L	1	..	..
33	8-30-07	Nk. J	..	..	1
34	9-16-07	J. C. K	1	..	..
35	9-21-07	" M	..	1	..
36	10- 5-07	" H	..	..	1
37	10-20-07	" D	1	..	..
38	1-20-08	" J	..	..	1
39	2-18-08	" H	..	1	..
40	4-22-08	" M	..	..	1
41	5-26-08	" L	1	..	..
42	7- 1-08	" A	..	..	1
43	7-14-08	Nk. G	1	..	..
44	8-16-08	J. C. C	1	..	..
45	8-20-08	" J	..	..	1
46	8-21-08	Nk. J	1	..	..
47	8-25-08	J. C. C	1	..	..
48	8-25-08	Nk. J	..	..	1
49	9-12-08	" J	..	..	1
50	9-13-08	J. C. F	..	..	1
51	9-13-08	" G	..	1	..
52	9-16-08	" G	..	1	..
	Totals.........	....	16	11	25

Similar comparisons might be made with the systems of the New York Interborough and the New York Edison, with this difference: the New York Edison system is 6600 volts and the neutral is grounded, while the Interborough is 11,000 volts and the neutral is grounded through resistance.

The fact that the Public Service voltage is highei than any of the above systems seems the most obvious explanation of the results, and the fact that the Chicago 20,000-volt figures are not as good as those from their 9000-volt system further bears out this conclusion; the answer being that as the voltage has been raised, the general factor of safety of cables, switching devices, protective apparatus, and the personal element of the operators, has lagged somewhat behind.

E. E. F. Creighton: All experience confirms the authors' statement that with rapidly growing systems the destructive effects and danger of breakdowns tend constantly to increase. There is a tendency to danger from another source which may be distinguished from that of mere increase of kilowatt capacity and mileage of cable. This danger is involved apparently in accidental coincidence of capacity and inductance with a surge frequency of harmonic when an accidental arcing ground takes place. There is a third condition, mentioned by the authors, namely, rises of potential when the circuit is operating normally. By normal operation is meant the absence of arcs between any phase and ground, or an isolated conductor. There is still a lack of complete experimental data, but I feel convinced that these three conditions outlined are quite distinct, although they may all exist simultaneously. Reviewing these conditions.

1. The troubles resulting from excessive power. A short-circuit of great power stores up electromagnetism which, if suddenly destroyed, produces excessive potentials. It is generally believed that arcs of thousands of amperes of alternating current have a tendency suddenly to interrupt the current. Incidentally, the excessive mechanical strains have a tendency to crack or otherwise injure insulation, combined with an accidental arcing ground.

2. The second condition is the old meace, resonance. That a mere increase in power and cable mileage may have less to do with the production of transient abnormal potentials was proved by some tests made by the writer in association with Mr. S. D. Sprong. The conditions of a cable system were such that the instantaneous value of the surges could be recorded with an oscillograph. Tests were made first with one cable, then we were surprised to find that the surges were less when a second cable was connected to the same bus-bar. When a third cable was added, however, the surges were again aggravated and increased in severity for every successive addition. The detailed results of these tests we hope to present later, when the work will be more complete.

3. The surges not generated by an arcing ground which travels around a circuit are surrounded with mystery. They are not directly connected with switching or accidental conditions, and by observation they frequently produce higher potentials on the system than accidental arcing grounds. The probable source of the oscillations is the arcs on the low-voltage circuit.

In some tests made several years ago on the system of the Detroit Edison Company operating under normal conditions we found potentials of 125 per cent every few seconds, potentials of 150 per cent to 175 per cent every few minutes, and potentials of over 200 per cent twice in one day.

It is not possible commercially to insulate every conductor from its adjacent conductor sufficiently to carry the maximum potential strains of the system. For example, a turn of a transformer normally carrying 10 volts cannot always be insulated to carry the line potential of say 10,000 volts, yet it is possible to get such a potential there. Fortunately such potential differences do not occur frequently in actual practice, and on the end-turns, where they usually appear, the insulation is exceptionally reinforced.

L. T. Robinson: In view of the reference to some limitations of the oscillograph used by Messrs. Junkersfeld and Schweitzer, and the accompanying statement that " For all practical purposes the apparatus is quite satisfactory," I do not feel that the remarks are entirely justified.

In reference to the free periodicity of the vibrators, it is quite possible to produce moving systems in which the current passes through the strips having a frequency of nearly double that which was used or, roughly, 10,000. However, this is accomplished at such a sacrifice of strength and size in the moving parts, that it is not desirable for industrial use. Further, with few exceptions any investigation that requires a frequency in the vibrators of over six or seven thousand would require a frequency of an entirely different order.

The authors say that the vibrators cannot be made absolutely dead beat. I feel that this must be due to a misunderstanding of what can readily be accomplished, as the tests on vibrators shown in Figs. 1 *a* and *b*, indicate over-damping in some instances.

It is true that the self-induction of the vibrator circuit can never be entirely nil. This is no less true of any circuit which can be arranged. The inductance of the vibrators themselves is so small in comparison with any arrangement of conductors which can be used to connect it to the circuit under test, that this limitation, if it has any real significance, does not relate specifically to the oscillograph. In fact it is quite possible that the vibrator itself, due to the action of the moving strips in the magnetic field, could be represented as a capacity rather than as an inductance; that is, the current through this circuit as far as the vibrator only is considered may possibly be leading by an imperceptible amount.

The sensitiveness of the apparatus can be improved somewhat, but if a different class of problems entirely are to be attacked a sensibility of an entirely different order is required, and the small amount which can be gained by sacrificing strength and dimensions of moving parts is not worth while. The limited time-interval of application, I believe, should not be considered a

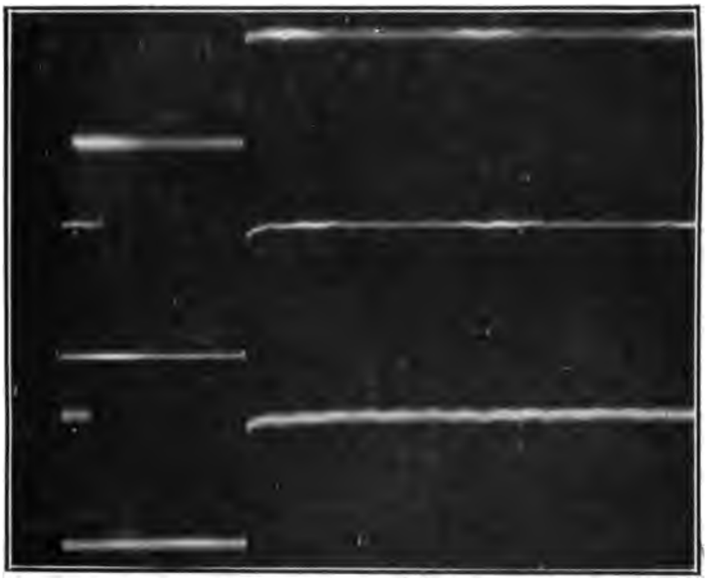


FIG 1.—*a*. Overdamping. ————>
Three vibrators using separate zero lines

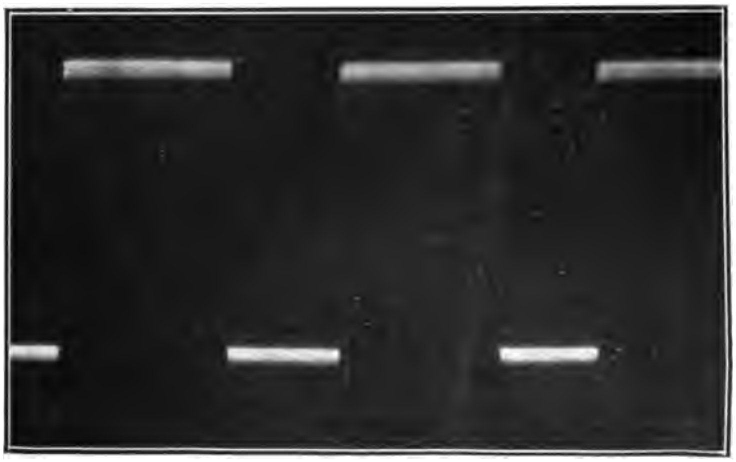


FIG. 1.—*b*. Critical damping. Single vibrator. ————>

The time scale is such that the portions of records shown in *a* and *b* correspond to about 1.25 sec.

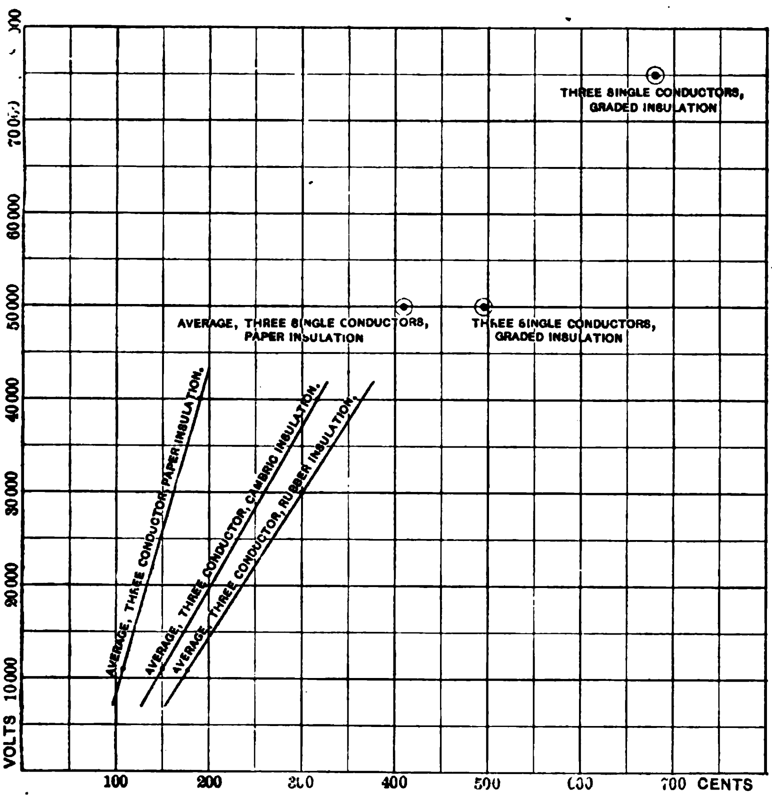
shortcoming at all. To meet conditions of continuous application for comparatively long intervals, special devices for moving a film to receive the image have been devised. They could easily be made and would, undoubtedly, be successful. The trouble here is not one relating to mechanical or electrical details, but to the fact that to operate a film continuously, of the usual width, and at speeds ordinarily employed, would cost about $500 per hour, a prohibitive amount.

I believe the reason why no spark-gap discharges were obtained simultaneously with the taking of an oscillogram was due to the fact that the potential rises represent a very small amount of energy. When the oscillograph is connected the discharge passes through it, thus preventing any rise of potential, and fails to record on the film, either because of its very short duration, or the very small current in the instrument at the time.

Henry Floy: The experience with 20,000 volts here cited and the testimony that could be adduced regarding 25,000-volt cables show that even at these higher voltages such cable is reliable. Above these potentials, we have, as far as I know, no commercial installations in this country. The authors have touched on the questions as to whether it is practicable, and whether commercial conditions will require in the near future, the use of cables at higher potentials. Some figuring has been done, particularly by engineers in England, tending to demonstrate that 25,000 volts is as high a potential as it will commercially and financially pay to use in connection with cables, indicating that if higher voltages are attempted the cost of the cables will be so great as to make preferable the installation of cables in greater number operating at lower potentials.

With reference to high-tension cables for alternating-current service, the manufacturing companies are ahead of the operating conditions; that is to say, the manufacturing companies have for years been ready to furnish cables for higher potentials than the operating companies were ready to use. For instance, at present it is possible to obtain, a cable designed for 75,000-volt operation, tested for a much higher potential; but I do not suppose that any one is quite ready to install a 75,000-volt underground distributing system, although possibly such cables could advantageously be used at the end of an aërial line or under water where special conditions require special construction.

As regards the commercial, financial, and practical use of higher tension cables; that is, those designed for potentials above 25,000 volts, the speaker has been investigating this matter lately and has obtained within the last two weeks from a number of the largest cable manufacturers, prices on high-tension cables for 11,000-volt, 25,000-volt, 35,000-volt and 50,000-volt service, having conductors of No. 0000 B. & S. gauge; also, not so recently, quotation on 75,000-volt cable of 50 square mm. conductivity. The 50,000- and 75,000-volt cables are single-conductor; all the other cables are three conductor, each separately insulated, laid up under one insulating jacket and covered with lead sheathing $\frac{1}{8}$ in. in thickness and containing 3 per cent tin. The thickness of the insulation varies slightly with the different manufacturers, the specification calling simply for time-tests of double normal voltage. The curves shown represent present-day average prices for the different cables indicated. The curves were obtained by plotting the prices quoted for the several characters of insulation



at the voltages named and drawing lines, to represent the average, through the points as plotted. The speaker assumes it is fair to conclude that curves obtained in this way correctly indicate the cost of cables at any voltage throughout their length. The price indicated for three 50,000-volt paper insulated single-conductor cables is the average price; that for graded similar cables is the quotation of a single maker; the same with regard to the 75,000-volt cable, except that the cross-section is of 50 square mm. instead of No. 0000.

Where No. 0000 conductors are intended for 50,000-volt service or above, the thickness of the insulation becomes so great with three-conductor cable that it is impossible to put the completed cable through the leading machines at present available. It may be possible to develop machines of larger cross-section, but I believe at present about 3 in. is the largest cable that can be lead sheathed. This means that for the higher voltages requiring thicker insulation, single-conductor cables must be used which means increased cost compared with three-conductor cables. This is to the disadvantage of the higher voltages, but is compensated for by the fact that fewer spare cables are required as a reserve.

The conclusion that the speaker comes to from a consideration of these curves is that 25,000 volts is not the commercial limit for underground cables. There is no reason why the same commercial advantages cannot be obtained by increasing the potential to 35,000 or 40,000 volts, where the type of cables are three-conductor, because the cost increases in the same ratio as the potential both above and below 25,000 volts. Similarly, for certain conditions it will be commercially advantageous to use 50,000 or even 75,000-volt cables.

John W. Lieb, Jr.: I would like to say a word or two on the contribution made by Mr. Merz. As several of the speakers have said, the local conditions in this country are so different from those that prevail abroad, that it is very difficult, without giving them most careful consideration, to determine which is the better type of construction to use.

My own experience with underground construction abroad dates from 1882. In that year quite an extensive underground system was laid in Milan, Italy, using Edison tubes. It took just about six months in that particular locality to destroy a wrought-iron tube laid underground. The accumulated detritus of many centuries, together with the poor drainage and sewerage system, made these tubes subject to corrosion to such an extent that it showed conclusively that the Edison tube system which was in successful practical use in this country was not adapted to the new conditions, and as a result after 1885, steel-banded, lead-covered cables were used exclusively.

A few years later there was laid in that city one of the first 2,000-volt alternating current underground systems, using concentric cables. This type of cable, lead covered with steel band armor, has given splendid results abroad and is the type most extensively used.

The type of cable advocated by Mr. Merz is very much better suited to European conditions than our American drawing-in system, as they have to deal largely with built-up communities where they can more easily and readily foresee the probable future requirements.

Philip Torchio: Seven years ago Dr. Steinmetz gave us the mathematical theory of surges on high-tension systems. Since then the oscillograph applied to high-tension cable systems has confirmed graphically the values deduced from formulas and the numerical values of line and power constants, making the results of these investigations of great benefit to the operating and designing engineer alike. It is interesting to compare these oscillograph records with similar data obtained a few years ago by David on a considerably smaller system in the south of France and by Patchell in London. Messrs. Junkersfeld and Schweitzer have also experimented on a 20,000-volt line, which is double the voltage previously experimented upon. Their conclusion, that 20,000-volt cables do not act differently from lines at lower voltages, is not only valuable for its industrial application but also for the practical confirmation of the theories that have been advanced and their applicability to wider limits. For this the Institute is greatly indebted to the authors.

As a contribution to the experimental data of this paper the writer wishes to present a record of disturbances in the high-tension cable system of The New York Edison Company during the last few months. The history of all cable disturbances of this company for the first five and nine years was presented to the Institute by the writer in June 1903 and October 1907 respectively.

These two communications described the two types of instruments to give immediate indication of every " high-tension ground " on the cables or system. The first is an electrostatic ground indicator with suitable contacts and alarm bells, and gives indications of low resistance grounds only. The other is a selective ground indicator and indicates the lowering of insulation in advance of a dead ground, and also designates on which particular feeder (averaging 80 or 90) the ground is developing or has developed. Reference in the table is made to each type of instrument respectively by the designation, "electrostatic ground detector," and, "selective ground indicator."

From the theoretical point of view the above records are interesting in showing the existence of the time-element in the breakdown of insulation, the effects of momentary grounds on the system, etc.

This record, as well as the previous ones reported, includes abnormalities of all kinds however unimportant or inconsequential they may have been from an operative standpoint or in their effects on the system as a whole. A large proportion of the so-called " disturbances " or " defects " referred to were not on the Edison system at all but were located on the premises of large customers, to whom high-tension current was supplied.

As illustrating the value of these selective ground indicators from an operating standpoint, it may be of interest to cite one case of cable breakdown which occurred in January 1907, before the selective ground indicators were installed. One phase of a cable grounded and some sixty odd feeders had to be taken out of service, one at a time, before the grounded cable was did not agree with the performance of these apparatus. The same condition exists now with these transient phenomena. The phantom circuit of distributed capacity and inductance with which most of the literature deals, is a uniform circuit of uniformly distributed capacity and inductance, which oscillates with a definite frequency. Is it very different from the circuit existing in practice, which is a combination of circuits of different constants joined to the massed inductance of generators, transformers, etc., and in these complex circuits the voltage rise may greatly exceed double value.

I think that this investigation could be extended still further by inserting spark-gaps across sections of the circuit, across inductive devices inserted in series in the circuit, as current transformers. A short spark-gap, with a tell-tale paper, connected across the current transformer at the terminal stations and middle stations of the high-voltage system would give an indication of the direction in which the disturbance passed, and how far it extended. Furthermore it would be important if one could get an indication not only of the voltage, but also of the frequency of these disturbances, and of the volume of the high frequency oscillation of these transient phenomena. I believe that this can be done by combining a condenser with the spark-gap. If across the circuit a Leyden jar is connected in series with an air inductance, and the spark-gap is shunted across the inductance, such a system takes a current and produces a voltage across the spark-gap depending on the frequency and voltage. As the plain spark-gap gives us the voltage, the combination of the condenser gap and ordinary gap would give an indication of the frequency; and as the volume of the discharge through such a condenser circuit depends on frequency and voltage, a fuse in series with the condenser circuit would give an indication of the volume of the high frequency oscillation.

I do not think that the burning off of the needle points in the ordinary spark-gap is a satisfactory indication of the volume of the oscillation, because the current following the discharge is also the normal machine current. The result depends to a considerable extent on the point of the generator wave, but a spark-gap and a condenser in series would exclude the machine current. The practical importance of the investigation of frequency is that perhaps the destructive effect of such a high voltage depends very largely on the frequency In such a cable system the high voltages may be fairly moderate in their frequency, perhaps not more than a few thousand cycles at the highest, or they may be of extremely high frequency, many hundred thousands of cycles. In the latter case they would be local, and even at considerably more than double voltage they would be practically harmless; with low frequency they may extend over a very considerable part of the system and be most destructive.

The greatest assistance in studying what may occur in the circuit is afforded by the oscillograph. This instrument bears the same relation to the electric circuit that the indicator does to steam engine practice. The oscillograph should enable the electrical engineer to determine all that takes place in the electric circuit; he should be able by means of it to recognize a danger signal, and know when the disturbance is harmless. That is, he must become familiar with the meaning of the different wave shapes.

In the oscillograms of the current there are quite a number of shapes. For instance, in Fig. 7 there is the characteristic effect of the hysteresis cycle of the transformer. In Fig. 5 there is shown the effect of the line intensifying higher harmonics. In this particular case it is the fifth harmonic. It is gratifying to see in all these oscillograms that there is nothing to indicate any dangerous condition in the circuit except the voltages at

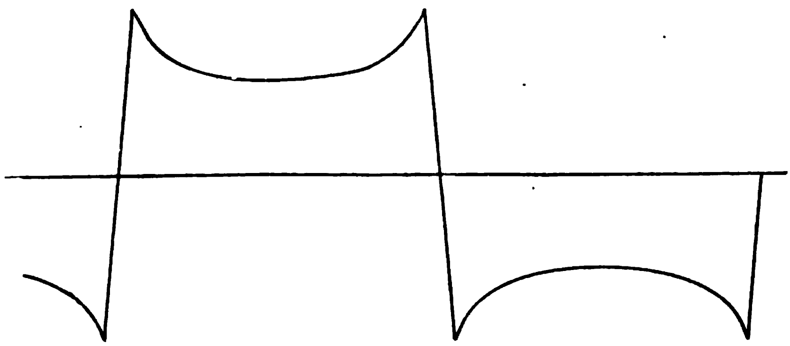


Fig. 1—Theoretical Arc Characteristics

7 and 8 in the two curves in Fig. 6. They look suspicious, and should be further investigated.

One of the most destructive things in a cable system is an arc, and it is important for the engineer who is using the oscillograph to be able to notice the first sign of an arc characteristic. This characteristic is in most cases very definite, it is a double peak of voltage, a characteristic double peak. Occasionally there are double peaks in generator waves, especially in three-phase machines between the neutral and the line. In that case there is no danger, but when a curve like that in Fig. 1 is shown, then there is something in the system in the form of an arc, which is likely to produce destructive voltages. The characteristic is a rapid rise and rapid dropping off, to two sharp peaks separated by a wide valley. The first peak is frequently far higher and the second one may almost disappear, as in Fig. 2. Even the magnitude of the danger of producing destructive voltages by the arc can be recognized from the curve, by the pointedness of the peaks, and occasionally in the extreme case one sees the first peak resolved into a local oscillation, and the valley then more flattened out, as in Fig. 3.

I hope that the authors will extend the scope of their inves-

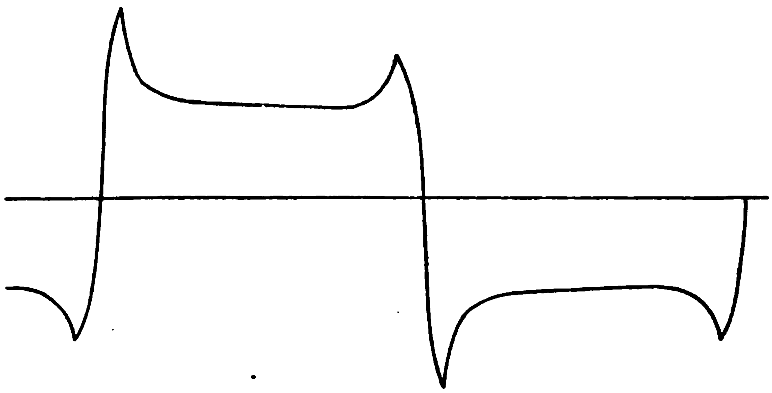


FIG. 2—Arc Characteristics derived from Oscillograms

tigation and give the frequency and the volume of the discharges. I think that at some future time the oscillograph will be a general adjunct in all systems. The oscillograph will be at hand so that it can be immediately utilized when anything

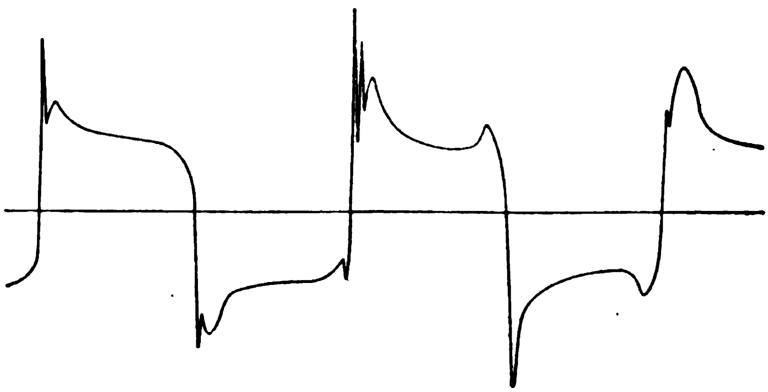


FIG. 3—Arc Characteristics derived from Oscillograms

goes wrong—a generator runs away, a machine burns up, or a short-circuit occurs. If these records could be taken at the time they would be extremely important in helping us to avoid these disturbances.

E. O. Schweitzer: In answer to Mr. Fisher's query regarding the meaning of "double-working voltage:" this means double the normal voltage between phases, and between phases and ground; that is, if the normal operating voltage is 19,100 volts between phases and 11,000 volts between phases and ground, the test voltage is 38,200 volts between phases and 22,000 volts between phases and ground.

With reference to Mr. Fisher's suggestion that two and a half times the normal operating voltage be used in making potential tests on new lines on account of the fact that our spark-gaps showed potentials of 25,000 volts for a line operating normally at 19,100 volts between phases and 11,000 volts between phases and ground. We do not consider this advisable, as this double voltage test is applied for a period of one minute, whereas the surge potential lasts for but a small fraction of a second. The strain on the insulation being a function of both the voltage and time, we, therefore, consider this double voltage test sufficient for the purpose.

With reference to our statement of 11,000 volts, mentioned several times at the bottom of Fig. 4. This statement is correct, as it is the star potential of the line that operates at a delta potential of 19,100 volts, which latter voltage is referred to as coming under the class of 20,000-volt lines.

Regarding the footnote stating that when the generators at Fisk Street were grounded through a 2.6 ohms resistance, a voltage rise of 30 per cent more was obtained, than when there was one machine on the system grounded without appreciable resistance. This may be explained by the fact that the voltage to ground of each phase becomes unsymmetrical, due to the drop in potential with large currents through the resistance. The neutral of the generator including the resistance is, therefore, displaced from the true neutral position.

Peter Junkersfeld: I have seen Mr. Merz's "solid" system and the conditions under which it operates, and I believe that under those conditions many of us would do what he has done. For the large cities, however, Mr. Merz himself admits that the drawing-in system is often a necessity. Under most conditions in this country, however, I must disagree with Mr. Merz.

As to Mr. Fisher's comment, I will answer that the insulation of the 20,000-volt 00 cable is $\frac{9}{32}$ in. wall over each conductor, and $\frac{6}{32}$ in. wall over the three and is of paper. The cable is installed mainly in 3.5 in. duct, with square holes, although some two or three miles is installed in a duct that has 3-in. circular openings. The cable is a trifle large for a 3-in. duct, and is intended for a 3.5 in. square duct.

I agree with Mr. Fisher that workmanship is very essential on the joints, but there must be not only good workmanship, but also good compound, and good methods to insure good results.

Our experience with paraffin has not been satisfactory. In making joints it is, of course, absolutely necessary to exclude air, and as is stated in the paper, the joints are filled with a high-grade compound, and this compound, very hot, is poured slowly so that the probability of any air or voids is reduced to a minimum.

Mr. Stott and others have mentioned the increasing danger when the generating capacity becomes so great. In our system the rated capacity of the three stations connected to the system is 100,000 kw. for the Fisk Street station at this time, 28,000 kw. for the Quarry Street station, and 18,000 kw. for the Harrison Street station, making a total of 146,000 kw. We operate the system in two sections, but even with this precaution the possible amount of power that can be delivered instantaneously from either of the two sections is very large indeed because of the instantaneous short-circuit current of the turbine-driven alternators.

Mr. Clark made some reference to the breaking down of the cable to ground. That has been our experience for several years. If the cable must break down at all, we would rather have it break down to ground first. The neutral is grounded, and we use three single-pole relays. The usual experience is, if one of the conductors goes to ground, that at least one of the other two conductors remains continuous and a test can then be made quickly and the fault located in a short time.

Regarding the comparison between the insulation of the 9000 and the 20,000-volt cables. That is largely a matter of local conditions. The 20,000-volt cable has not as large a factor of safety but it does not need it. The 9000-volt cable does, as it supplies a district in the central part of the city.

Regarding Mr. Floy's statement of higher voltage cables, I do not want to dispute the figures he has given, but cables for 35,000 or 50,000 volts will require special duct or other construction. I think that it will probably be some time yet before we see any very extensive 35,000 or 50,000-volt underground systems. It is difficult to justify them commercially, even if they could be installed and possibly operated with reasonable success.

Ralph D. Mershon (by letter): The authors suggest that, as transmission voltages increase there may be instances where it will be advantageous to build transmission lines after the manner of protected bus-bar construction. This suggestion is rather an interesting one. It is especially interesting to me, because about two years ago I made some estimates on the cost of building a transmission somewhat along the lines I understand the paper to suggest. It occurred to me that if we should build a transmission line with comparatively short spans and with supporting structures just high enough so that there would be no chance of an arc between the lowest point of the line cable and ground, and then build along and over the whole line a casing or continuous house of wire netting, the cost might not be prohibitive, and a number of advantages would result, amongst them being immunity from lightning troubles and ease of repair. An estimate was made with reference to a rough design I prepared in which the line cables were to be about 6 ft. from the ground at their lowest point; the height of supporting structure and length of span being such as would, for this clearance, result in the greatest economy. The idea of the wire netting resolved itself into a high fence of this material, on each side of the line, and for the wire netting roof were substituted a number of grounded cables, supported upon the line structures parallel to the line wires and over them. These grounded cables were to serve the double purpose of protection against lightning and protection against baling wire and similar troubles. The cost of this design was compared with that of a tower line of the same capacity and, while the former was somewhat the greater, the difference in cost was not such as would be prohibitive where large amounts of power are to be transmitted.

If such a construction were used, it would be necessary at road crossings, either to go overhead on high structures or underneath through tunnels. The former could be carried out without great difficulty, maintaining practically the same construction on the higher supports as that used near the ground; or the latter could be adopted where the contour of the ground rendered it possible to take care of the matter of drainage.

I should not be surprised if, in time, some such construction as that suggested were resorted to for transmission lines carrying very high voltages, especially as it seems to me not unlikely that as voltages go higher, the insulator will develop to a form and a size comparable with that of a transmission line pole. With the construction suggested, then, the three-line conductors will be best carried upon three separate lines of poles, each pole being, for the major portion of its length, an insulator.

Of course, instead of the grounded cables and the wire fences, we might substitute sides and a roof of corrugated iron which would not only afford the protection previously mentioned, but also protection against the weather, rendering the insulation problem less expensive. In such case, however, the damage resulting from an arc to ground or between the line cables would probably be more serious than in the construction above proposed.

H. W. Peck (by letter): About one and a half years ago suspicion was aroused that there were high-tension surges on a 13,200-volt underground transmission system in Baltimore, and oscillograms were taken under various conditions to determine the actual presence and causes of the surges. The system, at that time, comprised four 2000-kw., 13,200-volt star-wound generators commonly run with the neutral of one machine grounded through an iron-grid resistance; about 24 miles of 0000 three-conductor, paper insulated, lead-covered cables; four 13,000-volt, synchronous motors driving 60-cycle generators, and a number of step-down transformers, supplying synchronous converters. The oscillograms were taken at one sub-station located about 4 miles from the generating station. No rise of potential was discovered at the time of synchronizing an additional generator at the power house or a 1000-kw. synchronous motor at the sub-station, nor at the time of taking off either of these machines. The only condition observed which did give a rise in potential was the connecting of the dead line to the system or to a single generator with which we were testing. With the neutral of the test generator not grounded we obtained a rise in potential of 180 per cent of the normal. With the neutral of the generator grounded through heavy copper wire we obtained rises of potential of 150 per cent, 300 per cent, and 400 per cent of normal.

Further investigations were made with the oscillograph of the overhead distribution and in so doing one observation resulted which is perhaps of value in connection with such investigations. While adjusting for a certain test and observing the form of curve by looking at the image on the ground glass and shaking the head at right angles to the motion of the light there was observed a very irregular wave form. Investigation disclosed no apparent cause for this irregularity and repeated attempts were made to duplicate the same with the film, resulting finally in getting something very similar to that which had been observed. This suggested the value of throwing a reflection of the oscillogram by means of a vibrating mirror upon a screen and observing the reflection for a long period of time, noting whether irregularities did exist and if so endeavoring to ascertain any peculiar conditions obtaining at that time.

Discussion on "High-Potential Underground Transmission." Boston Section, November 18 1908

A. E. Kennelly: This whole subject is one of great importance, because cities are growing and more wires have to be installed, while wires already in existence have to be put underground. The underground construction involves great expense, and the questions of the construction, preservation, and operation of these cables are of immense importance from every standpoint. The subject is full of interest.

N. J. Neall: I should like to ask Mr. Elden what proportion of the total disturbances to the Boston Edison distribution system from all causes is created by cable failures and what is their nature.

L. L. Elden: Our records go back for only two years. The failures in the transmission system are the smallest item in the way of disturbances that we have. Our overhead systems give us considerably more trouble than our cable systems, and I may say that such cable troubles in the transmission systems as have happened during the years 1907 and 1908 have almost entirely resulted from cable newly installed. In six of the cases the cable was clearly defective and was replaced by the manufacturer. In the remaining two cases recorded the troubles were due to mechanical injuries to the cable, for which the manufacturers could not be held responsible.

One of the most serious features resulting from cable breakdowns is the interruption of a considerable capacity, which at times of maximum load is usually supplied over each individual cable.

The normal maximum operating capacity of the standard 0000 three-conductor cable at 6600 volts for peak-load purposes is 2500 kw. In order to prevent the interruption of this capacity from interfering with the service, it is customary to install two or more transmission lines to each sub-station, these lines being operated at all times individually at the sub-station ends, each carrying a part of the station load. Under these conditions the duplicate line is always ready for use, and in case of trouble the operator may quickly transfer to the spare line without any appreciable disturbance to the system in the direct-current districts and only a momentary disturbance in the alternating-current districts.

One very serious factor has to be dealt with by all companies operating cable systems in cities where the return currents of railway systems are present. This is true with us, although a very careful watch is kept on all districts so affected.

M. V. Ayres: The company with which I am associated has about 2.25 miles of three-phase, lead-covered cable in three sections hung up on poles like telephone cable. This is a 25-cycle, three-phase, 13,200-volt transmission. The cable is paper-insulated and lead-covered. One of these lengths of cable has been up since the spring of 1903. The others were put up not long afterward. We have very little trouble with them. Most of the troubles have been due to the fact that they have been hung on poles and run through trees, and are therefore subject to various kinds of accidental injury. We have had hardly any trouble that could be laid to the cable, the splices, the end-bells, or anything of that kind. We had one breakdown in a splice during a lightning storm which I think was due to poor workmanship. The latest trouble with that cable is typical of the kind of trouble we have. A trolley wire broke and pulled over in such a way that it charged the iron bracket supporting the trolley wire, and the current leaked to the cable sheath. Of course the cable sheath was grounded. Enough current passed not only to burn a piece out of the lead cover, but to set fire to the pole. The fire continued for fully 15 min. before the cable broke down. I arrived while this burning was going on, but could do nothing to stop it. I called up the power house ; they had not noticed any trouble then, but did shortly afterward.

This brings to mind the fact that I have been amazed by the number of minor injuries that can happen to that cable without resulting in transmission trouble. We have had holes burned in that cable many times without any serious results. Our practice has been, when we have found a hole to make a temporary patch on it, with tape, and then at the earliest opportunity make a wipe of lead over it. After that there is no further trouble. Some of these holes have existed a long time without resulting in any transmission line trouble. I have faith in the excellence of paper-insulated cable in all ordinary transmission work, although I have had no experience with underground work of that kind.

G. W. Palmer, Jr.: The Old Colony Street Railway Company has had some very gratifying experiences in connection with the maintenance of high-tension transmission cables underground. They have had operating in a quiet way for the last 5 years some 15 or 16 miles of 13,200-volt cables through which is transmitted almost all the current necessary to feed the entire Old Colony system. This system covers the greater part of eastern Massachusetts except Boston, and in some cases the energy is transmitted 50 or 60 miles from the main station. Since the cable was installed there has not been a single instance of disturbance of operation from any trouble. In fact the cable part of the system has shown itself to be the most reliable of any, more reliable than the overhead line of the sub-station or the main station equipment. About a year after a certain section was put in a joint blew out. It did so without making any disturbance which could be noted at the main station or substations, and the occurrence was not known until the manhole was visited ; although the conductor was bared, the cable was still operating at 13,200 volts. I think the cause of our good fortune with the cable installation has been the extremely high class of insulation used and the care with which the cable was put in. The cable was built to stand a factory test of 40,000 volts after installation, although that test never was applied to it.

N. J. Neall: In the work that I have been called upon to do at various times cable design and operation come into very close connection with lightning protection apparatus. I remember a case which came up some time ago on a newly installed line consisting of approximately 1.5 miles of 25,000-volt cable. It was necessary to go through a city underground. It had not been in operation very long before the lightning season began, and in this particular locality lightning was rather frequent and severe. The cable began to break down periodically almost in synchronism with the lightning storms. The manager, who was very anxious about the continuity of his service, called upon the cable manufacturer immediately to explain why he had those troubles. The cable manufacturer claimed it was the fault of the lightning-arresters; while the arrester man wanted to say that the cable man was at fault, he only said there was a reasonable doubt. As a result the operator was asked to provide a full system of inspection of the lightning protection by means of tell-tale papers and by special spark-gaps placed at various parts of the cable system.

I am sorry I do not know the result of these observations. I do know, however, that the system was much improved, but I have half a suspicion that it was largely for the reason brought out to-night—that the cable became seasoned, and that all these joints were finally brought up to concert pitch. I doubt very much whether static troubles were to blame, save in the first case. I mention this incident merely to show that it is desirable to have some definite information on these points, and that it is absolutely impossible to refute the statements of the cable manufacturers when defending something weak in connection with the cable joints, unless there is so aggravated a case of bad workmanship that the trouble can be found right there. I do not want it understood that I think lightning protective apparatus is perfected; there is much yet to be hoped for.

It has been the practice that all cable shall be tested at certain over-potential; I make it a point to protest against this requirement, because I do not think it is instrumental in bringing out entirely the quality of the cable. My experience has been that a breakdown usually occurs after a ground, or more often after a short-circuit on some part of the system, perhaps · xternal to the cable system, particularly if it be connected to an overhead transmission system. I think, therefore, that any such overcharge test on the cable itself is likely to impair it needlessly; that it does not indicate its ability to combat the sudden strains just mentioned.

The chances are that with the present state of manufacture, the quality and dielectric strength of the cable can be reasonably determined without subjecting it to this high strain. A very good test of the strength of the disturbance might possibly be obtained by an equivalent spark-gap measurement. A standard length could be subjected to static discharges, and the opening of a gap in shunt to the cable that would just take these discharges could be noted. This would give some measure of ability to absorb or to pass those operating disturbances which have all the characteristics of high frequency in their initial effect. I have never made any experiments on cables under these conditions, but it appears to me that the method could be adopted to give some guide to the behavior of the cable under those circumstances where there are " rushes " and not steady rises of potential to deal with.

Dugald C. Jackson: It is the ambition of the Commonwealth company, as I understand it, to put itself in a position so that there will be no extensive generation of primary power in Chicago except that carried on in its own stations. The realization of such an ambition would be of real advantage to the city, and its people, and has been so recognized by the city council. The Chicago city council has therefore taken the ground that small franchises for the electrical generation and distribution of power shall not be hereafter granted, and that such existing franchises should be extinguished as far as possible. The idea seems to be that every small station is a disadvantage to the city because it fouls the atmosphere, and it serves part of the best and most profitable territory at competitive rates and thus forces the larger company into a position where it cannot reduce its rates to a satisfactory low average for the entire territory of the city. Many small electric plants usually exist in the large cities, plants furnishing energy to one or two large office buildings and the like. The people operating such plants become ambitious and perhaps endeavor to obtain a franchise to cross the street. That means supplying another block. Ultimately that may develop into a four-block plant. Franchises of this kind are being refused in Chicago for the reasons already stated. The Commonwealth company has been required, however, to put its rates at figures which are thought to be reasonable, accompanied by a sliding scale of reduction for a certain period of years.

The Commonwealth company has already gone so far in its ambition to become the great primary power purveyor for all uses in the city, I understand, as to make a bid for the generation of power for drawing all the trunk-line railroad trains into and out of the city of Chicago which the railroad managers arrange to handle by electric power. This is quite a large project. Whether the Commonwealth company will succeed in persuading the great railroad interests that it is to their advantage to buy power is yet to be seen, as none of the large railroads has yet undertaken to draw trains into Chicago by electric power. That the the intention of the Commonwealth company is likely to succeed is indicated by the fact that the company is already supplying much power to electric street and elevated railways in Chicago.

The great stations that are to-day operating in Chicago, Boston, and other cities, were virtually out of the question under the old basis of reciprocating-engine generation, but the ease of operation is apparently as great with a 50,000-kw. steam-turbine station as it was with a reciprocating-engine station of not exceeding 12,000 or 15,000 kw. capacity. A few years ago it was thought that 5,000 kw. was near the limit of construction for a generating unit. Now 5000-kw. turbines are commonplace. If something very much larger than we have now in the way of generating units and generating stations does not materialize, the ambitions of the Chicago company will result in half a dozen stations; with 30,000-kw. units, however, perhaps the Chicago company could do the work they attempt to do with two or three stations.

A second factor which is brought out in this paper is the lack of analytical knowledge in regard to dielectrics and insulations. Although we may know the dimensions and kinds of the materials of which a cable is manufactured, we cannot compute with certainty its electrostatic capacity. The computations made are largely by rule-of-thumb. In other words, the science of cable construction lags behind the science of generator construction. By generator construction I mean the entire system beginning with the device that consumes the coal and ending with the device that delivers the electric power to the transmission lines.

Insulation is to-day truly one of the great problems of the electrical arts. We need an answer to the question: what causes dielectric properties, and why are some materials insulators and others conductors? Not until we begin to approach the answers are we likely to have that analytical knowledge of the electric transmission of power which is needed to make electric power of universal service.

The oscillograph has clearly been serviceable in the Commonwealth power house. I have thought for some years that the oscillograph might be of pretty nearly as much value to the transmission man as is the wattmeter. It has not been many years since the power transmission man thought that a wattmeter was a supernumerary. He went by ammeter or voltmeter, but he finally found it necessary to follow the dictates of analysis and take to the wattmeter and its indications. He now looks upon the oscillograph as a supernumerary, but the oscillograph in certain forms is coming to be an instrument of commerce as well as of research.

L. L. Elden: I would like to refer for a moment to Professor Jackson's remarks on the Commonwealth system of Chicago. Those who are not operating these large cable systems, I think have no realization of what happens at the generating station when a defective cable develops in a location in close proximity to the station. Upon the occasion of a recent burn-out of a turbine in one of the Chicago stations when operating with a very large generating capacity, the cables forming the generator leads between the generator and the switchboard were not only punctured, but were so seriously affected mechanically that the conduit containing the cables was also badly damaged. From this, one may judge what problems are to be met in cable construction in the future if the large stations continue their present rapid rate of growth; the result will probably be the division of the generating stations into sections in order to reduce the capacity concentrated on one bus-bar. This is being done in Chicago at the present time.

The New York Edison Company operates the largest generating capacity in the world on a single bus-bar, the peak load last year being in excess of 120,000 kw. The concentration of this capacity in a short-circuit within or adjacent to the station produces conditions very difficult to control; it is probable, therefore, that in future means will have to be provided to limit the amount of current that can be concentrated in a fault.

This company has in service a device invented by one of their engineers which has been described in one of the papers presented to the Institute. This device performs the duty of detecting breakdowns or defects in individual cables a sufficient time in advance of an actual short-circuit to permit the disconnecting of the defective cable from the system before serious trouble occurs. The actual record of the use of this device on their lines indicates a successful application to practice, as in answer to recent inquiries it was stated by the engineers that with the exception of four cases to date, the indication of trouble on cables had been given sufficiently in advance to permit of the disconnecting of the defective cable before it actually broke down. It would seem then that further perfected apparatus would prove of great value to all companies operating large cable systems.

Professor Jackson referred to the rapid development of the systems of the large companies. I recall that 10 years ago this month the Boston Electric Light Company built their station at L Street and planned for six 1500-kw. generators. At the rate of development in progress at that time it was believed that sufficient capacity was being provided for a long period in advance, yet within 3 years it was necessary to add two more units equivalent to 50 per cent of the original installation. Contrasting this with the present conditions, it is interesting to note that the Boston Edison Company is now regularly installing individual units cach of a greater capacity than six of the original 1500 kw. generators above referred to.

Referring to high-tension cable tests, I do not know that our experience quite agrees with that of the Old Colony system. Our experience indicates that the method usually employed of connecting the cable to the system without tests almost always results in some disturbances or troubles before the cable gets down to a working basis; and while the experience of other companies who make high-potential tests shows that they practically clear their cables of these troubles before they are actually placed in service on the system, there is a considerable difference in the practice of the large companies in making high-potential tests. Some arrange to apply 30-min. tests at double working pressure after the cable has been installed, while other companies reduce both the time of application and the limit of pressure applied in their individual practice. It is generally the practice to omit tests of double working pressure after the initial test of the cable at installation, all further tests being confined to pressures slightly in excess of the regular working pressure of the cable, such tests being made only after a cable has been repaired, due to the development of poor insulation or in actual breakdown.

Several different types of apparatus are used for the purpose of making these tests, two of the most common being a motor-generator or a static transformer. The Chicago company uses a motor-generator arranged for operation at variable voltage, their practice being to raise the voltage on a cable under test very gradually until they have reached the limit which their experience indicates as satisfactory. After applying the test for such time as they deem necessary, the cable is then connected directly to the main system. In but one or two cases have faults developed after such tests have been made.

The New York Edison Company uses static transformers for similar purposes, arranged for the gradual application of the test voltage to the cables in practically the same way as that done by the Chicago Company with their motor-generator equipment.

The Boston Edison Company is at present contracting for a transformer testing set to accomplish the same purposes, which equipment will probably be installed during the coming year. The company is advised by the cable manufacturers that the duplication of such tests as above described will be perfectly satisfactory from their standpoint, except that there are some objections to the application of the double-potential test for anything except a very short period of time, after the initial test of 30 min. has been made when a cable is first installed.

In nine cases out of ten, we have had very satisfactory operation with our relays. We are using the bellows type relay, and I think our experience has been similar to that of other companies using this type of relay. Our experience indicates that frequent adjustments are absolutely necessary in order to keep the relay in adjustment, and such troubles as we have had where selective operation has been required have been found to be due to imperfect adjustment of the relays. Aside from these troubles, we have obtained very satisfactory operation, both on individual lines and on lines where selective operation has been necessary.

more than 20 years. Originally very crude and expensive to operate, they are now highly developed and satisfactory pieces of apparatus. Some of the first locomotives were made of street-car trucks weighted down. Many were of the single-motor type, with connecting rods or with chain drive. The motors were not enclosed, and the trolley was of the pantagraph type. For a long time the equipment was inadequate, largely on account of the narrow gauge as compared with street-railway work, and also because each locomotive was proportioned for particular service, which afterwards increased as the haul was extended. The equipment was based on about 5.5 h.p. per ton, which will stand an intermittent service fairly well, but is not sufficient for long hauls or on heavy grades where sand is used.

Modern locomotives have usually two series motors, with a total capacity of about 10 h.p. per ton of locomotive weight. These motors are rated at 75 degrees cent. rise in 1 hr., or in some cases are rated for 40-min. service. A rheostatic controller with grid resistance is used, and motors are operated in parallel on heavy pulls and in series for switching. The change is made by a four-point reversing cylinder. In a new controller for heavy work, each point on the controlling cylinder has a double-contact finger and a separate blow-out coil. Arc headlights are largely used.

In many mines it is desired to gather the wagons from rooms to cross-headings by motors. For this service small locomotives weighing from 4 to 6.5 tons are used, and since it is not desirable to have trolley wire in the rooms, these locomotives are provided with cable reels. These reels are arranged automatically to reel out cable when running into rooms, and reel it up again on coming out. The cable is hooked over the entry trolley and supplies current to the locomotives. The reel may be driven by a sprocket chain, through a friction clutch, from the axle of the locomotive or from the shaft of one motor. The reel may also be operated by a coiled spring, if the cable be not too long, or by a small series motor connected permanently in circuit. Where grades are very heavy against the load, a wire-rope reel, with a small independent motor, is sometimes provided. In this case, the locomotive remains at the top of the grade, lowers the empty car by gravity and hauls up the loaded wagon by the wire rope.

Main-line four-wheel locomotives are not often made heavier than 14 tons, on account of the heavy rail required. When more power is needed, six-wheel, three-motor locomotives are used, of from 15 to 20 tons weight, or two four-wheel locomotives are coupled together with a single four-motor controller, and with the brake rigging operated from one lever. These tandem locomotives are sometimes so arranged as to be easily disconnected and worked separately when desired.

Electric coal-cutters were first of the "cutter-bar" type, in which the "bits" or small chisels were staggered around the surface of a horizontal cylinder, which was rotated and forced under the coal, making an undercut of a width and height equal to the length and diameter, respectively, of the cylinder cutter head. The next type was the chain-breast machine, which is very largely in use at present. This type comprises a stationary rectangular frame and a moveable frame, carrying the motor, gearing and cutter head, travelling within the stationary frame. The cutter head is the base of a long isosceles triangle, around which the chain is driven by a sprocket at the apex. This chain carries a number of cutters or bits, about one-third being uppers, one-third lowers, and one-third centers. The chain is fed forward into and under the coal by a rack-and-pinion feed, the cut being about 4 or 5 in. high and 44 in. wide. Both types of machines, after making a cut to the desired depth, are shifted side-ways; the operation is repeated until the face of the room is undercut completely across, after which the coal is blasted down.

A new type of machine is an electrically driven puncher. This puncher consists of a vertical motor with gearing, and a cylinder and two pistons, one of which is driven by the motor, the other carrying the cutting-tool or pick. The rear piston, driven by the motor, compresses air on its backward stroke, and the front piston, carrying the tool, is forced back by atmospheric pressure. At a certain point the rear piston passes a port which admits the compressed air between the two pistons. As the front piston is free to move, it is driven forward, causing the tool to strike the coal with great force. Should the pick not strike the coal, the blow is absorbed by an air cushion at the front of the cylinder. This machine is mounted on a pair of wheels, like the air puncher, and is operated in substantially the same way.

Another type of under-cutting machine is similar to the chain-breast machine, except that after making the first undercut, instead of withdrawing, moving sideways, and repeating the forward cut, this machine remains under the coal and is drawn sideways across the room, cutting as it goes. This type is especially adapted for cutting wide rooms or long faces, and is called a "long-wall" machine.

Machines have also been designed to cut and load the coal, without blasting. One of these is made up of two discs or cutting wheels about 4 ft. in diameter carrying bits in the periphery, mounted side by side and rotated in opposite directions. These are fed into the coal and undercut it, carrying the cuttings to the center, where a conveyor chain picks them up and feeds them back into a car. At the same time, two puncher machines, mounted above the wheels and one at each side, on swivels, are used to break down the coal as it is undercut, the cutter wheels throwing the coal to the center whence it is carried back by the conveyor. This machine as first designed is operated by compressed air.

Another machine has a double cutter head, like two standard chain machines, one above the other. The double head is swung in a vertical plane, describing an arc of about 180 degrees, like a steam shovel, except that the cutting is done on the down ward stroke. This machine cuts the entire vein into fine slack, which is preferable for coking.

Electric mine pumping does not differ essentially from power pumping on the surface, except that the water is usually acid, and that grit and fine coal must often be handled. In the geared-type plunger pumps, the valve lift is increased to allow the passage of fine coal, etc., and the cylinders, plungers, valve seats and stuffing boxes are made of acid-resisting material. In some extreme cases cement-lined pumps with wooden plungers are used, or the entire pump is wood-lined. Where the water is not too strongly acid, and other conditions are favorable, turbine pumps are used, eliminating valves, packing, gears, etc. Rotary pumps of new design are also coming into use where the water does not contain much grit.

One of the first electrical mining plants in the Pittsburg district was that at Grindstone, near Brownsville, Pa. The generator was a bipolar machine of about 60 kw., driven by a Corliss engine through a countershaft. It is still in daily operation. Cutter-bar mining machines were operated.

The two largest plants at that period were those at Essen, near Carnegie, and Scott Haven, above McKeesport, both of which are now operated by the Pittsburgh Coal Company. The Essen plant supplied two mines, and consisted of three bipolar, 250-volt, 150-kw. generators, belt-driven. A number of mining locomotives were used, these having but one motor, carried lengthwise in the locomotive frame, with bevel gears at each end. On account of this positive connection to all driven wheels, and also on account of the steel-tired wheels, these locomotives had a very high draw-bar pull, but they were expensive to operate on account of the difficulty of holding the bevel gearing in line.

The Scott Haven plant, installed in 1895, supplied four mines, and consisted of four belted, 550-volt, 100-kw., four-pole generators. The switchboard was a skeleton oak frame, with round copper bus-bars, the switches being front-connected. On account of the high potential, a metallic circuit was provided for the mining machines as being safer for the men, but this circuit quickly became grounded in the mines. All the apparatus was later put on a grounded circuit with rail return.

The locomotives in the Scott Haven plant were of the two-motor type and mechanically of modern design. The motors were of about 23-h.p. rating on 10-ton locomotives. The armatures were cross-connected and were very difficult to rewind. These motors were replaced in 1898 by 35-h.p. motors for narrow gauge use, originally designed, the author believes, for the Boston Elevated Railway. One of these locomotives has still in service the motor installed in 1898, the armature never having been taken out.

The installation at Scott Haven included some of the earliest motor-driven fans and pumps. Practically all of these are still in use.

One of the first direct-connected plants was installed in 1897 at Moon Run, about 5 miles West of McKees Rocks. This plant was first used for haulage alone, and included one 165-kw., 250-volt generator, driven at 240 rev. per min. by a center-crank engine. Another unit was installed later with a side-crank engine, but before this was in place the center-crank engine shaft broke in the disc. It was operated for several weeks after breaking in two, until the other unit could be started.

One of the first alternating-current central plants for coal-mining purposes was installed in 1902 at Gary, W. Va. This plant first contained two 400-kw., 6600-volt, 25-cycle generators, driven by twin simple engines with independent governors.

The exciting equipment consisted of one 25-kw. vertical engine-driven unit and one 25-kw. generator direct-driven by a 40-h.p. induction motor at 720 rev. per min. All oil switches were in brick cells back of the switchboard, with remote manual control, the feeder switches being of the automatic overload release type with time limit relay. The generator switches were non-automatic.

The longest transmission was originally 3 miles. Three substations were built, each containing one 150-kw., 275-volt converter with step-down transformers and the usual switchboard equipment. Each sub-station also contained step-down transformers for induction motors at tipples, etc. Direct current from the converters was used for haulage only, supplying main-line and gathering locomotives of the cable-reel type, and the larry equipments for the coke ovens. These larry equipments included one motor with a reversing controller, resistance and trolley, and the motors were duplicates of those on the smaller locomotives. Each larry could haul one trailer.

Induction motors are used on single-stage, two-stage, four stage and five-stage turbine pumps, for coke oven supply; on crushers in sizes of 75, 200, 300 and 400 h.p. at the tipples; on mine fans, for which purpose two-speed, changeable-pole, 100-h.p. and 150-h.p. motors are used; for driving deep-well pumps and small compressors for air lift, with silent-chain drive; and for driving picking tables, car hauls, conveyers, etc. at tipples.

This plant was later increased by adding two 750-kw., 6600-volt generators, driven by cross-compound Corliss engines at 125 rev. per min. At the same time the original twin simple engines on the smaller units were changed to cross-compound, giving better regulation and economy. Within the past year one 1000-kw. steam turbine on 140 lb. pressure has been added, also one 1000-kw. low-pressure turbine, using exhaust steam from the engines. Condenser equipment, including three large cooling towers, has been provided for the turbines.

Nine sub-stations are now in operation, and three 500-kw. step-up transformers, 6600 to 22,000 volts, have been installed for supplying mines to be opened about 12 or 15 miles distant.

A very economical plant has been installed near Fairchance, Pa., utilizing in the boilers the waste heat from the coke ovens. A large flue is built back of 100 bank ovens, with smaller flues connecting each oven, and a boiler plant is located at the center of each section of 50 ovens. Thus each boiler plant draws from 25 ovens each way, making a travel of about 350 ft. for the hot gases. The engine room is half way between the boiler plants, and contains four 400-kw., 2300-volt, 25-cycles, three-phase generators, driven by cross-compound engines. By means of five sub-stations this plant now supplies seven works, operating locomotives, larry equipments, car hauls, conveyers, fans, pumps and coke-drawing machines. Up to the present time it has been found necessary to operate but one boiler plant, but extensions are under way which will absorb the entire capacity of the plant, one of the additions being a 1000-cu. ft., 1000-lb. motor-driven compressor for supplying air locomotives in a gaseous mine. When the demand for power increases, it is expected to install a low-pressure turbine with condensers, which would give about 1000-kw. additional when operating three engine units.

At Ellsworth, Pa. it was expected ultimately to install an alternating-current central station, and belt-driven synchronous converters were installed for temporary use as three-wire direct-current generators. A neutral was obtained from the collector rings through the secondaries of small 200-volt transformers, and this neutral was permanently grounded to the mine tracks. Locomotives, machines, and pumps were operated at 275 volts between either main lead and the track, the load being balanced as nearly as possible, while outside motors, on fans, etc. are connected across 550 volts. It was found that straight star connections to the neutral from the collector ring through transformers would not work, as the converters bucked badly until interconnections were made to neutralize the flux in the transformers due to unbalanced direct current. It was also clearly demonstrated that the old style converters are much better adapted for use as direct-current generators than the modern design. The first ones installed were built 7 or 8 years ago and operated well with proper attention. One or two new ones could not be operated successfully at more than one-half rated capacity.

Three-wire, direct-current plants are also in operation in West Virginia, using railway type generators, with derived neutral through balancing transformers. One of these plants, at Minden, W. Va., operates three 300-kw., 600-300-volt generators, driven by cross-compound engines, and supplies four mines. The apparatus in use includes 20 locomotives, 16 machines, 21 pumps and 14 fan, car-haul, and miscellaneous motors.

The three-wire system has the advantage of being less dangerous than that using 550 volts pressure; the return or bonding need not be so good, and motors, etc. stand up better. The two voltages are also useful for variable-speed requirements. It is objected to on account of the care required in balancing the circuits. As a transmission problem, it is, with perfect balance, a 600-volt metallic circuit, and therefore requires just twice the copper—neglecting the difference in bonding—as a 600-volt, grounded-return system, or half the copper as compared with a two-wire, 300-volt grounded system. The balancing is most effective when a large number of small units are operated. The unbalanced current at Minden rarely exceeds 15 per cent.

In Woodward, Alabama, a plant has been installed at the furnaces of the Woodward Iron Company, taking steam from the furnace boiler plant, normally fired by blast-furnace gas. The plant includes three 400-kw., 3500-volt, 25-cycle, three-phase generators, with cross-compound engines. At or near the furnaces two two-stage turbine pumps, delivering 3000 gal. per minute each against a 135-ft. head, are operated for furnace use, while two 100-kw. motor-generator sets supply 250-volt direct current to shop motors, small locomotives handling ore wagons, and a travelling crane handling hot iron on the casting floor.

At No. 1 coal mine, about 2.5 miles away, the current is taken underground in a three-conductor, varnished-cloth insulated, steel-armored cable. Four thousand feet inside are two 700-gal. triplex pumps chain-driven by two 150-h.p., 3300-volt, induction motors and working against a 550-ft head. A few thousand feet farther in is an endless rope haulage, chain-driven by a variable-speed, 3300-volt motor, running at 200-480 rev. per min., with a controller and secondary resistance. About two miles from the pit mouth is an underground sub-station, containing one 200-kw. converter with step-down transformers, etc., supplying 275-volt current to mine locomotives. A number of small triplex pumps, operated by 440-volt induction motors, are also in use.

At No. 2 coal mine, about two-thirds of a mile farther from the power plant, current is taken underground approximately two miles to a duplicate sub-station, also supplying locomotives, and up an air shaft to a 50-h.p., 3300-volt motor driving a mine fan. On account of the frequent labor troubles in that district, it was not advisable to take any power overhead, except along the railroad to the coal mines, where the line could be patrolled.

The plant is now being enlarged to provide power to the ore mines, three miles from the plant in another direction, to operate hoists, crushers, pumps, and air compressors for rock drills.

The six 150-h.p., 3300-volt motors on turbine pumps, triplex pumps, and motor-generator sets are duplicates. The underground cable is suspended from a messenger wire, with frequent loops for expansion and repair. There have been several rock-falls on the cable, most of which have caused short-circuits, but repairs are easily made. The armor is protected against corrosion by a layer of jute and asphalt, but the miners have cut a good deal of this off for kindling to start fires. They also stole about 40 gallons of oil out of the substation transformers one night, for use in their lamps.

These mines go in on a slope of about 20 per cent for the first mile, after which the vein flattens out. There is only about 4 ft. of headroom. A good deal of the underground apparatus was skidded down on the track rails.

At Ehrenfeld, near Johnstown, is a plant in which belt-driven converters were first used for generators, supplying an underground substation through step-up transformers from 165 to 5600 volts. This substation contained, besides the necessary step-down transformers, one 250-kw., 275-volt converter, operating on the trolley in parallel with the direct-current side of the double-current generators. This was the first alternating-current plant operating underground substations. Larger direct-connected generators have since been installed. It was the intention to use the original generators as converters later.

At Yatesboro, Pa., there is a successful installation, operating a number of motor-driven compressors, fans and sub-stations. The station equipment includes two 1500-kw., 60-cycle turbine units, generating three-phase current at 6600 volts. In two substations are installed 300-kw., 550-volt converters running at 900 rev. per min. and in a third a motor-generator set, with an induction motor driving a 300-kw., 550-volt generator. Three motor-driven fans are in use at different mines, one with a 250-h.p. motor, belt-driven, one with a 250-h.p. motor and silent-chain drive, and a third with a 400-h.p. motor direct-connected through a flexible coupling. There are also, at the various mines, eleven 1400-cu. ft. duplex compressors, belt-driven by 250-h.p., 6600-volt induction motors. One interesting feature of this plant is that these high-tension induction motors are all started by throwing directly on the line.

At Windber, Pa., there have been operating, until this year, eight direct-current power plants supplying 12 mines. Two of these plants, those at No. 36 and No. 40 mines, are efficient installations with cross-compound condensing Corliss units. water-tube boilers, etc., but the range of distribution was such that the loss in transmission overcame the efficiency in the plants.

In most of the mines locomotives only were operated, at 550 volts, and the transmission was from 1.5 to 2.5 miles. Compressed-air " puncher-type " mining machines and compressed-air pumps were used, with steam engine-driven fans. It was decided to centralize the power, to do away with all the direct-current power plants, except No. 36 and No. 40, and to limit their field to a radius not exceeding 1.5 miles. The boilers from some of the dismantled steam plants have been assembled and a plant installed at No. 35 mine, in nearly the center of the field. The equipment includes one 400-kw., 6600-volt, three-phase, 25-cycle generator, driven by a cross-compound engine, and two 1000-kw. steam-turbine units to operate at 140 lb. pressure and 1500 rev. per min. One additional turbine unit, probably of 3000-kw. capacity, will be installed early next year.

The 400-kw. engine-driven unit, with two 200-kw. converters, was purchased some years ago. These converters with one additional, will be installed at No. 38 mine and also supply No. 39. At No. 33 mine and also to supply the front end of No. 34, will be located two 150-kw., belted-type, 550-volt generators. formerly used at No. 30, now to be direct driven by 225-h.p., 6,600-volt synchronous motors at 500 rev. per min. In this substation will also be installed one 1800-cu. ft., 2-stage air compressor, direct-driven by one 300-h.p., 6600-volt, synchronous motor at 187 rev. per min. In a substation at the back end of No. 34 mine, also supplying No. 31 mine, will be located one 300-kw. converter and two direct-driven 1800 cu. ft., 2-stage compressors.

At No. 41 mine electric puncher machines are supplied with 250-volt current from a 100-kw. generator driven by a 150-h.p. 440-volt induction motor at 750 rev. per min.

At No. 42 mine two 300-kw., 550-volt converters are installed, to operate locomotives in two drifts, one in each vein of coal, and also to feed into the back end of No. 36.

At No. 35 shaft, two miles back from the pit mouth is located a large fan to ventilate No. 35 and parts of No. 36 and No. 40. This fan will be driven by a 400-h.p., variable-speed, 6600-volt motor, direct-connected through flexible coupling, to operate at 200 to 285 rev. per min. A substation here contains two 300-kw. converters, with space reserved for one more, also six 150-kw. transformers, with a 6600-440-volt ratio supplying two large pumps at the shaft bottom. A drainage level was located across mines Nos. 37, 40, 35, 36 and 42, all the water above which will flow by gravity to this shaft. Here will be placed two two-stage turbine pumps, each of 4000-gal. capacity per minute against a 330-ft head, and each will be driven by a 500-h.p., 440-volt induction motor at 730 rev. per min. These pumps will replace over 70 compressed-air and steam pumps.

A sub-station will be located at the rear end of No. 37 mine, and contain one 300-kw. converter, and later one motor-driven compressor. This converter will work in parallel with No. 40 direct-current plant, supplying the outside haulage, etc.

An induction motor has been designed for the electric coal puncher, or "pneumelectric" machine, and it is expected, if this proves a success, to replace the compressed-air punchers wherever found practicable. On account of the large investment in machines, pipe, etc. some mines will still be operated by air, as No. 33, 34, 31, 37 and 40, and wherever necessary for best economy motor-driven compressors will be located near the work, displacing steam-driven machines.

The output of these mines is normally about 18,000 tons per day. The cost per ton has been slowly but steadily increasing from year to year, as the mines advanced, and transmission lines and water lines lengthened. It is expected to increase this output as desired to over 20,000 tons per day, and to reduce the cost at least 6 cents per ton. This cost should then increase but very slightly for probably the next 5 or 10 years.

The cost of the electrical equipment, including foundations, buildings and installation, will approximate $250,000, and the total cost of improvements, such as shafts, drainage tunnels, etc., will amount to about $600,000.

DISCUSSION ON "ELECTRICITY IN MINES." PITTSBURGH SECTION, NOVEMBER 10, 1908.

F. L. Sessions: I should like to suggest, what is not entirely original, a basis for the rating of mine locomotives as follows: the motor capacity shall be equal to a stated horse power per ton of locomotive weight per mile per hour at a stated speed in miles per hour, at 75 degrees cent. rise of motor temperature as measured by thermometer after a 1-hr. run.

Or, as an alternative which will refer strictly to the locomotive performance at the draw-bar: the rating of an electric mine locomotive shall be based upon the horse power per ton of locomotive weight per mile per hour at a given speed in miles per hour developed at the draw-bar continuously for an hour, with a rise in temperature of any part of the motors of 75 degrees cent.

W. A. Thomas: The exhaust-steam turbine will be of considerable importance in the near future. A number of plants have been equipped, and many more low-pressure turbines might advantageously be installed where the exhaust steam from reciprocating engines is being wasted. In the case of one coal company, since putting in exhaust-steam turbines 1,000 kw. have been recovered on four units, whereas formerly not more than 200 kw. in heating value were obtained from the exhaust of the engines. This economy is bound to become a marked feature in general work as well as in coal-mining operations.

The standard of laws for the use of electricity in mines, that has been considered in the establishment of laws in Pennsylvania is similar to that established in New South Wales. Electrical men who are associated with mining operations should make themselves familiar with these laws.

Probably one-third of the restrictions and special limitations in the New South Wales laws are on account of defective insulation on the wires, and as good insulation is used there as can be obtained in this country. In this country it is usually considered that up to 600 volts the wires should be left bare. In establishing rules in this country I believe it will eventually be decided that bare wires should be used and that we should not trust to insulation.

I agree fully with Mr. Sessions in his ideas on standard ratings. I think that concerted effort should be made on the part of the manufacturers to get mining men who are interested in electrical matters to comprehend more fully what an electrical rating is, and then come to a common basis, as it is confusing to an operator if he is not acquainted with these matters.

H. W. Fisher: The destruction of the lead covering of cables by acids in mines has been repeatedly brought to the attention of the manufacturer of cables. At present we are experimenting with a form of cable which is protected by the use of a high melting compound which is practically acid proof. In making this cable the strand is first coated with the compound, so as to prevent moisture from entering the cable along the strand in case the ends are not especially well protected. Then varnished cloth is applied to about the necessary thickness for the voltage in question, after which a layer of this special compound is applied, then one or two layers more of varnished cloth, and over these the usual braids.

We also make a rubber-insulated cable which is finished with a layer of special black compound and varnished cloth and braid as above specified. These forms of cables have only recently been put on the market, but sufficient time has not elapsed for us to know definitely if they are giving satisfaction.

A paper presented at the 231st meeting of the American Institute of Electrical Engineers New York, November 13, 1908.

ELECTRIC HEATING

BY W. S. HADAWAY, JR.

It may assist in a better understanding of the terms used in discussing electric heating to state at the outset that the words heat, potential, etc., are employed more specifically than has generally been found necessary in discussing heat effects apart from electricity. Heat, like electricity, is assumed to flow from a surface of high potential to one of low potential. Quantity or volume of heat is analogous to quantity of electricity. Unit quantity is here figured as the British thermal unit, though for technical purposes it is, of course, the calorie. Temperature corresponds to potential. Heat potential is as true an expression of potential as is impressed electromotive force; it should not, however, be confounded with static potential, a term used in discussing energy of gas. It would materially simplify thermal nomenclature if a word were coined to designate heat potential in the same manner that electromotive force expresses electrical potential.

The term "electric heating" is commonly used to express the frittering down of the higher potential energy of electricity into the lower potential energy of heat. There is no well defined line of demarcation; the result is inseparable from the conveyance or utilization of electricity in any degree. The expression is, therefore, a meaningless one from the heat engineer's standpoint and it fails accurately to express the performance of useful work by the transmission of heat energy to a distance by means of electricity. The expression, "electricity, a factor in a telethermic system," would be a better definition, but for obvious reasons the shorter and simpler expression is employed.

Heat energy is to be regarded as a commodity that can be generated or controlled in convenient form and distributed and sold, and in which electricity is used either directly as the high-potential heat factor or indirectly as agent. The distribution and application of heat energy by electricity to arc and incandescent lighting, to metal welding and forging, and to electric furnaces for various industrial operations are illustrations of results already attained in electric heating and are properly discussed under the broad interpretation of the subject.

The feature of the subject we are to consider is the practical adaptability of commercial electricity for heating purposes and for performing useful work under the conditions imposed in general industrial and domestic life, apart from the specific adaptations previously mentioned, and particularly in connection with other heat distributors for a multipotential heat supply.

These applications may be roughly divided into two classes according to the degree of concentration of the heat energy used. The first class includes low-temperature heating, generally diffused and in large volume; the second includes high-temperature heating, generally localized and in small quantity.

In heating on a small scale no adequate classification is possible; it is the blending of high-potential heat energy into low-potential heat work for many useful operations that primarily suggests the feasibility of heat transmission by electricity. An arbitrary line of temperature demarcation for heating on a large scale has been drawn at 250° fahr.

The closest common analogy to the differences between low-potential, large-volume and high-potential, small-volume heating is the distinction between volume and pitch in sound. As examples of the first class may be cited the heating of rooms and of water at atmospheric pressure; examples of the second class are electric ovens, sad-irons, and soldering-irons.

The electrical unit or kilowatt-hour is the equivalent of 3412 British thermal units. For practical purposes the work in hand is to determine the useful work in different lines obtainable from this number of heat units at a fair average cost of supply. There is involved indirectly the determination of relative efficiency between electric and other methods of heat generation, distribution, and application.

To show what the heating efficiency of different methods of application may mean, the following experiments, reviewed in the London *Electrician* for Nov. 2, 1894, are cited. The determinations were the relative heating values of combustion and electricity by resistance. The same wires were heated, first in a flame and then by electric current. It was found that with a platinum wire held in a flame less than 0.5% of the thermal energy produced in the flame was transferred to the wire, while 90% of the electrical energy used appeared as heat. In another case an iron bar weighing about 20 lb. was heated by charcoal and then by electricity. In the former case about 0.75% of the thermal energy was transferred to the bar and in the latter case 88%. It is evident from the description that the calorimetric methods employed were rough, and the results are quoted more as qualitative than quantitative determinations.

It should be observed that the foregoing illustrations are examples of the second class of heating applications; figures tabulated farther on will show results obtained in water heating by various methods of heat application, which are as true examples of efficiency tests as the preceding but in the first class of heating. The heat capacity of the work performed is sufficiently large to express the relative differences in combined efficiency, and, as we should expect, the lower the temperature of the work, the larger the area heated; and the greater the capacity the less the difference in ratio between heating by combustion and by electricity. We may readily determine the conditions where the ratio is an inverse one.

In room-heating apparatus on continuous run we may estimate that one watt-hour, 3.41 thermal units, will heat one square foot of common radiator surface through 1.26 degrees fahr.; that 1000 watt-hours or 3412 thermal units will heat the surface approximately 126 degrees fahr. above the room temperature; that is, for room heating from 85 to 110 watt-hours are practically the equivalent of steam at low pressure condensed by one square foot of radiator surface with the difference between the room and the radiator temperatures as above stated. This, of course, takes no account of the heat capacity of the apparatus, which is practically eliminated by the imposed conditions of continuous running. We may safely assume that a fair average price of the electrical unit from large steam electric stations is 6.7 cents. It would therefore cost 0.65 cents average to run one square foot of direct radiation surface for one hour, or the electrical unit would keep about 10.3 sq. ft. of radiator surface at the temperature difference noted for one hour.

Direct comparison of this value with practical steam heating is difficult, owing to the variables involved. We may, however, compare it with a central station steam distributor as quoted by Unwin in his work on the " Development and Transmission of Power," referring to a New York steam company.

The unit of heat used by this company is defined as the " kal ", which is stated to be the heat required to evaporate one pound of water from 100° fahr. at 361° fahr. or at 85 lb. pressure per square inch. One kal is therefore about 1110 thermal units. On a sliding scale the charge is stated to be 70c. per 1000 kals to small users, and 40c. per 1000 to large users. As the price of 6.7c. per electrical unit is to a fairly large user, we shall compare it with the 40c. per 1000 kal rate. For 40c. the consumer obtains 1,110,000 thermal units from the steam station, compared with 20,400 thermal units from the electric station, a ratio of nearly 54 to 1, or practically 50 to 1 as a loss must be figured in the case of steam, dependent upon the temperature at which the condensed water is allowed to escape.

It is interesting to note in passing that while the relative efficiencies of generation are about as 12 to 1, the commercial rates quoted are as 50 to 1. The ratio of the cost of heat energy from the electric station to the cost from the steam station is therefore practically four times as great as the ratio of the relative costs of production. This disparity should be considered in discussing the influence and importance of the load-factor as discussed by Crompton in his paper on " Electric Energy."

Clearly, the steam-driven electric light station ranks low as a heat distributor for house warming on any considerable scale. And yet notwithstanding the great disparities in the cost of heat energy on a large scale, the uses to which small electric air heaters are put are surprisingly numerous. We are all more or less familiar with the use of gas stoves for room warming, but it has been found by wide experience that the electric air heater for auxiliary room warming, for bath and dressing rooms, for libraries, cabins on yachts and steamships, in mild climates in early morning and evening, etc., serves its purpose with fair economy, and without vitiation of the air and by-products always evident from gas.

It is then on a comparatively small scale and in intermittent service that the electric air heater is useful when power is derived from the steam electric station and when the heat supplied is derived from the electric resistance. It is interesting to note that the relative costs of copper and structural steel differ in nearly the same proportion as the generation costs of electricity and steam. And while we use steel as the frame-body, and copper to round out and embellish the structure, so may the relative value of electricity to steam be regarded in heavy quantitative heating work.

This discussion does not include heat storage systems from hydroelectric installations which have been described briefly by Mr. Chas. E. Waddell*. Where an equipment is utilized for only 50% to 60% of the day, and water storage is not required, there are opportunities for broadening out electric heating applications by absorbing the energy of the plant in heated water, which may be employed in combination with high-temperature heaters at periods where under other conditions the loads would overlap. It is evident that this method of supplying the low-temperature component of a heating system must be resorted to before electric cooking which includes hot water in considerable volume, can be generally successful.

While considering the use of electricity in diffused heating on a large scale, we should not overlook the deductions made by Lord Kelvin " On the Economy of the Heating or Cooling of Buildings by Means of Currents of Air " and published in the Glasgow *Philosophical Proceedings* Vol. III for Dec. 1852 and followed by mathematical demonstrations in the Cambridge and Dublin mathematical journal for Nov. 1853. Quoting from these articles:

> In the mathematical investigation it is shown that according to the general principles of the dynamical theory of heat any substance may be heated 30 degrees above the atmospheric temperature by means of a properly contrived machine driven by an agent spending not more than about 1/35 of the energy of the heat thus communicated and that a corresponding machine or the same machine worked backward may be employed to produce cooling effects, requiring about the same expenditure of energy in working it to cool the same substance through a similar range of temperature. When a body is heated by such means about 34/35 of the heat is drawn from surrounding objects and 1/35 is created by the action of the agent; and when a body is cooled by the corresponding process the whole heat abstracted from it, together with a quantity created by the agent, equal to about 1/35 of this amount is given out to the surrounding objects.

We have thus introduced refrigeration as well as heating by electricity as agent. In editing his mathematical and physical papers, Lord Kelvin added the following interesting note under the date of June 26, 1881:

* Proceedings A. I. E. E., July 1908, p. 1197.

The method of cooling air in unlimited quantities described in this article has been realized by Mr. Coleman, first in refrigerators used for the distillation of paraffin and after that in the Bell-Coleman refrigerator for carrying supplies of fresh meat from North America to Europe; in a great refrigerator recently sent out for the Abattoir at Brisbane, Queensland. The Bell-Coleman machine sends large quantities of air cooled to 10° or 20° C. below the freezing point into the chamber to be kept cool; and the general temperature of this chamber is thus maintained at the desired point.

The method of heating air described in the article remains unrealized to this day. When Niagara is set to work for the benefit of North America through electric conductors it will no doubt be largely employed for the warming of houses over a considerable part of Canada and the United States. But it is probable that it will also have applications, though less large in other cold countries to multiply the heat of coal and other fuels, and to utilize wind and water power (with aid of electric accumulators) for warming houses.

In Anderson's treatise on the "Conversion of Heat into Work,"* will be found descriptions of compressed air refrigerating machines made on the principle above described, and in Professor Peabody's work on the "Thermodynamics of the Steam Engine," pages 464 and 465, will be found tests on a Bell-Coleman refrigerating machine. The use of electric motors in refrigeration has been under consideration of central station men for some time, being advocated by well known engineering authorities; but so far as the writer can learn nothing whatever has been accomplished in the use of motors for heating purposes.

Lord Kelvin's deductions on using electricity as agent for warming purposes have been advisedly introduced into this discussion. His conclusions may prove to be wrong until more perfect thermal insulation is developed, but to anyone familiar with thermodynamics and electrical development the possibility of using a positive force to control and direct a less definite form of energy must appeal with singular emphasis.

It may be that the electric furnace using low-grade carbon in high-temperature combustion chambers will give the desired results. Tests indicate that the only obstacle to this latter combination is in making the apparatus "fool-proof." The need for more economical use of fuels and the fact that electricity by resistance heaters is an ideal high temperature component make the development of the low-temperature factor imperative for a complete combination.

When electricity is derived from water power we might ex-

* Third edition, pages 189 and 194.

pect a far wider availability of energy from heat of resistance for applications requiring heat in large volumes than when electricity is derived from steam power. Careful deductions show that for house warming the cost of a horse power year must not exceed $6.00 in order to compete directly with high grade coal at $5.00 per ton.

Statistics compiled by Professor Geo. F. Swain show that the average cost of the mechanical horse power-year from water power in this country is about $10.00; to this must be added electrical generation and transmission charges, which make it seemingly impossible to realize a figure for electrical energy delivered that will not be greater than the cost of heat directly from fuel. The compensating advantages in regulation and control, combined with the improved load-factor, especially in combination with thermal storage work, warrants the extensive use of the heat of resistance for house warming at least within restricted areas. In the case of hydraulic plants with large water falls, where the capital charges are relatively low, we may expect many extensive applications of room heating on a successful basis.

In his paper on "The Cost of Steam Power",* Mr. C. E. Emery cites the case of Lawrence, Mass., where with moderate fall the cost of a horse power-year from water power is figured at about $25.00, or practically the same cost as power generated from steam. In the practical treatment of this question, local conditions will control the breadth of applicability, including in these local conditions cost of electric power, cost of coal delivered, extremes of climate, average humidity, etc.

We have still another source of electrical energy to consider in connection with low-temperature diffused heating, in which power is derived from gas, oil, and air engines. In a lecture before the Royal Institution in April, 1893, on the "Utilization of Energy," Professor Kennedy places the theoretical efficiency of coal gas at 80%. Of this a gas engine, he says, utilizes from 22% to 32%. For diffused heating it is apparently more economical to distribute the gas directly than to attempt to convert its energy into electricity. Attention is further called to the paper by Mr. Williams before the Society of Arts, Boston, Mass., on May 10, 1894, in which comparative costs of coal and various fuel gases are given, and in which it is shown that the gas equivalent of coal may, under most favorable circumstances, cost eight or ten times as much as coal.

* Transactions A. I. E. E., March 1893.

The consideration of the relative values of gas and electricity for heating purposes is a matter that can only be inadequately discussed here, for the problem is a complicated one and demands the most thorough consideration. We must not underrate the facility with which gas may be stored, the economy of rapid and continuous generation, and available heat from combustion and discharge of by-products, whereas in electric lighting and electric heating every unit of light and heat secured represents an appreciable loss or depreciation of the energy converter.

There are many advantages to be considered on the other side such as localization, immersion of heater in working chamber, as in ovens, sad-irons, and water heaters, freedom from bi-products, etc. In electric heating and cooking for domestic uses the real difficulty is how to provide a hot-water supply economically. The same thing is true in the use of gas, though in a lesser degree. On a broad scale it appears easier to arrange for this on a multipotential heat supply system than on any system using gas.

In his treatise on gas engines, Donkin shows examples of these engines of large size furnishing power at rates less than can be obtained from other prime movers. For high-potential heat distribution from isolated plants this form of energy transmitter is generally available; but it may be seriously questioned whether in isolated plants for domestic use the hot water supply as usually provided can be economically heated, unless of course the combined economies permit the use of individual gas-heated water supplies.

It may be safely concluded from the foregoing illustrations that we cannot afford to use electricity for general diffused heating purposes on a large scale. It is proper to point out, however, that practical experience shows that these deductions may lead to unsafe conclusions. There are many striking analogies in heating development to the working out of the electric motor.

If we were to take the case of an engine in a factory and substitute for it an electric motor we would not expect, with our present knowledge of this subject, to effect marked economies. We know that we depend upon the elasticity of the small motor to produce results and that the combined load of a number of individual units reduces fixed losses and increases output. It is the same in heating. We must consider the individual applications, and the combined results of these will be far different from any considerations controlling a fixed system. One of the reasons contributing to the slow growth of electric heating development is the necessity in many cases of devising apparatus to do work more effectively than could be done by the old methods.

On the other hand, the collective strength of these new methods has given the work a standing, in the industrial lines particularly, which constitutes an advance in heating development. It has further been shown that results may be obtained which make the cost of operation of secondary importance than if considered solely on basis of thermal values.

The British thermal unit is the equivalent of 778 foot-pounds of work, and we would obtain as much heat by applying a brake directly to the fly-wheel of the engine or prime mover as we obtain from the heat of electrical resistance, which so far as its results go may be considered as friction. If we use as an average thermal value of a pound of coal 14,500 B.t.u., we can secure 9400 units in a good steam boiler, or practically 65%.

There are many cases in practice in which this efficiency is exceeded, but Thurston's figures give 65% as a fair average value. The fuel efficiency of the steam engine rarely exceeds 10%, and the heat incapable of conversion into high-potential energy represents about nine-tenths of the whole, notwithstanding the fact that over six-tenths of the thermal energy of the coal are obtained in useful work in the boiler.

It is therefore to this point we should look for our large volume heat supply, and we find in actual practice many illustrations which show that no great engineering difficulties are encountered in distributing the boiler energy and also that of the exhaust steam from the engine for useful work at a distance. The New York steam company's plants already referred to are distributing energy by live steam for heating and power purposes at about 80 lb. pressure. The leakage and radiation losses are stated to be estimated at about 15%, yet with this loss the heat energy can be delivered on a successful and economical basis.

The work of Birdsill Holly since 1877 in carrying out central station heat supply is too well known to require explanation. It is important and interesting in connection with our problem since it shows that central station diffused heat supply is entirely feasible. We are now able to formulate a multipotential heat supply system in which the steam, either live or exhaust, or both, is used for low-temperature work, and electricity from generators in the same station is employed for all work requiring temperatures in excess of that of the steam; that is, from about 250° fahr. upward. In domestic work the steam heat would be employed for house heating and water heating for bathing, laundry, and cooking purposes. Electric heat would be used for lamps, ovens, broilers, small stoves, sad-irons and all purposes requiring a high temperature localized heat. And a feature of such a combination not to be overlooked is the facility with which ventilation and heat regulation can be secured, since all the elements are at command to force an abundant supply of heated pure air through the building to be warmed. There are no new or untried features in such a combination; practically all of the elements necessary for the work are already employed though in a somewhat disjointed form so far as complete flexibility of heat supply from one center of distribution is concerned. If we refer to the *Electrical Engineer* for April 5, 1893, there will be found a description of the Springfield (Ill.) electric light and steam distribution plant. In operating this plant the exhaust steam is used for warming buildings, but there appeared to be no systematic effort made to carry out a multipotential heat supply.

As bearing upon the historical development of this matter the author quotes from a letter from the American District Steam Co., of Lockport, N. Y., in 1894:

We observe that you recognize the futility of electricity competing with steam for diffused heating, but there is a wide field for electrical companies in connection with their exhaust. In blocks of buildings where power is required to run elevators, or for other purposes, the proprietors will run their own power if they have to do their own heating, but if the electric company can heat the building it can get the power at a good profit and there is a profit in both heating and power.

We have just constructed a heating plant for the Rochester (N. Y.) Gas & Electric Co., who are doing just this thing; they start off with 5,000,000 cu. ft. of space to heat in manufacturing and business blocks, and secure the running of several engines in the same buildings. They supply the steam from their stations in underground pipe and power by wire. They could not get the power without doing the heating. We think you will find this a fertile field for investigation.

Citations of events in 1893 and 1894 are made advisedly as having a bearing on the development of the art. It was found that while a business could be built up along these lines that there was a great deal to do in developing heating appliances to make them sturdy and effective, and that it would be necessary to choose the industrial isolated plant as a more immediate field for both heat engineering and appliance development.

Careful study of the question of multipotential heat supply, from both engineering and business standpoints, can hardly fail to give the utmost confidence that results of the greatest value can be obtained along this line. A total fuel efficiency exceeding 35% for both low- and high-temperature heating is certainly attainable, and probably a still higher efficiency may be secured from operations on a large scale. It is in this direction that we must look for the best immediate results of electric heating in domestic life. Electrical engineers realize that the ordinary light and power station is wasteful as a heat generator and distributor, and one of the ways to improve these conditions is to introduce a basis of supply in which the load-factor is a larger percentage of the station capacity and in which low-temperature steam distribution may also be utilized. This study deserves the greatest possible care and while data are occasionally available there is still much to be learned.

In the modern apartment house we find it possible to effect a comparatively high distributive heat efficiency. The rooms are heated by low-pressure steam or hot-water radiation and the hot water for laundry, bath-room, and cooking purposes is also supplied.

Electric lighting is available, but low heat efficiency is found in cooking except in cases where gas is available. No one can question the value of gas as a fuel in cooking; we may however take the ground that when a boiler plant is required for room and water heating, and the electric plant for lighting, the use of gas in cooking is unnecessary and comparatively expensive, for it is a form of heat supply not as effectively localized as electricity and is accompanied by many disagreeable features.

In these houses electricity is already in use instead of alcohol for heating conveniences, also for heating sad-irons for laundry work and pressing; and it appears to be a question of time only when electric ovens and other cooking utensils will replace the range or the gas stove on a considerable scale. The apartment house supplied with its own boiler and electric lighting plant, approaches more closely the proper conditions for electric heating than any other mode of domestic life, as all the conditions essential to economical working are already present.

The foregoing deductions have been introduced to indicate broadly wherein the true economical basis of electric heating lies. Like all general considerations they are qualitative rather than quantitative in character. We find in industrial and commercial practice a great number of heat applications, either at high or moderate temperatures, in which close regulation and facility of control are necessary and where electric heating on some scale is applicable.

The introduction of electric lights and motors has brought into use in central stations and isolated plants a vast amount of electrical generating apparatus, so that at the outset our problem is largely one of applying electric heating apparatus on circuits already established, primarily for other purposes. On these circuits we are usually able to supply apparatus to do the work required better than it is at present accomplished. First cost and cost of operation are not the sole considerations in any apparatus; in addition to the cost, the collateral advantages derived from its use must be considered.

In domestic work there have been large numbers of household conveniences sold, notably electric sad-irons. Perhaps novelty was in many cases the reason for first purchase and use, but once used the value of the method of heating is well appreciated and extensions are made to more and different work. In this slow and conservative way electric heating in households is progressing, and there are indications of a more rapid general reception and use.

There are some installations in which all or nearly all the cooking is done by electricity. These are generally successful and economical, according to the degree to which extensive water heating is eliminated. Laundry and pressing irons are found almost uniformly satisfactory. Some of the devices used in the dining-room are counted great conveniences when electrically heated. When it comes to the kitchen the same feeling of cleanliness and refinement of methods as for the dining room is expressed and hoped for, but the housewife is not so intimately in contact with the apparatus and ignorant manipulation is encountered, which, however, generally does no other harm than to add to the expense of running. But fortunately the manipulation and control of the devices is so simple and saves the servant so much work that an honest effort is usually evidenced to use it economically.

In industrial work the progress of electric heating has been more rapid than in domestic applications, with the exception of the electric sad-iron. There are cases where heating by electricity is found cheaper than by fuels, setting aside all collateral advantages. In electric cars the more equable distribution of heat; on a satisfactory basis of effective warming is sufficient to employ the method and apparatus even at considerably greater cost than former methods of heating.

We have before us for the future two methods for the distribution of heat; the first, a multipotential heat system in which steam and electricity simultaneously are the means of transmission; secondly, a constant-potential heat system in which electricity is employed as an agent for diffused heat work in large volume, and the heat of resistance, either directly or with the additional heat of combustion of carbon, is used for high temperature purposes. The first is a combination of well known engineering practices and represents elements in successful use at the present time.

The second method is more difficult, as it includes new methods of applications and untried apparatus. For the present we have to approximate the probable final conditions by using materials at hand so far as they will go with reasonable economy, using every energy to broaden the general conception of the help and convenience arising from central station heat supply.

It has been thus far assumed that the development of electric heating depends solely upon engineering conditions without reference to the state of the art of the appliances themselves. Unfortunately the lack of sturdy resistances of high rates of heat impartivity has materially retarded commercial progress. Factors of safety have been too low and rates of working too slow. We are still without a satisfactory heat insulation and in practically the same position as the electrical industries prior to the use of mica.

A circular gas burner will impart heat at a rate equivalent to 35 watts per superficial sq. in. Electric stoves made for 110 volts will work about one-third this rate. This is a serious hindrance to electric cooking and high temperature applications. Far better apparatus could be made for lower voltages, and in this respect the state of the art closely resembles the situation of tungsten and tantalum lamps which are simply high temperature heating applications.

Since Dr. Lindeek's paper on " Material for Wire Standards of Electrical Resistance " was presented before the International Electrical Congress in Chicago in 1893, the only alloy that has been offered for resistance purposes is the nickel-chromium product; that is, in fifteen years, notwithstanding the marked development in electrical furnace work, there appears to have been but one new alloy brought out in commercial form.

Attention is called to these points to emphasize the need of further research work in materials and processes for good commercial work. The heaters on the market to-day are greatly improved over the product of fifteen years ago but there are still wide gaps to be filled. Localization of heat at high temperatures with negligible heat capacity are the prime requisites of an electric heater. Without adequate heat insulation, with resistance material of comparatively low specific resistance and high voltages in commercial practice the average design is far removed from the ideal.

DISCUSSION ON "ELECTRIC HEATING." NEW YORK, NOVEMBER 13, 1908

Charles E. Waddell (by letter): Dividing heat applications into low-potential quantity and high-potential intensity types is a very expressive way of classifying two general conditions. In high-potential heating problems it has been demonstrated that the cost of electrical energy has little or no bearing on economy. Other factors enter that more than compensate for any discrepancy in calorific value between electricity and fuels; for example, electric glue cookers, laundry irons, and apparatus in printing and embossing plants. The field for this type of heating apparatus is decidedly small as compared with the field for low-potential applications; and it is this latter class that presents diverse conditions and offers fascinating opportunities for specific solution. Elusive in its nature, difficult to measure, and next to impossible to insulate, heat is in many respects a more subtle and difficult form of energy to control than is electricity. When comparing electric heating with fuels in the realm of domestic heating, the fact must never be lost sight of that by means of electrical measuring instruments the quantity of heat delivered by the electrical apparatus can be accurately determined; whereas in the case of fuel apparatus, while the calorific value of the fuel itself may be known, the efficiency of such apparatus is so variable and uncertain that its economy is largely a matter of conjecture.

It is doubtless true that the steam power plant, by reason of its low efficiency, is forever debarred from furnishing electric heat in competition with fuels. In the case of a thickly settled district, the use of exhaust steam for heating has been demonstrated as profitable and desirable; and, as Mr. Hadaway suggests, conditions may be such that it might be profitable to generate electricity for high-potential applications and use the exhaust steam for quantity work. It would appear that the scope of such a plant might be widened by applying the electric heat in districts too remote from the generating station to justify the installation of underground steam mains.

While the thermal value of a kilowatt-hour is very low (3412 B.t.u.) and electricity as a heating agent suffers by comparison with fuels, nevertheless it will not do to dismiss the whole subject as one unworthy of serious consideration. It must be remembered that the world's coal fields are being rapidly depleted, that natural gas areas are limited, and that wood as a fuel has even now virtually disappeared. Sooner or later electricity must necessarily become a factor in the field of heating. Where electricity generated from water power is obtainable, and where storage of the water through the night hours is impracticable, electrical energy for heating can be obtained at figures closely competitive with fuels. The problem in such cases becomes one of storing the energy of the water power in the form of heat, which brings up the all-important feature of heat insulation. Supplying electricity for heat as a by-product, with a view to maintaining a high load-factor on a plant, either steam or water, is a subject that will bear careful investigation in the majority of small steam and all hydroelectric plants.

In discussing boiler efficiencies, two factors are involved; first, the evaporative efficiency of the boiler; secondly, the all-day, all-season efficiency. Assuming a pound of coal has a calorific value of 14,000 B.t.u. and that under favorable conditions a given boiler plant can deliver 70 per cent to 80 per cent of this amount, the same plant in every day service seldom approximates 50 per cent of the calorific value of the fuel. As an instance of this, reference is made to the published data of a number of plants furnishing exhaust-steam heating where the annual average number of pounds of water evaporated per pound of coal is but seven and a fraction. These data are not specific as to whether they mean the evaporation is from and at 212° fahr.; and since vague in this particular assume the heat units per pound of water to be 1000, the delivered efficiency of the fuel is then seen to be but 7000 B.t.u. Here are cases where ostensibly the most expert supervision and attendance is provided; where the system is in a measure automatic; where the connected load in radiation is many times the capacity of the plant, hence a presumably high load-factor; and, finally, the demands are diverse, all tending to uniformity in load and consequent economy in fuel. If seven pounds of water is the best average result of large systems, the probable performance of the heating apparatus in an ordinary residence or apartment house is doubtless at an efficiency that is but a small fraction of the theoretical calorific value of the fuel.

Another point which I have had impressed upon me is that not only the quantity but the intensity of heat in domestic applications is essential to success. Formerly I attributed the asserted inferiority of the large, low-temperature electric radiator as compared with a small open fire to the idiosyncrasy or psychical peculiarity of the individual rather than to any physical difference. More recently I have revised my views, and I have found that in many instances a smaller quantity of heat, with which was blended an even smaller quantity of high temperature radiation, is extremely grateful. If electric heating is to be introduced upon a large scale into house warming, it will be necessary to accord the art of heating and ventilating more scientific attention than is now usually bestowed upon this class of work. Unquestionably, of the quantity of heat annually radiated in domestic installations the greater portion is wasted. Since the thermal value of a kilowatt-hour is fixed and not possible to increase, the general installation of this method of heating will be accompanied by economies in application.

The power engineer burns a pound of coal in a manner that will extract the greatest number of heat units in the shortest possible time. The heating engineer, while desiring the maximum number of heat units, endeavors to prolong the time as much as possible. As electrical heating apparatus is capable of delivering a uniform heat through any duration of time and at a maximum efficiency, it possesses an advantage that makes for economy.

The argument that heat obtained from electricity equivalent to that obtained from fuels is so high in cost as to be practically debarred from general use is scarcely logical in view of modern experience. The incandescent light, for instance, though more costly than either candles or kerosene lamps, has in a large measure supplanted both.

Finally, there is one other class of quantity work to which attention has not been directed and on which some remarks may be pertinent. In drying materials on a large scale—lumber, cotton yarn, etc., the heat that is supplied is merely a means artificially to accelerate a natural process. With a moderate amount of heat applied where needed and a skilful use of air currents, an apparatus more efficient than ponderous steam apparatus appears possible.

W. N. Ryerson (by letter): Mr. Hadaway has pointed out the limitations in the use of electricity for maintaining an equable temperature in buildings during the season of cold weather, and at the present time his remarks are undoubtedly to the point. There are instances, however, in which comparison between the use of electricity and other heating agents on the basis of cost leads to erroneous conclusions. An instance in point is the office building of the Ontario Power Company at Niagara Falls which was equipped, under the plans of the architects, with a low-pressure steam-heating system, having an auxiliary hot-air system with forced draught; the idea being to maintain a pressure in all of the outside rooms sufficient to prevent cold air coming through the cracks in the window frames, etc. The air in this auxiliary system was intended to be heated by a steam coil in the discharge of the circulating fan. Before this sytem was put in use, the writer designed an electric heater to take the place of the steam coils, and after having been in use for the last two winters it has been found that the low-pressure steam system is entirely unnecessary, a comfortable temperature being maintained in the coldest weather by the electrically heated hot air. It is true that the steam-heating system in this building is a part of a much more extensive one installed throughout the transformer building, of which the office forms an L, but it is not necessary to maintain a high temperature throughout the remainder of the building, with the exception of the switchboard room, which is heated by direct electric radiators. During the last winter it was found that the use of the boiler was only necessary for 25 days, and that during the remainder of the time the electric heaters in the office building and switchboard room were sufficient for all purposes.

The cost of this heater was $388.00 and it consumes 85 kw. (including power necessary for fan motor). In a plant of this size the cost of this amount of energy is negligible, whereas the natural gas and wages of boiler attendants form very real quantities. This heater supplies 72,000 cu. ft. of air space in a building exposed on three sides.

For the present and until some material capable of delivering heat at a higher rate than any resistive metal now known is found, the principal application of electricity will be in articles where the heating must be closely confined. The superiority of these articles is well recognized.

W. S. Andrews: It was my privilege to be associated with Mr. Edison while he was developing his low-tension, multiple-arc system of electrical distribution in the early 80's of the last century. I remember a statement that he made in 1881, to the effect that he expected to utilize the general distribution of low-potential electrical energy not only for public and private lighting purposes, but also for operating electric motors to drive the machinery in mills and factories, and for electric traction; furthermore, he confidently expected to make electricity generally serviceable for the cooking of food, and, to a limited extent, for the warming of dwellings.

We all know that Mr. Edison's prediction in regard to the use of electricity for lighting purposes was very quickly realized, and also that not many years thereafter the electric motor was developed to a considerable degree of perfection, and largely used. Heating and cooking by electricity, however, have failed to show a similar advancement, and a brief discussion regarding the probable causes of this delayed development may therefore be timely and profitable with the view of bettering existing conditions.

Let us consider the use of electricity for performing ordinary cooking operations. As Mr. Hadaway says, the real difficulty consists in providing a hot-water supply economically. At the price of 5 cents per kilowatt-hour, the cost of heating a gallon of water from faucet temperature to boiling at an efficiency of say 80 per cent is a little more than two cents, which to many people would be a prohibitive price, especially as the same results can be accomplished with gas for very much less. Apart from the question of water heating, however, the operations of baking, broiling, frying, toasting, etc., by electricity can be accomplished at a reasonable expense, when electricity is sold at the rate of 5 cents per kilowatt-hour, according to my own personal experience, taking into due consideration, as Mr Hadaway. points out, the collateral advantage of being able to raise utensils to a high temperature with localized heat, thus keeping the kitchen much cooler in hot weather than is possible when using a coal or gas range. For several years past I have used electricity for cooking purposes in my home during the summer months. Water only was heated by a gas arrangement attached to the kitchen boiler, but all other heating and cooking was done by electricity at an average cost of $4.50 per month. I estimate that the average cost of heating water by gas amounted to about $1.50 per month, making an average total expense of $6.00 per month for cooking three meals per day for a family of four. This outlay also included the heating of water for laundry work and the use of an electric iron, so I do not think it can be considered as extravagant. My experience shows that an ordinary breakfast, including cereal, coffee, toast, and broiled meat, sufficient for two or three people can be cooked by electricity with an expenditure of about one kilowatt-hour, the various devices averaging as follows, on a liberal basis:

Cereal cooker	600	watts	for	20	min.	=	12,000	watt-minutes.
Coffee percolator	600	"	"	20	"	=	12,000	"
Bread toaster	600	"	"	10	"	=	6,000	"
Meat broiler	1500	"	"	20	"	=	30,000	"
							60,000	"

Or one kilowatt-hour at a cost of 5 cents.

These instances show that good service can be rendered at a reasonable expense by existing devices, but I quite agree with Mr. Hadaway in his statement that we are still far removed from ideal conditions.

In a recent lecture on this subject Dr. Steinmetz went to the root of the whole matter, in discussing the possibilities of greatly reducing the cost of electricity by improving the station load-factor, aiming to get a steady and uniform load throughout the 24 hours of the day, when he said:

> When we have accomplished that, electric power will be much cheaper than anything else and then the end will come for gas and kerosene. And that time *will* come some time and we will probably see it.

Mr. Hadaway cites some favorable cases, such as first-class apartment houses in large cities, where steam heat, hot water, and electricity are supplied to tenants, and where, therefore, electric cooking may be performed economically provided only that the price of electricity is reasonable. It is nevertheless generally true that the cost of heating water by electricity for domestic purposes, at ordinary rates for current, presents an almost insuperable obstacle to its common use, excepting in a small way, such as for hot-water urns, coffee percolators, and similar table devices.

I believe with Mr. Hadaway that the field of industrial application will eventually be found equally if not more extensive and profitable than the domestic side. There are thousands of operations where heat is now applied to various parts of machines in a cumbersome and unhandy way by steam and gas where electric heat would be not only more cleanly and hygenic, but also more economical in the long run, on account of the especially favorable feature of being able to localize the heat where it is required, and other considerations covered by a variety of special conditions.

In regard to the heating of air in buildings, Mr. Hadaway has shown in a general way that the present cost of electricity when supplied from large generating stations makes extended applications prohibitively expensive. In a few special and isolated places it is applicable also for emergency cases where a maintenance or rise of temperature may be the means of saving valuable property, as in conservatories and greenhouses. where the value of these appliances is very evident.

H. P. Ball: The feature of the electric heating subject in which I am most interested is the design of the apparatus. Mr. Hadaway gives some comparative figures, stating that a circular gas burner will impart heat at a rate equivalent to 35 watts per superficial square inch, and electric heaters about one third of this amount, or about 12 watts to the square inch. He does not say how this gas burner was measured, but it might have been an annular surface, the surface of the flame, or the area of a circular disc; in any event, my experience would indicate that this rate can be exceeded. It is perfectly feasible by using modern materials, such as the new nickel-chromium metal and infusible insulating materials, to operate with watts running up as high as 20 on open stoves, and as high as 25 or even 30 on stoves which are used in connection with percolators or water heaters.

It appears that high temperature is the immediate field for electric heating devices, with the wide distribution of heat as something for the future, perhaps, but to-day the business to be had is in sad-irons and other devices where the heat is concentrated in small areas.

I call attention, therefore, to the absolute necessity of getting away from easily fusible materials, and adhering to the use of very high melting resistances, mica as the insulator, on account of its extreme thinness and high ground test, standing from 1000 to 1500 volts per mil of thickness, and metals, such as aluminum-bronze, which will not readily oxidize in the air at high temperatures.

As to the problem of heating rooms, the ideal arrangement would be the use of an exposed resistance conductor in the form of a thin but wide band, so as to have a maximum radiating surface, causing the conductor to operate at the lowest temperature; for it is a well known fact that if a conductor is surrounded with insulating material the conductor will run at a higher temperature for the same dissipation than if it were exposed in the open air.

In a sad-iron or water heater it is necessary to enclose the resistance, and in doing this it must be supplied with a maximum surface by using flat ribbon resistances. A material with a minimum thickness must also be used, such as mica, in order to get the heat out of the conductor into the thing we are trying to heat with the least possible thermal drop.

In a sad-iron the unit can be run at a very high current density on account of the specific capacity for heat of the surrounding iron; but in the water heater it must be restricted to lower densities, and care must be exercised to see that the surrounding material is of a minimum thickness, in order that the electrical energy and time taken in raising the temperature of the device to its working temperature may be the least possible.

Max Loewenthal: The author discusses the relative efficiency of steam, gas, and electricity and arrives at the almost self-evident conclusion that electrically generated heat is not so efficient for ordinary domestic or industrial operations as steam. He furthermore shows that the same is true when we compare electricity with gas for heating or cooking purposes. We have, however, many figures to prove that electrically heated appliances are as efficient, and frequently more efficient, than the same appliances heated by other means.

Take, for example, the case of an oven. The efficiency of an ordinary cooking-stove using solid fuel is only about 2 per cent, 12 per cent being wasted in obtaining a glowing fire, 70 per cent going up the chimney, and 16 per cent being radiated into the room. In a gas-stove, considering that the number of heat units obtainable from the gas at a certain price is but small compared with solid fuel, the ventilating current required for the operation alone consumes at least 80 per cent of the heat units obtained by burning the gas. In the case of an electrical oven more than 90 per cent of the heat energy can be utilized; and thus, although possibly 5 to 6 per cent only of the heat energy of the fuel is present in the electrical energy, 90 per cent of this, or 4.5 per cent of the whole energy actually goes into the food. Thus the electrical oven is practically twice as economical as any other oven, whether that oven is heated by solid fuel or by gas.

Furthermore, the comparative operating cost of electric and gas cooking depends upon two conditions; the relative rates for gas and electric heat units, and the relative heat efficiencies of gas and electric apparatus. A third quantity, the effect produced by the different rates and modes of heat applications in the two classes of utensils, may effect the efficiency slightly, but the existence of this effect is not yet verified. Starting with the heat of coal, which may be fairly estimated as 12,000 B.t.u. per pound, we compute the relative efficiency of the heat conversion as follows:

Gas	Electricity
1 pound of coal produces 5 cu. ft. gas. 5 cu. ft. of gas contain 3000 B.t.u.	1 pound of coal produces 0.25 kw-hr. 0.25 kw-hr. = 853 B.t.u.
Efficiency of heat conversion is	Efficiency of heat conversion is
$\frac{3000}{12000} = 25$ per cent.	$\frac{853}{12000} = 7.1$ per cent.

$$\frac{\text{Efficiency electrical heat conversion}}{\text{Efficiency gas heat conversion}} = 28.4 \text{ per cent.}$$

With manufacturing processes of equal cost per pound of coal converted, it is apparent that an electric heat unit must cost nearly four times as much as a gas heat unit, but with present processes the relative rates are:

Gas	Electricity
$1.00 per 1000 cu. ft. 1 B.t.u.0.000167 cents.	$0.10 per kw-hr. 1. B.t.u. 0.00293 cents.

$$\frac{\text{Electric B.t.u. } 0.00293}{\text{Gas B.t.u. } 0.000167} = 17.5$$

It is known that the efficiency of electrical apparatus is about four times that of gas, and consequently, as the gas utensils require four times as many B.t.u., the above figure of 17.5 is reduced to 4.4. If, then, the rate for electricity is reduced to one-quarter of that assumed, or 2.5 cents per kilowatt-hour, this figure of 4.4 is changed to 1.1 and we have practically identical operating costs. This ratio has been borne out by numerous tests.

One of the most interesting figures contained in Mr. Hadaway's paper is the one relating to room heating.

For room heating from 85 to 110 watts are practically equivalent to steam at low pressure condensed by one square foot of radiating surface, etc.

I may add that in practice we figure about 1.5 to 2 watts per cubic foot to be heated, under average conditions.

I do not believe that the heating element of the future will be altogether in the nature of a wire, a metallic paint, etc., but will probably be of a refractory character, such as a carbide, or a silicate of carbon. This will stand a red heat without apparent oxidation and is mechanically stronger than anything heretofore employed.

Charles P. Steinmetz: When using electric energy in producing heat energy for heating, cooking, baking, etc., we are confronted by the enormous handicap of the inefficiency of the thermodynamic engine. As electric power at the switchboard of a station we get only from 8 to 10 per cent of the energy of the fuel burned under the steam boilers, and have lost 90 per cent or more. When converting again this electric energy into heat energy, we can get only the same very small percentage of the heat energy of the fuel that is contained in the electric energy, and this greatly handicaps the use of electric energy in those cases where large amounts of heat are required, as in water heating. Therefore we always hear electric heating engineers refer to the use of gas or hot water or steam for the cases where large amounts of heat are required, as in water heating, or house heating, and reserving the use of electric heating for those applications where the heat capacity is small, as in cooking and baking, or for intermittent use, or where the convenience and cleanliness of electric heating outweigh other considerations. We look to future improvements in the production of electric power, and to improvements in the load-factor, to bring the cost of electric power down to such values as to compete in economy even in such fields as house warming, water heating, etc.

It is this statement of the inefficiency of electric heating, due to the inefficiency of the conversion of the heat energy of fuel into electric energy, that I desire to challenge. It is true that each kilowatt-hour of electric energy contains only a definite number of units of heat energy, 850 calories, and that the heat energy produced by the conversion of the electric energy into heat can therefore never exceed 850 calories of heat energy per kilowatt-hour, or about 8 to 10 per cent of the original heat energy of the fuel. The problem of electric heating is, however, not to produce heat energy from electric energy, but to raise the temperature of the air or some other objects by increasing the quantity of heat contained in them. This does not necessarily require producing the quantity of heat; it can be done by raising an existing quantity of heat to higher temperature, or higher heat level. For instance, to fill a vessel with water, the water could be produced by burning hydrogen gas in the air, but it would be more economical to fill the vessel with water by using the energy of the burning hydrogen to pump water from some lower level into the vessel. In the latter case, of the water in the vessel only a small part is produced; most of it is taken from an otherwise unavailable supply, a lower level.

In the thermodynamic engine, we supply a quantity of heat, at higher temperatures. Of this, a small part, say 10 per cent, is converted into mechanical energy, the remaining 90 per cent given off, "rejected", as heat at atmospheric temperature. The thermodynamic cycle, however, is reversible, and by performing it in the reverse direction, we could take the 90 per cent of heat at atmospheric temperature; that is, from the air, or the water supply, and by the conversion of the 10 per cent of mechanical energy into heat in the reversed cycle raise the other 90 per cent of heat from atmospheric to the higher temperature, and thus supply again the 100 per cent of heat at the same theoretical temperature at which they were supplied to the thermodynamic engine in the direct cycle. In this case we would supply at the higher temperature 10 times as much heat as corresponds to the consumed electric energy; but this heat is not produced from electric energy, 90 per cent has merely been raised from a lower level, lower temperature to a higher temperature. It is thus possible to supply, at the moderate temperature rise required by electric heating devices, many times more heat than the calories of heat energy corresponding to the consumed electric power, a larger quantity of heat even than the total number of units of heat energy of the fuel consumed in producing the electric energy, without infringing the law of conservation of energy. It is this possibility which Lord Kelvin referred to in the statement quoted in Mr. Hadaway's paper.

With a relatively moderate expenditure of electric power, such a thermodynamic engine could be operated between two temperatures, one above and one below atmospheric, and thus be used to heat the houses in winter or cool them in summer. Air, taken at atmospheric temperature, is heated by adiabatic compression; its heat is abstracted; then the air is expanded and thus cooled below atmospheric temperature, and heat supplied to it until atmospheric temperature is restored. Thus between compression and expansion, heat is given off above atmospheric temperature, and between expansion and compression heat is taken in below atmospheric temperature; that is, cold given off, and as the work done by the expansion supplies most of the work of compression, electric power has to supply only the differences; that is, a part of the energy representing the heating or cooling effect. Obviously, in summer the higher temperature, in winter the lower temperature, would be rejected by exhausting into the air and taking in a fresh volume of air at atmospheric temperature. Such a thermodynamic heating apparatus should be able, even with the present cost of electric power, to make the cost of electric heating comparable with that of direct heating by fuel.

Townsend Wolcott (by letter): In discussing Mr. Hadaway's paper, Dr. Steinmetz makes some statements about thermodynamics which, whatever he may have meant to say, are, as he has actually made them, misleading. First, assuming a thermodynamic engine with an efficiency of 10 per cent, as he does, the first great loss of temperature fall, and consequent dissipation of energy, is between the temperature of the fire and that of the boiler, if the apparatus be a steam engine; or between the temperature of combustion and the maximum allowable temperature of the cylinder, if it be a gas engine. In the steam engine, the temperature of the fire is at least 1000° fahr., while that of the boiler is less than 400°. It is not true, therefore, that, by reversing the cycle, the original temperature can be restored to the whole amount of heat, even theoretically; for the fall from fire to boiler temperature is a completely non-reversible process, and entirely outside of the reversible cyclic process. To be sure, if we were able to work down to absolute zero, all the energy of heat of any finite temperature would be available, and the drop between fire and boiler would be of no consequence; but, with a finite lower limit of temperature, such as Mr. Steinmetz uses, this drop is of the greatest importance. Secondly, if the expansion be carried below atmospheric temperature, as Mr. Steinmetz says it is, efficiency is sacrificed; it is precisely because the expansion is isothermal at atmospheric temperature, in the apparatus described by Lord Kelvin, that the efficiency is high.

Mr. Hadaway's quotations from Lord Kelvin's paper of more than half a century ago may be a little startling to those who are not familiar with thermodynamics; first, on account of a doubt as to whether the statements therein contained are quite true, in view of the fact that, upon superficial examination, there appears to be a regeneration of dissipated energy; secondly, because, if true, the principle does not appear to have been fully utilized. Upon investigation, however, all doubts are dispelled and the whole subject is illuminated. I have not seen the mathematical investigation published with the original paper, and to which Lord Kelvin refers in the passage quoted by Mr. Hadaway; but, as the numerical statements contained in the passage check satisfactorily, even when using the most modern values for the natural constants involved, it is, perhaps, of no great importance that I have not. Let us follow through the

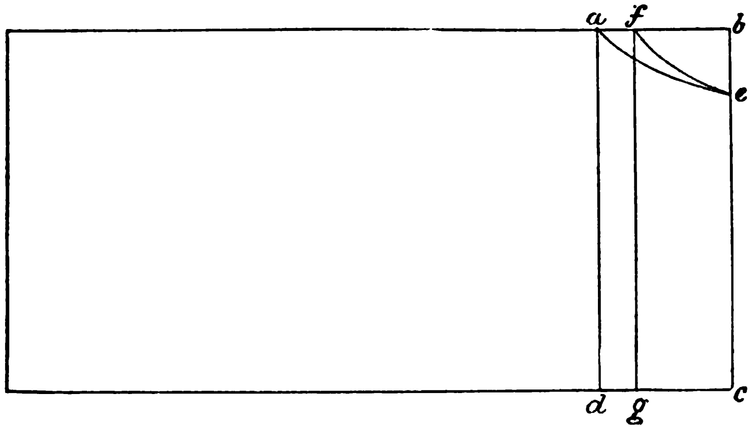


Fig. 1

process described by Lord Kelvin, using, in the interest of simplicity and perspicuity, the ideal cylinder of Sadi Carnot, which is frictionless and has walls which can be made alternately perfect conductors and insulators of heat, and the ordinary pressure-volume diagram. The values of the natural constants will be taken from the fifth edition of Peabody's "Thermodynamics of the Steam Engine." First suppose that the piston be withdrawn to 0.82 of full stroke, and further suppose that, at this point, the cylinder contains exactly one pound mass of air, at the normal pressure of 2116.32 lb. per square foot, which we will designate by p_1. So far, no work has been done. Now suppose the intake valve closed and the piston drawn out to full stroke, the temperature of the air in the cylinder being maintained constant by conduction through the walls of the cylinder. At the end of the stroke, the pressure will be $0.82p_1 = 1735.382$ lb. per square foot $= p_2$ The volume at p_1 we will call v_1 and at p_2

we will call v_2. $v_1 = 12.84375$ cu. ft., $v_2 = 15.66311$ cu. ft. The work of isothermal expansion is

$W_i = p_1 v_1 \log_e \frac{v_2}{v_1} = 5394.186$ ft-lb., and is represented on the diagram by the area, *a, e, c, d*. The total work required to move the piston from p_1 to p_2 against the atmospheric pressure is $p_1 (v_2 - v_1) = 5966.668$ ft-lb., represented in the diagram by the area *a, b, c, d*. The difference of these areas; that is, the area, *a, b, e*, represents the external work supplied, $W_0 =$ 572.482 ft-lb. The cylinder is now insulated and the piston allowed to go back until the pressure in the cylinder is again p_1; the volume at this point $= v_3$. The compression is adiabatic, and the temperature, of course rises. Assume the absolute zero as $-459.5°$ fahr., the specific heat of air at constant pressure as 0.2375, the mechanical equivalent of heat as 778 ft-lb. From these values we obtain that for the ratio of the specific heat of air at constant pressure to that at constant volume, $\kappa = 1.40593$. The initial temperature, $T_1 = 50°$ fahr. $= 509.5°$ absolute. The final temperature $= T_2$. From the laws of adibatic compression, $\sqrt[\kappa]{\frac{p_1}{p_2}} = \frac{v_2}{v_3} = 1.15160$, therefore, $v_3 =$ 13.601183 cu. ft.; $\left(\frac{v_2}{v_3}\right)^{\kappa - 1} = 1.0589711$.

$$T_2 = \left(\frac{v_2}{v_3}\right)^{\kappa - 1} T_1 = 539.54572° \text{ absolute} = 80.04572° \text{ fahr.}$$

This checks with Lord Kelvin's figure. From these figures we have $1 - \left(\frac{v_3}{v_2}\right) = .131643$, which checks with Lord Kelvin's statement of the amount of compression. The work of adiabatic compression is $W_a = \frac{p_1 v_3}{\kappa - 1} \left(1 - \frac{T_1}{T_2}\right) = 3948.718$ ft-lb.: this is represented on the diagram by the area, *f, e, c, g*. The work which the atmosphere is capable of doing on the compression stroke is represented on the diagram by the area, *f, b, c, g*, and equals 4363.721 ft-lb. The difference between this and the work of compression = 415.003 ft-lb., and is represented on the diagram by the area, *f, b, e*. As this work may be, at least theoretically, stored so as to be available for use in the next expansion, it is only the difference between it and the work originally required to pull the piston out which is permanently absorbed. This difference, represented on the diagram by the area, *a, f, e*, equals 157.479 ft-lb. The equivalent in work of the heat required to raise one pound of air from T_1 to T_2, and at the same time expand it from v_1 to v_2; that is, to heat it at constant pressure, is 5551.698 ft-lb. As this is a little more than 35 times the work absorbed, Lord Kelvin's numerical results check completely. However, the casual reader of Lord Kelvin's paper finishes with a more or less distinct idea that some dissipated energy has been reclaimed; or, at least that the Bell-Coleman refrigerating apparatus is more economical than some others. I do not mean to intimate that Lord Kelvin had any such idea, but simply that the ordinary reader is inclined toward that opinion, and it is to make the truth regarding this matter clear that I have gone through the foregoing calculation in so much detail. The regeneration of dissipated energy, or the flow of heat from a cooler to a warmer body, without an equivalent flow of heat from a warmer to a cooler body, is contrary to the second law of thermodynamics. The number of proposed machines intended to operate in defiance of this law would probably make a respectable second to the number of proposed perpetual motion machines; but one kind does not operate any better than the other. It will be observed that the described operations do not constitute a complete cycle, as the air has not returned to its original state. Suppose we complete the cycle by allowing the air to cool off to its initial temperature and pressure, and inquire how much of the heat energy is available for conversion into work, by any imaginable device. The portion of the heat delivered at the temperature, T_2, which is available for work in a heat motor operating between the temperatures, T_1 and T_2, is $1 - \frac{T_1}{T_2}$. But, as we cannot withdraw a finite quantity of heat from the air in the cylinder without lowering its temperature, we must assume an infinite number of elementary cycles between the fixed lower temperature T_1, and the variable upper temperature, T, and integrate between the limits, T_1 and T_2. Thus, allowing for the specific heat of air.

$$\text{Available energy} = 0.2375 \int_{T_1}^{T_2} \left(1 - \frac{T_1}{T}\right) d\,T =$$

$$.2375 \left((T_2 - T_1) - T_1 \log_e \frac{T_2}{T_1}\right) = 0.2024569 \text{ B.t.u.} = 157.511 \text{ ft-lb.}$$

which is pretty accurately equal to the absorbed work, as calculated in the foregoing. Had more places of figures been employed in the calculations, the agreement would have been closer. We see from this investigation that no dissipated energy has been regenerated or recovered, and that the only energy which can be, even theoretically, recovered from the cycle is just what is put in. An examination of the records of tests of ice machines in Professor Peabody's book shows that the Bell-

Coleman apparatus is no more economical than apparatus using ammonia and other volatile liquids, but this does not in any way invalidate Mr. Hadaway's contention that the heating of houses by electric motors and Kelvin apparatus would be more economical than by rheostats, even allowing for the inevitable losses in a practical machine. If the limits of the temperature range were extended, so as to boil water or cook, the economy would diminish and, ultimately, disappear altogether.

W. S. Hadaway, Jr.: Concerning some of the criticisms of the paper: it has been my effort to make it very conservative. The logical way to look at the matter is to conserve the fuel supply so far as it can be made to go. From a business standpoint, and it is really the business standpoint that determines the engineering features, it has been shown conclusively that a demand for electric heating apparatus has been created which must be met. It has become in many ways not so much a question of what the efficiency is as of results attained.

It has been thought desirable in connection with the practical side of the problem to point out some of the difficult questions yet to be solved. I thank Dr. Steinmetz for elaborating the Kelvin method of securing heat in large quantities at low temperature.

In the construction of apparatus the one important factor which remains to be solved at present is the question of heat insulation. If we were building ships in which we had to figure a certain percentage of tonnage capacity set aside for pumps to keep the ship afloat, we would be in relatively the same position as is the manufacturer of electric heating devices to-day so far as energy inputs are concerned. We have no practical method of confining heat at high temperatures.

I was asked a question as to how the statement that the gas burner corresponded to something like 35 watts per square inch was derived. That statement is based on laboratory work, in which gas stoves of ordinary construction with annular burners and with outside dimensions of approximately 6 in. are compared with an electric heater upon which a vessel of the same size was used on the two devices. It should be added, however, that the limitation of the watts per square inch is not necessarily as low as 12 watts, because on low voltages apparatus has been successfully used up to as high as 40 watts per square inch. The voltage situation, however, particularly on 220 volts, is a difficult proposition to handle, because apparatus built for that voltage has one fourth the resistance cross-section of apparatus built for 110 volts, and the electrical insulation has also to be somewhat increased.

The logical development in voltages is towards a decrease rather than an increase in the same way that the logical development of quantitative work in heating is the centralization of all the heating values at one point and the distribution of them over short distances to secure the highest fuel efficiencies in combination with the advantages which the electric heating devices give.

A paper presented at the 232d meeting of the American Institute of Electrical Engineers, New York, December 11, 1908.

THE LOG OF THE NEW HAVEN ELECTRIFICATION

BY W. S. MURRAY

SYNOPSIS OF PAPER

Introductory

1. Object of paper.
2. Value of mistake.
3. Value of ignorance.
4. Facts—necessary to stable conclusions.
5. Effect of experience on decision.
6. Relation of steam versus electric traction.
7. The new chain—its old links.
8. The public.

Major Faults

1. Power house.
2. Insulation.
3. Circuit-breakers.
4. Line.
5. Locomotives.

Minor Faults

1. Generator burn-outs.
2. Distribution.
   a. Multiple-track circuits.
   b. Independent track circuits.
3. Contact wire.
4. Hard spots.
   a. Hangers; *b*. section breaks *c*, deflectors.
5. Signal wires.
6. Locomotive current collectors.

Log of Operation

1. Electric passenger service.
2. Electricity versus steam.
3. Train-minute delays; *a*, table; *b*, chart.
4. Serious failures—over 300 train-minutes.
5. Engine repairs—*a*. Table showing engines in shop.
6. Engine capacity—*a*. Table showing trailing loads in excess of guarantees.
7. Engine - mileage — electricity and steam compared.

The Electrification in Relation to Matters Other Than Traction

1. Telegraphs and telephones.
2. Public safety.
3. Foreign wires.

Commentary

1. Lightning protection.
2. Grounds.
3. Tell-tales.
4. Train heating and lighting.
5. Fatalities.
6. Single- versus double-catenary construction.
7. Cross-catenary versus bridge bents.
8. Single-phase operation.

Conclusion

1. Alternating current and the engineer.

New York, New Haven and Hartford Railroad Company.

THE LOG OF THE NEW HAVEN ELECTRIFICATION

BY W. S. MURRAY

The object of this paper is to bring the actual operation of the New Haven single-phase electrification as closely as possible to those who are truly interested in its merits and faults. If the writer can feel that this object has been accomplished, the reward for the effort will be sufficient.

In advance of freely opening up the subject without mental reservation, it may be well to say that he is not unmindful of the adverse construction and criticism that will be placed by some upon the facts he intends to present. Having carefully weighed this, however, he has concluded that it should not be permitted to stand in the way of, or militate against, the sounder and analytical judgment of those intent upon a fair consideration of the facts as they exist.

The first intention was to make the subject of this paper " The Errors Made in the Electrification of the New Haven System ", but realizing what capital such an " admission " would make for those few who are inclined not to be fair-minded on this important matter, the title has been changed, and the body of the paper built upon a description of the faults, major and minor, that have been made, and their effect, by log-sheet reference, upon operation. After all, the citation of " how not to do it " is far more valuable than " how to do it ". It may be at some sacrifice of pride to discuss one's errors, yet there is something so valuable in the exposition of an error that this, in itself, is ample compensation for its acknowledgment. There seems to be, too, something altruistic about a mistake or " wrong "; for in its death there is born a " right ", and one can never really get on intimate terms with his mistake until its rightful complement has been substituted. After a railroad has changed from a steam to an electric schedule, the natural question immediately follows: "Is it a success?" The answer to this question involves almost an infinity of analytical judgment. Electrical engineers have no greater professional or moral responsibility before them than the presentation of all available data, in which will be included information of pertinent application in the solution of the vast electrification problems of to-day. To those entrusted with the responsibility of such decisions is due every possible help and light. Engineering conclusions are based on 5 per cent. theory and 95 per cent. practice; practice being made up of experience and an acceptance of existing allied conditions. We want nothing unless we know all about it before it is ours. If we are wise, we should not be so particular about its merits as its demerits. Perfection cannot exist. This applies to electrification as to other things.

The duty assigned to the engineers of the New Haven railroad was to provide for the electrical operation of its trains. At the early period of April 1, 1905, when we settled down to this responsible task, the data in the field, upon which to base real conclusions, were about 5 per cent, in comparison with the experience now available. If "Ignorance maketh the brave", I can only say that I am more than thankful we were so ignorant at the outset. I do not believe that this remark will need any explanation here. Could we have read the history of our errors, rather than have made them, I question whether we would have had the temerity—though I confess our convictions would have been the same—to ask the president of the New York, New Haven & Hartford Railroad Company to put up with the criticism, abuse, and unfair remarks that have been made by those ignorant of the facts, and by others who, perversely, were intent upon their own selfish, narrow course with manifested abandonment of all purposes to acquaint themselves with the nature, the scope, and the possibilities which lay easily within their knowledge, merely by the asking. To-day, I am thankful to say, this negative attitude is giving away to the alarm made manifest by undisputed facts, which demand their own recognition. I have no personal desire to make any other engineer think as I do. If he sees the facts as I see them, then we will vote alike. To those who may be interested in my conclusion in regard to the New Haven Railroad electrification, I can simply say that, if I were in favor of its use three years ago, now, standing on the more stable ground of experience, particularly in regard to the department of faults, I am doubly in favor of it to-day. I shall, of course, be interested in those who share this opinion after reading this paper, but I shall be doubly interested in them if their conclusion has not, in the slightest degree, been influenced by my own, but has been formed entirely upon the contemplation of the faults severally cited. And, let me quickly add, I shall be equally interested in those who honestly disagree with this opinion.

Unlike steam traction, where the number of links in the delay chain is but one, electric traction has its delay chain composed of three links; namely, the power house, line, and locomotive. A failure in any one of these links may produce train delay or delays. Measured by the degree of their seriousness, these links, in the order of their effect on delays, may be mentioned in the order given. This is, of course, immediately apparent, in that the power house affects all operation; the line, a considerable portion of the trackage, involving, possibly, several trains; and the locomotive is usually confined to its individual troubles, only infrequently affecting following trains. This order naturally has application to all forms of electrification, direct current, single phase, or polyphase.

In the conception of the form of power house, line, and locomotive to be used in the New Haven system, ignorance and lack of experience led those pertinently interested in its success to believe that while the chain of power generation, and its transmission and utilization for traction, was of a new character, its links, however, were made up of principles long recognized and reliable. They were right in this conclusion, except that it did not include certain phenomena which could not have been anticipated, due to the combination of these old principles in the form of this new chain. For example, there was nothing particularly disturbing about an 11,000-volt 25-cycle, three-phase power house, from which was to be taken most of the power from one of the phases generated; or of a 300-ft., 11,000-volt, single-phase transmission line, from the terminals of which the same voltage was to be distributed, east and west, along the right-of-way of a railroad; or of a locomotive with a transformer installed upon it to take 11,000 volts and step it down to 600 volts to supply 300-volt single-phase railway motors. Power houses of this character have been designed the world over, except perhaps that single-phase current in the amount to be utilized had not been used elsewhere. Transmission lines of 60,000 volts had been in constant use. And, except that there was a moving contact in connection with the current that was to go to the step-down transformers on the locomotives, transformers of many times the capacity and voltage were in universal use. The single-phase motor, indeed, may be said to have been new so far as the size required for the New Haven service is concerned, but tests made upon smaller units demonstrated beyond peradventure its tractive qualities and showed its characteristic curves to be closely similar to those of its prototype, the direct-current motor. This reasoning, in close consideration of each class of apparatus, *per se*, has proved itself correct, and such irregularities as have existed in the initial electric service which the New Haven road offered to the public, while deserving the condemnation which the public can unfailingly offer, have been due to the ramification of a series of faults that have developed entirely outside the zone of previous experience. The delays (and public criticisms of them) have followed even to this late day, when the electric service is far better than the steam service it has replaced. It has occurred to me that certain engineers, who are unbelievers in the single-phase system, may interpret this paper as an apology. To them I would say, as I do to the public, it is an apology for having delayed any of their appointments. To others, it is needless to add that the citation of these troubles is given in the hope that they will, as they easily can, avoid them in the future.

The Serious Faults

Power house. The electric power supply for the New Haven road is derived from four 11,000-volt steam turbine generators. three of which have an electric capacity of 3750 kilovolt-amperes single phase; the fourth unit consisting of a 6,000 kilovolt-ampere, three-phase generator, which can also supply single-phase current to the system.

Although the generators as originally designed were made exceptionally strong, and particular attention paid to their insulation, due to the necessity of grounding one phase, it was found that the utilization of so much single-phase current from a three-phase star-wound generator produced a stray magnetic field completely out of the path of normal lamination. As a result it was impossible to develop for continued operation more than 66 per cent. of the normal rating of the generators. Overloads of any character produced abnormally rapid heating, making such operation dangerous, although the generators were guaranteed to carry 50% overload for two hours, and 100% overload for two minutes, in order to meet the sudden drafts of currents required for a schedule such as exists on the New Haven road. Indeed, at the very start the actual drafts of current showed that the generators must meet imperatively the guarantees as to normal and overload capacities if the electrification were to be successful. These, then, were the power house conditions that confronted the New Haven road on beginning the work of propelling alternating-current trains. The power house being the heart of an electric system, to add more would be to say less in describing a critical situation.

I trust it is not to be my pleasure to meet the prophet, who says, "This could have been anticipated." Suffice it to say that after three unsuccessful attempts at complete correction, each, however, affording some constructive results—months being absorbed in the dismantling and readjusting of the parts of these generators—the final attempt was successful and the generators are to-day operating in the power house, fulfiling the guarantees mentioned previously. But this last mentioned fact is insignificant when compared with the valuable information that has been derived, which will permit all other generators to be manufactured without the fault described.

Line insulation. The years of experience which we have had in the study of insulating various voltages led to what was considered a very conservative insulation in the various parts of the line. Messenger cables had to be insulated from the intermediate trusses and at their anchor-bridge termini. The trolley copper conductor was suspended from the messengers and had to be insulated at points where it entered the oil switches on the anchor bridges. Trolley wires had to be insulated from each other at section breaks. On curves, both messengers and trolley wires had to be strained over the center of the tracks through the agency of pull-off posts at the side of the tracks, the pull-off wire being insulated between tracks and the pull-off column itself. Feeder wires had to be insulated from their supporting cross-arms and at points where they pass under highway bridges. None of these problems in insulation had the appearance of an especial character and, indeed, did the road-bed provide traffic only for electric trains, the problem would have been simplicity itself. A pressure of 11,000 volts being the prevailing dielectric strain, the problem was to provide sufficient insulation at all the points mentioned above.

Of the effect of steam locomotive discharges upon insulators, there was no initiative by which to be guided, and it became necessary to decide upon the factors of insulation that would be required. It was thought that ample provision had been made: it proved otherwise. Experience has proved that, in places, just double the amount is required. It was quickly noted that the greatest number of insulator failures occurred wherever the insulation was subject to the direct blast of the steam locomotive. To correct the difficulty, therefore, it was found necessary to double up on anchor insulators. The intermediate messenger insulators proved adequate and it was not found necessary to increase the impregnated stick insulation between trolley wires at curves, but wood stick insulators had to be added in series with the moulded material insulator between the pull-off wire and pull-off post. The original insulators on the anchor-bridge switches were made of moulded material and for them was substituted porcelain. It was not necessary to change the feeder insulators on the catenary bridge struts. While very little trouble has been experienced with the form of insulation used for supporting the feeders under highway bridges, it is anticipated that trouble will follow if this is not changed. The present form consists of the corrugated spool-type insulator, for which there will be substituted a regular porcelain double-petticoat insulator.

To-day, instead of line failures being the rule, they have become the exception.

Circuit-breakers. The momentary energy involved in a short-circuit produced upon a line fed by high-power, high-speed turbines is very great. Under the sub-title "*Power house*" it was stated that the generators were operating under their guaranteed capacities. Internal heating, due to stray magnetic field, was the cause of the generators failing to meet their designed capacity. This heating was completely cured by the simple addition of a short-circuited winding surrounding the rotating member of the generator, similar to that used in the well-known squirrel-cage type of induction-motor rotors. It is interesting to note here, however, that while the heating is entirely eliminated by this short-circuited winding, its effect on the occasion of a short-circuit is to allow more current to flow. This tendency, however, is controlled by a method later to be described. In the New Haven system, as the current from the power house was fed directly to the line and from there to the locomotives without transformation of voltage by transformers, the inductive element to counteract the surging current was practically negligible; under these conditions there resulted short-circuits which no circuit-breaker apparatus then designed could be relied upon to take care of.

Here it is expedient to point out the very marked difference in operating conditions between circuit-breakers used on a system that is grounded and one that is not. In either case, it is, of course, good practice to ground the frames of all circuit-breaker apparatus. In the case of the grounded system, this virtually brings one terminal of the generating system directly to the frame of the circuit-breaker; with the other terminal of the system connected to the jaws (contactors) of the breaker, it is readily seen that on the occasion of a heavy current surge, due to short-circuit, should the arc extend itself through the walls of the oil tank (wherein the circuit is broken), this would be a direct leakage to ground, establishing a short-circuit around (or in shunt) to the circuit-breaker jaws, which action, of course, renders the circuit-breaker useless, and subjects it to the damaging effect of the arcing locally produced until interrupted by another circuit-breaker in series with the circuit. On the other hand, where the system is not grounded, even should the walls of a circuit-breaker be broken down, as explained above, there would be no electrical connection (unless the neutral be grounded) between the circuit-breaker frame and the generator, and thus no return path for the current. I desire to draw particular attention to this point, for while the circuit-breakers used in our case open short-circuits, since the current surges have been practically eliminated, I believe that the lining of the oil tanks should have an especially high insulation factor on grounded systems.

The failure of circuit-breakers, either in the power house or on the line, naturally produced train delays of large or small magnitude. It was difficult to believe that these large circuit-breakers were incapable of taking care of the short-circuits, and some time was wasted in thinking this way. Therefore, we reluctantly but surely arrived at the conclusion that the conditions would have to be changed.

The remedy was simple. Instead of feeding the main line

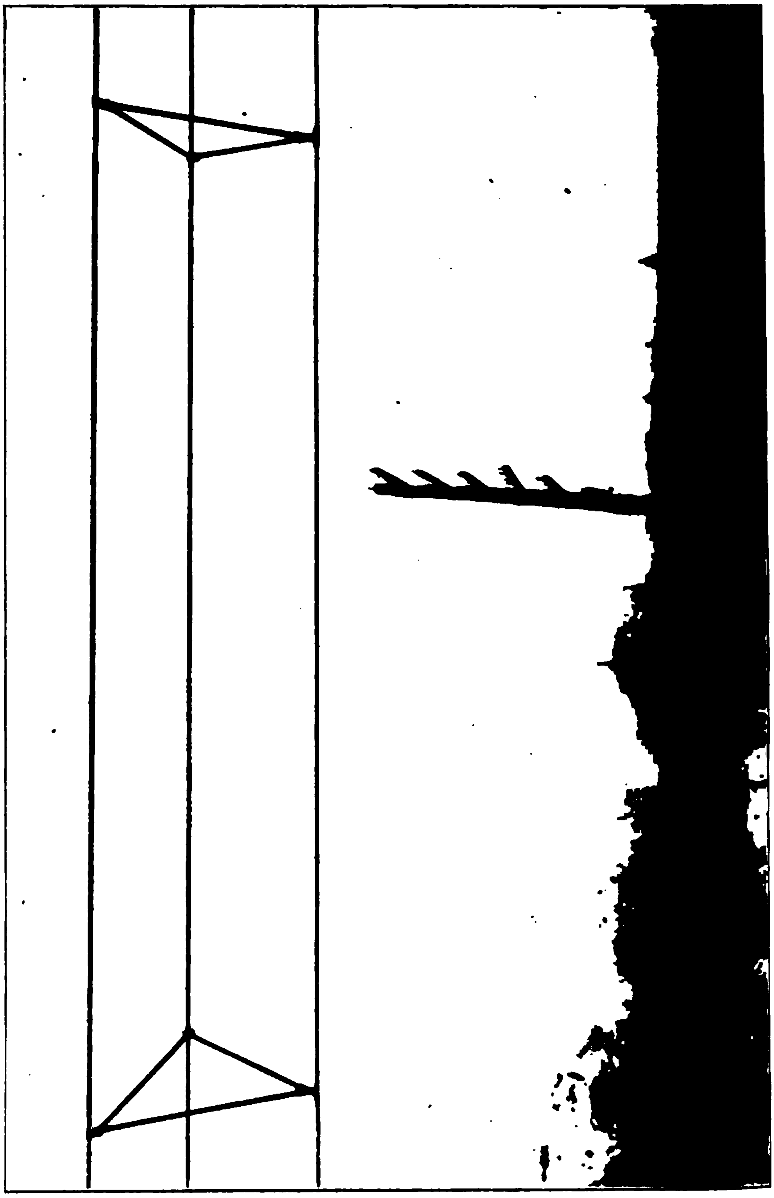


FIG. 1.—Original copper trolley suspension

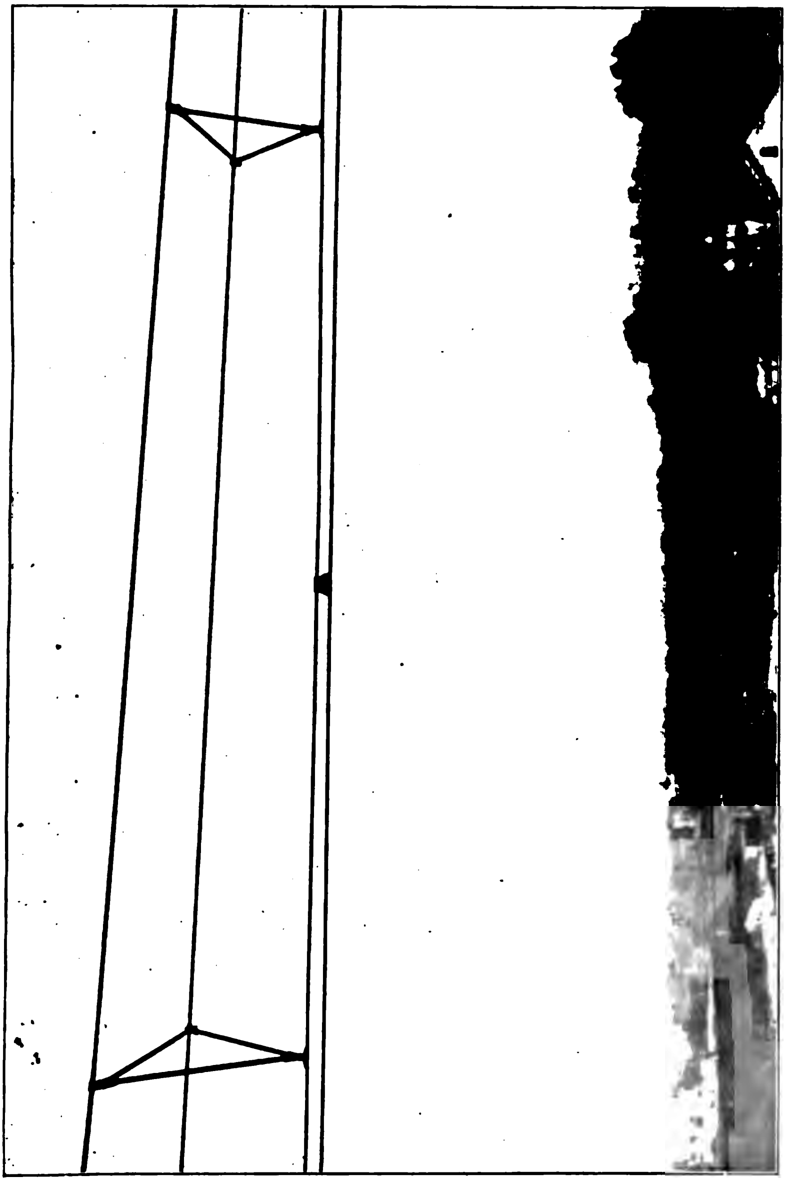


FIG. 2—Auxiliary steel contact wire suspended from original copper trolley wire

with a direct transmission straight from the power house bus-bars to the trolleys directly opposite the power house, the current was fed into the line over feeders connected to it at Portchester and Stamford. By the introduction of this ohmic resistance, amounting to not more than 2 per cent. normal drop on the system, we were immediately released from the disastrous effect of short-circuits on our circuit-breaker apparatus. Instead of losing as many as a half dozen circuit-breakers in a day, not that many were reported out of commission in a month, and, of course, they were not damaged to the same extent as the others, nor did they cause any serious delays.

For the feeder resistance, above described, there have since been substituted impedance coils, installed in the leads of the generators. These coils act as shock absorbers, protecting the generators. Later, it is to be expected that there will be installed a circuit-breaker across the terminals of these impedance coils, which, for normal operation, will shunt the current through them, the breakers opening under stress of abnormal flow of current and automatically closing when normal conditions are restored.

Trolley wire. In the month of May, 1908, it became evident to us that within at least one month from that date, if some change were not effected in the contact wire, the New Haven electric service would cease. While this truth was so plain, it may be best described by the fact that daily reports were showing that the copper trolley wire was breaking at various points; and where it was not broken it had become so badly kinked at the hanger points that it was impossible to operate electric locomotives upon the line without serious arcing. This resulted in violent surging on the locomotive transformers, and, at times, on account of the extremely poor contact of overhead shoes on the line, in reducing the voltage to such a low value as to prevent a supply of power sufficient to enable the locomotive to perform its schedule. An examination of the hard-drawn copper trolley wire throughout its length proved that even after only a few months' operation upon it, its cross-section had been so materially reduced as to point to its short life with a continuance of operation upon it. Especially was this true in the vicinity of the many low highway bridges where the trolley wire approaches the bridge on a two per cent gradient. This fault and dilemma were indeed serious. The cause of the difficulty was perfectly apparent; namely, the hard spots in the line which existed at the hanger points.

Many suggestions were offered. None of them, however, offered the speedy installation that was paramount. Mr. McHenry, vice-president of the New York, New Haven & Hartford Railroad Company, made the suggestion that an auxiliary wire be suspended from the present copper wire (Fig. 2) by clips at its midpoint between the hangers, and followed it with the suggestion that this auxiliary wire be made of steel, of the same cross-section as the 0000 grooved hard-drawn copper wire above it. It took two weeks for the manufacturer to draw two miles of this wire. It was installed immediately upon its receipt on the main line between Port Chester and Harrison. On the night of its completed erection a special seven-car train with two locomotives was operated upon it for several hours. Previously to the installation of the steel wire there had been installed a section of hard alloyed wire suspended in a manner similar to that of the steel. The electric train was operated upon them both, officials from both the railroad company and the contracting company being on hand to note their comparative merits. It was the consensus of opinion that there was less sparking on the hard wire, and the general tendency was towards adopting that rather than the steel. Though admitting that the operation was better, the steel seemed to be of an entirely satisfactory commercial nature, and all present finally concurred in this conclusion. It is undeniably true that hard alloyed wire would, from a purely operative point of view, be the better of the two, and yet the commercial aspect, which would naturally include its cost, had to be considered, particularly in reference to so large an immediate order as one involving 100 miles of single-phase electric trackage. Again it is important to note that the steel, besides having the advantage of being a cheaper, harder, and stiffer wire, also possesses a lower coefficient of expansion and higher elastic limit, especially valuable characteristics for the service desired. We found that we would be considerably delayed in the delivery of the steel, but an immediate order of 20 miles, followed by another for 40, was placed, and as much hard alloyed wire put up as could be obtained between the date of the conclusion to use the auxiliary wire and the final arrival of the first shipment of steel. The auxiliary wire construction on the main line, as described, prevails throughout the whole system, except at the approaches of and under a few very low highway bridges, where the contact system consists of two wires strung in the same horizontal plane.

The New Haven trains have been operating now on the auxiliary wire for several months, and absolutely no kinking has been noted at the hanger points, with the attendant result of a smooth and almost sparkless overhead contact.

Locomotives. There were originally purchased 35 locomotives, which was considered an adequate number to take care of the New Haven passenger service. These locomotives, considered *per se*, were rated on a half-unit basis. That is to say, the half unit was designed to handle about 75 per cent. of our trains, the remaining 25 per cent. to be handled by two units. Only a short experience in commercial operation revealed two important facts: the first one of a very encouraging nature; the second, decidedly otherwise. The first was the proof that the two main parts of the locomotive; namely, the transformer and motors, had sufficient capacity to more than handle the manufacturer's guarantees. The second was the discovery that many of the auxiliary electrical and mechanical parts of the locomotive equipment were not of equivalent capacity. The strength of the chain being measured always by its weakest link, it was immediately seen that the locomotives would be able to handle trailing loads in excess of their guarantees if the auxiliary parts were made of sufficient capacity to furnish the necessary current for the overload conditions. It was simultaneously apparent that more locomotives would be required to provide for an increase of train service and the reduction of time schedule, and an order was promptly placed for six additional ones; before accepting their design, however, a careful survey was made of all the component parts of the locomotives at hand, in order to determine the changes necessary to be incorporated in the six new engines. To accomplish this it was found necessary to make a number of electrical and mechanical changes, the nature of which is apparent in the following tabulation:

MECHANICAL	ELECTRICAL
Air Reservoir System. Revised.	*Ammeters.* Revised.
	Battery. Charging relay installed.
	Blowers. Commutator slotted
	Waste rear bearing.
Boilers. Installed.	Armature laminations ground.
	Bearing raised above center.
	Motor winding revised.

MECHANICAL

Stack hood removed.

Bell Rope. Installed.

Blowers. New design installed.

Guide on present damper rod.

Extra guard on shutter.

Bolster. Reinforced on truck.

Brakes. New release spring.

Cylinder leads lengthened.

Check nut on adjuster.

Compressor. Drip pans installed.

Exhaust pipe installed.

Air-cooled cylinder head.

Check-valve. In whistle pipe

Gauge. On oil tank.

ELECTRICAL

Controller. Bushing riveted.
Shoe and trolley switch installed.

Circuit-breaker. Fused.
Tank bolt secured.
New type installed.

Compressor. Commutator slotted.
Armature laminations ground.
New governor installed.
Bearing raised above center.

Cut-out Boxes. New drums.

Fuse. In alternating-current heater circuit.
New auxiliary fuse box (wooden.)
Auxiliary motors separately fused.

Grounding switch. Installed.
Control revised.
Shutter in cover.

Heater switch. Shield installed.

Heater bus-bar line. Reinsulated.
Couplers repaired.

Headlight. Adjusted properly.
Porcelain switch installed.

Motors. Holders reinsulated.
New cross-connecting leads.
Housing asbestos lined.
Linen tape on commutator end.
Side commutator covers perforated.
Perforated S. I. commutator covers.
Field clamp bolt secured.

Motor-generator. New switch.

Preventive coils. Revised.
Supporting frame revised.

Direct-current relay. Connection to "—S" switch.
Washer installed.

Resistance. Insulated.
Capacity increased.
Connected ahead of motor.

Switch group. Revised shunts.
Revised studs.
Revised arc shields.
Copper reverse switches.
One-turn blowout coils.
Air ports enlarged.

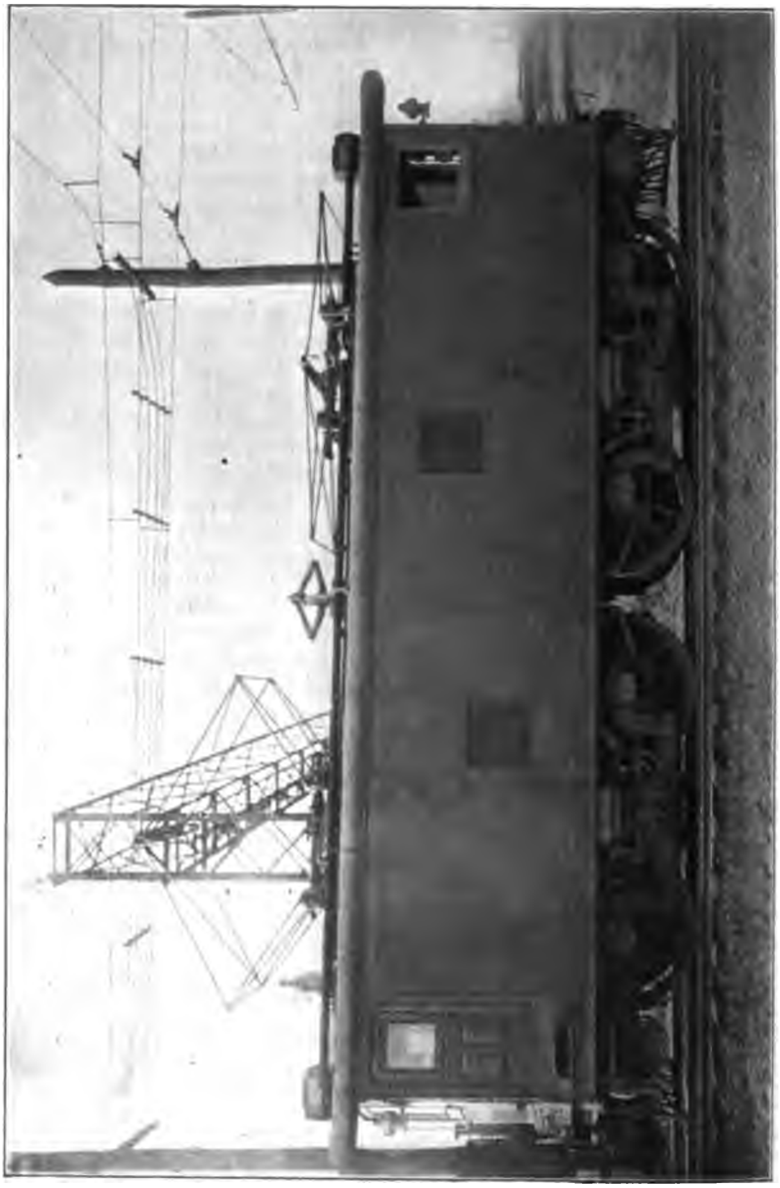


Fig. 3—Locomotive without pony trucks

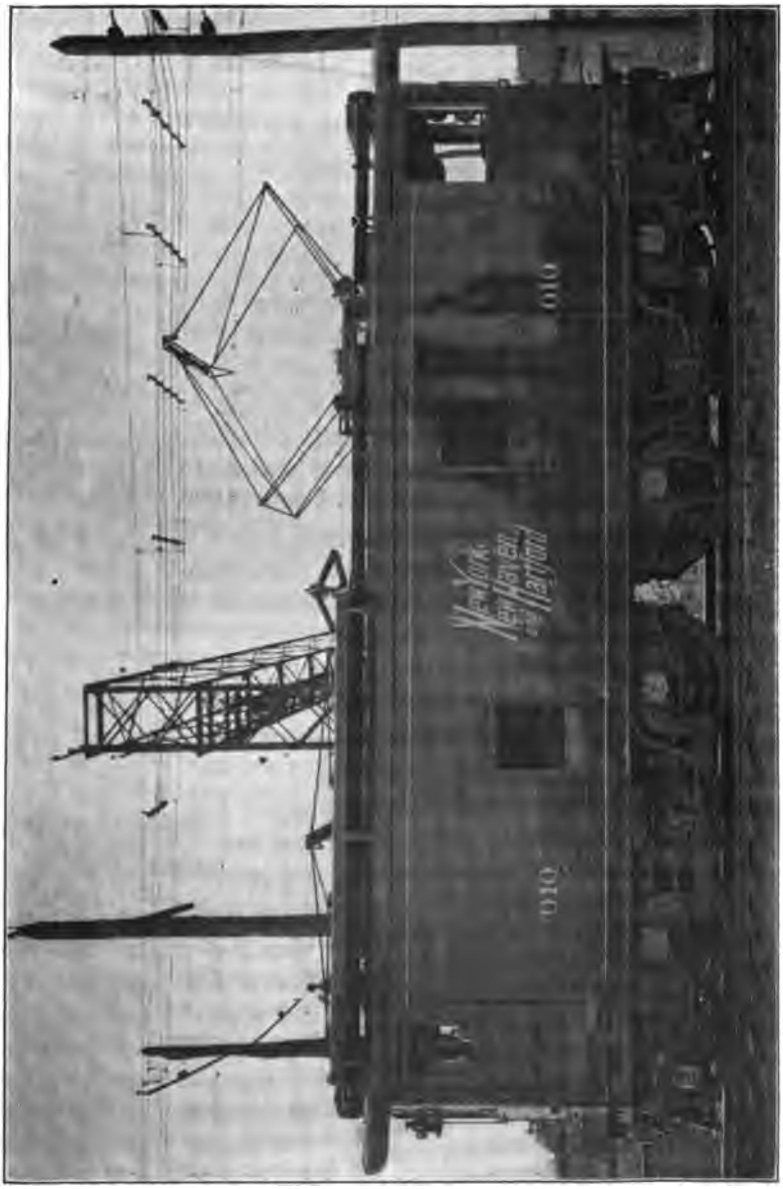


Fig. 4—Locomotive with pony trucks

Mechanical

On steam line.

Journal-box. Revised gibbs.

Quill-plate. Washers installed.

Running-board. On roof.

Semi-elliptic springs. 14 leaf.

Safety-valve. On control resistance

Safety-chain. Made heavier.

Torque rods. Heavy installed.

Uncoupling lever. Revised.

Pony wheels. Installed.

Valve. On oil line to boiler.

Electrical

Alternating current frames insulated.
Cylinder aligning set-screws.
Reinforcing wooden strip.
Interlocked reverse switches.
Fitting alternating current switches.
Group covers perforated.
New B. O. coil pole piece alternating current.

Switch. Block on direct current case.
Main switch, connection modified.
Canopy switch handles insulated.

Shunt remounted. Direct current ammeter.

Trolleys. Control revised (alternating current).
Side bearing springs (alternating current).
Insulate back cross-rod (alternating current).
Shield for high tension lead (alternating current).
Spring shortened (direct current.)
Insulator filled and painted.

Temperature Indicators.

Third-rail shoes. M. I. bracket.
Pin replaced by bolt.
Jumper installed.
Shoe levers cut off.

Fuse boxes—Refitted.

Wattmeter. (Auxiliary) installed.

The most important electrical changes made were in the switch-groups and brush-holders of the motors. The former lacked carrying capacity and the latter sufficient insulation. To these shortcomings were due the greatest number of our first train delays. The most important mechanical changes necessary were the reinforcement of the truck bolsters and installation of pony wheels (see Figs. 4 and 3, respectively, for locomotives with and without pony wheels). The especial reference to these electrical and mechanical changes must not be construed as diminishing in any way the force of necessity of the others, as they were all considered absolutely necessary in order to preserve a low cost of electrical and mechanical maintenance.

With the exception of the installation of pony trucks, the six new electric locomotives arrived within five months of the date of their order. In their design were included all of the changes above specified. To be noted here is the marked value of the spring type of armature and field suspension begun with the New Haven locomotive motors, thus making flexible the entire motor suspension. Indications already predict that this arrangement in combination with the pony trucks will reduce materially the track and locomotive maintenance and repairs. To-day the reconstruction, as above described, has been effected on over 90 per cent. of our locomotives.

This last and serious fault with which we had to contend completes the major difficulties that were constantly threatening the regularity of electric service.

It seems to be the time and place, here, to draw attention to a point in design concerning the New Haven locomotives that has been so persistently misrepresented by those who seem to have been ignorant of the facts. The specifications upon which the locomotive units were purchased, as stated hereafter, were that each unit would handle a normal trailing load of 200 tons. The writer, by careful measurement of the weights of all the trains (trailing loads) in the New Haven service, found that they averaged 212 tons. It seemed good engineering if 75 per cent. of the service could be handled by locomotives rated upon a basis of 200 tons trailing load, that that would be the correct locomotive unit size; using two units for the remaining 25 per cent. of the trains. To-day, three years after this decision, we find that 73 per cent. of our trains can be handled with single units, 27 per cent. requiring two units. The percentage is slightly different from the original, as the service is slightly heavier.

Coincident with the use of the first double-unit locomotive trains there was started a rumor that the New Haven locomotives did not have the capacity for which they were designed. To dispel this idea forever, I judge the best argument is to refer to Table X, wherein are stated the trains whose weights were in excess of the trailing loads which the locomotives were designed to carry.

I trust that the above clears up any erroneous ideas con-

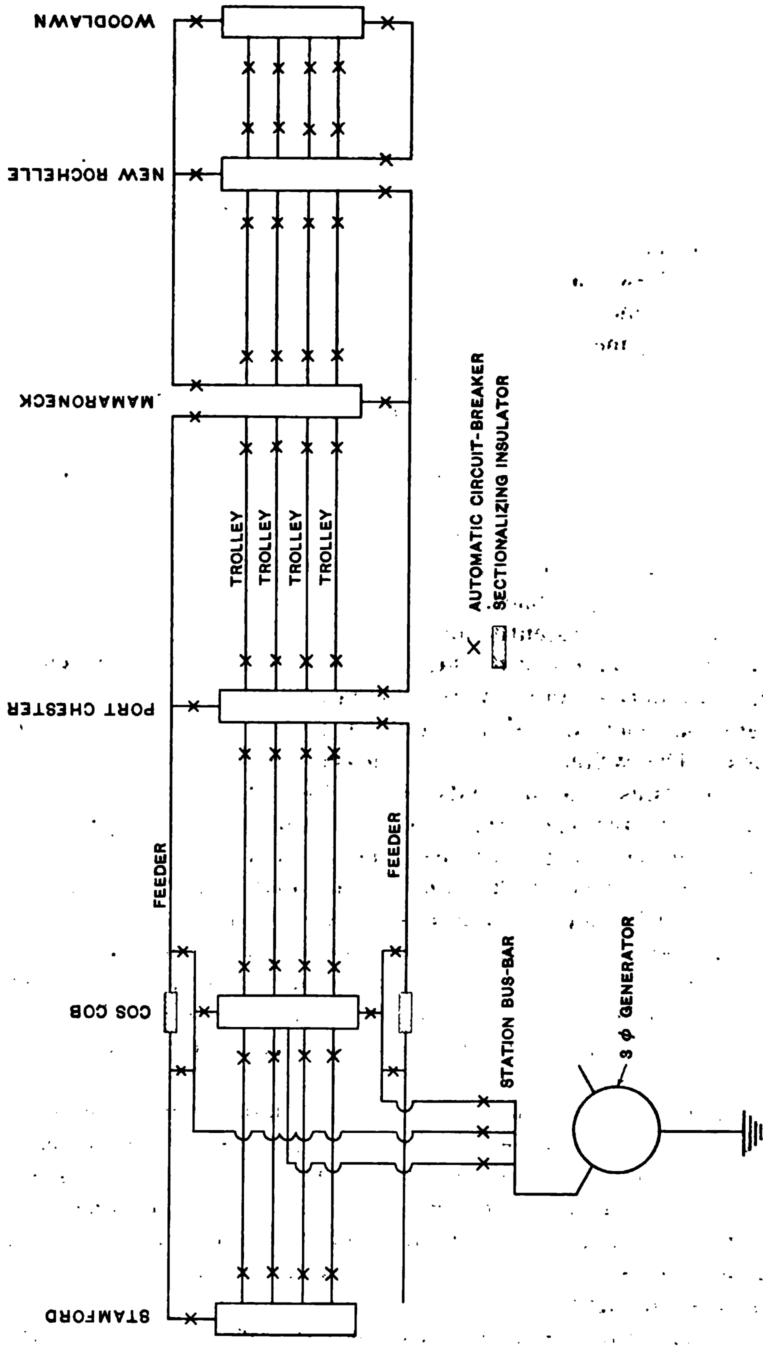


FIG. 5—Independent track circuits

cerning the capacity of the New Haven locomotives, and will justify the future double-unit trains entering and leaving the Grand Central Station.

Recapitulating, I believe my brother engineers will agree that with the coincident difficulties described in our power house, line, circuit-breakers, trolley wire, and locomotives, the. work of instituting electric service was not exactly laid over a bed of roses. These are the serious faults the system possessed and the description of our method of elimination of them that I am glad to be able now to place before you. We cannot blame the public for making the complaints they have against the New York, New Haven & Hartford Railroad Company's electric service, for to the traveling public there is nothing so exasperating as a train delay. It is fitting here also to say how appreciative the engineers of the New Haven road feel for the calm and generous suspension of judgment on the part of those engineers who have reserved opinion until it could be based on facts, which time only could so demonstrate as to permit presentation. There are many other little faults and difficulties here and there throughout the system, but the five described, though not fundamental in their effect, since they have been removed, were none the less the most serious.

The Minor Faults

Generator burn-outs. Previously to the rearrangement of the feeders or the installation of the choke-coils in the power station, the short-circuits that were experienced on the line naturally had a deleterious effect on the generators. The prodigious rush of current naturally produced severe strains in the armature windings, strains whose mechanical intensity was made manifest by the movements of the coils. Therefore, during this period of violent short-circuits, frequently the generator coils were grounded and burned out. Since the rearrangement of feeders or the installation of choke-coils, the violent short-circuits have disappeared and with them the damaging effects on the generators. It has been found advisable, from time to time, to connect different generator terminals to the line. By this arrangement the three phases of the generators are in turn worked equally.

Distribution. Although the arrangement of the line in its present feed and trolley connections (Fig. 5) effects the highest economy in loss of power by transmission, it is questionable to

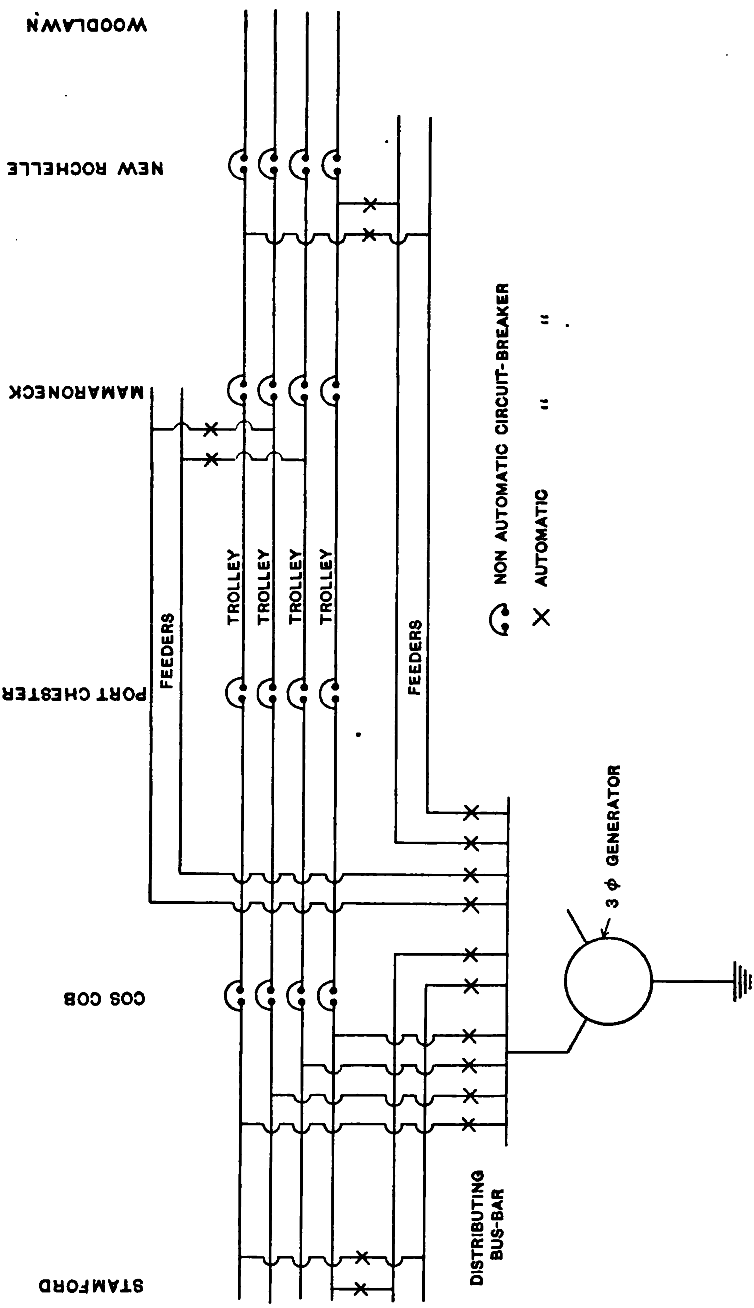


FIG. 6—Multiple track circuits

my mind whether this economy is worth some of the disadvantages which accrue from this arrangement. Until we had experienced the heavy current surges, due to short-circuits, it seemed reasonable that, should a short-circuit occur on the line, the two circuit-breakers which immediately fed that short-circuit would be the ones to respond. We were quite correct in this conclusion, but did not go far enough, as other circuit-breakers seemed to feel it their duty to relieve the situation and their location with respect to the correctly involved breakers was quite varied. Of course the ideal condition would be to have only the two breakers adjacent to the short-circuit open, as by this arrangement the voltage would remain on all trolley wires except the one in which the ground is involved. This operation can be effected, and at the time of this writing an effort is being made to obtain these results in practice.

A method of line transmission (Fig. 2) less efficient than that described in the previous paragraph (using the same amount of copper for each case) is one in which each track has its own individual voltage supplied to it and is kept separate from the other trolleys.

Contact wire. While the auxiliary wire, as previously described, was the panacea of our difficulties, and has given an excellent account of itself since having first been placed in operation, it is fair to believe that an even more sparkless operation can be obtained from an auxiliary wire which, instead of being attached by a rigid clip to the overhead copper conductor, is simply suspended from it by a loop, thus permitting the contact wire to rise slightly as the upper pressure of the pantagraph shoe travels along its surface. The contact wire should be anchored by turn-buckles to permit slacking and straining of the wire in winter and summer seasons, respectively.

Hard spots in line. Irregularities in the contact wire may be due to two causes. First, those that exist in the wire as it is manufactured; secondly, those formed on account of hard spots. The former can be taken out; if not, they are usually ironed out in the course of the operation of the shoe on the trolley. The three principal sources of hard spots are hangers, section breaks, and deflectors.

1. It has been shown in the foregoing how the hanger hard spots have been eliminated.

2. The original main-line section break consisted of a rectangular impregnated wood frame suspended under the anchor

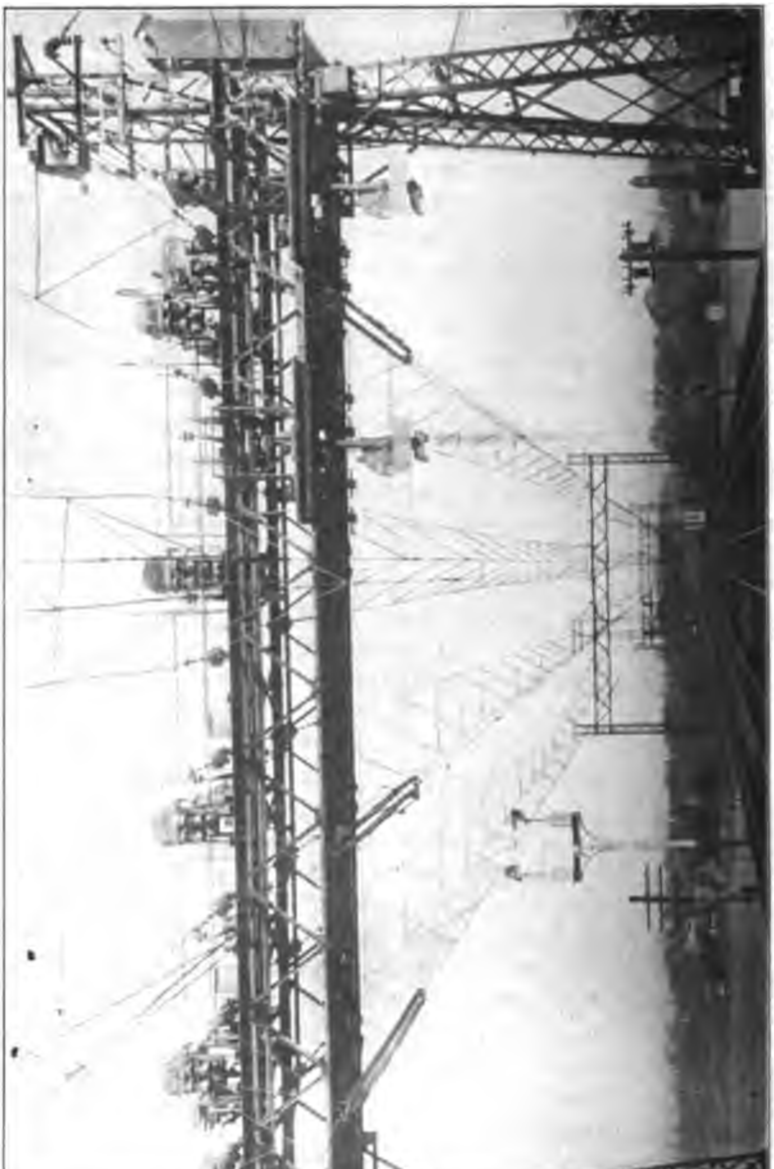


FIG. 7—End-on view of one new and three old section breaks

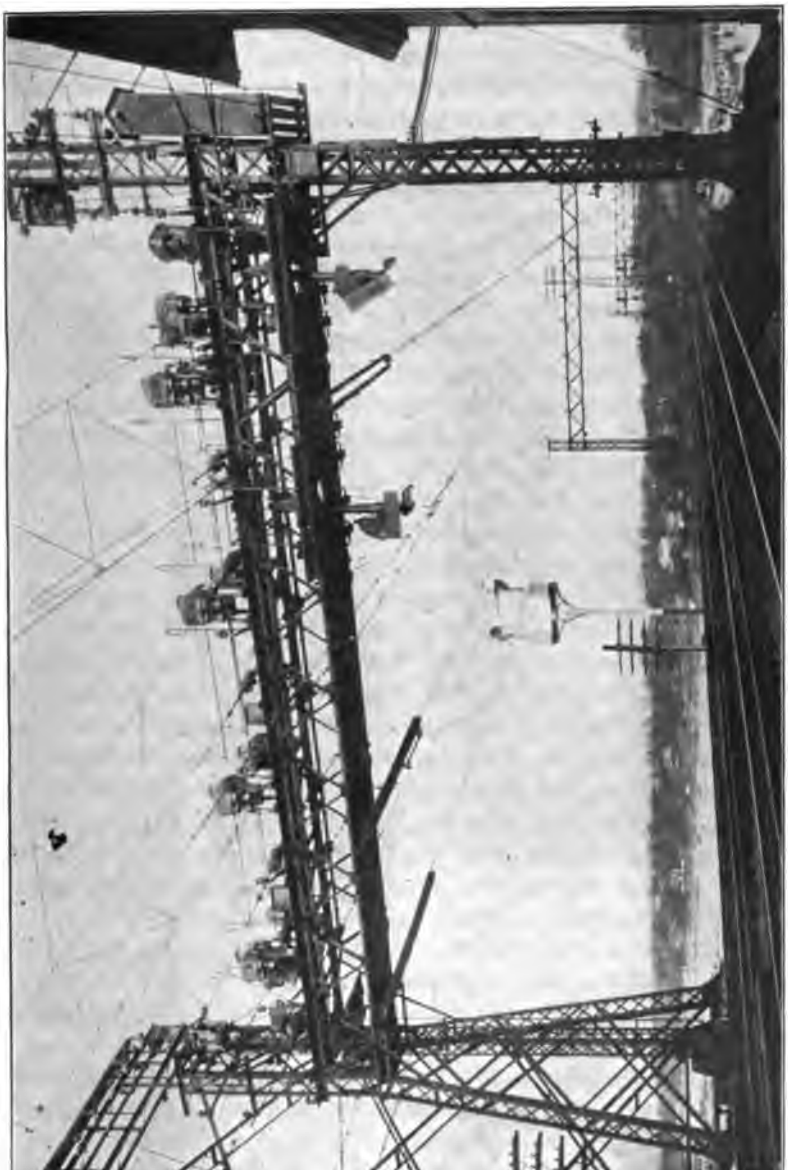


Fig. 8—Three-quarters view of one new and three old section breaks

bridges. The trolley wires were carried on opposite sides of this rectangle lapping, of course, a sufficient distance in the center to permit a short period of simultaneous contact of the shoe with both wires, thus preserving continuity of current to the locomotive in passing from one section to another. It was found that these rectangular section breaks were prone to get out of shape, introducing a very rough spot in the line, its effect being, on the passage of the shoe, to draw long arcs which frequently extended themselves to ground. Our new section break, Figs. 7 and 8, containing all the flexibility obtained by the auxiliary wire, has been constructed by simply staggering the messenger insulators of each trolley wire 16 in., thus permitting the anchor bridge trolley wires of the same track to pass at this distance. Each trolley wire is dead-ended to a wood impregnated stick insulator, supported by the messenger of its opposite section. By this arrangement practically all inertia is taken out of the section break and a sparkless passage between sections is obtained.

3. The auxiliary wire construction has been, of course, applied to the deflectors, and this has greatly reduced the tendency to spark at these points. In order to prevent the pantagraph shoe from engaging the wrong trolley wire at turn-outs on diverging tracks, the mesh construction of the overhead frog has up to this time been the only solution, but this method seems unduly cumbersome and even a greater improvement is anticipated than that secured by the application of the auxiliary wire.

Signal wires. The catenary bridge struts carrying the cross-arms for the by-pass or feeder wires, likewise carry the cross-arms upon which are installed the signal wires. The voltage placed on these signal wires is entirely distinct from the propulsion voltage, it being 2200 volts and 60 cycles. The proximity of the wires of these two systems has been the cause of short-circuits between them, resulting in double failures. It would seem that as the signal system should be so completely separated from the propulsion system, it should be installed on an entirely separate pole line. If, however, right-of-way conditions make it absolutely necessary that the supporting structures of the catenary system should support also the signal wires, then the two circuits should be made as distinctive as possible, the propulsion system having its wires on the cross-arms on the railroad side of the struts, the signal wires being installed on the opposite.

Locomotive current collectors. An efficient pantagraph shoe has proved itself a very difficult problem. The present cost is about .06 of a cent a locomotive-mile. We have made various experiments with aluminum, phono, copper, and steel rigid and spring-supported pantagraph shoes. While this feature does not present a serious aspect, it is none the less a most interesting study. Shoe life is also seriously affected by the amount of soot deposited by the locomotives upon the overhead wire. While we have obtained mileages varying between 600 and 1500 per shoe with various types used, other roads of lesser speed and not subject to the effect of locomotive stack discharges, have obtained as high as 25,000 shoe-miles.

The Log of Operation

Electric passenger service. So great was the demand for electric service that in July, 1907, long before we were ready, as the records of our operation have proved, the first commercial service was begun between New Rochelle and the Grand Central Station. With all speed possible the electrification was pushed eastward and local service was established from Port Chester in August, 1907. Local service from Stamford followed in October, and finally, on July 1, 1908, all through and local passenger trains were under electric schedule between Stamford and the Grand Central Station. It is, therefore, beginning from this date of July 1, 1908, that I have compiled data of our operation as taken from our records. We are not proud of it, but we believe that you now know of some of the obstacles that stood in the way of its immediate betterment at that time.

Comparison between steam and electric service. It had been the hope of the writer to be able, even at this early date, to make an absolute comparison between electric service and the steam service which it has replaced between Stamford and New York, including all passenger trains. On account of the speed restrictions that have been introduced on the electric service until all locomotives have been reconstructed with the new pony truck equipment, it is impossible to compare train-minute delays, for the reason that steam locomotives are permitted to make up time while to electric locomotives this privilege is denied. Later, when this restriction is removed, and especially after a period of time more extensive than that offered by the present comparatively short period of operation, these data can be presented. Particularly worthy of notation here in com-

TABLE I.

TRAIN-MINUTES DELAY DURING JULY, 1908.

Day	Power house	Line	Locomotive	Total	Cause of delay
1	..	..	40	40	Third-rail shoe. Air hose, third-rail. Train-line hose. One motor of locomotive grounded. Third-rail shoes.
2	..	..	34	34	Third-rail shoes.
3	..	..	20	20	Change-over switch direct-current fuses
4	..	..	3	3	Pantagraph.
5	..	..	..	..	No delays.
6	..	15	..	15	Circuit-breaker.
7	..	..	..	..	No delays.
8	..	57	3	60	No power in direct-current rail.
9	..	..	15	15	Direct-current switch grounded.
10	..	13	..	13	Oil circuit-breaker. No power in direct-current rail.
11	..	..	..	..	No delays.
12	..	..	16	16	Control current from storage-batteries low.
13	..	..	..	..	No delays.
14	..	..	..	..	Over 300 train-minutes.
15	..	4	72	72	Circuit-breaker. Preventive-coil ground. Pantagraph. Third-rail switch. Motorman failure.
16	..	..	..	..	Over 300 train-minutes.
17	..	12	29	41	Insulator on foreign line broke. Change-over switch alternating-current coasting over gap at new bridge.
18	..	..	7	7	Direct current fuses. Stopping on gap at bridge.
19	..	34	47	81	Line circuit-breaker. Pantagraph.
20	..	..	..	..	Over 300 train-minutes.
21	..	43	15	58	Line circuit-breaker. Transformer grounded on locomotive.
22	..	..	44	44	Alternating-current controller. Pantagraph.
23	..	..	41	41	Pantagraph.
24	..	..	3	3	Alternating-current controller.
25	..	..	124	124	Pantagraph. Direct current fuse. Alternating-current controller. Change-over switch. Motor grounded.
26	..	..	..	..	Over 300 train-minutes.
27	..	..	..	..	" " " "
28	..	..	30	30	Pantagraph. Grounded blower.
29	..	8	19	27	Circuit-breaker (line). Transformer-lead ground on locomotive.
30	..	16	..	16	Circuit-breakers.
31	..	..	21	21	Third-rail shoes. Alternating-current switch. Pantagraph.

TABLE II.

TRAIN-MINUTES DELAY DURING AUGUST, 1908.

Day	Power house	Line	Locomotive	Total	Cause of delay
1	..	5	3	8	Circuit-breakers (line). Third-rail shoes.
2	..	..	7	7	Heavy train.
3	..	..	12	12	Third rail shoes. Air hose. Pantagraph.
4	..	..	..	..	No delays.
5	..	12	10	22	Pantagraph. Motor ground. Third-rail fuses.
6	..	..	..	..	Over 300 train-minutes.
7	..	..	..	..	No delays.
8	..	..	..	..	Over 300 train-minutes.
9	..	43	..	43	Circuit-breakers.
10	..	220	28	248	Circuit-breakers (line). Locomotive circuit-breakers. Alternating-current switches. Resistance lead.
11	..	57	26	83	Circuit-breakers (line). Change-over switch. Pantagraph.
12	..	5	2	7	Line circuit-breakers open. Hot tire.
13	..	..	..	..	No delays.
14	..	19	..	19	Insulator.
15	..	..	6	6	Control (Alternating-current). Direct-current circuit-breaker.
16	..	64	..	64	Insulator.
17	..	3	43	46	Circuit-breaker open (line). Ground. Direct-current shoes. Pantagraph. Preventive coil ground. Direct-current relay.
18	..	1	13	14	Insulator. Line. Air hose. Third-rail shoe.
19	..	..	..	..	No delays.
20	..	..	..	..	" "
21	..	..	8	8	Hot box. Alternating-current control.
22	..	1	13	14	Insulator. Line. 18 in. water direct-current zone.
23	..	5	..	5	Insulator. Line.
24	..	5	15	20	Ground on signal wires. Pantagraph.
25	..	32	41	73	Open circuit-breaker (line). Insulator (line). Hot box. Grounded motor. Third rail. Third-rail shoe.
26	..	..	73	73	Grounds developed by flood direct-current zone
27	..	..	64	64	" " " " " " "
28	69	..	1	70	Turbine failure. Third-rail shoe.
29	63	..	22	85	Turbine failure. Engine dispatcher. Grounded motor.
30	..	..	6	6	Heavy train. Low-voltage direct-current.
31	..	..	14	14	Grounded compressor. Low-voltage direct-current.

TABLE III.

TRAIN-MINUTES DELAY DURING SEPTEMBER, 1908.

Day	Power house	Line	Locomotive	Total	Cause of delay
1	..	..	4	4	Compressor. Pantagraph.
2	..	..	9	9	Air hose.
3	..	15	18	33	Line ground. Alternating-current switch.
4	..	..	26	26	Alternating-current control. Low-voltage direct current Alternating-current control. Third-rail shoes—Ground direct-current switch.
5	..	4	..	4	Open circuit-breaker.
6	..	..	3	3	Low-voltage direct-current.
7	..	..	10	10	Third-rail fuses.
8	..	..	..	..	Over 300 train-minutes.
9	..	3	..	3	Line ground.
10	..	..	..	..	No delays.
11	..	..	7	7	Direct-current switch ground.
12	..	..	19	19	Motor ground.
13	..	2	5	7	Line ground. Low-voltage direct-current. Pantagraph.
14	..	..	8	8	Pantagraphs. Alternating-current relay.
15	..	..	8	8	Preventive coil. Third-rail gap.
16	..	..	4	4	Preventive coil.
17	..	..	28	28	Locomotive wheels off track (pony truck).
18	..	..	..	..	No delays.
19	..	..	9	9	Preventive coil. Signals.
20	..	..	1	1	Third rail shoes.
21	..	..	2	2	Third rail shoes.
22	..	..	10	10	Hot tire.
23	..	..	32	32	Ground on reversing switch.
24	..	..	3	3	Hot box. Direct-current shoes.
25	..	20	..	20	Circuit-breaker (line).
26	..	..	..	..	Over 300 train-minutes.
27	..	16	..	16	Grounded insulator.
28	..	4	8	12	Signal wires. Pantagraph—Third-rail shoe.
29	..	3	10	13	Open circuit-breakers. Locomotive jumper grounded.
30	..	4	..	4	Line ground.

TABLE IV.

TRAIN-MINUTES DELAY DURING OCTOBER, 1908.

Day	Power house	Line	Locomotive	Total	Cause of delay
1	..	..	18	18	Nipple broken on train line pipe.
2	..	..	..	..	No delays.
3	..	..	3	3	Motor ground.
4	..	14	1	15	Splicing ear. Direct current shoes.
5	..	..	7	7	Pantagraph. Alternating-current switch.
6	..	..	4	4	Change-over
7	..	..	9	9	Alternating current switch. Air hose.
8	..	..	10	10	Direct-current fuses. Third-rail shoes.
9	..	22	4	26	Circuit-breaker open. Direct-current switch. Direct-current shoes (third rail).
10	..	61	..	61	Open circuit-breakers.
11	..	..	..	..	No delays.
12	..	..	9	9	Pantagraph. Motor ground. Brakes sticking.
13	..	..	4	4	Direct-current relay. Pantagraph. Low direct-current voltage.
14	..	3	..	3	Open circuit-breaker.
15	..	15	..	15	Line ground.
16	..	..	..	..	Over 300 train-minutes.
17	..	..	9	9	Pantagraph. Control, direct-current.
18	..	..	..	..	No delays.
19	..	..	10	10	Direct-current circuit-breaker. Direct-current control.
20	..	4	..	4	Insulator.
21	..	3	..	3	Open circuit-breaker.
22	..	..	5	5	Grid resistance.
23	..	..	10	10	Air hose.
24	..	..	3	3	Third-rail shoes.
25	..	..	..	..	No delays.
26	..	70	19	89	Grounded insulator. Hot box.
27	..	54	19	73	Hot box. Insulator breakdown. Line ground.
28	..	..	63	63	Direct-current resistance ground. Hot box. Pneumatic control.
29	..	31	82	113	Man failure. Grounded feeder. Direct-current circuit-breaker. Motor ground. Direct-current switch ground. Alternating-current switch. Third rail gap.
30	..	..	42	42	Direct-current switch ground. Man failure.
31	..	..	12	12	Direct-current relay. Motor flash-over. Pantagraph.

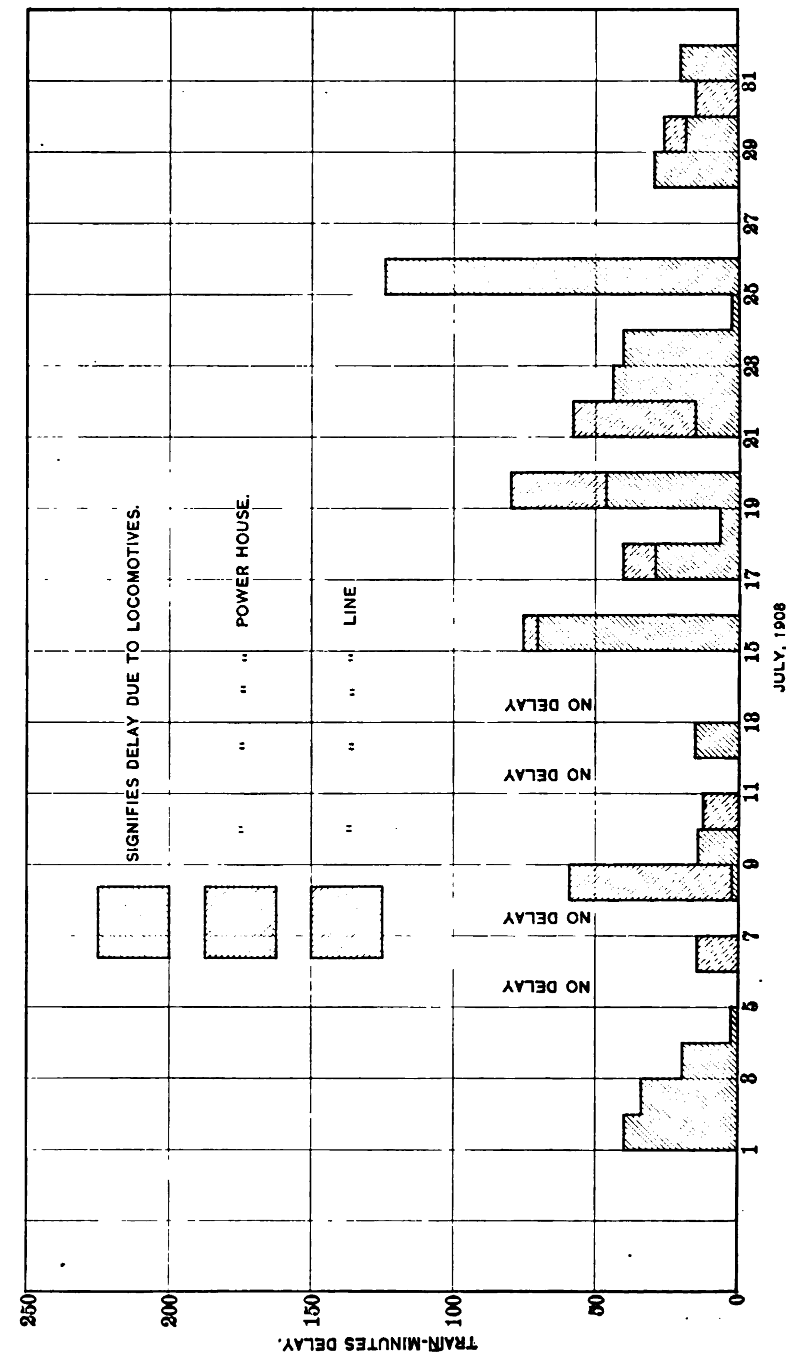


Fig. 9—Individual and collective train-minute delays in electric zone due to failures in power house, line, and locomotives

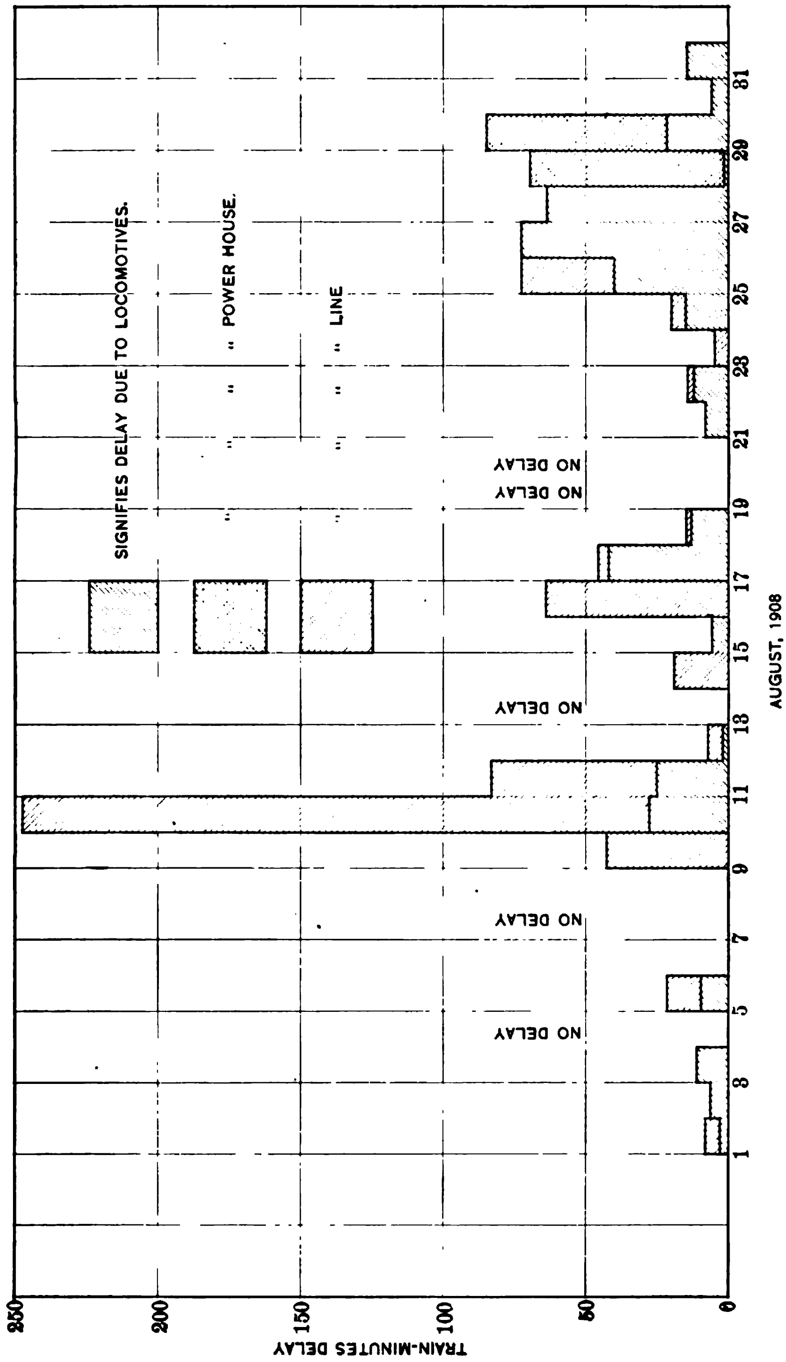


FIG. 10—Individual and collective train-minute delays in electric zone due to failures in power house, line, and locomotives

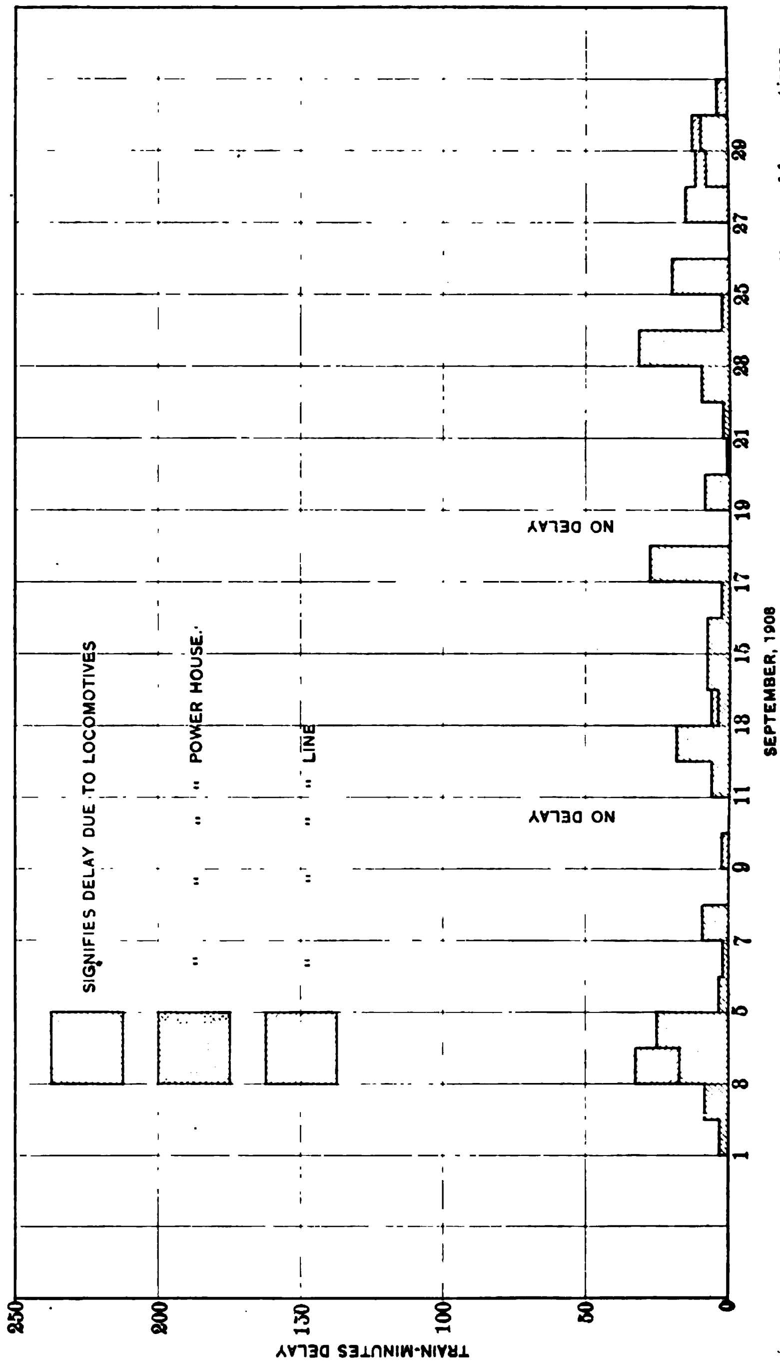


FIG. 11—Individual and collective train-minute delays in electric zone due to failures in power house, line, and locomotives

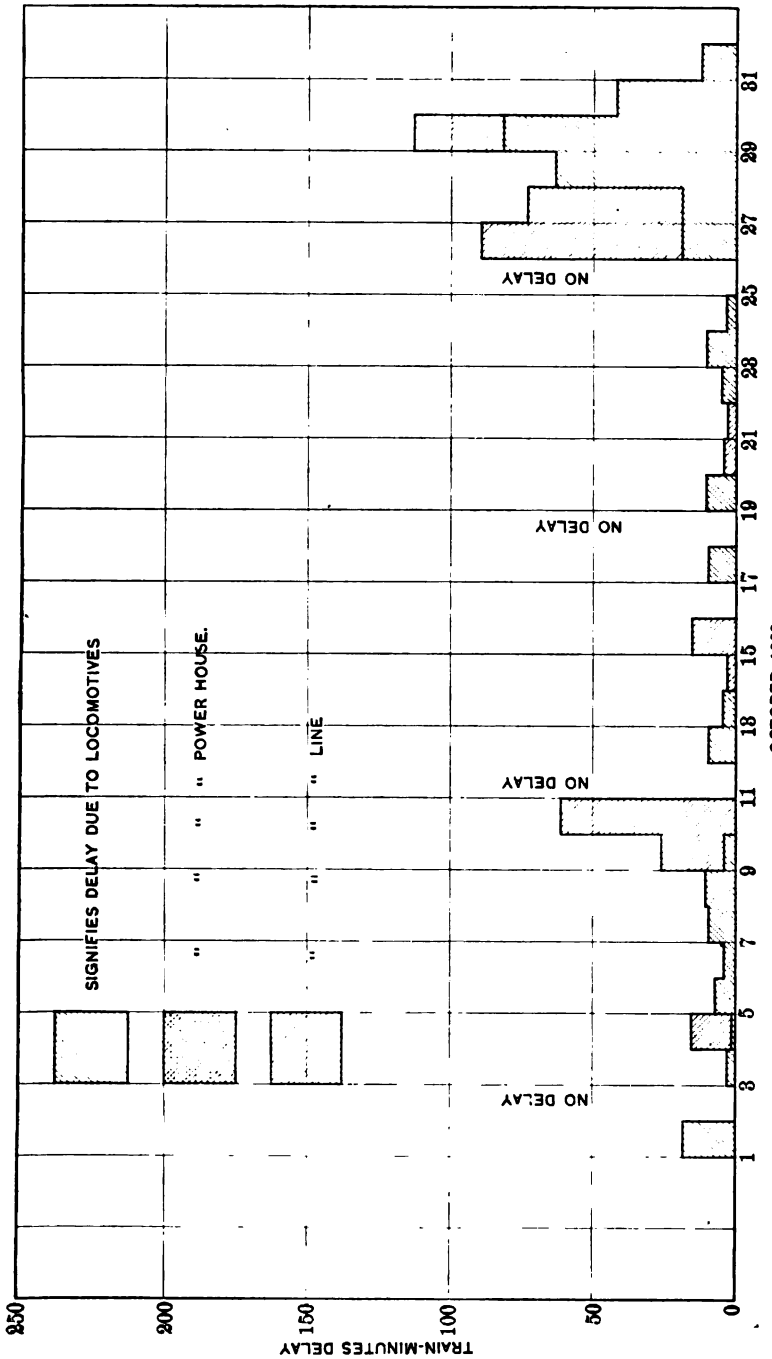


FIG. 12—Individual and collective train-minute delays in electric zone due to failures in power house, line, and locomotives

paring steam and electric passenger service, is the fact that while the operation of the power house does not differ from that of ordinary practice, there is a marked individuality (which initiative always brings), in connection with the operation of the line and locomotives. While every credit is due the present organization charged with the responsibility of line and locomotive operation, it cannot be forgotten that these two main links of the system are essentially new in their adaptation to the heavy conditions presented by the New York, New Haven & Hartford train service. Operation of this character is so far beyond the scope of previous experience that it was absolutely impossible to foresee and prevent all contingencies. The methods for the prevention of failure and delay were necessarily based upon the results of actual, practical operation.

Individual and collective train-minute delays in electric zone, due to failures in power house, line, and locomotive. Tables I, II, III, and IV are self-explanatory, and the charts shown in Figs. 9, 10, 11, and 12 that immediately follow represent graphically the data included in the tables.

The segregation of these failures based upon train-minute delays offers a quick and interesting comparison. It is to be noted that the last two months indicate a great improvement over the two preceding ones. From the chart have been taken the train-minute delays that have amounted to over 300 minutes per diem, which are treated separately in the following paragraph. My reason for doing this is to make the data inclusive of ordinary operating days, in order to get the monthly comparison of train-minute delays for what might be termed ordinary service. It is interesting in reviewing the " cause of delay " in this table to note that 90 per cent. of the causes are of an inconsequent nature, and might be reasonably expected when considered in the light of an initiative service.

Serious failures in system causing over 300-minute delays. It is to be noted that these failures are indicated by the dates of July 14, 16, 20, 26 and 27; August 6, and 8; September 8 and 26, and October 19. A description of the cause of one applies to all of them, except those of July 16 and October 19. In every instance, excepting the two dates mentioned, the tie-up was due to simultaneous failure of several circuit-breakers, owing to a short-circuit, thus temporarily disorganizing the distributing system, by preventing the electrification of trolley wires.

On July 16, the White Mountain Express left the rails just

TABLE V.

Locomotive Daily Reports, July, 1908.

Date	Recon-struc-tion	Experi-mental & test	Trucks in-stalled	In-spec-tion	Re-pairs	No. of trains	Cause of repairs
1	1	..	..	2	1	136	Grounded motor.
2	..	..	..	..	..	142	
3	..	..	..	..	..	148	
4	..	HOL	IDAY	..	..	..	
5	1	..	..	..	1	81	Loose tire.
6	1	..	..	2	1	140	Loose tire.
7	1	..	..	2	..	135	
8	1	..	..	2	..	135	
9	1	..	..	1	2	135	Trucks (1) grounded switch group (1).
10	1	..	..	2	..	135	
11	..	SATU	RDAY	..	..	..	
12	1	..	..	..	3	69	Direct-current shunt burned out (1). Tramming truck (1). Change-over switch grounded (1).
13	1	..	..	1	..	135	
14	1	..	..	..	2	135	Broken axle (1). Changing spring-hangers (1)
15	1	..	..	1	3	135	Broken axles (1). Short-circuited commutator (1). Grounded transformer (1).
16	1	..	..	3	1	132	Broken axle.
17	2	..	..	..	2	135	Trucks (1) slipped tires.
18	..	SATU	RDAY	..	..	..	
19	2	1	..	1	2	68	Hot box (1). Changing motors (1).
20	2	..	..	..	3	135	Trucks (1). Burned-out transformer (1). Change-over trouble (1).
21	3	..	1	..	4	135	Transformer grounded (3). Blower and compressor motors grounded (1).
22	3	..	1	..	..	135	
23	2	..	1	..	2	135	Motor trouble (1). Trucks.
24	2	..	1	..	1	135	Trucks.
25	2	..	1	..	1	137	Motor leads burned off.
26	..	SUN	DAY	..	..	..	
27	2	..	..	..	1	133	Motor leads burned off.
28	2	1	1	..	..	133	
29	2	..	1	1	1	133	Side motion.
30	1	..	1	1	1	133	Trucks.
31	1	..	1	..	2	135	Trucks (1). Loose tires (1).
Total	38	2	10	19	34		
Avg.	1.41	0.074	0.37	0.71	1.26		

TABLE VI.
LOCOMOTIVE DAILY REPORTS, AUGUST, 1908.

Date	Recon-struc-tion	Experi-mental & test	Trucks in-stalled	In-spec-tion	Re-pairs	No. of trains	Cause of repairs
1	1	..	1	..	1	141	Trucks.
2	..	SUN	DAY	..	..	..	
3	2	..	1	..	2	135	Truck springs (1). Blower motors (1).
4	2	..	1	1	..	133	
5	1	..	1	1	1	134	Changing motors.
6	1	..	1	2	1	131	Changing motors.
7	..	..	1	..	2	133	Compressor grounded (1). Trucks and reconstruction.
8	1	1	..	..	2	136	No. 1 Main Reservoir leaking. Trucks and reconstruction (1).
9	..	SUN	DAY	..	..	..	
10	1	..	..	..	1	135	Trucks and reconstruction.
11	1	..	..	1	1	133	Trucks and reconstruction.
12	1	..	..	..	3	133	Grounded direct-current relay (1). Loose tires (1). Trucks and reconstruction (1).
13	1	..	..	..	1	133	Trucks and reconstruction.
14	1	..	..	..	1	134	Waiting for wheels from Readville.
15	1	..	..	..	1	138	Waiting for wheels from Readville.
16	..	SUN	DAY	..	..	..	
17	1	1	..	1	2	133	Direct-current shoe cables grounded. Waiting for wheels from Readville (1).
18	1	..	..	..	3	133	Circuit-breaker trouble (1). Direct-current shoe cables grounded. Waiting for wheels from Readville (1).
19	1	..	..	..	1	133	Waiting-wheels. Readville.
20	1	..	..	1	1	133	Changing compressor.
21	2	..	..	1	..	133	
22	2	..	..	2	..	137	
23	..	SUN	DAY	..	..	..	
24	2	..	..	..	1	135	Grounded motor.
25	2	..	..	..	2	133	Grounded motor (1). Grounded armature (1).
26	2	..	..	..	9	133	Grounds, due flood, N. Y. Central zone.
27	2	..	..	..	5	133	M-1 switch-group grounded. Grounds, due flood—N. Y. Central zone (4).
28	2	..	..	..	2	133	Grounds, due flood—N. Y. Central zone.
29	2	..	..	..	2	141	Grounded motors (2).
30	..	SUN	DAY	..	..	..	
31	2	1	..	..	..	143	
Total	36	3	6	10	45	3502	(Total daily trains).
Avg.	1 39	0.11	0.23	0.39	1.73	135	(Average).

TABLE VII.

LOCOMOTIVE DAILY REPORTS, SEPTEMBER, 1908.

Date	Recon-struc-tion	Experi-mental & test	Trucks in-stalled	In-spec-tion	Re-pairs	No. of trains	Cause of repairs
1	2	..	1	..	..	137	
2	2	..	1	1	1	135	Grounded motor.
3	2	..	1	1	..	134	
4	2	..	1	..	1	139	Grounded motor.
5	1	..	1	..	1	148	Grounded compressor motor.
6	..	SUN	DAY	..	..	..	
7	..	LABO	R DAY	..	..	..	
8	2	..	1	..	..	143	
9	2	..	1	..	..	138	
10	2	..	2	..	..	138	
11	2	..	1	1	1	137	M-1 Switch grounded.
12	2	..	1	..	1	140	M-1 Switch grounded.
13	..	SUN	DAY	..	..	..	
14	2	..	2	..	..	129	
15	2	..	2	..	..	126	
16	2	..	1	1	..	126	
17	3	..	2	1	..	126	
18	3	..	2	1	..	126	Trucks.
19	3	..	2	..	..	130	
20	..	SUN	DAY	..	..	..	
21	3	..	2	..	1	130	Change-over switch grounded.
22	2	2	2	..	1	127	Change-over switch grounded.
23	2	..	2	..	1	126	Change-over switch grounded.
24	3	..	3	..	..	126	
25	2	..	2	..	1	126	Sharp flanges.
26	3	..	2	..	..	128	
27	..	SUN	DAY	..	..	..	
28	3	..	2	..	..	128	
29	3	..	2	..	..	126	
30	3	..	2	..	..	126	
Total	58	2	41	5	10	3295	(Total daily trains.)
Avg.	2.32	0.08	1.64	0.20	0.40	132	(Average.)

TABLE VIII.

LOCOMOTIVE DAILY REPORTS, OCTOBER, 1908.

Date	Recon-struc-tion	Experi-mental & test	Trucks in-stalled	In-spec-tion	Re-pairs	No. of trains	Cause of repairs
1	3	1	2	..	..	126	
2	3	..	2	..	..	126	
3	3	..	2	..	..	129	
4	..	SUN	DAY	..	..	..	
5	3	..	2	..	..	126	
6	3	1	2	..	..	124	
7	3	..	2	..	..	124	
8	3	..	1	..	1	125	Sharp flanges.
9	3	..	2	..	..	125	
10	3	1	2	..	..	129	
11	..	SUN	DAY	..	..	..	
12	3	..	2	..	..	125	
13	3	1	2	..	..	126	
14	3	..	2	..	..	124	
15	3	..	2	..	..	124	
16	3	1	2	..	..	124	
17	3	..	2	..	..	126	
18	..	SUN	DAY	..	..	..	
19	3	..	3	..	..	125	
20	3	1	2	..	1	124	Turning flanges.
21	3	..	3	..	..	126	
22	3	..	3	..	..	124	
23	3	..	2	..	1	125	Grounded motor.
24	3	1	2	..	..	126	
25	..	SUN	DAY	..	..	..	
26	3	..	3	..	..	125	
27	4	..	2	..	..	124	
28	4	..	3	..	..	124	
29	4	..	2	..	..	124	
30	3	..	3	..	..	124	
31	3	..	3	..	..	126	
Total	84	7	60		3	3380	(Total week-day trains.)
Avg.	3.10	0.26	2.22		0.11	1250	(Average.)

TABLE IX.

ELECTRIC LOCOMOTIVE-MILEAGE DATA FOR SEPTEMBER 5, 1908.

No. of Engine	No. of runs made	Main Line Mileage
01	In shop	
02	7	238
03	7	230
04	7	206
05	4	136
06	In shop	
07	4	111
08	3	102
09	8	256
010	8	264
011	5	170
012	7	196
013	8	245
014	6	188
015	8	247
016	5	162
017	In shop	
018	5	136
019	10	340
020	6	188
021	8	264
022	7	204
023	9	306
024	4	136
025	7	222
026	8	264
027	9	306
028	8	272
029	6	196
030	6	188
031	8	272
032	6	187
033	3	102
034	4	135
035	8	272
036	4	136
037	7	238
038	6	196
039	7	230
040	7	238
041	8	272
Total	248	8051
Average	6.5	212

Main line mileage does not include switching.
Maximum mileage made, 340.
Average mileage, 212.

east of Greenwich station, tying up both east-bound tracks, and causing excessive train-minute delays. Except for the electric rail-bonds which suffered destruction, no other electrical apparatus was disturbed. One of the electric locomotives attached to this train was immediately returned to service, the other following it the next day after light repairs were made to its pilot and third-rail shoe mechanism.

On October 19, although the impedance coils in the generator station were in action, a ground occurring on the bus-bars of the anchor bridge directly outside of the power station, in combination with a defective circuit-breaker, produced a short-circuit which destroyed the operating mechanism of the breaker in question. This unfortunately occurred at a time when temporary connections had been made between the power house and line; on this account a serious delay was experienced in restoring the voltage to the line.

It is interesting to note that since the complete inauguration of electric service, the serious failures in the system have been reduced from five to one.

Engine repairs. Under " faults " is described the necessity in detail of our engine reconstruction. Since 100 per cent. service has been established, it has occurred to me that the shop reports concerning the daily number of engines which are out of service on account of repairs or for other reasons, would be interesting. On account of the reconstruction being done on the locomotives, it has been found necessary to give up on an average three electric locomotives at Stamford and three at New Haven, the new bogie trucks being installed on the engines at New Haven. Thus out of 41 locomotives we have had only 35 to handle the complete passenger service. This has resulted in requiring that the New Haven locomotives handle the service with virtually no spare engines. An examination of Tables V, VI, VII and VIII, under columns " Inspection " and " Repairs ", indicating how few have been the engines in the shop for the past three months, is an attest of this requirement.

The capacity of the electric locomotive. The capacity of the electric locomotives was based upon their ability to handle a trailing load of 200 tons in local service with stops averaging those that exist between stations from New York to New Haven. As a matter of fact, the station stops between Stamford and New York average very much greater than between New Haven and New York, and as very quick turns are made

TABLE X.

TRAILING LOADS IN EXCESS OF LOCOMOTIVE GUARANTEES.

Date	Service	Train No.	Locomotive No.	Trailing train weight	Percentage over weight
10/18	S. Ex.	59	07-024	538	7.6
	S. Ex.	18	04-09	623	24.8
	S. Ex.	1	038-041	604	20.8
	S. Ex.	2	04-027	682	36.4
10/19	S. Ex.	21	038	284	13.6
	S. Ex.	15	028	327	30.8
	S. Ex.	45	040-041	561	12.8
	S. Loc.	215	08	236	18.0
	S. Ex.	63	030	302	20.8
	P. C. Loc.	181	041	348	39.2
	S. Ex.	269	034	277	10.8
	S. Ex.	1	06-036	674	34.8
	S. Ex.	29	01	280	12.0
	S. Ex.	2	04-09	609	22.0
10/20	S. Ex.	2	03-08	762	52.0
	S. Ex.	1	030-033	684	36.8
	P. C. Loc.	181	040	314	25.6
	N. R. Loc.	151	023	344	14.7
	S. Loc.	252	020	235	17.5
	S. Ex.	11	07-024	560	12.0
	S. Ex.	15	013	400	60.0
	S. L. Ex.	260	034	273	9.2
10/21	S. Ex.	21	07	331	32.3
	S. Loc.	287	04	223	11.5
	S. L. Ex.	260	03	271	8.4
	S. L. Ex.	246	038	283	13.4
	S. Ex.	15	040	332	32.7
	S. Loc.	216	026	256	28.0
	S. Loc.	252	033	275	37.5
	S. Loc.	206	038	257	28.5
	N. R. Loc.	151	010	338	12.7
	S. Loc.	207	025	262	31.0
	P. C. Loc.	181	09	297	18.8
	S. Ex.	1	032-037	698	39.5
	S. Ex.	29	010	280	12.0
	S. Ex.	2	07-032	817	63.6
10/22	S. Ex.	21	010	328	31.2
	S. Ex.	55	040	317	26.7
	S. Loc.	287	026	226	13.0
	S. L. Ex.	260	02	285	14.0
	S. L. Ex.	246	016	274	9.6
	S. Ex.	245	037	279	11.6
	S. Ex.	15	034	391	16.4
	S. Loc.	252	034	281	40.5
	S. Loc.	222	011	222	11.0
	S. Loc.	206	037	251	25.5
	N. R. Loc.	151	030	343	14.3
	P. C. Loc.	181	033	302	20.8
	S. Ex.	1	06-038	668	33.6
	S. Ex.	29	028	271	8.4
	S. Ex.	2	06-038	700	40.0

Date	Service	Train No.	Locomotive No.	Trailing train weight	Percentage over weight
10/23	S. Ex.	21	013	339	35.6
	S. Ex.	55	012	320	28.0
	S. L. Ex.	246	02	263	5.2
	S. Ex.	15	010	377	50.8
	S. Ex.	11	05–06	542	8.4
	S. Ex.	56	024	313	25.2
	S. Loc.	252	027	269	34.5
	S. Loc.	215	09	241	20.5
	S. Loc.	206	02	233	16.5
	S. Loc.	207	03	245	22.5
	P. C. Loc.	181	028	318	27.2
	S. Ex.	1	017–041	747	9.6
	S. Ex.	2	03–011	853	70.4
10/24	P. C. Loc.	190	012	281	12.4
	S. L. Ex.	292	010	258	3.2
	S. Ex.	15	019	333	33.2
	S. Ex.	24	06	396	58.4
	S. L. Ex.	288	02	258	3.2
	P. C. Loc.	176	022	298	19.2
	P. C. Loc.	172	037	290	16.0
	S. Loc.	212	041	245	32.5
	N. R. Loc.	151	029	323	7.7
	S. Loc.	207	06	246	23.0
	P. C. Loc.	181	033	299	19.6
	S. Ex.	1	019–036	624	24.8
	S. Ex.	29	08	271	8.4
	S. Ex.	2	026–030	819	63.9

at both termini, the service may be said to be more severe under these conditions than under the guaranteed conditions of purchase. It is interesting to note in Table X the trailing loads, taken at random, hauled by the New Haven electric locomotives for the week ending October 24, 1908.

Comparative steam and electric engine mileage. September 5, last, Labor Day, offered an interesting day to note what mileages could be made by the electric locomotives. Table IX gives an individual record for each of the 38 electric locomotives in service that day.

In obtaining an average for all electric locomotives of 212 miles, it is to be remembered that this was made for all classes of service, express, express-local, and local, and further that the mileages were made over three short terminal runs; namely, Stamford, Portchester, and New Rochelle to New York, the distances being approximately 34, 26, and 17 miles, respectively. As indicated in the table, several engines made eight runs, two of them nine, and one ten. It is difficult to get an exact comparison for steam locomotive mileages. All of the electric engines are confined to one division, while the steam locomotives do inter-division service. However, using the records of the 1906 steam service for Labor Day and considering steam locomotives doing mileage in and out of the present electric zone, out of 117 locomotives the following record is to be noted:

10	made	over 300 miles.	
20	"	between	200 and 300 miles
21	"	"	150 and 200 "
34	"	"	100 and 150 "
32	"	under 100 miles.	
117			

Thus with division limits double that of the present electric division, and with the additional advantage of interdivision runs, the electric mileage for this concrete case averaged 34 per cent. better than the steam mileage.

The Electrification in Its Relation to Matters Other Than Traction

Telegraph and telephone. Single-phase electrification affects telegraph and telephone systems whose wires lie parallel with and in close proximity to the railroad. The corrective for this disturbance has proved to be simple and not costly. Briefly described, it consists of compensating transformers whose secondaries are a part of the telegraph and telephone wires and whose primaries receive their voltage from pilot wires strung on the same cross-arms as those bearing the telegraph and telephone wires, and thus having impressed upon them the same voltage, by electromagnetic induction, as the telegraph and telephone wires. The transformer secondary voltage is approximately equal and opposite to the induced voltage on the telegraph and telephone wires and thus constantly compensates for it throughout all ranges of induction due to the single-phase wires.

An interesting commentary on the efficacy of the compensating transformer is that its use obviated the necessity of any change in the physical location of the telegraph and telephone lines within the zone of induction, and has thus been the means of removing what at first was rightly considered a very offensive attribute of the single-phase system.

Public safety. As no fatality to the traveling public has happened by reason of the high-voltage wires since the system has been in operation, this would indicate the safety involved in its construction. The catenary form of construction as applied to the suspension of the trolley wire virtually eliminates all danger of these wires falling. On the other hand, the feeder or by-pass wires are not suspended by messengers, and although the 300-ft. spans used in this construction are not uncommon, it has been considered a wiser policy to reinforce all such spans which are above passenger platform stations. It may be of interest to state also that the scheme of reinforcement is not to cradle or place supplementary wires in connection with this span, but simply to use a steel wire with a large factor of safety reinforced by copper for conductivity conductor and supported from the struts by insulators. The resistance to ground of this construction will be far in excess of the ordinary insulator used in the open construction, with the steel wire connected to it in such a manner as to make impossible the burning of this wire in two, in the case of the insulator breaking down.

Foreign wires. All electrifications are subject to foreign wire crossings. As in the case of suspension of high-voltage wires over passenger platforms, instead of using supplementary catenaries or cradles, the safer (and incidentally the cheaper) method of crossing is to use at the crossing heavier (and possibly guyed) poles, better insulation, and larger wires than are commercially necessary in the other parts of the foreign transmission line. High factors of safety with economy are obtained following this policy.

COMMENTARY

Lightning protection. An ideal arrangement of catenary construction in relation to lightning protection would be to have the overhead messenger system grounded, and from it suspended by insulators a secondary catenary system, to which in turn would be attached the contact wires. Even in the New Haven case, where the overhead messenger system is not grounded, lightning has given but slight trouble. This is probably due to the very great number of grounded steel trusses and struts projecting above the electrified wires. Lightning was the indirect cause of one of our delays, but played only a small part in the real cause, which can be attributed to the surge of current occurring at the time lightning caused a ground. At that

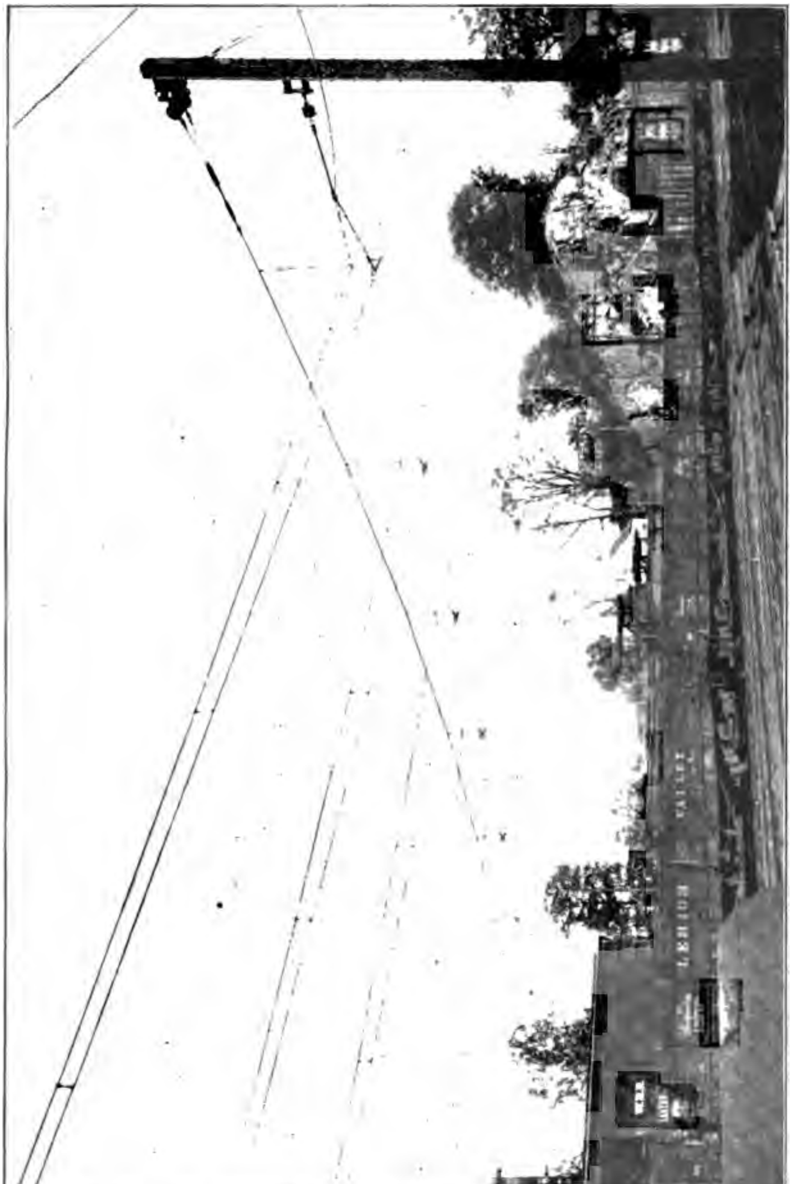


Fig. 13—Cross-catenary construction East Portchester yard.

time the feeders had not been rearranged, nor had the impedance coils been installed in the power-house circuits. On the occasion of this ground, due to the lightning, we lost several of the anchor bridge circuit-breakers; this naturally tied up the distributing system, preventing the proper restoration of voltage to the trolley wires. The conditions are now such that the circuit-breakers will relieve short-circuits due to this cause.

Grounds. Structures supporting high-voltage insulators should be grounded. Particularly is this true of structures foreign to the railroad company, such as highway bridges. This arrangement insures a prompt response of the circuit-breaker apparatus, and carries out the good practice universally applicable, that the material supporting the insulator be grounded. This principle has application in wood-car construction. Positive grounds between Pintsch gas pipes and both car trucks should be made to avoid any arcs being drawn under the car body. On two occasions the gas in the Pintsch mains of New Haven cars has been set on fire on account of connection with electrified wires. While the percentage is low, being two cars in about 400,000 during the period involved, with little damage in either case and that to the cars, it is a matter not to be ignored.

Many of our catenary bridges, of both the intermediate and anchor type, serve to support signals. Signal men have been entirely free from coming in contact with the high-tension wires, by the simple provision of grounded close-mesh screens interposed between the signal platforms and the high-voltage wires. The value of two grounds can be rated considerably higher than twice that of one.

Tell-tales. Tell-tales in connection with an overhead system offer a peculiarly difficult problem. It is quite apparent that the pantagraph current collecting device would get into either electrical or mechanical difficulties with the present form of tell-tales. We have experimented at some length in trying to produce an electrical horn that would be automatically sounded by an approaching train. There is nothing particularly difficult in getting the automatic action or in producing a noise. The superiority of the tell-tale over the horn, however, is that no matter how much noise the freight train is making, the tell-tale always notifies, while in the case of the horn, should the freight train be making more noise than it, the notification is lost. A horn if used for this service should be a large one.

As a matter of fact, the necessity of tell-tales is a bit of a relic of the barbaric past. In the days of hand brakes, the brakeman had to walk the roofs of cars, and low bridge warnings were necessary. To-day the braking is all done from the engine cab, except in yards where, of course, the overhead wires have their normal height of 22 ft. from the rails, permitting safe clearances for manual operation of the brakes.

Train lighting and heating. On account of the necessity of wiring for electric heating and lighting some 2500 coaches, it seems reasonable to retain the present method and apparatus of heating and lighting the trains in the electric zone. To accomplish this it was only necessary to supply each electric locomotive with a small steam boiler, used solely for train heating. The Pintsch gas system has been retained intact. Later, when the service is sufficiently extensive, all trains will be heated and lighted by electricity.

All multiple-unit cars are to be heated and lighted by electricity. The problems of heating and lighting a multiple-unit train and a locomotive train are quite different. In the former the current is fed into the train at several points, as many as there are motor cars; in the latter it can be taken only from the locomotive (if there are two locomotive units, of course, it can be supplied from both). Again, in the case of the multiple-unit equipment, the supply of current is along progressive points of the train-length, thereby not requiring train bus-bars of excessive copper section, while in the case of the locomotive train, the supply, whether from one or two units, will be at the head end of the train, thus requiring a transmission of considerable length. A train of 12 cars, for instance, would be between 700 and 800 ft. long. Allowing 24 kw., maximum, for heat and light per car, would require the delivery of about 300 kw. apparent power. An investigation of this department in electrification has brought out some interesting details, particularly in reference to locomotive transformer design and electric car jumpers.

Fatalities. As has been pointed out, we have had several fatal accidents, due to men coming in contact with electrified wires. We are none the less concerned that in every instance they were men employed by the company. In each instance the accident was due to carelessness or violation of instructions, and most of the accidents occurred during the period of construction. The operating period has provided a smaller number of these unfortunate events, and the records to-day now seem to indicate that the future will be free from this most regrettable feature.

Cross-catenary versus bridge-bents. The excellent results of a year's experience with cross-catenary construction in our Port Chester yard, Fig. 13, where as many as ten tracks are spanned, is tempting encouragement for its application to main-line work. Should the cross-catenary be used, anchor bridges should be spaced at close intervals, say one per mile. The question of the use of cross-catenary versus the bridge-bent on multitrack roads, however, depends largely upon the right-of-way conditions. Cross-catenary struts have to be guyed. In many instances right-of-way limits will require the guy to be placed on foreign land. If the additional cost of the foreign land brings the catenary bent up to or more than the cost of the bridge-bent it loses its object.

Single- versus double-catenary construction. Lack of operating data led the engineers of the New Haven road to take no chances with the overhead construction, and the double catenary was considered the safest. The adoption of single, double, and compound systems of catenary suspension is so intimately and definitely related to the number of tracks, the length of longitudinal spans, and the cost of transverse supporting structures and foundations, that the determining of the most economical type of catenary construction must necessarily include the consideration of these factors, and for this reason simple standardization adapted to all conditions is impossible.

Single-phase operation. The New Haven system provides that the volt manufactured in and leaving the doors of the power house, is the same physical volt that knocks at the doors of the locomotives. Thus the line is the single link that unites the power house and the locomotives. All such adjuncts as step-down transformers, synchronous converters, storage-batteries, and low-voltage distributing systems with their necessary attendant complement of help are dispensed with. The Cos Cob power house has the usual number of men for a station of its output, and the locomotives are operated by the electric locomotive engineer with the customary assistant present for emergency. This crew holds good for single- or double-unit trains. An emergency repair train is the guardian of the line, attending to all matters pertaining to its repair and maintenance. Including the night and day crews of the emergency train, the number of men employed to maintain the distribution system is nineteen. This covers about 100 miles of single track, including yards.

Conclusion. The writer will not use the conventional method of summing up and making recommendations. The paper has been written with the intent to lay before the members of the American Institute of Electrical Engineers a practical picture of the New Haven single-phase electrification, describing the first suit of clothes it wore and what it is wearing now. He is conscious of the innumerable little points of real interest that time and opportunity have not permitted him to insert here, but he trusts to the discussion to bring them out. The description of the faults and how they were handled will answer for the usual column of recommendations. In concluding, an interesting definition of engineering comes to mind; namely,

Engineering is the art of making a dollar earn the greatest interest.

The writer has intentionally omitted the discussion of the operating costs of a direct-current versus an alternating-current system; but, bearing in mind the preceding definition, it is his belief that in the electrification of steam roads to-day straight alternating-current traction is the agency through which that title can be earned.

History sustains the undeniable truth that alternating current is the preferred agent for the transfer of electricity where either distance or capacity is involved. A railroad involves both. Granted, therefore, that the alternating-current traction apparatus has received the trade mark of practicability, what further argument does it need in its favor?

In connection with the New Haven electrification I wish to speak of the privilege of association with Mr. E. H. McHenry and Mr. Calvert Townley. Upon Mr. Townley devolved the responsibility of decision as to the form of electrification selected. Neither the object nor the scope of this paper can include a description of the analytical course of procedure preparatory to the conclusion that the New Haven road would adopt the single-phase form of electrification. Suffice it to say that Mr. Townley's conviction to accept this responsibility were sufficiently strong to lay the foundation of this work. To Mr. E. H. McHenry, whose serious illness had prevented his filling the office of vice-president in charge of matters pertaining to engineering, fell upon acession to that office, the responsibility of ratification or disagreement with the policy as set forth by Mr. Townley. That ratification is better written in the details of the electric zone itself, as in each department of power house line and locomotive may be seen the betterments due to his suggestion. To him is due the credit of the practical issue of this electrification.

This paper would indeed be incomplete did I not refer to the splendid courage and indomitable pluck of our contractors, who through a fire of criticism and business depression perseveringly stuck to their belief in the principles they were advocating. They can now have the satisfaction of viewing something begun and something finished; for while there may be many improvements to accrue to the alternating-current, single-phase system, the trunk-line principles have by them been laid and demonstrated.

Discussion on "The Log of the New Haven Electrification." New York, December 11, 1908

President Ferguson: The American Institute of Electrical Engineers is to be congratulated in having as a member, a man who is so willing to come forward and frankly state the actual experience he has had in the construction and operation of so new, so important, and so untried an undertaking. I think we owe it to him to show our appreciation of the work he has done by giving full discussion to his paper: first, as to the major and minor faults that have been experienced in the system and of the corrections that have so far been applied, and of the possible corrections that may come in the future; secondly, as to whether this system, as designed and constructed, will fill the requirements of the entire electrification of the road from New York to Boston, as we understand is contemplated by the railroad; thirdly, as to whether this system as designed and constructed will fill the requirements of the electrification of terminals in our large cities, taking into consideration the handling of the through and suburban passenger traffic as well as the switching and hauling of the freight traffic.

Calvert Townley: I am strongly impressed by the extremely radical treatment which the author has given to his subject. Abandoning the usual course of emphasizing the advantages of the system adopted, he has passed over them with but scant mention, and has given in minute detail a list of the troubles encountered. The magnitude of the project itself, the amount of advertising which it has had, and the fact that it has been attacked continuously, not to say viciously, by those who disagreed with the plans adopted, or perhaps were interested in seeing them fail, demonstrates the author's courage in handling the subject as he has, but also emphasizes the radical character of his treatment. What the designing engineer wants to know, what the constructing and the operating engineers want to know—what indeed everybody wants to know who is undertaking a new project is, what troubles and difficulties he is likely to meet, and how can they be overcome. The advantages of one plan or another, the perfect features, so to speak, will, in great measure, take care of themselves. Therefore, where a system is already playing so important a part in the railroad world as is the single-phase system—destined to play a much greater part in the future—we are particularly fortunate in having a member who can present so full a log of its application to so important a project, and of its troubles and their remedies, as Mr. Murray has done in this paper.

It is to be noted that Mr. Murray has omitted the customary statement of conclusions usually drawn from the facts furnished in a paper. The conclusions, however, are none the less clearly indicated, even though they may not be stated. Those which seem to stand forth are:

1. This system was put in without any previous similar installation to pattern after.

2. Several of its fundamental features were either entirely new or had so new an application, that their previous use was not much of a guide.

3. Sundry and various defects developed as the installation progressed, resulting in interruptions to the service.

4. These defects have practically all been remedied, and certain other improvements have been made.

5. Five months after the complete substitution of electricity for steam, and while some of the improvements have still to be completed, the system has been demonstrated to be successful, and well adapted to the service for which it was designed. As an interesting fact it may be here noted, that at the present time the electric service is less subject to interruption than was the steam service which it has replaced.

From an engineering standpoint it may help us to realize the nature of the project undertaken, if I recapitulate some of the pioneer features that were included in this installation. For example: it is the first installation to generate single-phase current in large units at a fairly high electromotive force with a dead ground on one side of the generating system; it is the first installation to adopt, though not the first system to put into operation, the 11,000-volt, alternating-current trolley, with the rail return, without any transformer, converter, or other sub-station appliance whatever between the generator and the locomotive itself; it is the first alternating-current system to handle 800-ton trains at 60 miles an hour; it is the first installation to use an alternating-current, gearless, 25-cycle motor (it also uses much the larger motors of this type); it is the first installation to adopt a locomotive unit designed to fit the normal, instead of the heaviest, train service, and to take advantage of the multiple-unit feature of operating two or more locomotives together, when heavier trains require it.

In my opinion, the one radical thing that was done wrong in the New Haven electrification, and which was really at the bottom of the major part of the difficulties that have been encountered, is that the electrification was not begun soon enough. It is undoubtedly true that had more time been allowed for attention to the minor details of the equipment, and could work that had to be done under great pressure have taken its normal course, many of the minor troubles which can be traced to these facts would never have existed, and, furthermore, had there been time for a reasonable operating tryout, very many of the defects which have caused delays to service, would have been detected and remedied during the trying out period instead of in practical service.

In referring to the log of operation, Mr. Murray says, "We are not proud of it." I am not one who feels that any apology for the results obtained is in order. The combination of an installation embodying so many new features and an insufficient time in which to overcome difficulties, was recognized in the beginning, and it was felt that a superior plan, handicapped with some temporary troubles, was better than an inferior plan with fewer threatened temporary troubles, but many permanent disabilities.

I should feel that I had been remiss if I closed my remarks without paying tribute to the Westinghouse engineers for their part in this project. Their development of the necessary machinery and appliances alone has made the project possible. Without them this single-phase installation could not have been successful. They are rightfully entitled to a frank acknowledgment of their continued cheerful, resourceful, and courageous coöperation through the entire progress of this enterprise.

The keystone of the system is the locomotive. Without the single-phase, alternating-current motor the whole plan would become impossible. No one doubts but that a power house can be built successfully to generate current, and that a line can be constructed successfully to transmit single-phase alternating current at any reasonable voltage and over a wide range of quantities; but it was not until the single-phase motor was developed, that the simplification possible with this form of traction could be even considered. To me, therefore, the most interesting part of this paper relates to the performance of the locomotives. The log of operation showing delays, etc., is interesting, but I do not regard it as of particular importance from an engineering standpoint, as a hot bearing on a blower motor would as quickly bring about a delay to service as an entire breakdown of the locomotive itself. Some troubles with the minor parts may always be expected at the start in any large installation, but if the fundamentals are not impeached, success is not in jeopardy.

A study of the list of changes that were made in the locomotive originally furnished, and enumerated by Mr. Murray, discloses the interesting fact that while there were a number of electrical difficulties, a very large per cent of the changes made were mechanical, not only mechanical changes to mechanical features, but also mechanical changes to electrical features, most of the latter being on the minor parts of the equipment, which, naturally, had to receive the least attention when time was pressing. With the principal elements of the locomotive design; namely, the large gearless, 25-cycle motors, there has been practically no trouble. Their performance as to torque, commutation, and capacity has been most gratifying. It was soon demonstrated, in fact, that the motors and transformers had capacity for service in excess of that called for by their guarantees, and investigation showed that by somewhat increasing the capacity of some of the minor parts of the locomotive equipment, a machine of materially greater duty would be provided. For this reason several changes were made in the locomotive details, not primarily to correct defects, but so to increase the capacity of the parts modified as to provide an excess locomotive capacity. Mr. Murray mentions but briefly a feature of locomotive design, which from the standpoint of dollars and cents value to the operating road, deserves a place in the front rank. Perhaps he has passed by this feature so readily because his mind has been bent on telling of troubles, and this feature has caused no trouble. I refer to the flexible drive whereby the armature of the gearless motor is wholly supported on, and exerts its driving power through a set of spirally concentric springs. Trackmen have frequently been appalled because electric locomotives are so much harder on track than steam locomotives of even greater weight, and it has now come to be a generally accepted conclusion that this fact is due to increased impact because the armature dead-weight is supported without cushion, rigidly on the axle. The impact due to this additional dead-weight is something astounding. In the New Haven locomotives, the method of mounting the armatures, obtaining the cushioned effect, exerting the full driving power of the motors, and compensating for the so-called varying torque thereof, has been successfully accomplished in a way never before tried. The results in improved track maintenance cannot fail to be of far-reaching consequence.

Similarly with the line—while insulation troubles have been manifest, due first, to low bridge clearances, and secondly to steam locomotive stack discharges, the principal overhead difficulties have been mechanical; that is, the mechanical wear on the trolley wire and too great rigidity in its method of support. The serious line difficulty to which Mr. Murray refers, resulted from a combination of these mechanical troubles. They were not discovered for the first time in May, but had been manifest for a considerable period, and were being carefully studied. The extent of the trouble, however, could only be demonstrated by an extended tryout, because it was affected so vitally by the amount of stack discharge and by the rapidity with which the pantagraph shoes traveled over the trolley wire at high speeds. The addition of a steel wire flexibly suspended from the copper conducting wire, a wire to take the wear, was, however, the solution. With it there disappeared the last threatening difficulty that need cause any apprehension.

B. G. Lamme: Though I have been more or less intimately interested in all of the problems in this New Haven electrification, there were two parts of the total equipment to which I gave a great deal of my personal attention, namely, the design of the alternating-current generators in the power house and the main motors on the locomotive. The popular opinion, even among designers, was that the motor was by far the most difficult problem of the two, considering that it was the first large gearless single-phase motor constructed for commercial service. As for the generators, one engineer has said "anyone can build generators." However, from my own point of view, since borne out in practice, the design of the motors was a comparatively simple problem compared with that of the generators.

Considering first the main motors, there are 41 locomotives on this road, each with four gearless motors, wound for both direct-current and single-phase operation. I do not intend to discuss the design of these motors, but simply to indicate some of the records that have been made. In the motors themselves there were two points that were generally looked upon as sources of weakness; namely, the commutation (and life of the commutator) when operated on alternating current, and the use of preventive leads (or so-called "resistance leads"), between the armature winding proper, and the commutator. Many people have sounded a warning in regard to the dangers from the use of these preventive leads. It was also assumed that there would necessarily be severe sparking at the commutators of these motors, which was bound to injure them in a relatively short time. Let us look at the record on these two points:

As I stated, there are 164 main motors. The first equipment was put into commercial service the latter part of July 1907, and the last equipment, being part of the order of six, mentioned by Mr. Murray, was put in operation about July 1908. The records show that the total of 164 motors have averaged slightly over 40,000 miles each since being put in operation, the minimum mileage for any one motor being approximately 28,000. A range from 30,000 to 50,000 miles will cover the record of most of these motors. With the above mileage, the commutators of these motors are all in excellent condition, having a good polish and showing but relatively little wear. Virtually all of the motors on the first 35 locomotives have been dismantled at some time or other, for the causes mentioned in Mr. Murray's list of changes. At such times the commutators of these motors were cleaned up slightly, just as any other part of the motor would be cleaned up. The commutators were not turned unless they showed evidence of mechanical injury, generally from some cause external to the commutator itself. However, in commercial service these commutators have required no attention, and, as far as I could determine, none of them has ever been sand-papered while in service. At the present rate of wear on these commutators, there appears to be 15 to 20 years' life in each one of them. As there is but relatively little sparking, even when developing two or two and a-half times full-load torque, there is no real reason why these commutators should not make such good records.

Considering next the injuries to the motor. Mr. Murray's tables show reference here and there to motor injury, without stating just what kind of injury is meant. Simply to state that a motor is injured, without saying what part is injured, may give a wrong impression. It is generally considered, and correctly so, that an injury to the armature of a motor is a much more serious condition than an injury to the field or the connections. The records of the New Haven installation show that the injuries to the armature represent only a small percentage of the total injuries. In fact, the few damages to the armature have been largely from external causes, foreign substances, such as bolts, tools, water, etc., getting into the motor, or from unsoldering at the commutator or at the rear end, due to the extreme overloading, which has occurred in a few instances, as when pulling a heavy train with one locomotive with one pair of motors cut out. In a few instances these damages have resulted in an actual burn-out of a section of the armature, necessitating part rewinding.

In such cases I have been able to make a thorough examination into the condition of the winding on the armature, both in the main winding and in the preventive-lead winding. These examinations have been very interesting. They indicate clearly that the armature windings are showing no dangerous overheating from the heavy loads that have been carried, and also that the preventive leads are showing no more evidence of heating than the main windings. If any comparison could be drawn, I would say that of the two, the preventive leads were in the better condition. Furthermore, only two instances have been found where there was a breakdown in the preventive lead. In other words, the record for these leads is better than that for the main coils themselves. But this is partly due to the fact that the preventive-lead winding lies underneath the armature winding proper, and is thus mechanically protected to a greater extent than the main coils. From the standpoint of experience then, these leads have proved to be one of the most durable parts of the equipment; taking everything into account, their record may be considered exceptional. If the rest of the New Haven system had shown anything like as good a record, there would have been no occasion for the greater part of Mr. Murray's paper. I think the time has now arrived when the "preventive" leads, as they should more correctly be called, should stand on their own merits, for no amount of criticism can belittle the record which they have made in the New Haven service.

Now consider the generators. No defect has developed in them which was not, to a certain extent, foreseen and apparently provided for, based on the data at hand. However, on the completed machine, the first and most pronounced source of trouble was heating, not in the winding, but in the field or rotor structure, due to the pulsating reaction of the armature winding when carrying a heavy load in single-phase current. This reaction was known previously to building these machines, but on machines of smaller capacity it had not developed destructive tendencies. It was proved later that this was simply because it had not been tried out under the conditions which would develop its most harmful effects. This pulsating armature reaction may be analyzed in the following manner:

Consider the armature winding as a magnetizing coil fixed in the space and carrying an alternating current. This coil may be considered as setting up an alternating field fixed in space. For analysis, this alternating-current field, fixed in space, may be considered as made up of two constant fields of half value, rotating in opposite directions at the synchronous speed of the machine. One of these fields therefore rotates at the same speed and in the same direction as the rotor. The other field is traveling round the rotor core in the direction opposite to its rotation. This field may therefore be considered as equivalent to one fixed in space with the rotor running in it at double speed. This armature, carrying single-phase current, could be replaced by an external direct-current field, and if the rotor were run at double speed in this field, an equivalent result, as regards heating, should be obtained. This core thus becomes an armature core subject to a heavy induction at a high frequency.

When the first rotor was built, the structure was laminated as completely as mechanical conditions would permit. However, in the case of high-speed turbine-generators of very large capacity, it is almost impossible completely to laminate everything, due to the fact that the mechanical requirements call for rigidity in some of the structural features. Upon testing the first machine it was found that there was local heating, with heavy load, sufficient to create hot spots in the core; and in a comparatively short time in turn these hot spots damaged the insulation on the coils from the outside, thus causing grounds on the winding. As soon as this was noticed, an effort was made to eliminate these hot spots; but it was found, after several attempts, that as soon as one was eliminated others would show up in some different place as soon as a higher load condition was reached. It was evident, after considerable work had been done, that the correct remedy was not being applied to this trouble. It was then decided to take a bold step by attempting to eliminate all pulsating reactions from the armature, by putting a short-circuited winding on the rotor, of such value that a very large current could flow in it with but very little loss. It was the idea to damp out the field in very much the same way that the armature of a polyphase alternator will demagnetize, or kill its magnetic field, if the armature terminals are all short-circuited together. It is known that under this condition the armature current will rise to such a value that the field flux is practically eliminated. In order to maintain this condition indefinitely without overheating, it is only necessary to put enough copper on the armature so that the I^2R losses in it under this condition are within the temperature capacity of the windings. Working on this theory, a complete cage winding was placed on one of the rotors of the New Haven generators. This rotor had not been designed originally for this purpose, and it was therefore difficult to adopt the most suitable proportions in this winding, but what was put on, immediately showed in practice that a practicable remedy had been applied for this trouble. Meanwhile the new rotors designed for the application of heavy cage windings were under construction, and upon the installation of these, the field or rotor trouble all disappeared. It is interesting to note that the fourth machine installed, which has a 4260-kilovolt-ampere single-phase rating, has a solid steel core, in the surface of which the copper cage winding is embedded. As this winding completely eliminated the pulsating armature reaction, there was no further occasion for laminating the field as a protection from magnetic pulsations.

I might add that a number of the earlier tests, leading up to the design of the first New Haven rotors, were misleading, in the fact that turbine-generators were used for obtaining the preliminary data for single-phase operation and, in all cases, the machines had solid steel cores. These cores acted as dampers to a certain extent, and this in itself eliminated part of the pulsation. It thus developed afterwards, that in the very act of lamination to avoid the trouble, we had gotten into it deeper.

Practically all this work on the generators was done before the full electric service was established, and while only one or two generators were required to be operated at one time. With one generator running, there was apparently but little or no disturbance due to short-circuits on the system. As the service was increased and two generators put in operation, the effect of short-circuits became more pronounced. When, in June 1908, the entire electric service was established, and three generators were connected to the system, it soon became evident that there was some serious condition existing in the system, as indicated by the extremely violent shocks to everything in case of a short-circuit. This was particularly noticeable in the switching system, and, as Mr. Murray intimates, in the case of a short-circuit, all the switches in the system felt it their duty to jump in and open the circuit. This indicated an abnormal current condition. It was calculated that these machines would give possibly six or seven times full-load current on the first rush, in the case of a dead short-circuit, this excess current dying down to possibly two or three times normal full-load current. All indications were, however, that this current was being greatly exceeded, and therefore a series of oscillograph tests were made to determine the current rush when the lines were purposely short-circuited under various conditions. These tests indicated that under certain conditions each machine could give, at the moment of short-circuit, almost 5000 amperes on one phase, the normal full-load current being 340. With three machines in parallel, this would therefore mean that approximately 15,000 amperes could be delivered momentarily. This enormous current rush was sufficient to explain many of the difficulties, but this was not all the explanation. The oscillograph tests also showed that this short-circuit current would be maintained at almost its maximum value for a very considerable period, due to the cage winding on the rotors of the generators. Apparently this current at the first rush, was not appreciably greater than that on the machine before the dampers were added, but without the dampers the field was killed more quickly by this enormous current, so quickly that apparently the breakers did not open until the current had fallen somewhat. However, with the heavy cage winding on the field structure, secondary currents were set up in this winding, tending to maintain the field strength, and thus the current rush was maintained at almost full value for possibly 20 to 30 alternations. These oscillograph tests indicated very clearly that the armatures of these generators did not have nearly so great internal self-induction as our calculations indicated.

Meanwhile, the generators in the power house had been suffering from the tremendous shocks which accompanied short-circuits on the line. There is necessarily considerable local field around the end-windings of all these machines, and this stray field is especially large on machines with a small number of poles, and, in consequence, high ampere-turns per pole. These stray fields at the ends tend to exert a bending or distorting effect on the end-windings. In any given machine the distorting force varies as the square of the current carried by the coils. Our experience with the windings on these machines indicated that they were being subjected to enormous forces in the end-windings. The oscillograph tests gave an indication as to the amount of this force. As the machines could give about 15 times full-load current momentarily on short-circuit, the force acting on these end windings would be 225 times normal; in this case, therefore, these forces were so great that it became a serious problem to devise a type of bracing on the end-windings sufficient to withstand such a force. It should also be borne in mind that probably as many short-circuits came, in one day, on these generators, as the ordinary high-voltage, power-house generator is called upon to sustain in one year. While ninety-nine shocks out of a hundred might not be sufficient to do damage, yet if the shocks occur frequently enough, the hundredth one will soon be reached. In our endeavors to support these windings against movement, probably the most complete system of bracing ever applied to alternating-current generators was developed and used on these machines.

But in spite of this there was evidence of movement at times. It thus became evident that some method of limiting this short-circuit current to the value originally intended; namely, about six times full-load current, would have to be applied. This was done by placing an unsaturated choke-coil, or impedance coil, on the trolley side of each machine. This coil takes up a comparatively small voltage under normal operation, but in case of a short-circuit, the electromotive force generated in it is sufficient to limit the current rush to less than half the value it would attain without this coil. Thus as the shock on the end windings of the generators varies as the square of the current, it is evident that cutting this current in half would cut the shock to one-quarter of its former value, which, with the method of bracing used on these machines, would mean the difference between good and bad.

When these choke-coils were installed, the results on the power house were evident. The shocks on the machines were very greatly reduced, so reduced that we do not fear future trouble from this source. It is interesting to note that No. 4 machine; that is, the 4260-kilovolt-ampere generator, referred to before, was put in service a considerable time before the choke-coils were installed, and it went through the most severe short-circuits ever encountered on this system. Its armature winding has never shown any distress. This is partly because, in the design of this machine, the difficulties to be overcome were known, and the remedies could be applied in the most suitable manner.

An interesting point in connection with the use of the cage windings on these generators, is that the apparent regulation of the system has been improved. This was anticipated, but the actual result in practice was more pronounced than was expected. In installing new rotors for these machines with the heavier cage dampers, the inherent regulation of the generators was made somewhat poorer than before, partly in order to accommodate certain structural features in the rotor. It was anticipated that the cage winding with its damping effect would, to a certain extent, mask this poor regulation by making the machine sluggish as regards fluctuation in voltage with sudden variations in load. In practice it was found that, with the later rotors with their poorer inherent regulation, the average regulation of the system was considerably better than before, thus indicating that most of the disturbances in the voltage, when the old rotors were used, were due to sudden changes in load, while the slow variations were taken care of by the automatic regulators. With the new rotors the voltage changes are so slow, that the Tirrill regulator has plenty of time to act before any serious disturbance can take place.

It must be borne in mind that in one way this New Haven power-house installation was more difficult than anything undertaken heretofore, and that is, in the use of 11,000-volt generators with one terminal connected directly to ground. Taking this condition into account, together with the enormous current rushes with consequent shocks on the winding, and the single-phase operation of units of such large capacity, it may reasonably be claimed that this was the most difficult case of alternating-current generation ever undertaken.

L. B. Stillwell: The writer of the paper is just emerging from a long and most trying contest with difficulties upon a conspicuous stage. The installation for which he is largely responsible did not attain, during several months of initial operation, that degree of reliability which the traveling public has a right to expect, and, naturally, it has been subjected to severe criticism. It is probably as a result of this criticism that Mr. Murray's paper is so constructed as to convey an exaggerated idea of the shortcomings of the system, while it fails perhaps to emphasize sufficiently the far-reaching importance of the fact that the single-phase alternating-current system is in actual service in the operation of this very important terminal and that this operation is successful. Its success is shown by a reduction in aggregate train delays when the present service is compared with the steam service which it has replaced.

Papers which set forth fully and frankly not only successful achievements but also mistakes, are of especial value, and all who appreciate the difficulties which beset the path of progress, and particularly members of this Institute who, by training and experience, are qualified to form well-grounded and just opinions, owe it to the officials and engineers, who have laid before us these valuable data, to regard and discuss the facts judicially and without prejudice, and to interpret them fairly to the railway world and the general public. I am sure we shall all endeavor to do this, but I trust that a cautionary suggestion will not be regarded as irrelevant, for the Institute is under exceptional obligations for this paper, not only to Mr. Murray but to the officers of the New Haven Railroad Company who have authorized its publication, for an opportunity to weigh and utilize operating data of a kind not often published.

The engineering world is fortunate in the fact that the New York Central Railroad and the New Haven Railroad have not adopted the same electric system. Such an opportunity to compare the possibilities and limitations of the direct-current system and the single-phase alternating-current system is unprecedented, and, except in America, would be impossible. Whatever we may think of the wisdom, from an economic standpoint, of investing millions to learn from experience, the answers to questions which by German methods, for example, might be answered at much less cost (and there is much to be said on both sides of this question) we as engineers are in no position to complain of lack of opportunity to subject our ideas to the test of practice. If, therefore, we fail to agree with reasonable promptness upon fundamental specifications of that system of electric equipment, which we shall advocate as a standard in railroad practice to be kept in view and ultimately used to the exclusion of the many possible but less desirable alternatives, we shall never see electricity applied in the operation of railroads to anything like the extent which its possibilities justify from an economic standpoint. Unquestionably, each and every problem of electrification should be studied primarily with reference to local conditions and requirements. In any given case it is conceivable that such local conditions and requirements may properly lead to the adoption of a system other than that which would be chosen to solve the more general problem of electrifying not only a terminal and limited suburban zone, but a railroad division or a trunk line. But we are all convinced that during the next ten years railway electrification will not be limited to terminals and suburban operation, within narrowly restricted zones. In this light the decision of the New Haven officials to adopt, in the equipment of its New York terminal, a system applicable not only to terminals but also to trunk line operation, and the frank presentation of results attained, should assist powerfully in the prompt evolution of that system which is fittest, and therefore destined to survive.

When Mr. F. J. Sprague, with clear conviction and admirable courage, installed the multiple-unit system of train operation on the Southside road in Chicago, a most important advance was made in the art of railroading. The applicability of this system to trains requiring tractive efforts exceeding 50,000 lb., was demonstrated first in the New York Subway. The equipment of the New York Central has contributed to our demonstrated utilities an excellent third-rail construction. Incidentally it has demonstrated most effectively, throughout the steam-railway world, the fact that under the conditions existing within the zone electrified, electricity can do the work of steam and do it better.

The New Haven installation, substituting 11,000 volts for 600 volts and eliminating sub-stations, opens the door to possibilities of electrification of railways, with its assured improvement of service and economy of operation, over zones far beyond the reach of the 600-volt direct current.

Under the conditions of respective development of the direct-current system and the single-phase alternating-current system, the decision of the New Haven management to use the system adopted was a bold one; and its boldness was emphasized by the selection of the highest standard voltage for which suitable alternators had been constructed.

With reference to the plan adopted to improve the overhead trolley construction which converts this to a catenary construction with secondary suspension cable, I observed in Hamburg recently that the single-phase system in use in that city employs this method of supporting the trolley wire.

In electric operation of railroads the three great links of the chain between coal pile and draw-bar are the power house, the locomotive, and the system employed to transmit power from power house to locomotive. The power house and its equipment as constructed for alternating-current operation is practically identical with that constructed for direct-current operation. The only differences are that one phase is transmitted instead of three phases, and that one side of the circuit is grounded. The fact that the New Haven road has experienced an unusual number of delays, due to power-house trouble, is due to errors in the design of the generators and in the high-potential circuit-breakers. It seems that these errors could have been avoided by the exercise of that degree of care and foresight which an installation of such importance should receive. Of the ten electrical failures which occurred from July 1 to October 31, 1908, and involved delays exceeding 300 train-minutes in each case, eight were due, as Mr. Murray says, "to simultaneous failures of several circuit-breakers due to a short-circuit." This apparently might and should have been prevented by previous investigation of the ability of the circuit-breakers selected, to function under the conditions of service to which they were applied. Of the other two cases of serious failure, one apparently was due in part to a defective circuit-breaker; the other was a case of locomotive derailment.

So far as four months' operation may be taken as a guide, there is nothing in the record which is discouraging as regards performance of the electric equipment of the locomotives. If we except the delays due to circuit-breaker failures, the record is a good one.

It will add to the value of the paper if Mr. Murray will supply, for purposes of comparison, the record of train delays in steam operation during, say, the corresponding months of the preceding year. It is to be hoped also that the engineers of the New York Central will follow the example set by Mr. Murray and lay before the Institute a log of the New York Central electrification showing results attained as compared with preceding steam practice. This would afford an opportunity for accurate comparison of the results attained by the two contrasted electric systems in the operation of these important properties.

An incidental problem, which at one time appeared rather serious, but which apparently has been solved in an entirely satisfactory manner by the engineers responsible for the equipment of the New Haven Railroad, is the protection of existing telegraph and telephone systems from the effects of induction due to power currents.

Operation thus far has covered too short a period to justify final conclusions in respect of the reliability and safety of the high-potential overhead trolley and feeders, especially as the system has not been called upon to meet the conditions imposed by snow and sleet. If Mr. Murray will supplement his paper six months from now by a contribution giving the log of operation during the coming winter, and if some one representing the New York Central will favor the Institute with similar information, further conclusions of importance can be drawn.

E. B. Katte: If Mr. Murray could tell us in some detail of the troubles which led to the failures that caused the delays, it would help to a better understanding of the situation. He mentions two instances where no power was available on the direct-current third-rail. On July 8, the erecting men dropped a signal-mast across two tracks and broke down the third-rails, and there was a delay to trains while getting the third-rail up again. A few days later, on July 10, there was another delay of 13 minutes to the New Haven trains, due to no power on the third-rail. The third-rail was knocked down by a freight wreck, and there was some delay to traffic in clearing the tracks and in building up the rail. I think I can say that at no time has there ever been a lack of direct-current power available to apply to the third-rail when the third-rail was standing, and further, that when the third-rail has been knocked down due to a derailment or a wreck, it has been our usual experience that the third-rail has been re-erected and ready for service as soon as the wreck had been cleared and the track rails made ready for traffic.

H. P. Davis: In the operation of a railroad one condition stands ahead of all others; namely, the trains must be kept running and the schedule maintained at almost any cost. In this connection, Mr. Murray has presented, in a most striking way, a condition of vital difference between electric and steam operation of a railroad in the comparison which he has made of the delay chain. The importance of these conditions and the respective bearing of each of the links in this chain have received only slight attention in the literature of railroad electrification, but to those engaged in this work, it has been recognized as of vital importance, for the success of any large electrification is predicated wholly upon the infallibility of each link of this chain.

The usefulness of the electric locomotive is only realized with continuous and unfailing source of current supply. Realizing this, the working out of the problems presented in the power house, and particularly in the feeder and trolley system, are the most important presented to-day for our consideration.

As Mr. Murray points out, none of the separate links of this chain presents new principles, but when moulded together, some very astonishing characteristics have developed in the combination, when applied on a railroad system like the New Haven.

One of the preceding speakers has discussed most admirably the problems presented in the power house. It is my belief, however, that insurance in this respect really resolves itself into the question of sufficient reserve capacity. The really troublesome member of the chain is the link between the power house and the locomotive. The answer to the question: what is the best way to distribute current at high potential to a trolley wire over a busy railroad, with four or more parallel tracks, increased in places to several times this number, and connected at intervals to busy sidings and shifting yards?—is yet to be forthcoming.

For economy of conductors and high efficiency in transmission, the New Haven system will be hard to improve on. For in this system the 11,000-volt generators are connected directly to the line without the use of intermediate apparatus, thus eliminating all losses due to apparatus of this kind. The question whether it is the best for continuity of service and for operation with a minimum of train delays in case of trouble, time only will tell and is now, of course, open to argument and discussion.

The troubles encountered with the New Haven distributing and trolley system can be divided into two classes.

1. Those which have been constructional.
2. Those which have been a matter of operation.

Difficulties in the constructional features of the New Haven line have occurred, as pointed out by Mr. Murray. I look on this, however, as an unavoidable penalty, consequent to any pioneering installation, resulting from lack of knowledge and inability from previous experience correctly to forecast the conditions to be met.

It must be admitted that the problems encountered on this railroad are not simple, nor within the scope of ordinary or established practice. Many conditions have arisen impossible to foresee and guard against. It is only by encountering these difficulties that experience can be gained and data obtained to correct the constructional faults. Constructional difficulties, so long as they are not fundamental, do not affect the delay chain except to the degree of time sufficient to investigate the troubles and provide means to correct and eliminate them. The flexibility of the distributing and trolley system in operation is, however, of crucial and lasting importance in the delay chain.

Under the subject, " Distribution," Mr. Murray uses the word " disadvantages " in referring to this system. He should have used the word " difficulties." These have now been largely cleared up and have been shown to be only incident to a complete knowledge of what has been taking place in the system. In the last sentence of the paragraph above referred to, Mr. Murray says: " This operation can be effected, and at the time of this writing an effort is being made to obtain these results in practice." Since the preparation of his paper, most satisfactory results have been obtained with the scheme which has been developed, and as it introduces something new and likely to become general practice in power distribution, I shall briefly describe it.

Fig. 5, is a diagrammatic representation of the scheme of the New Haven trolley and feeder connections. By reference to this it can be seen that if selective operation of the circuit-breakers installed at each end of a trolley section can be obtained, in case of trouble in the section only the length of single track in that section will be affected. No delay to trains will occur, as they can be shunted on one of the adjacent tracks, except in case of not more than a single train, which might be in the section at the time of trouble. In principle this closely approximates the more advantageous feature possessed by the steam unit in confining delays to individual trains in case of trouble. Attention is particularly called to this to show that careful thought was given to this feature of the problem in planning the installation, and that its importance was recognized.

In practice, the failure of the line to operate as laid out was for a considerable time extremely puzzling. The destructive action of short-circuits on the circuit-breaking apparatus, on the generator windings, on the insulation of the line, and the failure of the circuit-breakers to be selective, could not be explained by any previous experience. A careful and thorough study of the nature of these disturbances by means of the oscillograph showed current surges of great severity, both as regards amount of current supply and the duration of the surge. It was shown, for instance, that with three generators connected to the line, it was possible for them to deliver at times as much as 120,000 to 150,000 kilovolt-amperes single-phase, and to maintain it for a period approximating one second.

It is not surprising, that circuit-breaking apparatus set for instantaneous operation failed, and that it was not selective in action, for the automatic trips on the trolley breakers were set to act at currents so low as to be out of all comparison with the amount supplied, and as the feeder and trolley arrangement is a series-parallel one, there was more than enough current in the distributing system to trip them all over the line. It was apparent, as soon as it was realized what the trouble was, that:

1. The generators must be provided with some sort of a shock absorber to protect against the first rush of current at times of short-circuit.
2. The opening of the circuit during times of heavy current surges must be prevented, because of the possibility of violent voltage surges from such operation with resulting destructive effect on the insulation of the system.
3. The current supply in the distributing system on short-circuit must be controlled, and automatic operation of circuit-breaking apparatus must be prevented until this condition was established.
4. The line sections must be made selective.
5. Corrective apparatus should be simple and automatic.

To meet the first condition, choke-coils were placed in the trolley phase of each generator, the coil being permanently in circuit. This coil is designed to cut down the maximum short-circuit current values to about 40 per cent, and the resulting mechanical shock to about 16 per cent.

To limit the current to be opened in the distribution system, and to make the selective action possible, was a more complicated problem and has been solved in the following manner.

Ohmic resistances have been placed in each of the three feeder leads going to the distributing system. These resistances are normally short-circuited by circuit-breakers. These circuit-breakers are electrically operated, and are automatic in opening and closing, being controlled by a single underload and overload master relay, so installed that the entire station single-phase output passes through it. The automatic tripping coils of the circuit-breakers on the overhead feeder and trolley systems are normally cut out; in other words, these circuit-breakers are normally non-automatic; the automatic operation being controlled from the power house.

In the event of a short-circuit or overload on the system exceeding a certain predetermined current, the master relay above referred to operates, causing the circuit-breakers short-circuiting the resistances to open. The resistance is thus automatically inserted in the feeder circuits. The value of this resistance is sufficient to reduce the current to a certain fixed and predetermined value. As soon as the resistances have been inserted in the feeder circuits, a feeder wire connected with all of the trolley and feeder circuit-breakers is then automatically supplied with potential. This at once makes the circuit-breakers automatic throughout the feeder and trolley system.

Each of the trolley sections terminates at anchor bridges. These bridges serve the double purpose of anchorages for the trolley structure and of sub-stations for current distribution to the adjacent trolley sections. Current is fed from the power house to the bus-bars on these bridges by feeders supplemented by the entire trolley system. The whole distributing system is, therefore, multiplied at each anchor bridge. Each end of a trolley section connects to the bridge bus-bar through a single circuit-breaker. In case of a short-circuit in a section, it is evident that these two circuit-breakers will have the total current flowing divided between them. As the remainder of the system is in multiple with the bridge bus-bars, these two circuit-breakers will be carrying much more current than any others.

With this circuit condition established, it is only necessary that the power-house resistance shall be so proportioned that it will allow sufficient current to flow to trip the two circuit-breakers in the trouble section, and yet limit the amount of current to less than enough to trip any of those in parallel in the remainder of the system.

With the current reduced to this definite amount, it divides through the system properly, concentrating at the point of trouble. The amount of current is, as pointed out, less than the amount required to operate any of the circuit-breakers, except those adjacent to the point of trouble. As soon as these two breakers open, the troublesome section is disconnected, and the current in the power house will at once drop to normal value. When this happens, the master relay at the power house will act again, and cause the circuit-breakers shunting the resistance to close and short-circuit it. At the same time, the feeder wire connecting to the automatic trips on the circuit-breakers of the trolley system will be opened, thus leaving the whole outside system again non-automatic. This entire cycle is accomplished automatically (without the use of delicate or complicated apparatus), power never being removed from the line. The whole cycle takes only about two seconds.

If it happens that one or both of the circuit-breakers at the ends of a trolley section fail for any reason to operate, and thus do not clear the line, it is arranged that the main circuit-breakers at the power house will operate and open the entire circuit. To accomplish this, the main power-house circuit-breakers are provided with time relays of the constant time type, set so that they will not act until a sufficient period has elapsed for the trolley and feeder current breakers to open. This condition is, however, of rare occurrence.

The use of a scheme of this kind, in which ohmic resistance is thrown in series with the distributing system at the first rush of current to a short-circuit, has other advantages beside that of making the system selective; for it effectually prevents much line disturbance directly traceable to violent opening of heavy currents by circuit-breakers, and to making circuit again under these conditions. As the trolley and feeder circuit-breakers are normally disconnected, the circuit cannot be opened at the first rush of current. The resistance being inserted in circuit first, the current is lowered to a value which is sufficient to trip only the circuit-breakers on each end of the trouble section.

When the circuit-breaker does act, it is called to open a practically non-inductive circuit, so that voltage surges cannot be produced, thus safeguarding the insulation of the system. For the same reason when power is thrown on the line the resistances act to damp out any surging effect. This arrangement provides a method of limiting line disturbances; for no matter how severe the short-circuit, or how great the amount of power available in the generating station, the circuit-breaking apparatus cannot act under conditions which will destroy them or cause destructive effects in any part of the system. It also has the advantages of being entirely automatic in action and effectually overcomes, without service interruption, effect of flashes, which are common in a system like the New Haven. In a case of this kind, when no permanent ground exists, the automatic insertion of the resistance into the line will usually cause the flash to be extinguished without interruption of service and without harmful effect. This method is simple, automatic in action, and most effective in controlling and regulating destructive line effects, and will, I think, either in this form or in some modification, be used quite generally in future in distributing systems of this kind.

Philip Torchio: In the paper, the main cause of the considerable troubles due to circuit-breakers is ascribed to the fact that the type of breaker used had the oil-tank grounded. The author suggests that the lining of these tanks should be improved. I would ask why, in the corrective changes introduced to overcome this deficiency of the circuit-breakers, no consideration was given to the obvious types of circuit-breakers which have the oil-tank insulated from ground. I would also like to ask why, in a single-phase electrification, three-phase generators were used in preference to single-phase generators? Would it not have been preferable to adopt single-phase generators? By using single-phase the company would eliminate, of the numerous generator troubles, those troubles which seem to have caused repeated rebuilding of the field of the generators on account of local heating resulting from the magnetic distortion due to the single-phase load upon three-phase generators. This magnetic distortion would have been considerably less with a single-phase generator.

C. P. Steinmetz: The importance of to-night's paper and discussion consists in the explicit and concise information which it gives as to the progress in the development of the electrification of a part of one of the country's trunk lines by means of the single-phase alternating-current motor. It is especially gratifying to see that the statements which have been made by unbiased engineers based upon theoretical considerations, have now been verified by practical experience, and that heavy railroad work can be handled by single-phase, alternating-current motors, though obviously not with the same high drawbar pull per ton of locomotive weight, and, possibly, at least for the present, not with quite the same reliability of service.

This I believe establishes the single-phase alternating-current motor as one of the pieces of apparatus by which the future electrification of our country's railway systems will be accomplished. The drawbar pull per pound of motor weight of the single-phase, alternating-current motor must necessarily be lower than that of the direct-current motor, because in the alternating-current motor the magnetic field pulsates between zero and a maximum. The same motor, when energized by direct current, with the same maximum magnetic flux, would give 41 per cent more output. It is important, therefore, to know that in those cases where direct-current motors cannot be used; for instance, where heavy train units at infrequent intervals traverse long distances, the work can be done by alternating-current motors. I believe when the time arrives to electrify our transcontinental railways, this will be done by means of the alternating-current motor.

My conclusions from the evidence and data now available are the same as I have expressed before; that those railway problems which cannot be handled by direct-current methods must be solved by means of the alternating-current motor. But where direct current can be used equally as well as alternating current, the higher weight efficiency; that is, the greater drawbar pull per pound of motor weight afforded by the direct-current motor, necessarily makes this motor preferable from the electrical engineer's point of view.

It is interesting, then, to observe that in those immediate railway problems which can be handled by alternating-current motors as well as by direct-current motors, the decision whether one or the other type of motor should be used has shifted from the electrical engineer to the railway engineer or the railway manager. This decision will be determined by the economic question whether higher weight efficiency, that is, higher draw-bar pull per pound of motor weight and reliability of service, should be sacrificed because of the possibility, at some future time, when the electrification of railroads for long distances is proposed, of being able to extend the same system of operation over remote parts of the road, and also to other roads where direct current can not be used; or whether the higher weight efficiency of the direct-current motor, and the at present greater reliability of service, compensates for the possibility that at some future time with the advance of the electrical industry and the extension of railroad electrification, we may have to replace the locomotive equipment, or change locomotives at the end of the direct-current zone and the beginning of the alternating-current zone.

Minor M. Davis (by letter): It is possible that Mr. Murray's remarks about the effect of single-phase currents upon telegraph and telephone wires will be misunderstood. Transformers are by no means completely corrective, although to some extent they are beneficial. My understanding is that the transformers on the New Haven road are applied only to single-wire operation and that they do not neutralize the disturbing effects of the single-phase electric current sufficiently to permit the operation of duplexes, quadruplexes, and printer circuits upon the wires to which Mr. Murray refers. This means that one-half or two-thirds, or even a larger proportion, of the carrying capacity of the telegraph wires is destroyed, notwithstanding these transformers. To apply these transformers, several otherwise idle wires must be carried upon the poles, and these wires and transformers are objectionable. They complicate single-wire equipment and prevent the operation of multiplex systems. Although Mr. Murray refers to the pilot wires and transformers as "not costly", they mean considerable expense. It is more or less of an open question, as I understand it, whether it is the duty of a corporation to prevent its electric current interfering with the current of another company, or whether it is the duty of the latter to provide its own protection. I understand the highest court in New York state has ruled in one of its decisions that if a party:

> Collects for pleasure or profit the subtle and imperceptible electric fluid, there would seem to be no great hardship in imposing upon it, or him, the same duty which is exacted of the owner of the accumulated water power: that of providing an artificial conduit for the artificial product, if necessary to prevent injury to others.

Certainly there is no disposition on the part of the telegraph companies to retard in any way the advancement of the art of applying power. They are willing to exercise all reasonable patience, and they would be glad to have the power companies adopt a simple and inexpensive remedy for these disturbances; but although the writer and his associates have given much attention to the subject they know of no remedy which is at the same time simple, inexpensive, and complete.

B. A. Behrend: The single-phase generators have been a source of great trouble, the cause of which was not immediately recognized. The heating of the units due to the variation in the armature reaction was overcome by low-resistance circuits placed in the rotor surface, and acting as the secondary winding of a transformer. This winding, however, reduced the effective reactance on sudden short-circuits, resulting in great increase of the momentary short-circuit currents, and in disturbing forces corresponding to the square of these currents on the armature conductors. Both these difficulties were minimized by reactance coils placed in the feeders. I should like to ask Mr. Murray what reasons have prevented the adoption of two-phase generators in the New Haven case, and what are, in his opinion, the objections in general to two-phase generators for this class of service.

The difficulties experienced with the collection of current are not specifically connected with the single-phase system, but they are, at present, an essential part of all systems of electric railroads employing high potential. An inspection of the line and the flashes produced by imperfect contact, which I had an opportunity of making since the steel wire has been installed, indicated that great improvements have been made in that direction.

The difficulties experienced with the single-phase locomotives and the motors in particular, are the vital issue. Through the courtesy of the New Haven road I had the opportunity of observing the motors of a heavy two-engine train between the Grand Central station and Stamford, Conn. The operation on both direct-current and alternating-current power is very much alike, as far as sparking is concerned. In fact, the eight motors operated very well and without undue sparking, both in starting, and at about 65 miles an hour. No unbiased observer of experience with electric-traction apparatus can close his eyes to the fact that, if such results can be obtained in the first large plant of this nature, very much better results may be predicted for the system in the future.

The temperature rise on alternating current is naturally greater than on direct current. It would be extremely interesting to obtain comparative figures from Mr. Murray on this subject. This matter is intimately tied up with the question of efficiency to which Mr. Murray has alluded, without, however, going into figures.

H. F. Parshall (by letter): In 1901 I was appointed by the Board of Trade a member of a tribunal, presided over by the Hon. Alfred Lyttleton, to determine whether or not such an alternating-current system as proposed by Ganz was suitable for the underground railways of London. The matter was most thoroughly investigated from every point of view. I came to the conclusion, which conclusion was supported by the umpire, that the alternating-current traction system was not the best for working a heavy passenger service. My reasons were that the alternating-current motor is at serious disadvantage as compared with the direct-current motor during the period of acceleration; that from the nature of things it is inherently heavier and more complicated, and must be more expensive as regards first cost, and vastly more costly to maintain; that the sole advantage incident to the use of an alternating-current motor is in the supposed saving in the first cost of the collector system; that when a high-tension system is safely installed it must be more expensive than a third-rail system—consequently the whole reason for the existence of an alternating-current system disappears in the final result. Since 1901, material progress has been made in the design of alternating-current motors, and the greatest credit is due to those engineers who have brought the alternating-current motor to its present high state of perfection. Nothing, however, has occurred that would justify me in departing from my original conclusion. In the beginning it was suggested that the alternating-current motor would not have a commutator, and that this evil inherent to a direct-current motor would be dispensed with. In the final result, however, a commutator 50 per cent bigger than the direct-current commutator has been found necessary. The perfected alternating-current motor weighs about 40 per cent more, has all the objectionable features of the direct-current motor and practically twice over, on account of the construction of the stator. The air-gap of this motor in practice may not exceed 60 per cent of that of an efficient and well-designed direct-current motor. Higher armature speeds have generally been found necessary. With the features above mentioned inherent to alternating-current motors, the first cost of a train equipment, as also the maintenance, must be nearly double that of a direct-current equipment of like capacity. Further, I doubt in the heaviest class of electric traction installation, such, for instance, as that in the New York Subway, whether the alternating-current motor can be made to do the work satisfactorily at all, over a period of time. In addition to the above well-known features of the alternating-current motor the use of a train transformer has to be considered. What ultimate size and form this transformer may take in heavy, high-speed work remains to be determined, since with a jumping contact the strains on this transformer are practically incalculable.

I think it must be conceded by all, that the alternating-current train equipment, *per se*, is inferior as regards cost and maintenance to a direct-current train equipment, and that it gives no tractive advantage. The advantage, then, must be external to the train, and confined to the system of transmission, or more properly the collector system, since the transmission system would naturally be the same either with direct current or alternating current. As a matter of experience it has been found that the cost of a high-tension overhead system properly installed is greatly in excess of that of a third-rail system, and having regard to the exceedingly numerous difficulties, as put in evidence by Mr. Murray's paper, it cannot be contended that the action of an overhead high-tension system is anything like as satisfactory as the third-rail system. Considering all that has gone on in the New Haven system up to September 5, 1908, which appears to be the termination of Mr. Murray's log, there is absolutely nothing to indicate that during 1909 some such series of occurrences are not again to be expected. All that has occurred may be expected to occur again in some form or another, since electric traction apparatus does not manifest all its faults during the first few months of operation.

The absolute lack of commercial data as regards the cost of the installation, cost of operation, and cost of maintenance vitiates any conclusion that may be drawn other than that the system when compared with such a system as has been installed by the New York Central is entirely unjustified by the practical results. Some two years ago I was shown a diagram of the feeders and electrical system of the New Haven Railroad. I could only state that it transgressed every established principle as to safe and reliable working, and that if the system as a whole were to be in any way justified, there was no excuse for starving the system of feeders to such an extent, and mixing up what might be properly termed the collector system with the transmission system. In my judgment many of the troubles that have been experienced would have been avoided had the alternating-current system been installed in accordance with accepted alternating-current practice, with a high-tension system of transmission, say from 20,000 to 30,000 volts with sub-stations at the different sections and a trolley voltage not exceeding 5,000. The cost need not have been greater, and the system would have been far and away safer, in that the different sections could be independently controlled. Further, the volume of stray current in the earth causing electrolysis and induction troubles would be greatly lessened. The system has apparently been designed on the assumption of the undisputed right to the wholesale use of the earth as the return circuit.

As regards the difficulty experienced with the generators, the New Haven line is not unique in this respect. With a feeder system, however, designed as above, it is doubtful whether the generator windings would have been displaced, although the use of a train transformer and a jumping trolley contact may be expected to introduce unexpected phenomena. Looking at the alternating-current system broadly, it seems to me that its advocates are more interested in the electrical than the mechanical aspects of the electric traction field, and its opponents are those who have had the longest experience in traction matters and are not inclined to experiment with the fundamental elements of a traction system, considering the sole and total advantage that can seriously be suggested is cheapness. Why a railway should spend its tens of thousands per mile endeavoring to make a system as safe as possible, and then attempt to effect a minor saving by the introduction of a system bringing with it fatal and most experimental elements, is beyond my comprehension. Mr. Murray states that there have been no fatalities so far as the public is concerned, nor would one expect such fatalities considering the short time the system has been in operation and the overhauling to which it has been subjected. In the fulness of time, however, there may be collisions, derailments, or some such occurrence, in which the overhead system will be disarranged, at the same time as passengers debouch from the trains. The result of an occurrence of this description will in my judgment sound the death-knell of the alternating-current system so far as a passenger service is concerned. It is stated that there have been several fatal accidents to the employees. The number is not given although I have been informed it is considerable, and in the nature of things it must be so. Men are required to maintain a good train service, and throughout the world it is recognized, sacrifice their safety to the public needs. When, however, an element of certain death so insidious as a system of high-tension conductors is made an accessible and working part of a railway system, nothing but a long list of fatalities can be expected. No amount of rules and regulations will ever suffice to make a system, having inherent fatal elements, safe to the employees, in fact, having regard to the exigencies occurring in normal railway practice, rules and regulations in general may be taken as existing in form and not in substance.

I regret to have to make use of Mr. Murray's expression in stating as my conclusion that his paper furnishes conclusive material of how not to install a traction equipment, and it appears to me the only way he could possibly justify his belief as regards the commercial usefulness of the system would be by the publication of such commercial data as would show that the alternating-current system as regards first cost and cost of operation has such material advantages over the direct-current system that all such occurrences as are recited in this paper are as nothing considering the proved commercial advantages.

To summarize, I may say I am responsible for the electric traction installation of the Central London Railway. While this was one of the first systems to employ multiphase transmission with synchronous converters and a direct-current collector system, none of the troubles instanced by Mr. Murray has been experienced, and since the installation opened there has never been, all told, an hour's delay caused by any electrical failure; in fact the whole service has been characterized by the utmost regularity and freedom from trouble either electrical or mechanical. Since the installation was designed, considerable improvement has been made in motors, so that with the interpole construction higher voltages are practical. Also with the under-running protected third-rail system these higher voltages can be safely employed without risk either to the public or to the staff of the company. Having regard to the history as regards reliability and freedom from trouble with a third-rail system, as also the wider field over which it is applicable on account of the above improvements, I am of the opinion, so far as the working of passenger services over the widest districts is concerned, that the direct-current system of electric traction is superior to the alternating-current system for dense traffic movements, and in the majority of cases in which electricity can properly compete with steam. That there are problems in connection with freight haulage over long distances in which the multiphase system possesses commercial advantages, I do not feel in a position to deny, although an examination of the individual cases which have been submitted to me has not in my judgment made this an indisputable fact. As regards the single-phase, alternating-current system there may be special applications to which it is commercially applicable. I have given the deepest study and thought to various problems, but in each case the use of the single-phase system has proved illusory. I do not wish to go further than to state that I have not found a problem to which it is properly applicable, considering all the facts as I now see them. I have recently pretty thoroughly investigated railway matters in the United States and, considering the inherent properties of the different classes of apparatus, my conclusion is that the direct-current system of electric traction is superior to all others, and is likely to remain so for many years to come.

A. H. Armstrong: The enthusiastic coöperation of the engineers of the railroad company and the manufacturing company has brought this alternating-current installation to its present engineering success, and nothing but admiration can be expressed for the courage and ingenuity shown in overcoming the difficulties met with in operation.

It is my understanding that the alternating-current single-phase system was adopted in the New Haven installation in order to afford an opportunity to demonstrate the suitability of that system to the possible further electrification of the road. It is further my understanding that preference was given the system chosen, for the reason that it was considered to be a cheaper system as to first cost than either the 600 or the 1200 volt direct-current systems, proposals upon which were also received.

As pointed out in the paper, the overcoming of difficulty after difficulty met with in operation, is positive proof of the progress made in perfecting the complete electrical and mechanical equipment. Material changes have been made both in the construction of the locomotives as well as in the generating and distributing system, so that the final installation as now in operation is widely different from the original plans, both as to engineering details as well as to first cost of the apparatus finally installed. Certain changes were made in the generators, in the switchboard controlling apparatus, etc., in order to obtain increased capacity and reliability, these changes presumably resulting in a somewhat increased cost over that of the original plans.

The trolley construction as now installed embodies certain improvements, as pointed out in the paper. A considerable departure must have been made from the original plans in order to account for the large difference in the estimated cost of this part of the installation, before and after its completion. In the December issue of the *Street Railway Journal*, 1905, Mr. Westinghouse publishes his estimate of $12,436 per mile as sufficient to cover the cost of contact line on a four-track road between Woodlawn and Stamford. Presumably this figure covers the same labor and material, estimated to have actually cost nearly $55,000 by Mr. Wilgus in his paper before the American Society of Civil Engineers. In view of the wide discrepancy between the cost of the trolley construction as proposed, and as completed, it would be instructive to know which of the two figures is nearer the actual construction costs.

The locomotive changes found necessary are most frankly stated by Mr. Murray. They have been extensive, and those listed are presumably the changes after the original locomotives were received, and developed defects in practical operation. The author also states that the locomotives have fulfilled their guarantees as to their capacity to haul 200 tons trailing. A log of the run giving the time required to haul 200 tons trailing with 45-second stops averaging 0.451 per mile would have been most interesting. If the present locomotives fulfil the guarantees as to performance of a 26 miles per hr. schedule with a 15 per cent margin in time, it may account for the increase in locomotive weight from 68 tons total, as proposed, to 102 tons as now in operation.

It is reasonable to suppose that the cost of these locomotive units has increased in at least a like ratio, although it is my experience that it costs considerably more to construct an alternating-current single-phase locomotive than one of equal weight equipped with direct-current motors. We are therefore impressed with the fact that in this completed New Haven alternating-current installation many changes have been found necessary, not only in the apparatus as originally furnished, but also in the original plans upon which the decision to adopt alternating-current motors was presumably based.

Published figures of trolley construction indicate that this item was estimated at but one-fourth the actual cost, while the increase in locomotive weight would account for at least an increase of 50 per cent in the cost presumably used in the original estimates. The large increase in cost of these two items may be accounted for, either by an error in the original estimates, or by the assumption that continued development of the alternating-current single-phase system has called for more liberal construction, greater factors of safety, and a readjustment of ideas previously formed.

While preference may originally have been given the alternating-current motor for this installation, with the limited knowledge of its operation then considered, I find it hard to justify Mr. Murray's present enthusiasm for a system that appears to be in many ways unsuited to the New Haven conditions. In this connection, I will state that the weight of the locomotives as now installed approaches fairly close to the weight of the locomotive which I estimated was required to perform the service originally specified.

No operating company can afford to submit to the traffic interruptions and delays outlined in Mr. Murray's paper, unless the object to be attained be of the most far-reaching character. In view of the fact that the alternating-current trolley construction as installed on the New Haven apparently costs 80 per cent more than a 600-volt third rail, and the alternating-current locomotives cost at least double for the same service performed, it is instructive to compare the operating results secured with the two systems.

Comparative Data Alternating-Current and Direct-Current Locomotives

	600 volt d.c.	New Haven 11,000 volt a.c. 600 volt d.c.
Weight total	94.5 tons	102 tons
Weight on driver	68.5 "	77 "
Number of motors	4	4
Total h.p., 1 hr. rating	2200	1000
Guaranteed trailing load	400 tons	200 tons

The locomotive weights given above, include pony trucks on both locomotives.

Having approximately the same total weight, a single New York Central locomotive unit has a capacity equal to two New Haven units, although the motors of the latter are cooled by forced ventilation, while in the New York Central motors natural ventilation is employed.

Much has been said to-night about the New Haven locomotives meeting their guarantees, and handling a trailing load of 200 tons, and also the wisdom of not adopting a unit of any larger capacity on account of the average weight of trains on the New Haven road being approximately 212 tons. I have drawn up a list of the trains drawn on November 21 by the New York Central locomotive unit, a single unit. This list comprises a majority of the trains handled that day which exceeded the guaranteed trailing load of 400 tons.

NEW YORK CENTRAL LOCOMOTIVE PERFORMANCE

Trailing Loads in Excess of 400 tons, Nov. 21, 1908

Train No.	Weight	Train No.	Weight
Special	475 tons	7	480 tons
"	570 "	71	471 "
54	416 "	Special	571 "
14	430 "	"	424 "
18	424 "	"	558 "
28	434 "	"	483 "
26	491 "	"	420 "
16	468 "	"	475 "
10	445 "	"	506 "
58	628 "	"	482 "
22	411 "	"	460 "
23	467 "	"	550 "
27	441 "	"	572 "
31	524 "	"	536 "
17	439 "	"	558 "
33	475 "	"	570 "
9	454 "	"	575 "
19	668 "	"	571 "
3	515 "	"	650 "
41	465 "	"	519 "
29	541 "	"	600 "
59	516 "	"	560 "
37	420 "	"	569 "

It is to be noted that every one of the above trains hauled by a single direct-current New York Central unit would have required two New Haven alternating-current units.

The reliability of the direct-current and alternating-current system is clearly illustrated in the following table of train-minutes delay for the months of July to October 1908, inclusive. The alternating-current figures are taken from Mr. Murray's paper and the direct-current figures from the New York Central operation. The figures for the New York Central operation are not official, but doubtless Mr. Murray can correct any error with his possible access to official records.

TRAIN-MINUTE DELAYS, CENTRAL AND NEW HAVEN, 4 MONTHS, JULY-OCTOBER, 1908

	New York Central	New Haven
July	41	2281
August	53	1611
September	18	893
October	48	910
Total	160 min.	5695 min.

The New Haven figures make no note of delays of over 300 minutes. The Central figures include delays due to locomotives or operators only, as during these four months there were no delays due to generating, transmission, sub-stations or third rail and feeder distribution systems, in spite of the alleged complication of such a system.

It is difficult to assign any exact field for alternating-current motors, but it is my conviction that under the conditions outlined in Mr. Murray's paper, the installation of the alternating-current system is a mistake, and that Mr. Murray has not shown any reason for his statement that if three years ago he was in favor of the alternating-current system for the conditions outlined, he is much more in favor of it to-day. In view of the fact that the generating station does not cost less, that the overhead trolley construction costs 50 per cent more than a third-rail construction, that the locomotives cost fully double that of corresponding direct-current locomotives of the same capacity, it is hard to find any justification for the enthusiasm displayed for the alternating-current system installed under the conditions outlined in the paper.

N. W. Storer: In his remarks, Mr. Armstrong makes comparisons between the New York Central and New Haven locomotives. A table is given in which the weight of the New York Central locomotive is stated to be 94.5 tons, while that of the New Haven locomotive is given as 102 tons. Following this table is the statement that the locomotive weights given include pony trucks on both locomotives. As to the exact weight of the present New York Central locomotive, I am unable to give any information, except that it has usually been given as about 97 tons, instead of 94.5. However, it is well known, and has been announced in the technical press that all the New York Central locomotives are to be reconstructed by changing the two-wheel trucks to four-wheel trucks, and that the weight of the modified locomotive is between 115 and 120 tons. Of course, as the New Haven locomotives haye been modified so much faster than those on the New York Central, their weight may be temporarily slightly in excess of the New York Central locomotives, but, to be strictly fair in the comparisons, Mr. Armstrong should have given the final weight of the locomotives.

Mr. Armstrong also states in his table that the guaranteed trailing load of the New York Central locomotive is 400 tons, and that of the New Haven is 200 tons. The trailing loads of these two locomotives should, of course, be compared in the same kind of service. The New York Central guarantee was based on express service with long layovers. The New Haven locomotive was guaranteed to handle a 200-ton trailing load in *continuous local service.* These locomotives are guaranteed to handle 250 tons in local service as far out as Port Chester, a distance of 25.6 miles from Grand Central station. They also handle 300 tons trailing load in local service to New Rochelle. a distance of 16.6 miles. In express service, which is the class of service on which the New York Central guarantee was based, the New Haven locomotive handles easily a load of 300 tons.

In Mr. Armstrong's table, therefore, there should be substituted a weight of not less than 115 tons as the weight of the New York Central locomotive, and 300 tons as the guaranteed trailing load of the New Haven locomotive.

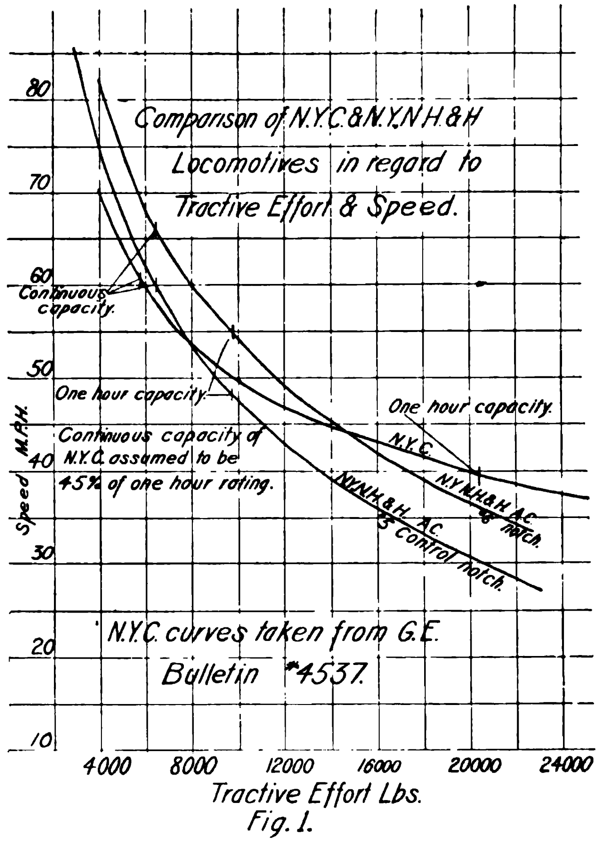


Fig. 1.

No one is more anxious than I to give the New York Central locomotive full credit. I believe that it is doing most excellent service. But what is the service? Simply switching—nothing more or less. There are a few trains that have to be hauled out to Wakefield, a distance of about 12 or 13 miles. The remainder of the trains are changed at High Bridge, which gives a distance of only about 6 miles for the locomotives to handle the majority of the trains. This is anything but trunk-line service. The distance is so short, and the service necessarily so intermittent, that the locomotives should be able to work at a load far in excess of the one-hour rating without overheating. They are, therefore, able to handle the heavy trains without double-heading, as was intended when the electric zone was laid out. It is a noteworthy fact that since the wreck of the Brewster express, these locomotives have never been run double-headed. It is also noteworthy that after that date the limiting speed on the New York Central electric zone was 45 miles per hr. until recently, when it has been raised to 50 miles per hr. Under these circumstances, it is impossible to make comparisons between them and the New Haven locomotives, which are and have been operating in all classes of service, running most of the time at very high speeds. If the New York Central locomotives should be reconstructed so as to make them safe for high-speed express service, and the company carries the electrification to the limits originally considered, there may be a chance to compare the two locomotives on a fair basis. At present to talk about the enormous trailing loads handled by the New York Central locomotives is misleading.

Mr. Armstrong mentions the great increase in weight of the New Haven locomotive over what he thinks was the original estimate. He is in error in regard to this point. The original estimate for this locomotive was 78 tons. The first one built weighed 89 tons. The electrical equipment installed in the first locomotive weighed 10 or 15 per cent more than the estimate, and has been increased very little since then. The additional weight which has been put into these locomotives has been almost entirely in mechanical parts, steam heaters, etc. One thing must be borne in mind: the electric locomotive of the future will be designed so as to have good riding qualities; and to secure them, the mechanical parts will form a much larger part of the total weight than was thought necessary or desirable two or three years ago. This feature in itself will bring the weights of the single-phase and direct-current locomotives much nearer together.

When discussing the question of cost, why is it that the increase in cost of the New Haven installation over and above the supposed estimates is so much under discussion, while in the New York Central installation nothing is considered except the cost of locomotives, and possibly that of a straight section of third-rail. I should be more than glad to see the total official cost of the New York Central installation side by side with that of the New Haven. Without such figures, it is useless to discuss costs. However, it may be noted in passing that the statement of Mr. Wilgus in a paper read before the American Society of Civil Engineers, that the cost of electric current must be increased from 0.58c. per kilowatt-hour at the power house to 2.6c. at the shoes of the locomotive to cover the cost of operation, investment, etc., foreshadows some enormous expenses.

Concerning double-heading, much has been said about the number of extra locomotives required for the New Haven Railroad, on account of being obliged to operate 25 per cent of their trains with two locomotives. In this connection, a query arises in my mind. Why is it that the New York Central finds it necessary to buy 12 additional locomotives, making 47 in all, while the New Haven Railroad can get along with 41 locomotives of such an extremely small capacity as they are supposed to have, in handling all of their traffic as far as Stamford, it being borne in mind, of course, that the New York Central operates a large number of multiple-unit trains. Let us have some actual figures in regard to the New York Central installation, both in regard to cost of installation, cost of operation of power stations, sub-stations, distributing system, etc., and in regard to the mileages, etc., of their locomotives.

O. S. Lyford, Jr. (by letter): In developing its plans for electrification, the Erie Railroad Company has adopted the method advocated by one of the speakers. It commenced its work on a moderate scale on a branch line with 34 miles of track and 6 motor cars. This may seem at first to be insignificant compared with the New Haven system; but the mileage is 30 per cent of that of the New Haven, the number of motive power units 15 per cent, and the capacity of each motive power unit 40 per cent. The line is operated with 11,000-volt, single-phase trolley. It was constructed during the operation of the road by steam locomotives, and the division is now being operated by both steam and electric motive power. This line has been in operation 18 months with the following results:

Safety. One man lost his life during the first few days of operation by coming in contact with the trolley wire. This was an employee who had been carefully instructed but "forgot." There have been no other personal injuries to employees or to the public due to the electrical equipment.

Reliability. In October 1908 there were no detentions chargeable to the electrical equipment. In November there were four detentions of this character: one due to control trouble, two due to hot motor bearings, one due to breakage of air-compressor crank shaft. Three of these detentions were of a character preventable by more rigid inspection.

In December there had been no detentions up to the date of the meeting.

It should be noted that only one detention was due to a strictly electrical cause, and none was due to high voltage or to the single-phase features of the equipment.

It is believed that this record of nearly 2.5 months will compare favorably with any other electrically operated road. This record, however, was obtained only after the equipment had been in process of "tuning up" for many months. In the 12 months ending Oct. 31, 1908, there were 120 detentions chargeable to the electrical equipment, which detentions totaled 4800 minutes. These include a number of serious detentions due to the 90-mile, 60,000-volt transmission line over which power is received from Niagara Falls for the operation of this road. The record is much better than that of the steam equipment replaced.

Maintenance. There have been no transformer troubles, either on the cars or in the sub-stations. Motor troubles have been very few. There were no mechanical troubles with the contact wire. The wear on this wire up to August 1908 was from 0.004 to 0.009 in., at which rate it would be between 20 and 30 years before the wire would be worn back to the supporting clips. Stretch of the copper wire and abnormal wear at hard spots will no doubt result in a life materially less than this.

There has been no apparent deterioration of the galvanizing of the messenger cable and other galvanized parts, but such painting as was done on the overhead work has not lasted.

There have been no material troubles or expense caused by locomotive gases.

The life of the pantagraph shoes average about 15,000 miles.

Cost of operation. The operating cost under the usual transportation and maintenance headings of the Interstate Commerce Commission classification, averaged during last summer about 18 cents per car-mile.

It is difficult to make a fair comparison of the New Haven log of operations with a similar log of other existing electrical service. The records of such roads as come under the writer's observation vary widely, and the variations are largely due to conditions independent of the character of the equipment.

Multiple-unit service with trains of more than two motor cars shows up better than single-unit service, owing to the fact that many of the faults of car equipment can occur without causing detentions, the other equipment carrying the train through on time.

Some schedules are operated under conditions well within the speed limit of the equipment, so that time can be made up after a detention. On other roads the schedule is practically up to the speed limit.

In some schedules there is considerable layover of trains at one or both ends of the line, while in others, where the conditions permit and the equipment is worked hard, the layovers are short, and a car trouble cannot be remedied in time to prevent delays possibly affecting a number of trains.

If trains are operated on short headway, the train in trouble may be helped through by a succeeding train, thus minimizing the number of train-minutes delay, whereas with infrequent service a comparatively insignificant car trouble may show up badly in the log.

Equipment troubles increase with the speed, particularly troubles with the moving contact.

In an analysis of the troubles which actually occur in electrical operation, whether direct current or alternating current, it is apparent that the power station, transmission lines, substations, and distribution system, whether third-rail or overhead, cause but a small proportion of the train detentions. The record of the third-rail is especially good in this particular. Most of the detentions are due to trouble on the cars, and of these by far the largest proportion results from defective control.

In the light of this experience, it is believed by the writer that the New Haven troubles will soon be confined mainly to the locomotives, and these will not be serious. The forced use of the combined alternating-current and direct-current systems is an abnormal burden on this operation. In view of the fact above stated, that is, that most of the train detentions are caused by the control, it may be anticipated that the New Haven operation will not do quite as well in the long run as a straight single-phase service. The use of multiple-unit motor cars will help to improve the log.

The Erie record above quoted, gives no indication that detentions due to locomotive troubles with a straight single-phase system will be materially greater than with the direct-current system.

The maintenance cost is another matter. The greatly increased cost of the electrical equipment of the locomotive or motor-car must necessarily involve greater cost of repair and maintenance. The savings effected in other parts of the system must carry this burden.

W. N. Smith (by letter): The oral discussion of Mr. Murray's paper was so far given over to the particular merits of single-phase railway apparatus, and especially to contrasts between alternating-current and direct-current locomotives, that certain other equally important features of the paper were overlooked.

It was stated that the single-phase motor is the keystone of the system, and controversy upon this feature has been plentiful. The writer submits, however, that although the single-phase motor may figuratively be called the keystone of the single-wire, high-voltage trolley system, the whole system is a means to an end, the end being the minimum cost of motive power which it is possible to effect by means of this system of working conductors.

Comparing the relative amount of dependence placed upon each element of the electric motive power system as a whole, in the last analysis the chief element is the working conductor. In another sense also the working conductor is the fundamental point of departure in a railway system; for in the nature of things it is the means whereby power is supplied to a moving load through a moving contact, instead of to a stationary load as in all other varieties of electric power application. Of the three links in the chain, therefore, the one that is most truly characteristic of railway conditions is the working conductor.

The writer believes it will be conceded by all engineers who have had to carry any real responsibility for bringing a railway line into operative condition, and keeping it there, that the worst interruptions they have to guard against are those due to failures of the working conductors and the power distributing system. That part of the system which is housed in power house and sub-stations, while equally vital in importance with the line, is nearly always present in duplicate, so that failure of one piece of apparatus is quickly made good by substitution, and it is also always possible to duplicate the feeder distribution system; but working conductors, whether trolley or third-rail, are rarely installed in duplicate, and their failure usually means cessation of service until the defect can be reached and repaired.

An examination of the four months' record given by Mr. Murray, as well as records of several other heavy electric traction systems, both alternating and direct-current, show clearly that while the greatest number of train detentions is due to trouble with car apparatus, the detentions due to line trouble or other derangements of the power system are of far greater average duration than those due to car failures.

It is stated by Mr. Murray that it became evident at one time that electrical operation would cease if some change in the contact wire were not effected; but nowhere in the paper is it implied that such a result was at any time impending, by reason of the defects in locomotives or generators. It is difficult to see how greater emphasis could be placed on the necessity for any single element.

As between the merits of the alternating-current and direct-current systems for any given train service on a given road, there is no longer any doubt as to the possibility of determining in advance which system can be installed and operated for the least money; but, with respect to other considerations than those of cost, it is fortunate that engineers now have available the results of the New Haven Railroad experience to assist their judgment in solving heavy traction problems.

Reference has been made to the boldness of the New Haven Railroad in trying such a newly developed system on so large a scale. In the writer's opinion the boldest part of the enterprise was the adoption of a previously undeveloped system of line construction. Unlike the other links in the chain, this one could not be put out of commission for days at a time nor could it be taken into a shop over night and brought out in the morning. It had to be maintained *in situ* over a very frequent traffic and repaired only with the current off, and for the most part from a work train. The engineers who maintained the line in the face of all adverse conditions should be credited with the performance of a task which, however easy to conceive and calculate in advance, was the most difficult to carry out of any, in the maintenance of the whole system.

The means adopted by the New Haven engineers to overcome trolley difficulties seem to have been successful. There is no doubt that the conditions of catenary trolley operation on the New Haven road are so severe, particularly by reason of the frequency of very low bridges, that the fact that there is now a reasonable certainty of continuous operation bodes well for the permanency of the improved methods adopted. Experience on other single-phase systems where traffic and clearance conditions are less exacting, shows that the devices composing the system of working conductors and moving contacts are adequate and reasonably reliable. For railway systems where clearance conditions are severe, and requirements for continuity of service are the most exacting, it is probable that further improvement in mechanical detail of the working conductors and moving contact device will be developed to ensure even greater smoothness of operation. The improvements which experience has shown to be necessary in the manufactured apparatus comprised in single-phase generating and locomotive equipment, seem to have been well taken care of; but the apparently simpler yet more fundamental problem, of improving the moving contact system, still presents a field for the exercise of inventive genius.

Philip Dawson (by letter): On broad principles I agree with the conclusions at which Mr. Murray arrives, in which he pins his faith for general railway electrification, of urban, suburban, and long-distance lines, to the single-phase system. It is the very serious inconvenience possessed by the direct-current, third-rail system which I believe has brought about the introduction of the single-phase system. Had it not been for the latter system, the conditions are such that the directors and officers of the London, Brighton, and South Coast Railway Company could never have seen their way to adopt electric traction, if adopting it had involved the use of the third-rail system.

The form of construction is the double catenary, details of which are shown in Fig. 1. Figs. 2, and 3 illustrate the trolley wire after it has been put up and whilst it is only temporarily supported, before being finally fixed to the clips and droppers which support it from the two catenaries. The droppers are solid sherrardized steel wire, the shorter ones having a loop at each end, the longer ones consisting of two parts looped in the middle. This arrangement gives great flexibility for the trolley wire in the vertical plane, whilst ensuring great stability in the horizontal plane.

The catenaries are of galvanized steel wire cable $\frac{3}{8}$ in. in diameter. Each catenary is independently fixed by means of a turn-buckle and insulator, and supported from a saddle-piece resting on the top of the main insulator. The sag in the catenaries is calculated so as to produce absolute balancing at the supports. The principal insulator has to bear a dead weight which amounts to 850 lb.

All of the insulators are of the best quality porcelain. All of them have been tested, the principal ones to 65,000 volts, and the secondary ones to 25,000 volts for 30 minutes; 10 per cent of the insulators were tested mechanically. The whole construction of the catenaries, girder bridges, insulators, etc., is calculated so as to result in a factor of safety of ten.

The trolley wire itself is round, with sharp grooves at the sides into which mechanical clips fix, and its area is 0.197 sq. in.; its tensile strength being in all cases superior to 22 English tons, per square inch. (1 ton = 2240 lb.)

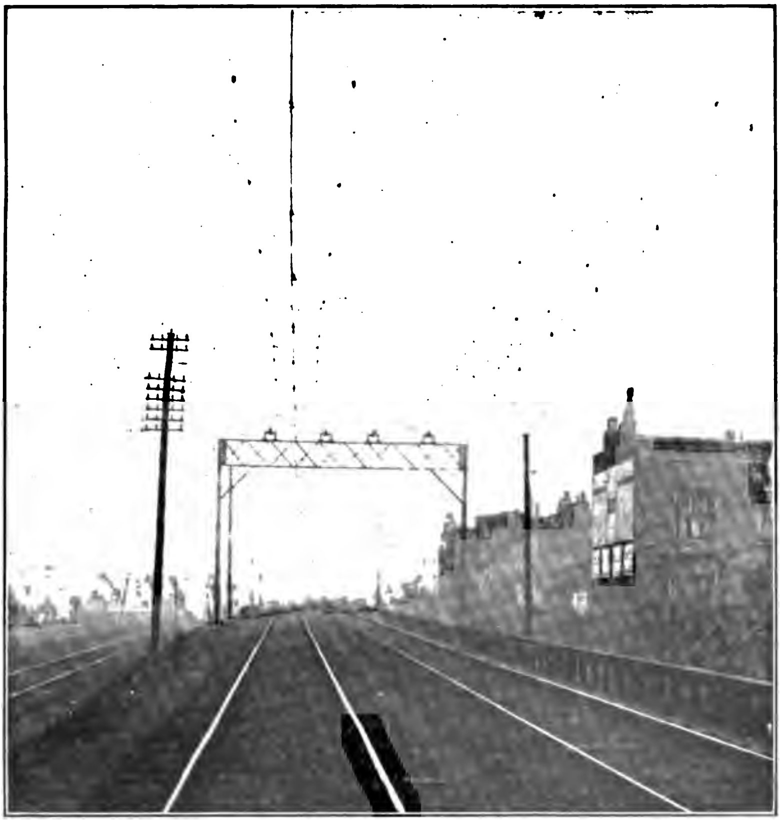


Fig. 1

In order to prevent the hammer blows from which the New Haven line has suffered, the whole construction has been designed so as to make the clips and the droppers from which the clips are supported as light as possible in comparison to the weight of the trolley wire itself.

Three years ago, at the time the London, Brighton and South Coast Railway installation was planned, I made the most careful investigation; and it was then that I decided on the flexible system instead of the rigid one originally adopted by the New Haven line.

The greatest span so far is 195 ft. The vertical projection of the sag of the catenary wire at 50 degrees fahr. for this span, corresponds to 6 ft. Contrary to continental practice, the trolley wire is drawn up and anchored at both ends with due

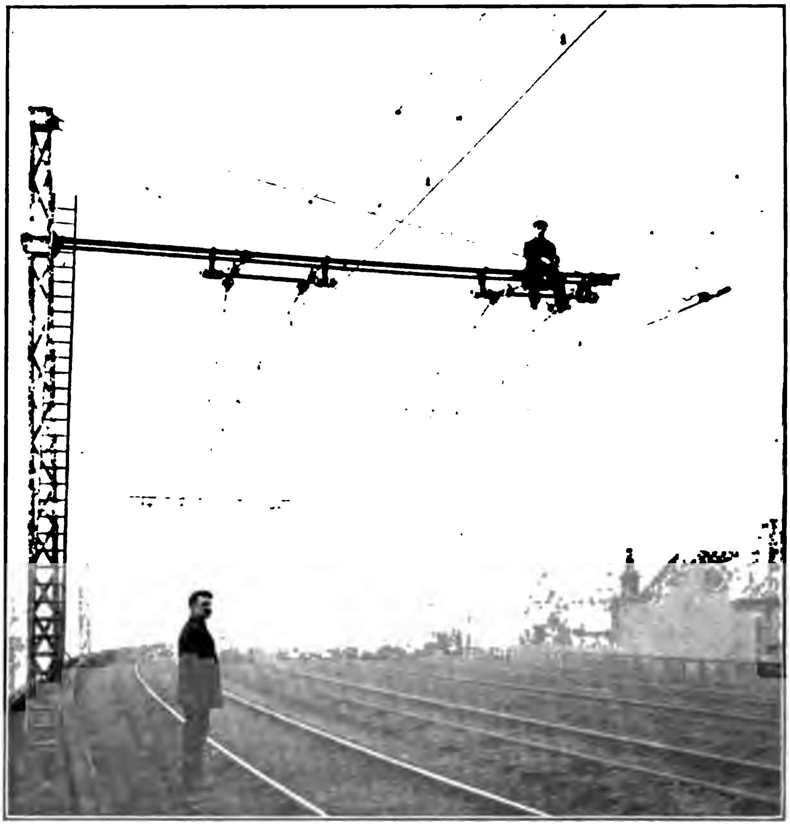


FIG. 2

regard for the lowest temperature to which it may be subjected, so that in the coldest weather the elastic limit should not be exceeded in the wire.

I have provided for the wire being staggered 18 in. altogether, or 9 in. on either side of the center line of the track, although there are occasions when I have diverted so as to go as far as 13 in. from the center line.

The normal height of the wire will be between 16 and 17 ft., whilst in the two terminal stations the height is 21 ft. At some of the lowest bridges the wire descends to a height of 13 ft. 10 in., or only 4 in. above loading gauge. The train service which will operate over this line at present is of a purely local character, and it is not likely that at the outset a speed of 45 miles an hour will be exceeded.

The extensions that will be proceeded with as soon as satis-

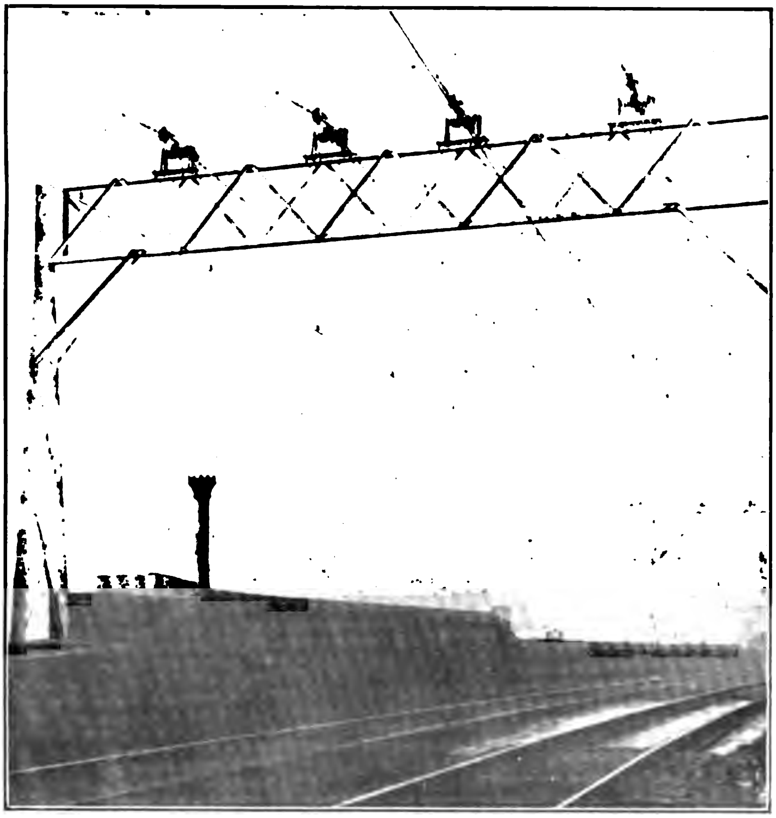


FIG. 3

factory results are obtained on the first portion now nearing completion may, however, involve the operation, not only of multiple-unit local trains, but of trains hauled by electric locomotives, which of course may obtain speeds considerably higher than that at present contemplated.

Careful consideration has led me not to adopt the pantograph type of collector, but to adopt a type of trailing bow collector. This collector consists of a tubular frame, the top of which is fitted with two bows, one forming the rigid portion of the frame itself whilst the other is hinged and sprung from it. It is believed that this method will ensure sparkless running, and operating with a pressure of from 12 to 15 lb. between the collector strip and the contact wire. There are two bows, one for operating in each direction. On the top of the collector bows are fixed renewable grooved aluminum strips, as it is believed that by this means the wear of the wire will be localized on the contact strip, virtually no wear of the trolley wire taking place. Furthermore, owing to the form of strip, and to the low pressure exercised by it against the trolley wire, the cost of the strip itself will not be a very large item in the working expenses.

Ivan Ofverholm (by letter): In Sweden we have had a trial road for testing single-phase details. These trials are now finished and a report made, which I hope will soon appear in American translation. It may interest the Institute to read of some details about our trials, and make a comparison between the experience gained by us and the New Haven officials.

The conditions in our power station were somewhat different from those in the Cos Cob station. We had four-pole generators, running 750 rev. per min. and driven by steam-turbines. In this case the fly-wheel effect of the steam turbine with its gearing (reduction 8500 to 750) was about three times as great as the fly-wheel effect of the generator; therefore, when a short-circuit came on, the stresses on the generator shafts were so severe that the shafts were twisted. For this reason the coupling between generator and turbine had to be made so weak that it would break when severe short-circuits came on. It was also proposed to make a kind of spring-coupling that would allow the generator to slow down when the momentary stress of a short-circuit came on; but as the cost of such a contrivance was considerable, we preferred to make a coupling that did break, but was easily replaced.

In the power station there were two generators but only one circuit-breaker, which at the same time kept the generators in parallel and connected to the line. With this circuit-breaker we have not had the least trouble.

In the beginning there was much trouble with the windings of the generators. They were often short-circuited. At first we thought this was due to some kind of high-potential stresses, and put in protective devices. As these did not seem to work at all, we studied the conditions more closely and found that the coils of the windings were bent in many places. We therefore got the idea that the short-circuits were due partly to mechanical stresses. We put in mechanically stronger insulation in some places where weak points were found; since then we have had no short-circuits in the generators, although there have been several very severe external short-circuits.

It interests me to hear that Mr. Murray thinks the single catenary contact line is just as good or even better than the double catenary. In Sweden we think that a contact line without any catenary is the best, provided the mechanical stress in the contact wire is kept constant at different temperatures, and the bow pressure so adjusted that the resultant stress in the contact wire (the sum of the normal stress in the contact wire and the bending stress resulting from the bow pressure) never exceeds the elastic limit. At present it is not so with ordinary trolley lines. In nearly all the cases we have considered, we have found that the elastic limit is exceeded; the fact that the trolley lines do not come down more often, is due to the fact that copper wire can stand bending above the elastic limit to a remarkable degree.

For steel wire the elastic limit is sufficiently high, and according to our researches such a wire can keep the internal stresses sufficiently constant at varying temperature without being readjusted. For this reason I think it was a good idea to put up a steel wire as contact wire on the New Haven road, but I wonder if it can stand the smoke from the locomotives. Our experience in this respect is that steel wire cannot withstand smoke. I know of at least one big installation with steel catenary wires, where these have had to be replaced by bronze wires.

As I have said, we think we can improve the present trolley line construction without catenary very much by using automatically operated tightening gear. We intend to make the distance between the suspension-points 25 to 30 metres. But we do not think that we can operate locomotives at high speeds on such a line with only one bow. It is our intention to use two bows placed 6 metres or more apart. The bow has a small (30 cm. high) top bow with little energy, supported by a pantograph. The arrangement we intend to use is an improved form of this bow.

Mr. Murray says that the insulators for supporting the messenger cables had to be doubled up. As far as I can see from Fig. 13 in the paper, these double insulators are placed in parallel. I know of other cases where insulators had to be doubled up, but there they were placed in series. I know from experience that this does not improve the conditions very much. The arrangement on the New Haven road is, I think, just the thing to be used, as that is the only reliable way by which a bad insulator can be put out of service by itself.

Mr. Murray also speaks of hard spots in the line, and the way to get rid of such things. We have had about the same experience, and therefore we will now make the line as movable as possible in a vertical direction, but have it anchored duly in a horizontal direction.

We don't intend to use any deflectors on our lines. We have made many trials in order to see how to get rid of the difficulties at the points where the wires are crossing each other, and have found that all these difficulties can be done away with by placing the wires in the proper position at the crossing.

As to the telegraph and telephone disturbances, which Mr. Murray says can be done away with by means of pilot-wires and series transformers, in the case of the New Haven Railroad the disturbances seem to be due chiefly to electromagnetic influence and not to the static charging effect from which our telegraph and telephone suffered so much. This can also be helped, but by other means, mentioned in Mr. Dahlander's report on the Swedish trials.

C. E. Eveleth (by letter): In explaining the failure of the oil circuit-breakers on the New Haven feeders, Mr. Lamme says that the short-circuit current of the generators reached for a few cycles a value of fifteen times normal, which was far in excess of the amount anticipated. Just why this was not anticipated is rather difficult to understand, as machines giving fifteen times normal current, or even twenty times normal current on short-circuit, are not infrequent; in fact, generators that give less than twelve times normal current for the first few cycles when short-circuited are rather the exception.

The oil switch has been generally regarded as a simple piece of electrical apparatus that can be built in almost any form, and still do its work satisfactorily, but experience shows that for successful design and application there must be at hand quantitative knowledge of the conditions to be met.

When associated with Mr. E. M. Hewlett in the spring of 1902 we were impressed with the evidence of enormous current rushes for the first few cycles when generators were short-circuited, and realized that these conditions should be analyzed if we expected to apply protective devices properly. Subsequently we investigated the magnitude of these effects and the action of oil-switches under short-circuit conditions in a series of tests involving several thousand dead short-circuits at various potentials. from 2200 to 75,000 volts, with and without transformers, transmission lines, etc. With the data so obtained we were put in position both to design switches of great energy rupturing capacity and to select the size and type suitable for specific requirements. In some instances this results in a rather more expensive switch than would be advocated by those who have not had such data at hand, and it is not surprising that they continually run into unexpected difficulties.

Simultaneously with the oil-switch tests, investigations were conducted concerning the requirements of reinforcement of the end-windings of generators. It may be interesting to know that in a number of instances we short-circuited individual generators at full excitation from one hundred to three hundred times.

In 1904 it was my privilege to build and test an oil circuit-breaker—which, by the way, did not have a grounded can, and contained less than three gallons of oil—that opened repeatedly on a single break current of 20,000 to 25,000 amperes at 2200 volts. This was equivalent to a rupturing capacity with four breaks in series, such as would ordinarily be used for a single-phase circuit of over 100,000 kilovolt-amperes. Under these conditions the circuit was ruptured without throwing oil or otherwise damaging the switch, except for some burning of the contacts. As these tests preceded by a considerable time the selection of the New Haven oil circuit-breakers, and as there are many oil circuit-breakers in service for feeders where the total generator capacity connected to the bus-bar exceeds 50,000 kw. it is apparent that circuit-breakers capable of handling the New Haven conditions had been built and might have been selected. If they had been adopted a large proportion of the serious delays would have been obviated.

The accompanying oscillogram shows the action on one of the short-circuit tests referred to. Attention should be given to the lower curve of Fig. 1, which shows a rise to more than double potential in the voltage of the phase not short-circuited. As this potential strain would be applied to all of the polyphase apparatus on the system, and furthermore as

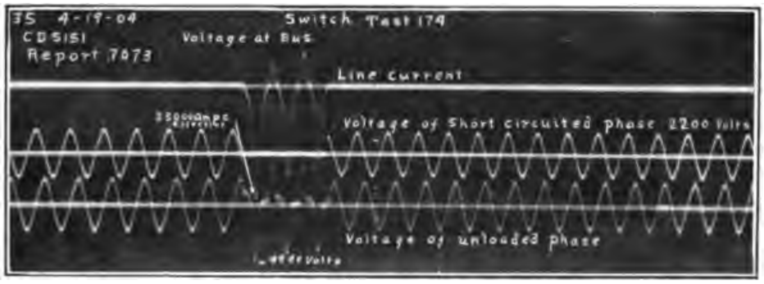


Fig. 1

there exist at all times badly unbalanced voltages whenever one phase is loaded more than another, especially with inductive load, and the voltage maintained by regulator on the loaded phase, there are some manifest disadvantages in trying to operate single-phase and polyphase apparatus from the same generator. For this reason it is my opinion that such operation is at a decided disadvantage as compared with straight single-phase operation on an undertaking of the magnitude of the New Haven system. I think it would have been better to supply separate polyphase generators for operation of shops and direct-current sub-stations.

W. S. Murray:* In connection with the discussion contributed by Mr. Townley, it is a pleasure to record the number of interesting points that he has brought out which the paper did not include. As I have been accused of being over-enthusiastic about the New Haven electrification by one author of a dis-

* The closure of the discussion was written about four months subsequent to the date of the paper.

cussion, and Mr. Townley has seen fit to feel that I am not sufficiently proud of it, between the two I am able to average a happy medium. As a matter of fact, my pride was inclusive of being willing to make a complete exposition of the difficulties, and in reviewing the paper, while recalling the many successes that could have been mentioned to offset the temporary failures, my mind constantly dwelt on the belief that " our friends ", who were so freely disporting themselves in the general condemnation of the system, were entitled to the actual facts of the situation. In this " deed of gift " I constantly steeled myself against introducing any ameliorating features of success in the system, for fear some slight prejudice would come to their minds. Indeed, I will never be quite able to determine whether the cause of the sudden secession of this voluble condemnation was due to their possession of these facts, or the reliability of train schedule on the New Haven system, which arrived about the same time.

In reference to the discussion of Mr. H. P. Davis. I am exceedingly glad that he has included so much detail in regard to the general operation of the distribution system. The fear of giving the paper too great length constantly suggested caution in digressing too far from the trunk-line principles involved, and as stated in the closing paragraphs, it was the author's hope that the details of construction and operation would be brought out by the discussion. Mr. Davis has brought out clearly the electrical and mechanical operating characteristics of the distribution system, and it may be said that except for the fact that the multiple arrangement of overhead wires in the distributing system brought about such an exceedingly high conductivity, the circuit-breakers would have functioned entirely in accordance with their anticipated plan of operation. The lowering of the current density on the occasion of short-circuits by the actuation of the auxiliary apparatus described by Mr. Davis has accomplished the degree of selectiveness which was the first aim in the conception of the system.

In connection with this type of distribution system, employing a voltage of 11,000, this is an opportune time and place to point to a suggestion that comes from the mouth of experience. While I do not think it is by any means impossible to design generators of the voltage and capacity, as installed in Cos Cob station, to withstand the shocks of short-circuits of the character that have been received by that station, the form of protective apparatus that has been installed to alleviate the electrical and mechanical strains incident to these short-circuits, certainly suggests the wise expedient of designing a low-voltage station (say not greater than 2300 volts) employing static transformers to secure the higher transmission voltages. It is to be noted in connection with the softening effect, as a result of the transformers being between the generators and the line, that the increased cost, due to transformer equipment, will be offset by a lower price per kw. of generators and switchboards, due to lower voltage and, further, that what loss may be suffered in virtue of the introduction of transformers will be greatly offset by the higher efficiency accruing from a low-voltage, instead of a high-voltage, generating system. I believe in connection with single-phase electrification, this arrangement in power house design should be effected.

Referring to Mr. Torchio's discussion and in answer to his three questions:

1. Concerning the grounded circuit-breaker frame. If the frames of the circuit-breakers were supported upon insulators, this would approximate the ordinary or ungrounded system conditions; that is, where no phases are grounded. But the earth not being an insulator, and as the frame of the circuit-breaker rests on metal (anchor) bridges, as explained in the paper, this brings one terminal of the generating system directly to the circuit-breaker frame; hence, the necessity of strong *lining* insulation. If the frames of the circuit-breakers were ungrounded, this would introduce danger in handling the switches; as it is not impossible to conceive of a connection being made between one of the switch points within the tank, and the iron tank itself, should an attendant be standing on the ground and touch the tank, he would be shocked. By the present arrangement of grounded circuit-breaker tanks this hazard is obviated.

2. Three-phase generators are used in order that power may be supplied to the polyphase motors operating in our power house and in shops adjacent to the line. Polyphase current will also be used in operating direct-current railway plants, whose prime movers, at present, are steam engines, which will, later, be replaced by polyphase motors.

3. Concerning the regulation of single-phase *vs.* 3-phase generators, it is quite evident that the use of single-phase generators would do away with the unbalanced conditions that obtain in the case of the 3-phase generation. However, the amount of unbalancing is extremely small, due to the copper-clad rotors on the generators, and, further, had single-phase generators been used this would not have permitted the advantages of power house, shop and railway plant drive, as described above.

In connection with the interesting discussion that Dr. Steinmetz has contributed, it is pleasing, indeed, to note his prediction that when the time arrives to electrify our transcontinental railways, the alternating-current single-phase motor will be the ways and means. His commentary on weight efficiency suggests a thought which I would venture supplementary to his reference to drawbar pull *vs.* weight of motor. I think it is wise to guard against the exclusive rather than the inclusive view. Given two terminals, say, 12 miles apart, between which electrified tracks are to handle a given traffic, if the fixed charges and operating expenses per ton mile are the same for either the alternating or direct-current systems, it is undeniably true that we can not escape the conclusion that the direct-current type of motor, on account of its lesser weight per pound of drawbar pull, would be the legitimate and logical selection of motive power, or in other words, all other considerations of the system being equated, these and only these are the circumstances under which the matter of weight efficiency is a ruling factor. Where we are considering 50 or 100 miles of dense trunk line operation, the weight efficiency, though having a bearing, takes its position along with the other considerations, but is assigned a proper degree, which becomes exceedingly minor in the consideration of heavy mileages and heavy power requirements. In charging up the alternating-current motor with more weight per drawbar pull than the direct-current motor, the difference can be represented in dollars and cents, but we must not end the investigation where it has just begun. The 12-mile railroad, above described, may have its power station at its center, and its traffic sufficiently light to require no rotary sub-stations, and possibly not very much copper in its distributing system. The situation takes on an entirely different hue, however, in making a decision upon the 50 or 100-mile heavy traffic proposition. If direct-current propulsion is to be used, this will require sub-stations and with it many pounds of low-voltage copper feeder. These items of sub-stations and feeder copper, like the increased weight of the alternating-current motor over the direct-current motor, cost money and must not be forgotten, and so when we formulate weight efficiency equations, while we are not forgetting to charge up the cost of the greater weight for the alternating-current motor, let us not forget to charge up the cost of the copper and sub-stations complementary to the direct-current installation. Outside of all this comes the careful consideration of the system's commercial efficiency, better stated as the ratio of operating expense to gross income and, therefore, except that weight efficiency be considered inclusive and charged with its relative value of importance in the system as a whole, its importance is only apparent and not real.

In reference to Mr. Minor M. Davis's contribution concerning the effect of single-phase currents upon telegraph and telephone wires, I would further supplement my remarks which I made on this subject on the occasion of its reading before the Institute, and say that it was the effort of the author to touch as briefly as possible upon the electrification, in its relations to matters other than traction. Each one of the subjects mentioned is one upon which much interesting matter, in detail, could be brought out, and in this effort at brevity I was conscious of the fact that this particularly important subject might have been treated in greater length. Too much credit cannot be given to the companies adjacent to our property, who were affected by our lines, for their coöperation in which both their efforts and our own were united to overcome the disturbance. As I review the two paragraphs upon this subject, I do not see any occasion to modify the statements that have been made, except to say that possibly Mr. Davis's position enables him to better judge of the cost of the transforming equipment necessary to effect the voltage compensation described.

Mr. Lamme is the father of a practical single-phase motor, and has brought to himself another responsibility in the nature of the dampened field 3-phase generator, which has been so well described in his discussion. However, as progenitor of these two types of apparatus, only the greatest satisfaction can come to him, in view of the uniform good behavior of both, thus making the new responsibilities at once both heavy and light. I am particularly indebted to him for the generous detail in which he has described both of these types of apparatus. Four months have elapsed since his discussion was written, and the short-circuit disturbances that occurred in the generators then described have since disappeared, and the mileages of the motors, instead of the figures as stated by Mr. Lamme, have become in some instances as high as 70,000 miles, the average mileage of all motors being probably 60,000 miles, and this with 60 per cent of the commutators still unturned. As a matter of fact, and as Mr. Behrend has pointed out, the operation on both alternating and direct-current power is very much alike as far as sparking is concerned, and as the number of miles over which a commutator operates before it is necessary to place it in the lathe for turning, is dependent almost entirely upon the degree of fineness with which it commutates, and as the mileage, as stated, is comparable with the ordinary direct-current practice, it is fair to assume that it is a matter of indifference whether the motor is commutating alternating or direct current. Mr. Lamme's interesting statements in regard to the strains produced in the coils of the Cos Cob generators on the occasion of short-circuits, leads me again to one of the fundamental reasons for the low-voltage generating station with step-up transformers. It cannot be denied that a short-circuit on a system of high conductivity produces an enormous rush of current. As previously mentioned, the interposition of the transformers between the line and generators offers a cushioning effect for this blow. Added to this valuable adjunct of protection, is the fact that in the low-voltage-wound generators the coils naturally increase in cross-section in proportion to the ratio of diminished voltage; that is, for the same output and the same number and length of coils, a generator of 2300 volts would be wound with coils whose cross-section would be 4.75 times the cross-section of the coils of an 11,000-volt generator. Thus the coil itself is inherently a strong mechanical device for resisting these strains, and, further, its tendency to puncture is greatly reduced on account of a reduction in potential between copper and iron in the ratio of 4.75 to 1.

In connection with Mr. Stillwell's reference to errors in generator and circuit-breaker design, I think Mr. Townley's statement that the " . . . one radical thing that was done wrong in the New Haven electrification, and which was really at the bottom of a large part of the difficulties that have been encountered is that the electrification was not begun soon enough ", is a complete and decisive answer on this commentary. Seldom, if ever, do we see the manufacturer test at the factory, at full load and full voltage simultaneously generators larger than 2,000 kilowatts. Rather do they elect to provide these two conditions separately, and accomplish virtually the same result, using only 5 per cent of the power that would be required in the former case. Similarly with the New Haven, it would have been quite impossible to have duplicated the operating conditions in the case of the generators or circuit-breakers. A temporary duplicate test system was impossible. The test would have been made on the original apparatus had it been possible, but time was not available, as has been pointed out in the paper under " electric passenger service ", in which an attempt was made to describe the pressing force that was constantly brought to bear on the New Haven engineers to begin electric service.

It is a pleasure, however, to read Mr. Stillwell's commentary that the four month's record of operation, as presented in the paper, was not a discouraging one, in the realization that the average for the train-minute delays during the last five months are 13½ per cent of the average of the four months cited in the log.

In response to Mr. Behrend's question in regard to the use of a 2-phase rather than a 3-phase generating plant, as applied to the New Haven conditions, I think this is a point deserving of a very careful explanation. Power house location has much to do with the choice between these two systems. In the case of a 2-phase system, it is natural to suppose, in order to secure the highest copper efficiency, that the point of divergence of the two phases should take place at the power house, if the power house is on the line of the railroad. Examining the location of the power house in the case of the New Haven, it is to be noted that about 18 miles of track lie west of the station, while about 3½ miles lie east of it. Thus it is quite clear that had the station been designed for two phases, they would have been very much unbalanced, especially as the train density is heavier per mile of track on the longer section, which, of course, would be a very unfortunate distribution of load in the windings of the generator.

In the operation of polyphase motors for shops and for street-railway-station drive along the line of the railroad, it would be necessary to intermingle the two phases of the system using the track for a common return of the currents of the two phases in quadrature. This would not introduce any features that are distinctly inferior to the present arrangement of the three phases throughout the length of the system. The question of voltage regulation for a 2-phase distribution, however, offers a decidedly serious aspect, as it is quite clear that it would be impossible to keep the voltages of both phases constant, unless the load on each was always the same. In the case of the 3-phase, we regulate the voltage of the propulsion phase, and the copper-clad rotors of the generators smooth out to a great degree the unbalanced voltages on the two other phases. I believe the deciding points against the 2-phase propulsion are the difficulties incident to loading the phases equally and maintaining voltage regulation.

The relative temperatures due to operation on alternating and direct-current, to which Mr. Behrend refers, I regret have not been obtained. The conditions of operation on the alternating-current zone and direct-current zone are so absolutely different that a comparative set of figures would be impossible, unless they were obtained under the auspices of test conditions. Later, this information may be obtained and will be gladly offered.

I note by Mr. H. F. Parshall's contribution that in 1901 he was appointed by the Board of Trade a member of the tribunal, presided over by the Hon. Alfred Littleton, to determine whether or not such an alternating-current system, as proposed by Ganz, was suitable for the underground railways of London. Immediately follows his conclusions that the alternating-current system was unfit for application upon the aforesaid railways, even though this date was previous to Mr. Lamme's paper on the Washington, Baltimore and Annapolis single-phase railway read before the American Institute of Electrical Engineers, at New York December 26, 1902,* which might have been said to be the entering wedge of the single-phase motor in its application to the propulsion of trains. Engineers of this country would surely look for a few grey hairs on motors of any design existing over a period of eight years, and so I do not wish to be counted out in the first ranks to congratulate Mr. Parshall in this eminent decision, which sounded the death-knell to the alternating-current motor for the subways of New York and the underground railways of the City of London. The London and New York subways offer about as much comparison to the New York, New Haven and Hartford Railroad Company's conditions as a steamboat does to a flying machine, and so I must leave this subject abruptly, and pass to a few concrete statements, which Mr. Parshall has made, and which seem to have been arrived at by projected hypothetical curves. A few brief counter statements may save time. The perfected alternating-current motor does not weigh 40 per cent more than a direct-current motor of the same capacity. If the air-gap of the alternating-current motor is of a character which permits the revolution of the armature in its bearings, and requires no greater maintenance than the direct-current motor, it does not matter how much smaller it is than the direct-current motor air-gap. Higher

* Transactions, vol. xx, p. 15.

armature speeds are a specific advantage rather than disadvantage, in view of this characteristic having excellent application to the direct-connected type of locomotive. Our experience, though possibly not as great as Mr. Parshall's, does not prove that the cost of the high-tension overhead system properly installed is greatly in excess of a third-rail system. It is possible that the overhead wires and bridges necessary to carry them may cost more than the third rail, though even these may be equated. I can even conceive of the overhead contact construction on an alternating-current system being more expensive, *per se*, in first cost, and yet more economical in use, thus again we have this unfortunate feature presented of only looking at a part of a system instead of its whole.

The alternating-current commutator is in itself heavier than the direct current, but practice and experience in this case has not substantiated Mr. Parshall's theory that the maintenance " ..... is nearly double that of a direct-current equipment of like capacity ". A few facts scattered in this nest of hypothetical reasoning may be useful, one among the number being that our present operating sheets show that our operating costs, *per se*, on the direct-current zone are just 40 per cent greater than that of the alternating current. At the time of presenting the paper on the " Log of the New Haven Electrification " it had been in commercial operation just four months, and I cannot refrain from quoting the last few lines of Mr. Parshall's discussion: " Concerning all that has gone on in the New Haven system up to September 5, 1908, which appears to be the termination of Mr. Murray's log, there is absolutely nothing to indicate that during 1909 some such series of occurrences are not again to be expected. All that has occurred may be expected to occur again in some form or another, since electric traction apparatus does not manifest all its faults during the first few months of operation ". Is not the first commentary upon this closing, encouraging eulogy a disappointment, that one is willing to make up his mind to condemn a system four months after its birth? Is this not a brief period during which to compose its obituary? Secondly, I can only say that " All that has occurred " *has not* " occurred again ", and whether it will in " some form or another " is so closely related to indefiniteness that analysis itself is duped, and prophesy becomes a helpless paralytic.

Mr. Armstrong opens his discussion with the following startling sentence, which contradicts everything that follows. " The enthusiastic coöperation of the engineers of the railroad company and the manufacturing company has brought this alternating-current operation to its present success and nothing but admiration can be expressed for the courage and ingenuity shown in overcoming the difficulties met with in operation ".

In thanking Mr. Armstrong for thus felicitating us, I am sure I will be pardoned for the number of exceptions that I must take to his subsequent remarks, in view of their almost complete inconsistency with the facts.

1. The New Haven specifications, as doubtless looked over by Mr. Armstrong, were drafted to make them inclusive of the operation of electric locomotives from New York to New Haven. The installation, therefore, of the alternating-current system of propulsion was not to *demonstrate* the suitability of that system to possible future extension of the road. It so happens that it has accomplished this purpose, but this is off the point, and the interesting feature of Mr. Armstrong's statement in the second paragraph of his discussion, is his full commitment to the 600-volt, or the 1200-volt, direct-current system, as the system applicable to the New Haven conditions, inclusive of the zone above described.

It is interesting to note the seeming pleasure in which Mr. Armstrong points to the " material changes " that were made to the locomotives and the generating and distributing systems. Following upon this is his statement of Mr. Wilgus's report of the cost at $55,000 a mile of our overhead construction. Why bring in Mr. Wilgus? Mr. Wilgus had nothing to do with either the estimates or the checking of them in actual practice. It is difficult to conjecture how he even arrives at these figures. Also how is this relevant to the paper ? It does not discuss costs.

2. This inconsistency comes to mind in viewing Mr. Armstrong's guesses on the weights of the locomotives. These are completely without foundation.

3. Mr. Armstrong refers to published figures on the trolley construction, which as a matter of fact have never been published, not to mention inconsistency No. 4, in which we find Mr. Armstrong increasing the weight of the locomotives 50 per cent, for which he has absolutely no basis, and in which statement he is again absolutely wrong. Mr. Armstrong advises that the present weight of our locomotive approaches fairly close to the weight of the locomotive which he estimated would be required to perform the service originally specified. Doubtless Mr. Armstrong's experience in the design of alternating-current locomotives would have permitted us to make a closer approximation. It has never been my pleasure to have seen an alternating-current locomotive of Mr. Armstrong's design, though this does not preclude the possibility of one being on the rails. The only sure conviction I have, however, is that neither he nor his associates offered to design one which, with an amount of capacity equal to that of New Haven locomotive, was lighter.

I confess to a slight agreement with Mr. Armstrong's statement that the object of the New Haven electrification was of a far-reaching character—at least, far enough to reach to New Haven. How Mr. Armstrong can commit himself to such a statement that the New Haven trolley construction costs 80 per cent more than the 600-volt third rail, and that the locomotive costs double the amount of a locomotive sold another company, without even a film of foundation for his statements, is really remarkable. To deny them flatly is, I judge, sufficient.

Such remarks draw one almost to the necessity of discussing comparative figures of the one system with the other, as it is not to be denied either within or without the ranks of the two companies interested, that the New Haven road knows the investment costs and operating expenses of both systems. Some day the dictates of progress may demand an exposition of these facts, but they can not be drawn out by baseless innuendoes of the character above cited.

Mr. Armstrong's tables in reference to weights and horse powers of the New York Central and New Haven locomotives are wrong, in respect to the data he assigns to the New Haven columns. Again we have the brilliant reference to the 1-hour performance of the locomotives, completely ignoring the continuous capacity for which the New Haven locomotives were designed. This is not better exemplified than in the table that follows, showing the list of train weights above 400 tons which the New York Central locomotive has handled. The table would have been more useful to an engineer had an asterisk been attached with the notation that none of these trains were hauled to a distance greater than 12 miles.

To Mr. Armstrong's generous offer that I correct any error concerning the New York Central company's train-minute delays in view of my access to their official records, in the table which appears on the closing page of his discussion concerning the train-minute delays on the two systems for the months of July, August, September and October, 1908, I would say that were an error discernible, I would trust that it would prove the train-minute delays less than the number he has cited. I will refrain even from checking Mr. Armstrong's statement of train-minute delays he charges for the New Haven system over the same interval. My acquaintance with the officials of the New York Central company leads me to believe that they regret this form of argument which Mr. Armstrong advances, knowing that their system, incorporating the principles of 20 years' experience, and in operation one year previous to that of our own, should be compared with the first four months of a system whose lines were so radically different from their own, and whose principles were of such an entirely different character. It is interesting to note, however, that the average for the train-minute delays during the past five months following the four months cited in the paper are 13½ per cent of the average during the period of operation the paper includes.

Since Mr. Armstrong's desire to compare operating statistics has crystallized into the train-minute delay table which he has published, doubtless a reply more to the point of his argument would be the statement of the fact that notwithstanding the main line New Haven electrification is inclusive of both alternating- and direct-current operation, with an average of 135 trains a day, the actual number of train-minute delays during the whole period of operation since 100 per cent service has been under electrical control, has not been equal to the train-minute delays that are charged up against the two straight alternating-current cars operating upon the New Canaan branch during two of the four months cited in the paper. I feel justified in making reference to this fact, in order to substantiate Mr. Armstrong's opposition to the single-phase situation, for on this branch the equipments are of a design better known to himself, but I am happy to be able to record the fact that our position is fortunate in that should the New Canaan situation fall a victim to a policy which I understand is being advocated, we will not have to *go back* to the direct-current form of propulsion.

While the intent of the paper did not include a desire to compare the electrification of the New Haven with that of the New York Central, it seems that this phase of the situation developed in the discussion, and Mr. Storer has offered some very pertinent and interesting inquiries, the answers to which doubtless will come later. The most interesting part of his discussion to me is the emphasis he lays on the necessity of making system comparisons rather than the comparison of the individual parts of systems to each other.

It is very interesting to note Mr. Lyford's statistics in regard to wear on the contact wire and to note his prediction of a life of between 20 and 30 years. This corroborates micrometer measurements made on the steel wire during the past nine months of operation.

I have been very much interested in Mr. W. N. Smith's analysis of the situation, and I think he has placed his finger upon the keystone of the arch of successful alternating-current operation when it rests upon the overhead line. As stated in the paper, the three great links in the chain of electric propulsion are the power house, line and locomotives. As Mr. Smith has pointed out, power houses and locomotives may be relayed, but the line must stand alone, at least over a specified single piece of track. His analysis is to the point when he shows that the train-minute delays were the greatest in duration when occurring by reason of line failures. It was with this point in view that the author of the paper made the statement that, "In the month of May, 1908, it became evident to us that within at least one month from that date, if some change was not effected in the contact wire, the New Haven electric servi e would cease." But by far the most pleasing feature of Mr. Smith's contribution is that with finger pointing to the single link that had to be intact to ensure electric service, and while assigning it to its great degree of importance, his mind dwelt at the same time upon what the whole system stood for as a means to an end, and which must have been inclusive of safety, reliability of service and economy of operation.

Mr. Phillip Dawson's contribution to the discussion, in the form of the description of the London, Brighton and South Coast Railway Company, which uses the overhead single-phase system, is of much interest. It is to be noted that on this construction the spans are very much less than in the New Haven. To me the most interesting piece of metal in the overhead system is the contact wire, and I note that on the London, Brighton and South Coast Railway, the trolley wire is drawn up and anchored at both ends. It is quite possible that the clips Mr. Dawson has described as supporting the contact wire will be sufficiently light to prevent kinking at the hanger points. This arrangement will be greatly fortified if, as Mr. Dawson states, speeds of 45 miles an hour will not be exceeded. The double messenger triangular form of hanger offering a more rigid form of construction, and a greater degree of inertia at the hanger points, shortly proved itself undesirable for the speed conditions that exist upon our lines, and I feel quite sure that even in the form of single catenary suspension, our conditions would not permit the form of construction as adopted and described by Mr. Dawson. In short, our experience with the overhead construction leads me into the following brief specification for an overhead contact wire.

1. One single steel stranded messenger cable with deflection corresponding to economical values fixing distance of span.
2. One horizontal solid copper conductor supported from steel messenger at intervals of 10 ft. with light tension on same.
3. One horizontal solid grooved steel (contact) wire supported by light clips (weight not greater than 9 ounces) from the solid copper wire at mid-points between messenger hangers. The tension in this wire not to exceed the elastic limit of the steel at lowest temperature.

This auxiliary or contact wire as described under (3), it seems to me has as much right for existence as the floor of a highway bridge. To make contact on the copper wire would be the same as running on the members of a bridge, which, while consistent with the past practice of the electrical engineer does not match up with the precedent established long before the electrical engineer had to design his bridges for electrical traffic. The steel wire offers a cheaper and more easily replaced floor, and the conductor system always remains intact with undiminished efficiency.

The double-bow pantagraph collector with renewable contact strips has been tried out on the New Haven system. We did not get a very great deal of encouragement out of this form of collector; however, with the varying heights which highway bridges have dictated the contact wire to be, and with a continual deposit of steam locomotive discharges upon it, not to speak of the very high-speed conditions our collectors are subjected to, these may possibly have qualified the success which I only trust will accrue in their use on Mr. Dawson's road.

In answer to Mr. Ivan Ofverholm's inquiry in regard to the effect of smoke of the locomotives on the steel contact wire, I would say that our experience in connection with this deteriorating influence that the electrification has to contend with at present, is not so great as I had anticipated. A careful observation on the steel wire during the past nine months does not indicate that the blast from the locomotives has had much, if any, effect.

In connection with Mr. Ofverholm's statement concerning the doubling up of our messenger cable insulators, I see that he has referred to Fig. 13 descriptive of the yard construction. He is quite right that the insulators in this case were placed in parallel. On this occasion, however, this arrangement was made in order to resist mechanical rather than electrical strains. The strain in the cross catenary, which supports trolleys over 10 tracks, is 14,000 pounds. To-day porcelain insulators capable of resisting a mechanical strain of 40,000 lb. and 40,000 volts are at hand, but it was decided that single insulators of the character shown in the picture, which were the only ones then available, would not be sufficient, with proper margin of safety, to withstand this strain, and a special yoke was constructed upon which were installed the two insulators, to which Mr. Ofverholm has referred. A reference to Fig. 8, though not as clear as if the picture had been taken closer to the insulator, shows two insulators of the same type in series, this being the arrangement throughout the system over the main line tracks where they are subject to the blast of the steam locomotives.

Concerning Mr. Eveleth's suggestion that had we adopted the circuit-breakers of his design a large portion of the serious delays would have been obviated: even granting that his circuit-breaker would have operated in the exemplary manner described, I feel sure that the oversight in not having adopted it will be excused in our having considered it a part of the detail to be supplied by a company which believed *first* in the system, before they believed in the circuit-breaker. We may yet have an opportunity to use some of Mr. Eveleth's circuit-breakers though I doubt if the results can at present be bettered.

A paper presented at a meeting of the Pittsburgh Section of the American Institute of Electrical Engineers, Pittsburgh, Pa., October 2, 1907.

METHODS FOR LOCATING TRANSPOSITIONS OF WIRES AND SPLIT PAIRS IN TELEPHONE AND TELEGRAPH CABLES

BY HENRY W. FISHER

Some of the telegraph companies demand that there shall be no transpositions of wires in any of the layers in a telegraph cable. By this is meant that the wires of every layer must have the same relative order at each end of the cable. There is usually a tracer, or wire covered with different colored paper from the rest in every layer, and by counting from this tracer any particular wire can be located at any point in the cable. If a wire is fifth from the tracer in a clockwise direction at one end of the cable, it will be fifth from the tracer in a counter clockwise direction at the other end of the cable. By this general plan it becomes possible to identify wires without resorting to some of the common methods of testing. However, in order to be able to place implicit reliance in a scheme of identification of this sort, not only must the cables be manufactured correctly but the jointing must be made by skilled men who are careful to connect the wires of each layer in consecutive order, and as an extra precaution make repeated tests at both ends of the jointed sections as the work of splicing progresses. Even after exercising the greatest care, a transposition may occur. The object of the present article is to show how a fault of this kind can readily be located by some methods devised by the author.

To do this, a ballistic galvanometer, battery, condenser, and discharge key can be employed, but the writer prefers to use an alternating current or continually reversed direct current, a wheatstone bridge or its equivalent, a condenser, and, as a current detector, a telephone, or, still better, a vibration galvanometer.

Instruments are now made which are suitable for this work, provided that the length of cable is great enough to obtain sufficiently discriminating values between the various tests made.

As the methods employed deal entirely with measurements of electrostatic capacity, which are ably described in many text and hand-books, the writer will here only briefly show by diagram and description, the arrangement of sensitive apparatus which was employed to verify the accuracy of the methods used.

Fig. 1 gives in diagrammatic form the connections and apparatus employed. R, R' and R'' are non-inductive resistances wound

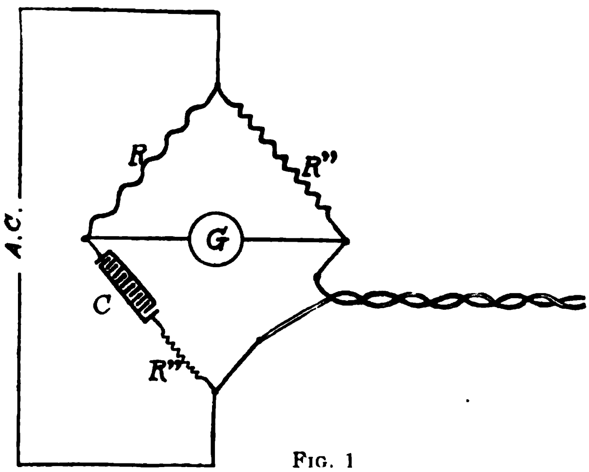


FIG. 1

also so as to have a minimum capacity. G is the vibration galvanometer, C a standard condenser, and A C the alternating-current source which, by means of an electrically driven tuning-fork arrangement, was kept at a frequency that gave maximum vibration to the galvanometer needle. The resistance R''' is needed to obtain the condition of balance or zero vibration of the galvanometer.

The plan of operation was to keep R' constant and vary R and R'' until the galvanometer ceased to vibrate. R''' does not enter into the calculation of capacity which is represented by the formula $\frac{R}{R'}C$. Therefore, as R' and C were constant, the relative capacities are proportioned to R, and as we deal here entirely with relative values, the figures given for the different tests are the R resistances required to give zero vibration to the galvanometer.

Telegraph cables. Before proceeding to the derivation of formulas, it is important to remember that the theory of relative capacities with different connections in bunched or paired telegraph or telephone cables is extremely complex. The formulas here given are not absolutely correct, on account of certain very small capacity effects that are ignored for the sake of sim-

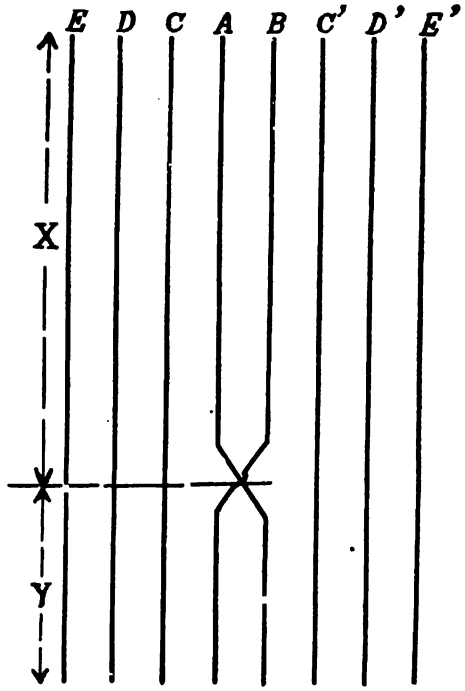


FIG. 2

plicity. They are probably close enough, however, because it is impossible to make cables of this sort, and not have in the capacities of different wires, variations of as much as five or ten per cent., which would be sufficient to cause great discrepancies, even if theoretically correct formulas were employed.

We shall first consider the simplest case; namely, that where adjacent wires in a telegraph cable are transposed, and after that we shall give general directions which are applicable to almost any condition of single transposition.

Fig. 2 represents a number of wires in a layer of cable. A

and B are the two wires which are transposed at a distance of X feet from the end where the test is applied and Y feet from the other end of the cable. In representing a test between two wires the letters designating each will be coupled together thus: The capacity between A and B will be spoken of as capacity $A\ B$.

The general plan of procedure is to determine the capacity between adjacent wires and also the capacity between wires C and E, the latter being selected so that the number of wires between C and E is equal to the number between C and B or between C' and A.

If the capacities between C and A, and between C and B, are then measured we can derive a formula which will give the approximate distance to the fault. In order to get good average results, it is advisable to make a number of measurements of similarly placed wires as is shown below. The first thing to do is to determine the relative capacity values of the following combinations of wires—$A\ B$, $C\ E$, $C'\ E'$, $C\ A$, $C'\ B$, $C\ B$, $C'\ A$.

$$\text{Let capacity } A\ B = a \tag{1}$$

$$\text{“ “ } \frac{C\ E + C'\ E'}{2} = b \tag{2}$$

$$\text{“ “ } \frac{C\ A + C'\ B}{2} = c \tag{3}$$

$$\text{“ “ } \frac{C\ B + C'\ A}{2} = d \tag{4}$$

Let L = the total length of the cable.

$$\text{The capacity per foot of (1)} = \frac{a}{L} \tag{5}$$

$$\text{“ “ “ “ “ (2)} = \frac{b}{L} \tag{6}$$

From which we get,

$$\frac{a}{L}x + \frac{b}{L}y = c \tag{7}$$

$$\frac{a}{L}y + \frac{b}{L}x = d \tag{8}$$

Solving equations (7) and (8) for x we have,

$$x = \text{the distance to the fault} = \frac{a\,c - b\,d}{(a-b)\ (a+b)} \times L \tag{9}$$

Now as $x+y = L$, we can substitute $y = L-x$ in equations (7) and (8) and get the following results,—

$$x = \frac{c-b}{a-b} \times L \qquad (10) \quad \text{from equation (7)}$$

$$x = \frac{a-d}{a-b} \times L \qquad (11) \quad \text{from equation (8)}$$

Equation (9) is the proper one to use because equations (10) and (11) do not generally give concordant distances, although their average is usually near the distance obtained by equation (9).

To give the above formulas a practical test, two wires were transposed at 130 ft. from the end of a cable measuring 280 ft. and the following values were obtained.

TRANSPOSITION No. 1.

$$a = 167$$

$$b = \frac{132.5+133}{2} = 132.75$$

$$c = \frac{148+145.5}{2} = 146.75$$

$$d = \frac{147.5+150.5}{2} = 149$$

Substituting these values in equation (9) we get,

$x = 128.9$ ft., or about 1 foot from the transposition.

Substituting in equations (10) and (11), the values of x are 113.6 ft. and 146.1 ft., or an average distance of 129.8 ft.

TRANSPOSITION No. 2.

$$a = 161.5$$

$$b = \frac{127+130}{2} = 128.5$$

$$c = \frac{138.5+141.5}{2} = 140$$

$$d = \frac{140.5+142.5}{2} = 141.5$$

Substituting these values in equation (9) we get,

x = 129.6 ft., or about 0.5 ft. from the transposition.

Substituting in equations (10) and (11) the values of x are 96.9 ft. and 168.4 ft., or an average distance of 132.6 ft.

As mentioned before, it will be noted here that the values found by equations (10) and (11) differ widely. This may be partly due to inequalities in manufacture, but is more likely caused by the formulas being only approximate. The average value of (a) as measured should be slightly too high, and this would make the results by formula (11) too great. In formulas

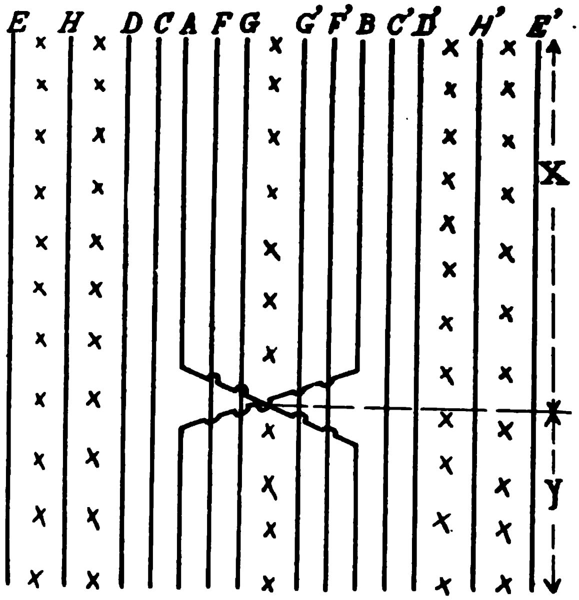


FIG. 3

(10) and (11) nearly equal terms are subtracted and this is always a source of error. This kind of test is necessarily of such nature that even slight irregularities in manufacture may cause great differences in the calculated distances. It is for this reason that so many measurements have to be taken, for we are enabled thereby to obtain average results.

We will next consider the case where there are several wires between the transposed wires, which, as before, should be marked A and B. The wire adjacent to B should be marked C' and that next to A should be marked C, as indicated in Fig. 3.

Then select and mark a wire E such that the number of wires between C and B is equal to that between C and E. Also select and mark a wire E' such that the number of wires between C' and A is equal to that between C' and E'.

The tests can now be made exactly as described above, with the exception that for capacity $A\,B = a$, the capacity between several pairs of adjacent wires near but not including A or B should be determined, and the average of these taken as (a) in formula (9). The wires generally measured are $C\,D$, $F\,G$, $F'\,G'$, $C'\,D'$. After the values of (b), (c), and (d) are found, the distance to the fault can be calculated by equation (9), and, if desired for the sake of comparison, by equations (10) and (11).

A separate set of values for b, c, and d can be found by selecting

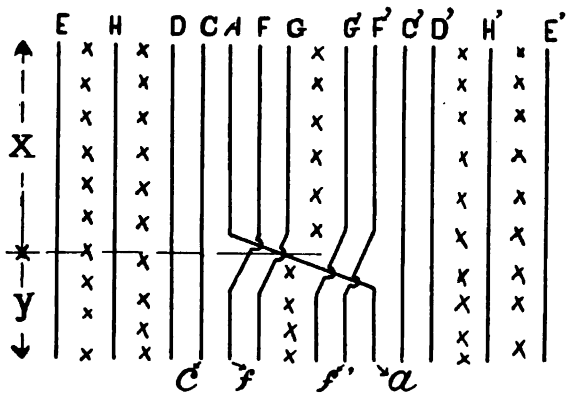


Fig. 4

the wire H, so that the number of wires between F and H equals the number between F and B; also the wire H', so that the number of wires between F' and H' equals the number between F' and A.

Then

$$b = \frac{F\,H + F'\,H'}{2} \quad c = \frac{F\,A' + F'\,B}{2} \quad \text{and } d = \frac{F\,B + F'\,A}{2}$$

With these and the previously determined value of (a) a check value of x can be found by equation (9).

Fig. 4 illustrates the case of a single displaced wire A. Here the number of wires between C and E and between C' and E' equals the number between C and a. Also the number of wires between F and H and between F' and H' equals the number

TABLE I.

RELATIVE CAPACITIES = R IN FIG. 1.

Test between wires	Fault No. 1 m	Fault No. 1 n	Fault No. 2 m	Fault No. 2 n	Fault No. 3 m	Fault No. 3 n	Fault No. 4 m	Fault No. 4 n	Fault No. 5 m	Fault No. 5 n	Fault No. 6 m	Fault No. 6 n
$C\,D$	412	416.5	421	426.7		384	37[illegible]	381	3[illegible]0	363	[illegible]	36[illegible].5
$F\,G$	419	427.5	419	422.5		38[illegible]	[illegible]79	384	368.5	3[illegible]0	[illegible]	[illegible]
$F'\,G'$	410	419				[illegible].5			3[illegible]7	380	[illegible]	[illegible]
$C'\,D'$	403	412.5	420	427		381	382	385	370	378	[illegible]	[illegible]
Average = a	411.3	419	420	425.4		379.6	378.7	383.7	360.4	373.8	367.8	378.4
$C\,E$	344.5	358.5	343	357.5		324	313	3[illegible]4.5	304	313.5		
$C'\,E'$	342	359	344	359.5		328	307	330.5	301	314		
Average = b	343.2	358.8	343.5	358.5		326	310	329.5	302.5	313.8		
$C\,A$	357	373	354.5	369		360	371	380.5				
$C'\,B$	353.5	369.5	362.5	377		357.5	365	377.5				
Average = c	355.2	371.2	358.5	373		358.8	368	379	310.5	320.5		
$C\,B$	405	413.5	402	411		353	3[illegible]7.5	345.5				
$C'\,A$	398	407.3	401.5	411		353	325.5	341.5				
Average = d	401.5	410.8	401.8	411		353	320.5	343.5	301	300.5		
$F\,H$	348.5	361.5	354	363.5		325	318	328			297.5	318.0
$F'\,H$	335.5	352.5	354	365		330	316.5	327.5			30[illegible].5	318.5
Average = b	342	357	354	364.3		327.5	317.3	327.8			300	314.8
$F\,A$	370	376	369	375		363	368	374.5				
$F'\,B$	356	369	366	373		355	373	380				
Average = c	363	372.5	367.5	374		359	370.5	377.3			[illegible].5	[illegible]
$F\,B$	408.5	417.5	407.5	414.5		354.5	334	340.5				
$F'\,A$	399	406.5	398	404.5		350	338.5	347				
Average = d	403.7	412	402.8	409.5		352.3	336.3	343.8			[illegible].5	[illegible]

m indicates that other wires in cable were free and not grounded.
n indicates that other wires in cable were all grounded.

between f and a. Having tagged the wires as just described, and as illustrated in Fig. 4, the same connections and tests as mentioned above can be made except those involving the wire B which does not exist here. Therefore only one-half the number of tests can be made to get c and d, and hence an accurate location of the fault cannot be expected.

Tables 1 and 2 give the results of tests on telegraph cables.

Fault 1 had 3 wires between A and B.
" 2 " 2 " " "
" 3 " 3 " " "
" 4 " 2 " " "

Fault 5 was a single displaced wire as illustrated in Fig. 4 and there were 4 wires between a and c. All the figures obtained are given in the tables, so that persons interested in this subject

TABLE II

No. of Fault	Length of cable	Actual distance to fault	Calculated distance to fault		Average error in feet	
			Other wires not grounded	Other wires grounded	Other wires not grounded	Other wires grounded
No. 1	1098 ft.	208 ft.	176.1 ft. 234.5 ft.	189.9 ft. 205 ft.	— 2.7	—10.5
No. 2	1098 ft.	208 ft.	238 ft. 252.5 ft.	237.6 ft. 226 ft.	+37.	+24.
No. 3	1021 ft.	557 ft.	——	569.8 ft. 581.5 ft.	——	+18.6
No. 4	1021 ft.	794 ft.	823 ft. 803 ft.	818 ft. 819 ft.	+19.	+24.5
No. 5	987 ft.	265 ft.	193.6 ft. 203.8 ft.	209.6 ft. 181.9 ft.	—66.3	—69.2

can study them, noting the change of capacity for different connections.

Table II gives the errors in feet; they are sometimes + errors and sometimes − errors, thus tending to show that inequalities in manufacture are probably largely the cause of the discrepancies.

Telephone cables. Telephone cables are made up of twisted pairs, and when they are jointed no attempt is made to connect the pairs in any definite order. In fact most of the telephone companies mix the pairs of the different layers up as much as possible, thereby making the capacity of the pairs more uniform and the cable more efficient for telephone service. On account of this method of installation, it is more difficult to locate split pair faults accurately.

Fig. 5 illustrates what is meant by a split pair. The wires C

and F are connected straight through the cable, but D and E are cross-connected. If those two pairs are used for telephone service there will be more or less cross-talk, depending upon the location of the transposition. Hence the need of locating the joint where the wrong connection has been made and rectifying the mistake. Some of the methods which have been used for locating split pairs are not reliable, giving only a rough approximation.

After a consideration of the former work presented in this paper, it will be seen that the same general methods may be employed provided we can make the capacity b fairly uniform throughout the cable, and then either measure it directly, or else eliminate it from equation (9). The first can be accomplished

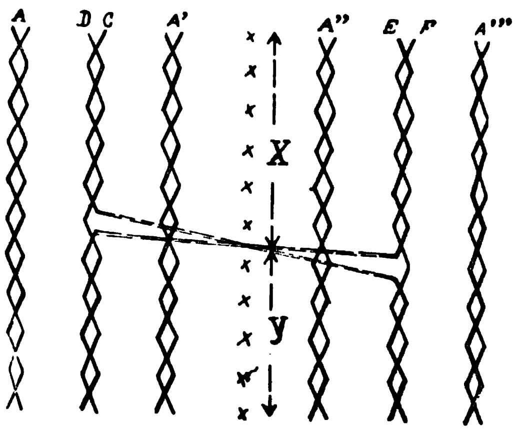


FIG. 5

fairly well by grounding all the wires in the cable except those used in the test, and b can be eliminated from equation (9) as will now be shown.

Referring to Fig. 5, A, A', A'', and A''' represent four good pairs. C, D, E, and F represent the split pairs, C and F being connected correctly, D and E being cross-connected. The capacity between the wires of the individual pairs A, A', A'', and A''' should first be measured and the average taken as the value of a. It is not absolutely necessary to take just four pairs, but enough should be measured to insure a good average result.

As in the case of telegraph cables, we will let the capacity

$$\frac{CD + EF}{2} = c \text{ and } \frac{CE + FD}{2} = d.$$

Now by referring to Fig. 5 we can say that the capacity b is equal to the capacity $C\,E$ throughout the distance x added to the capacity $C\,D$ throughout the distance y. Now as a is the capacity of a pair, we can say that a equals the capacity $C\,D$ throughout the distance x added to the capacity $C\,E$ throughout the distance y. This gives the relation $b = c+d-a$. Substituting this value of b in equation (9) we get

TABLE III
RELATIVE CAPACITIES = R IN FIG. 1.

Test between wires	Fault No. 1	Fault No. 2	Fault No. 3	Fault No. 4	Fault No. 5		Fault No 6	
	m	m	m	m	m	n	m	n
A	555	523	527	554	827	827	827.5	826.5
A'	537	530.5	540	576	832.5	833	839	838
A''	538	519.0	618	589	837	838.5	848.5	846
A'''	536	547.5	592.	566	830	830	831.0	829.5
Average a	541.5	530	569	571.2	831.6	832.1	836.5	835
$C\,D$	505	472	466	478	681	710	762.5	770
$E\,F$	468	485	504	498	686.5	716	766	773.5
Average c	486.5	478.5	485	488	683.7	713	764.3	771.8
$C\,E$	497	469	469	479	756.5	764.5	679.5	714.5
$E\,D$	466	480.5	490	492	747.5	756.5	674.5	707.5
Average d	481.5	474.8	479.5	485.5	752	760.5	677	711

TABLE IV

No. of fault	Length of cable	Actual distance to fault	Calculated distance to fault		Average error in feet	
			Other wires not grounded	Other wires grounded	Other wires not grounded	Other wires grounded
No. 1	586 ft.	293 ft.	305.6	——	+12.6 ft.	——
No. 2	586 ft.	293 ft.	303.1	——	+10.1 ft.	——
No. 3	586 ft.	293 ft.	302.3	——	+ 9.3 ft.	——
No. 4	586 ft.	293 ft.	297.3	——	+ 4.3 ft.	——
No. 5	1155 ft.	433 ft.	404	433.7	—29. ft.	0.7 ft.
No. 6	1155 ft.	759 ft.	797	765	+38. ft.	6.0 ft.

$$x = \frac{(a-d)\,L}{2a-(c+d)} \tag{12}$$

Which is the formula to use for a split pair in a telephone cable?

Referring now to Tables III and IV, faults No. 1, No. 2, No. 3, No. 4 were purposely made in 585 ft. of 203 pairs No. 19 cable.

There were two, three, four, and five pairs respectively between the faulty pairs of each fault. It is evident that the farther the faulty pairs are apart, the nearer correct is the fault location. Tests were not made in this case with all the wires except the test wires connected to the lead cover, or else a less error in feet would probably have been obtained. Faults 5 and 6 were in the No. 13 pairs of a combination cable consisting of 15 pairs No. 19, and 10 pairs No. 13.

It is a notable fact that the error in feet is much smaller when all the wires not used in each test were connected to the lead cover, than when all wires were free.

Some tests were made using a ballistic galvanometer and the discharge deflection method, and good results were obtained. This method would probably be used the most in practice, but there are testing sets where the resistance method and a telephone are used; these are especially suitable for locating faults of the kind herein treated. One of these designed by the writer is giving good satisfaction in commercial use.

Conclusions. Sufficient tests are not here given to enable anyone to tell how successfully such faults could be located in practice, but the writer is of the opinion that relatively better results might be obtained, because the general average of many sections of cable would tend towards capacity uniformity, especially in telephone cables. The tests on telephone cables here recorded average better than those on telegraph cables. The methods would probably have a wider range of usefulness in telephonic than in telegraphic engineering work.

BY-LAWS

OF THE

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

[*Adopted by the Board of Directors April* 22, 1904, *Amended February* 24, 1905, *March* 24, 1905, *February* 23, 1906, *May* 19, 1908, *and November* 13, 1908.]

The Constitution as amended May 21, 1907, authorizes the adoption of By-laws as follows:

ART. VII, SEC. 40. *The Board of Directors shall prepare and adopt a series of By-laws which shall govern its procedure and that of the committees, under this Constitution. Such By-laws shall be adopted or may be amended by a concurring vote of not less than twelve* (12) *members of the Board of Directors; provided, that the text of a proposed By-law or amendment shall be furnished to each member of the Board of Directors at least ten days before the meeting, at which a vote on the same will be taken.*

BY-LAWS.

MEMBERSHIP.

SEC. 1. Members, Associates, and Honorary Members, are authorized to use the following abbreviations indicating their grade of membership in the INSTITUTE.

Honorary, Hon. Mem. A. I. E. E.
Member, Mem. A. I. E. E.
Associate, Assoc. A. I. E. E.

The term member (when printed without a capital), where used herein, includes Members, Associates, and Honorary Members.

It shall be the duty of all members to call the attention of the Secretary to the improper use of INSTITUTE badges or symbols, or to claims made by non-members that they belong to the INSTITUTE. The Secretary shall thereupon investigate the facts and lay the matter before the Board of Directors.

SEC. 2. An applicant for admission as an Associate who is not personally known to three members of the INSTITUTE, may refer for one or more of his three references preferably to a member or members of any grade in the societies designated in Section 3, or to any citizens of standing in the community; and the members of the Board of Examiners may then, if satisfied of his merits, endorse his application in the usual manner.

SEC. 3. An applicant for transfer who certifies that he is not personally known to five Members may refer for one or more of his five references to an Associate or Associates of the A.I.E.E., or to one or more professional engineers of recognized standing who may belong to any of the following named societies: American Society of Civil Engineers; American Society of Mechanical Engineers; American Institute of Mining Engineers; The Canadian Society of Civil Engineers; The Society of Naval Architects and Marine Engineers, all of America; or, the Institution of Civil Engineers, the Institution of Mechanical Engineers, or the Institution of Electrical Engineers, of Great Britain; or of any approved American or foreign national engineering body.

SEC. 4. The names of applicants for transfer to the grade of Member, after being recommended for transfer by the Board of Examiners, shall be posted in the PROCEEDINGS, and final action thereon shall not be taken by the Board of Directors until one month after such posting, in the case of all applicants residing in North America, and until three months after such posting, in the case of all other applicants.

SEC. 5. Objection to the admission of any candidate for the grade of Associate, or for transfer to the grade of Member, must be accompanied by reasons for such objection, all statements to be treated as confidential.

SEC. 6. The privileges attaching to membership in the INSTITUTE, shall not be accorded to applicants for admission until they have been duly elected, and have paid their entrance fees and current dues.

SEC. 7. A prompt notification of his election as an Associate, accompanied by a statement of his initiation fee and dues, shall be mailed to each candidate by the Secretary. A second notice of his indebtedness shall be sent by registered mail 30 days (60 days in the case of members residing outside of North America) later to every candidate remaining in arrears. The Secretary may, at his discretion, cancel the election of any applicant remaining in arrears 30 days (60 days in the case of members residing outside of North America) after the mailing of the second notice.

SEC. 8. A member who has resigned in good standing, may, on the approval of the Board of Directors, be reinstated without payment of entrance fee, and shall be entitled to admission to the grade last held by him, after review of his professional record by the Board of Examiners. If the applicant so elects, he may retain his original date of election or transfer, by payment of all dues which would have accrued during the interim, and he will then be entitled to such INSTITUTE publications issued during such period, as are available.

SEC. 9. Any resignations received by the Secretary not later than June 1st of any year (September 1st in the case of members residing outside of North America), may, at his discretion, be considered as having been received as of May 1st preceding.

SEC. 10. The rights and privileges of each Honorary Member, Member, or Associate, shall be personal to himself, and shall not be transferable or transmissible by his own act or by operation of law.

SEC. 11. An Honorary Member, Member, or Associate, whose membership shall terminate for any cause shall have no right or interest in any property of the INSTITUTE or any claim or privilege in or against the INSTITUTE or pertaining thereto.

PAYMENT OF DUES.

SEC. 12. Annual dues are payable in advance upon receipt of bills dated May 1st, the beginning of each fiscal year.

SEC. 13. A bill for annual dues shall be mailed to each member on or about May 1st. On October 1st a second notice shall be sent to those in arrears. On March 1st following a third notice shall be sent by registered mail to those remaining in arrears, a copy of this section of the By-laws to accompany such notice. Any member remaining in arrears May 1st shall be placed on the Suspended List, shall receive no further INSTITUTE publications, and shall be debarred from enjoying its other privileges until reinstated. At its May meeting the Secretary shall submit to the Board of Directors a list of the delinquents, for its action thereon.

SEC. 14. The mailing of such bills or statements to the last address upon the office Mailing List, shall be considered a valid notice of indebtedness.

BOARD OF DIRECTORS.

SEC. 15. The regular monthly meeting of the Board of Directors shall be held on the same day as the regular monthly meeting of the INSTITUTE. The meeting may also be held in any month when no regular INSTITUTE meeting occurs, the date of such meeting and the hour of all meetings to be set by the President.

ELECTIONS.

SEC. 16. A Manager elected to fill a vacancy for an unexpired term shall be selected from those whose names appeared on the General Proposal List prepared for the last annual election.

SEC. 17. In preparing the Directors' Ticket, candidates shall be voted for by written or printed ballot. The names of candidates for Vice-president should be selected, as far as practicable, from different sections of the country.

SEC. 18. For the guidance of members in the selection of nominees for the annual election, there shall be published in the January and February PROCEEDINGS, each year, a summary of the nomination votes of the preceding year, containing the names of all persons having received at least three per cent. of the entire number of nomination votes cast, and also the names of all directors not included in this list and of ex-Vice-presidents and Managers who have held office at any time during the preceding five years.

SEC. 19. Nomination forms, as provided in the Constitution, shall be sent, upon application, to Associates who have qualified between the dates of February 1st and March 1st; and single copies only, of such nomination forms, shall be sent, upon request, as duplicates, to members who may have mislaid or destroyed their original copies. Nominations may, however, be written on blank paper if the regular form is not available.

OFFICE ORGANIZATION.

SEC. 20. The office organization shall include an Assistant Secretary and an Editor.

The Assistant Secretary shall assist the Secretary, and, in his absence, may act for him, and shall have such other duties as may be specifically assigned to him by the Board of Directors from time to time.

The Editor shall report directly to the Secretary, and shall, under the direction of the Meetings and Papers and the Editing Committees, have charge of the publication of the PROCEEDINGS and of the TRANSACTIONS.

JOHN FRITZ MEDAL.

SEC. 21. The representatives of the INSTITUTE on the Board of Award of the John Fritz Medal, shall be the President of the INSTITUTE, and his three immediate predecessors in office. Each year the president-elect, after assuming the active duties of his office, shall, on the third Friday in January, become a representative of the INSTITUTE on the Board of Award of the John Fritz Medal, and the functions of the senior past-president, in connection with this award, shall then cease and determine. In the event of the death or permanent disability of any of the representatives of the INSTITUTE on the Board of Award of the John Fritz Medal, the Board of Directors shall be empowered to select his successor from among the surviving past-presidents of the INSTITUTE, and such successor shall hold office for the term of the representative in whose place he is appointed.

ADDITIONAL DUTIES OF COMMITTEES.

SEC. 22. Each standing committee shall have a regular monthly day of meeting, as far as practicable. All resolutions or reports which each committee shall forward to the Board of Directors for action, must be signed by the chairman.

SEC. 23. In addition to the duties prescribed by the Constitution, it shall be the duty of the Finance Committee to cause to be prepared for the information of the Board of Directors, a balance sheet each year, to September 30th, showing the financial condition of the INSTITUTE, including the state of each fund which may be separately invested; also to submit to the Board of Directors at its October meeting a budget of estimated receipts and expenditures for the ensuing year beginning October 1st. It shall also be the duty of the Finance Committee to have the bank books and securities of the INSTITUTE examined, prior to the annual financial statement made in the month of May, and to certify to their correctness.

SEC. 24. It shall be the duty of the Library Committee to advise in regard to the purchase of books or the acceptance of donations for library purposes, on behalf of the INSTITUTE; to prepare an annual report for the year ending April 30th, showing the state of the library, and giving the names of all donors to the library for the preceding year, which report, when accepted by the Board of Directors, shall be printed, and form part of the monthly PROCEEDINGS.

SEC. 25. The Meetings and Papers Committee shall accept no paper from a non-member unless directly authorized to do so by the Board of Directors.

SEC. 26. In addition to the duties prescribed by the Constitution, it shall be the duty of the Editing Committee to exercise a general supervision over all publications of the INSTITUTE. It shall also call the attention of the Meetings and Papers Committee to papers which it may deem unsuitable for publication in the PROCEEDINGS or TRANSACTIONS, assigning specific reasons therefor.

SEC. 27. Each temporary committee appointed to arrange for the holding of the Annual Convention, or for any other purpose involving expense to the INSTITUTE, shall prepare an estimate of its probable expenditure and submit it to the Board of Directors for approval.

SEC. 28. The functions and duties of temporary committees shall be defined by the Board of Directors. At the beginning of each administrative year the Secretary shall transmit to all the members of each temporary committee whose duties may be continued, copies of the original resolutions constituting such temporary committee. It shall be the duty of the Secretary to call the attention of the Board of Directors to work done by any temporary committee outside the scope of the resolution or resolutions which constituted it.

SEC. 29. The expenditures of the INSTITUTE shall be regulated by annual appropriations which shall begin on the first day of October of each year. Each committee shall submit to the Secretary for the information of the Finance Committee in preparing its annual October budget, an estimate of its requirements for the ensuing year.

SEC. 30. No standing or temporary committee shall, under any circumstances, exceed its authorized appropriation without first obtaining the sanction of the Board of Directors.

SEC. 31. For the information and guidance of these committees the Secretary shall each month send to the chairman of every committee which has incurred expense during the preceding month, the following information:

1. Total amount of all appropriations authorized for the current year by the Board of Directors for such committee.

2. Amount in dollars and per cent. of appropriation expended during the preceding month.

3. Amount in dollars and per cent. of appropriation expended during the current fiscal year, including the preceding month.

4. The same information for the corresponding periods of the preceding year, when available.

SEC. 32. The President may invite any non-member to discuss any paper presented at a meeting of the INSTITUTE, and the chairman of any Section may invite any non-member to discuss any paper presented at the Section of which he is chairman.

PUBLICATIONS.

SEC. 33. The regular publications of the INSTITUTE shall include PROCEEDINGS, to be published monthly, and TRANSACTIONS, to be published quarterly, semi-annually, or annually.

SEC. 34. The PROCEEDINGS may have two principal divisions:

1. For current matter and INSTITUTE news;

2. For technical papers, discussions and other matter of an engineering character.

SEC. 35. The TRANSACTIONS shall contain such papers, discussions and other engineering matter previously printed in the PROCEEDINGS, as may be approved for publication in the TRANSACTIONS by the Meetings and Papers Committee and the Editing Committee.

SEC. 36. From the annual dues paid by each member, five dollars ($5.00) shall be deducted and applied as a subscription to the PROCEEDINGS for the year covered by such payment.

SEC. 37. The payment of the annual dues by any member shall entitle him to receive a bound copy of each volume of the TRANSACTIONS for the period covered by such payment.

SEC. 38. The paper or papers presented at the meeting in any month shall be published in the PROCEEDINGS, preferably in advance of the meeting.

SEC. 39. A paper, after being edited and revised for publication under the direction of the Meetings and Papers Committee, shall be returned to the author to obtain his consent to its publication as revised. In case the author shall not consent to publication in the form suggested by the said committee, and a revision acceptable to both the committee and the author cannot be arranged, the paper shall not be presented or published.

SEC. 40. A discussion or communication, after being edited and revised for publication under the direction of the Editing Committee, shall be returned to the author to obtain his consent to its publication as revised. In case the author shall not consent to publication in the form suggested by the said committee, and a revision acceptable to both the committee and the author cannot be arranged, the discussion or communication shall not be published.

SEC. 41. No paper, or written contribution to a discussion shall be given to the press for publication, in advance of its presentation, except by authority of the Board of Directors.

SEC. 42. No person shall receive monetary compensation from the reprinting of any paper or discussion presented before the INSTITUTE or before any Section or Branch without previous authorization from the Editing Committee.

SEC. 43. The Secretary is authorized to receive annual subscriptions for the monthly PROCEEDINGS, ending with the calender year, at the rate of $10.00 per annum for all countries to which the bulk rate of postage applies. To all other countries the price shall be $12.00 per annum, fifty per cent. discount to libraries.

SEC. 44. Existing bound volumes, or bound volumes hereafter published, may be sold to non-members at the following prices:

Paper covers, Vols. I to IV, each	$2.00
Paper covers, later volumes, per year	10.00
Cloth " " " "	11.50
Half Morocco " " " "	13.00

A discount of 10 per cent. from the above prices, either for volumes, subscriptions, or parts, may be allowed to publishers and dealers. Members and Associates shall be allowed a discount of 50 per cent. on parts or duplicate volumes to replace those lost or destroyed. They may purchase volumes issued prior to their election, when available, at $5.00 each, either in paper or cloth covers, or at $1.00 extra for half morocco binding.

SEC. 45. An Associate shall be entitled to receive the bound volume or volumes of the TRANSACTIONS, beginning with the volume including the corresponding half year in which he has paid his entrance fee, and such single copies of the PROCEEDINGS for the same period as he may apply for, and which are available.

SEC. 46. A member who has presented a paper to the INSTITUTE, which has been printed in the PROCEEDINGS, shall be entitled, upon application, to 15 copies of the paper, or, at the Secretary's discretion, of the PROCEEDINGS containing such paper, and to the same number of copies of the discussion, provided such copies are available.

SEC. 47. A catalogue of members shall be published annually. A directory, in which the names are grouped geographically (Members' names shall be distinguished from those of Associates by a difference in type), may also be issued. The Constitution and By-laws of the INSTITUTE shall be printed in at least one of these publications. Either the catalogue or the directory, or both, at the Secretary's discretion, may be issued as a part of the PROCEEDINGS.

SECTIONS.

SEC. 48. The principal work of a Section shall be the holding of regular meetings for the presentation and discussion of papers on matters relating to electricity, and to the allied arts and sciences.

SEC. 49. A Section of the INSTITUTE may coöperate with other local engineering organizations in the holding of joint meetings, and may invite members of such organizations to its meetings.

SEC. 50. A petition for the formation of a Section shall specify the territory to be included and shall be signed by not less than 25 members residing therein.

SEC. 51. The territory of a Section shall include the locality naturally tributary thereto, but shall not include territory so remote that members cannot readily attend meetings.

SEC. 52. A Section shall be known as: The (name of place) Section of the American Institute of Electrical Engineers.

SEC. 53. In the initial Section organization only petitioners shall have the right to vote.

SEC. 54. In addition to INSTITUTE members the Section by-laws may admit other persons interested in engineering, who shall have all the Section privileges except those of voting and holding office.

SEC. 55. The terms of office of the Section Chairman and Secretary shall coincide with that of the President of the INSTITUTE. They may or may not be eligible for reëlection as prescribed in the Section by-laws.

SEC. 56. The Board of Directors may at any time rescind the authorization of any Section and terminate its existence.

SEC. 57. Papers and discussions presented before a Section or Branch shall be the property of the INSTITUTE, and may be published only on authorization of the Editing Committee.

SEC. 58. The Executive or other suitable Section or Branch committee in charge shall be responsible to the Editing Committee for all Section or Branch papers and discussions. Such matter as is approved by such local Section or Branch committee, as suitable for publication in the PROCEEDINGS, shall be presented to the Secretary of the INSTITUTE for consideration by the Editing Committee, who in judging its suitability shall be governed by the same reasons as are applied to similar matter offered for presentation at INSTITUTE meetings.

SEC. 59. Section or Branch papers and discussions not published in the PROCEEDINGS may be published elsewhere, with the statement that they have been presented before such Section or Branch, provided that previous approval of such publication has been secured from the Section or Branch committee in charge, and from the Editing Committee. The Editing and the Section or Branch committees may authorize the publishing of a Section or Branch paper or discussion without reference being made to its presentation before the Section or Branch.

SEC. 60. It shall be the duty of the Secretary of the Section to send to the Chairman of the Sections Committee, on or before October 1st of each year, an estimate of the appropriation required from the INSTITUTE for the expenses of the Section during the year ending the 30th of the following September. The Sections Committee shall apportion among the Sections such appropriation as may be authorized by the Board of Directors, and shall notify the Secretary of each Section of the amount apportioned to it. Such amounts may be drawn upon only for the reasonable expenses of the engineering meetings of the Section upon presentation of original bills endorsed by the Section Secretary and for the transportation expenditures of the Section's official delegate to the Annual Convention.

STUDENTS.

SEC. 61. Any person pursuing a regular course of undergraduate study in electrical subjects, in any university or technical school of recognized standing, may be enrolled as a "Student of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS," as hereinafter provided.

SEC. 62. Each application for enrolment as a Student shall be endorsed by a member of the INSTITUTE. It shall be passed upon by the Board of Examiners, and submitted to the Board of Directors for action.

SEC. 63. The annual Student fee shall be three dollars ($3.00), payable in advance. The period of Student enrolment shall not exceed three years, and shall date from the first of January nearest the date of filing application.

SEC. 64. The annual fee of three dollars ($3.00) paid by each Student shall be applied as a subscription to the PROCEEDINGS for the year covered by such payment.

SEC. 65. Students enrolled under the provisions of these By-laws shall each be entitled to receive the regular monthly PROCEEDINGS and ordinary notices of meetings. They will be permitted to purchase the current TRANSACTIONS at $3.50 per year during the period of their enrolment.

SEC. 66. Student privileges may be withdrawn at any time by the Board of Directors.

SEC. 67. A Student changing his mailing address without notifying the Secretary shall be entitled to available duplicate copies of parts of the PROCEEDINGS upon payment of twenty-five cents for each part.

UNIVERSITY BRANCHES.

SEC. 68. Any member who may be connected with a university or technical school of recognized standing, having a course in electrical engineering, may, upon the approval of the Board of Directors, organize Student meetings, to be held under his direction, such Student meetings to be designated as (name of institution) Branch of the AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

SEC. 69. University Branches shall organize by appointment or election of an Executive Committee of any suitable number of which the said member shall be a member, and of which other members or the enrolled Students of the INSTITUTE may constitute the other members. This organization shall be subject to the approval of the Board of Directors. Other students in the electrical engineering course of the institution may attend the meetings, under conditions to be prescribed by the University Branch.

SEC. 70. The member, by whom the University Branch shall be organized, or the member who succeeds him on its Executive Committee, shall be responsible to the Secretary of the INSTITUTE, for the conduct of its affairs, and shall make regular reports to the Secretary, of the proceedings at its meetings.

SEC. 71. At the meetings held by University Branches the papers read and discussed may be either original papers, or may be papers read before one of the regular meetings of the INSTITUTE. The Secretary of the INSTITUTE may send to University Branches a limited number of advance copies of the papers of the INSTITUTE.

SEC. 72. The INSTITUTE shall publish, from time to time, a list of universities and technical schools in which University Branches have been organized, and brief notices of the proceedings of their meetings.

INDEX TO BY-LAWS.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS

REPORT OF THE BOARD OF DIRECTORS FOR THE FISCAL YEAR ENDING APRIL 30, 1908.

The Board of Directors presents herewith for the information of the Institute a report of its work during the year, of the various standing and special committees, and the financial standing of the organization.

The Annual Convention was held at Niagara Falls, June 25-28, 1907. The total registered attendance was 459. Thirty-two professional papers, including the President's annual address, the report of the Committee on a Code of Ethics, and the revised Standardization Rules, were presented and discussed.

The Board of Directors has held 11 regular monthly meetings and one special meeting during the year, and the Executive Committee one meeting.

Upon recommendation of the Finance Committee, and with the approval of the General Counsel, *advance* payments were made upon the Institute's share of the land occupied by the Engineers' building. The total amount paid was $81,000, covering nine out of the twenty annual instalments, up to and including July 1, 1916. These payments were made from the Land, Building and Endowment Fund.

Upon recommendation of the Finance Committee the Institute has purchased for investment $12,000.00, face value, New York City registered bonds drawing interest at 4½ per cent., which were paid for out of the current bank balance.

Board of Examiners.—Eleven meetings were held during the year, at which applications for election as Associates, enrolment as Students, and transfer to the grade of Member, have been reported to the Board of Directors as follows:

Recommended for election as Associates	1,588
Not recommended for election as Associates	8
Recommended for enrolment as Students	687
Recommended for transfer to the grade of Member	53
Not recommended for transfer to the grade of Member	23
Held for further information	8
Total number of applications considered	2,367

Sections Committee.—Prior to the adoption of the new Constitution a year ago the local organizations were divided into three classes: namely, Branches, University Branches, and Student Meetings. The total number of local organizations of all classes on April 30, 1907, was 33. Under the new Constitution the local organizations were divided into two classes: Sections and Branches. There are now 48 local organizations, 22 of which are Sections, and 26 Branches; an increase of 15 during the year. Of this number four are Sections, organized at Toledo, Cleveland, Norfolk, and the City of Mexico. The other 11 are Branches located in various parts of the country. In addition to the Sections and Branches organized during the year, the committee has received many inquiries from different cities respecting the organization of Sections. The reports of local secretaries show that 284 meetings were held at the different Sections and Branches during the year, with a total attendance of 11,604. At these meetings 204 original papers were presented, in addition to which 104 Institute papers were discussed. The increasing interest taken in section work as shown by the facts presented in this report is further indicated by the large number of visiting speakers who have at various times during the year been invited to take part in section and branch meetings. The results of the year's work are very encouraging, and there is every reason to believe there would be even a more satisfactory showing had all the local meetings been reported to the secretary of the Institute.

Building Fund Committee.—Notwithstanding the financial depression of the latter half of the year, the Building Fund Committee is able to report that the number of subscribers to the Fund has increased from 934 to 1,538. The amount subscribed during the year was $6,464.68, showing an average for the new subscribers of a little over $10.00 per capita. The amount collected and placed in the treasury of the Institute during the year by the committee was $10,780.23. The total amount now pledged by the efforts of the committee is $170,199.15, out of which $135,609.50 has already been collected. It will thus be seen that the Fund has made substantial and encouraging progress, and it is hoped that with the return of prosperity the remaining $15,000 required may be obtainable from the membership, so that the Institute may enter upon its second quarter of a century in 1909 absolutely free of debt. While the number of subscribers is already so large, it is believed that amongst those who have not yet subscribed, and especially amongst the newer members, there will be found enough public spirited men to extinguish the small amount of indebtedness not now provided for. The committee feels strongly the desirability of basing the subscription list as broadly as possible, and while it has been successful in securing some very large individual gifts, it has directed and is still directing its efforts more particularly in the line of smaller amounts.

Standards Committee.—The Standards Committe has held three meetings since its appointment. Through its members it has engaged in a number of conferences with representatives from other national associations, including the National Bureau of Standards, with a view to assisting in a movement for an international agreement as to standard of light. The work of recomputing a wire table is held in abeyance until the National Bureau of Standards announces the value of the temperature coefficient of resistivity of copper, which is being determined.

Library Committee.—The complete report of this Committee will be printed in the Institute PROCEEDINGS in the near future.

Committee on Bibliography.—The catalogue of the Latimer Clark

Library, which was presented to the Institute in 1901 by Dr. Schuyler Skaats Wheeler, is now ready for printing. A very large amount of painstaking effort has been expended upon this work, and the proofs have been carefully revised by many authorities both in this country and abroad.

Meetings and Papers Committee.—During the year this committee has arranged for eight regular and two special meetings and a meeting in honor of Lord Kelvin. The special meetings were on the subject of Forest Preservation and Technical Education, the latter under the auspices of the Educational Sub-Committee. A total of seventeen papers were read at these meetings.

The regular annual convention was held at Niagara Falls under the auspices of the last committee. Preparations are now being made for the coming convention at Atlantic City. About thirty-three papers, of which many are short, will be presented.

The committee has made a vigorous effort to publish the papers in the issue of the PROCEEDINGS immediately in advance of the meeting, and this result has been accomplished in all instances with one possible exception. It is believed that it is a great advantage for the membership at large to have the papers in their hands some days in advance of the meeting. The manuscripts for the convention papers have nearly all been received and will be published in the May and June PROCEEDINGS as far as possible.

It has been the practice of the committee during the present year to consider the various sub-committees as practically independent bodies, except so far as the arrangement of dates and the editorial revision of papers is concerned. The initiative in the choice of subjects, the soliciting and accepting of papers, etc., have been in the hands of sub-committees. This is believed to be the only practical method of conducting the work. The present arrangement of the sub-committees, if treated in a liberal spirit, seems to be a very desirable one.

Papers presented at the Section meetings have been considered by the same standard as the papers submitted for the New York meetings, and a number have been printed in the PROCEEDINGS during the year.

Educational Sub-Committee.—During the winter several meetings of this committee have been held in New York City. As a sub-committee of the Meetings and Papers Committee, the first function of this committee is to obtain papers on the subject of technical education, and to arrange for meetings. Accordingly a special meeting of the Institute was held on January 24, 1908, at which two papers on engineering education were presented. Eighteen members of the Institute took part in the discussion which followed.

The University Branches throughout the country were urged to take up the subject of technical education and devote a session to it, which many of them did. The chairman of this committee conducted the meeting on this subject at the University of Illinois. He also visited several other technical institutions during the year. The committee has arranged for the presentation of several papers on technical education at the coming convention at Atlantic City.

Railway Sub-Committee.—The Railway Committee during the past year has acted in conjunction with the Meetings and Papers Committee in obtaining papers and speakers on railway subjects. The committee suggests that one of its members be appointed on the Standards Committee, and that all matters relating to the standardization of railway apparatus or railway practices be referred to it by the Standards Committee. The field of usefulness of the Railway Committee appears to be threefold—acting in an advisory capacity to the Meetings and Papers Committee, the Standards Committee, and the High-Tension Transmission Committee, and while its activities have been largely limited to acting in conjunction with the Meetings and Papers Committee during the past year, it is probable that the continued existence of the committee will demonstrate the benefit resulting from broadening its duties.

Editing Committee.—Since September, 1907, there have been edited and published eight numbers of the PROCEEDINGS. The total number of pages of these PROCEEDINGS is 1494; of this total, 454 pages have appeared as Section I, 1040 pages as Section II. Of the 1040 pages in Section II, 580 pages were devoted to technical papers, and 460 pages to discussions. In editing the discussions, the Committee has endeavored consistently to make them approach the well established Institute standards of style as to substance and form.

This committee has conferred with the Meetings and Papers Committee in regard to the publication of Volume XXVI of the TRANSACTIONS. This conference has decided that the TRANSACTIONS for 1907, consisting of approximately 1850 pages of papers, discussions and reports, be printed in two parts. Part I is now at the bindery and Part II is in press. Both parts will probably be ready for distribution in July.

Law Committee.—The Law Committee held six meetings during the year. Its most important work has been the revision of the By-laws to conform to the new Constitution. In this work the committee was assisted by members of other committees and by suggestions and comments from Institute members. The proposed new By-laws are now in form for presentation to the Board of Directors.

Other matters to receive the attention of the committee during the year were: the discrepancy between the dates of the Annual Meeting as set forth in the Charter and in the Constitution; the resolutions of the past-presidents embodying several important suggestions; resolutions of the Toronto Section respecting an intermediate grade of membership in the Institute.

Code Committee.—The Code Committee has held five meetings during the year for the discussion of some of the more important changes which have been proposed for embodiment in the National Electrical Code. The committee requests that any member of the Institute who may have noted instances of ambiguity in the present rules of the code, bring the matter to the attention of Mr. C. M. Goddard, Secretary of the Underwriters Electrical Committee, 55 Kilby Street, Boston, Mass., preferably with a suggested modified wording to avoid the ambiguity.

The Code Committee was officially represented by its chairman at the meeting of the Underwriters National Electric Association, March 25 and 26; also of the meeting of the National Conference on Standard Electrical Rules on March 27.

Committee on Forest Preservation.—This committee has devoted its efforts largely to emphasizing the importance of forest preservation from the standpoint of the protection of water powers. As a result of the suggestions of the committee, meetings devoted to this subject were arranged by the local officers of the Atlanta, Boston, Minneapolis, Pittsfield, and Urbana Sections. Through the coöperation of the Meetings and Papers Committee Mr. Gifford Pinchot read a paper on the " Conservation of Natural Resources" at a special New York meeting of the Institute.

Through the courtesy of the Forest Service copies of the report of the Secretary of Agriculture on Southern Appalachian and White Mountain Watersheds were sent to all members of the Institute.

In accordance with its instructions the committee transmitted to the members of congress the resolutions on forest preservation adopted by the Board of Directors, and the Institute was represented at the congressional hearing on the Appalachian Forest Bill by Messrs. Schoen and Waddell, who have already presented their report. That many members of congress do not appreciate the importance of the protection of water powers has been conclusively demonstrated during the present session, and this committee feels that the Institute as an organization as well as the individual members should exert all legitimate influence to further legislation tending to protect the head-waters of streams on which important water powers are located. It is only by the active efforts of every one interested that any results can be accomplished.

International Electrotechnical Commission.—No meeting either of the International Electrotechnical Commission or of the United States subcommittee has been held during the year. A meeting of the International Electrotechnical Commission is contemplated for next fall, in London.

Various communications have been received from the acting secretary of the International Commission, and replies have been forwarded. In particular, a glossary of terms, A-G, prepared by the British special committee on nomenclature, was received in February, for consideration. It was transmitted to the Standards Committee, and received the consideration of that committee in March. The opinion and suggestions of the Standards Committee were communicated to the acting secretary of the Commission in London, under date of March 19, 1908.

Code of Ethics Committee.—A reprint of the proposed Code was mailed to all members in September, 1907, with a request for suggestions and criticisms, and a large number of very interesting and valuable replies were received.

Two meetings of the committee have been held at which the subject was discussed, but definite action was not taken, as all the members were not present.

Upon invitation of the Faculty of Science of Columbia University, the chairman made an address upon the subject before the graduating class.

Edison Medal Committee.—Competition for the Edison Medal under the terms of the original " Deed of Gift Creating the Edison Medal " having been demonstrated a practical failure, a new and substitute deed of gift has been executed.

By the terms of the new deed, the Edison Medal is no longer open to competition, but is to be awarded for " Meritorious Achievement in Electrical Science, or Electrical Engineering, or the Electrical Arts."

The next important work before the committee is to formulate new By-laws to govern its proceedings in consonance with the new deed.

Increase of Membership Committee.—This committee has been continuously active in bringing the advantages of Institute membership to the attention of desirable candidates. The number of applications for admission to the Institute as Associates received from October 1, 1907, at which time the present committee began its work, to April 30, 1908, is 874. No doubt a considerable number in addition will be received during the next few months, as a result of letters and printed matter already forwarded to prospective applicants whose names have been suggested to the committee.

Membership.—The present total membership and the increase during the past year are indicated below:

	Hon. Mem.	Mem.	Asso.	Total
Membership April 30, 1907	2	536	3,983	4,521
Additions:				
New associates	...	...	1,361	....
Transferred	...	43	....	....
Deductions:				
Died	1	1	21	....
Resigned	...	3	54	....
Dropped	...	2	126	....
Transferred	...	...	43	....
Membership April 30, 1908	1	573	5,100	5,674

Net increase in membership during the year......................1,153

Resignations.—The following Members and Associates have resigned during the year in good standing:

Members.—C. Batchelor, D. H. Fitch, George A. Redman.

Associates.—P. L. Anderson, C. A. Babtiste, C. D. Bailey, T. P. Bailey, F. P. Beach, T. W. Blake, J. H. Brown, J. S. Butler, P. G. Chace, E. H. Clarke, F. E. d'Humy, R. N. Dickinson, T. E. Doubt, G. F. Durant, J. M. Eglin, A. S. Fairbanks, T. Z. Franklin, C. Furgueson, W. H. Gallaher, L. T. Girdler, Louis Glass, H. C. Goldrick, Heatley Green, E. C. Hall, L. I. Hall, N. H. Heft, C. M. Heminway, Ho Hidetaro, J. S. Hill, Robert Hiltbrand, E. Hockett, A. C. King, A. H. Lidderdale, A. Marston, H. H. B. Meyer, S. A. Moss, L. C. F. Nautre, J. S. Neave, E. J. O'Beirne, A. G. Palgrave, C. C. Parks, H. A. Pharo, A. Pratt, L. W. Pratt, W. T. Ritchie, Bernard Rowntree, A. Hugh Scott, Oliver Shiras, J. F. Skirrow, B. K. Sweeney, H. P. Tewksbury, J. C. Walter, A. J. Wood, F. V. Young.

Total resignations, 57.

Deaths.—The following deaths have occurred during the year:

Honorary Members.—Lord Kelvin.

Members.—C. K. Stearns.

Associates.—F. C. Barr, U. S. Bell, T. V. Bolan, E. B. Brisley, E. Daniel son, L. J. Deremo, C. D. Howell, W. A. Kreidler, L. J. Magee, C. P. Matthews, L. K. Perot, W. J. Phelps, H. L. Price, O. R. Roberson, Paul Robinson, J. M. Simpson, J. J. Skidmore, L. W. Snowden, F. P. Spiese, D B. Watson, Charles Williams, Jr.

Total deaths, 23.

Delinquent.—Dropped as delinquent during the year, 128.

The average receipts and disbursements *per capita* for the past seven years, are shown in the following table:

RECEIPTS AND DISBURSEMENTS PER YEAR PER MEMBER.

During each fiscal year for the past seven years.

Year	1902	1903	1904	1905	1906	1907	1908
Membership, April 30th, each year	1549	2230	3027	3460	3870	4521	5674
RECEIPTS PER MEMBER:							
Entrance Fees	$1.16	$1.59	$1.65	$0.83	$0.75	$0.90	$1.21
Dues	10.06	9.01	9.33	9.30	9.47	8.95	9.36
Transactions, Sales, Advertising	1.54	1.79	2.11	1.70	2.15	1.81	1.83
Badges	.26	.35	.39	.28	.27	.30	.43
Interest	.24	.21	.18	.21	.13	.25	.18
	$13.26	$12.95	$13.66	$12.32	$12.77	$12.21	$13.01
DISBURSEMENTS PER MEMBER:							
Transactions	$3.50	$4.67	$3.43	$3.77	$3.33	$2.83	$3.51
Salaries	2.78	2.49	2.50	2.20	2.64	2.53	2.55
Meeting Expenses, incl. Sections	1.13	.87	1.16	.82	.68	.68	.91
Housing	.94	.65	.79	.75	.68	1.33	1.15
Library, incl. Salaries	1.85	1.38	1.39	.81	.69	1.37	.51
Postage	.51	.69	.66	.66	.58	.59	.65
Stationery and Miscel. Printing	.53	.96	1.01	.70	.78	.85	.80
General Expenses	.59	.52	.45	.54	.29	.43	.62
Badges	.19	.27	.35	.25	.22	.29	.40
Express	.15	.15	.28	.22	.23	.28	.14
Advertising					.36	.30	.35
Office Fittings						.14	.14
Total	$12.17	$12.65	$12.02	$10.72	$10.48	$11.62	$11.73
Credit Balance per Member	$1.09	$0.30	$1.64	$1.60	$2.29	$0.59	$1.28

LAND, BUILDING AND ENDOWMENT FUND.

Receipts.		Disbursements.	
Before appointment of Committee	$ 6,100.00	Paid United Engineering Society, acct. of contract	$ 8,000.00
Collected by Committee	135,609.50	Paid United Engineering Society, acct. of mortgage	99,000.00
Interest on balances	4,998.40	Paid United Engineering Society, acct. of interest	16,289.45
		Expenses of Committee	6,146.10
		Paid for Furniture and Fittings, Engineers Building	3,924.04
		Balance in bank, May 1, 1908	13,348.31
Total	$146,707.90	Total	$146,707.90

GENERAL FINANCIAL STATEMENT.

New York, May 13, 1908.

MR. H. G. STOTT,

President American Institute of Electrical Engineers,

No. 33 West 39th Street, New York City.

Dear Sir: The Finance Committee has during the past year conformed to the provisions of the Constitution and By-laws describing the duties of the committee. It respectfully submits herewith the report of the chartered accountants, Messrs. Peirce, Gimson & Company, who have audited the books of the Institute for the year.

The committee has personally examined the securities held by the Institute, and found them to be as stated in the balance sheet.

In accordance with instructions from the Board, your committee has invested a part of the Institute's surplus funds in high grade securities and in view of the prosperous condition of the Institute, believes that additional security and increased income can be had by continuing to follow this policy of investing surplus funds not required for immediate use, in high-grade bonds.

Respectfully submitted,

CALVERT TOWNLEY,
Chairman Finance Committee.

New York, May 14, 1908.

MR. CALVERT TOWNLEY,
Chairman Committee on Finance.

Sir: In accordance with your instructions, we have audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1908.

The results of this examination are presented in four exhibits, attached hereto, as follows:

Exhibit A. Balance sheet, April 30, 1908.

Exhibit B. Receipts and disbursements for general purposes for year ended April 30, 1908.

Exhibit C. Receipts and donations for designated purposes, also expenditures on same for year ended April 30, 1908.

Exhibit D. Condensed cash statement.

We beg to present attached hereto our certificate to the aforesaid exhibits.

Yours very truly,

PEIRCE, GIMSON & CO.,
Certified Public Accountants.

New York, May 14, 1908.

MR. CALVERT TOWNLEY,
Chairman Committee on Finance.

Dear Sir: Having audited the books and accounts of the American Institute of Electrical Engineers for the year ended April 30, 1908, we hereby certify that the accompanying Balance Sheet is a true exhibit of its financial condition as of April 30, 1908, and that the accompanying statements of Cash Receipts and Disbursements are correct.

PEIRCE, GIMSON & CO.,
Certified Public Accountants.

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

BALANCE SHEET APRIL 30, 1908.

EXHIBIT A.

ASSETS.

CASH:		
Land, Building and Endowment fund	$13,348.31	
Carnegie (Library) fund	3,168.19	
General Library fund	244.15	
Compounded Membership fund	5,600.00	
*Mailloux fund	1,017.35	
International Electrical Congress of St. Louis, 1904, Library fund	2,253.34	
		$25,631.34
†General cash in bank	12,225.71	
Secretary's petty cash on hand	500.00	
		12,725.71
United States Government Bonds 3s, 1918	8,000.00	
Premium on Bonds	320.00	
		8,320.00
New York City 4½% Gold Bonds, 1957	12,000.00	
Premium on Bonds	677.50	
		12,677.50
Equity in Societies Building (25 to 33 West 39th Street)	353,346.61	
One-third cost of land (25 to 33 West 39th Street)	180,000.00	
		533,346.61
Library volumes and fixtures	25,375.88	
Transactions	4,654.75	
Office furniture and fixtures	5,215.04	
Works of art, paintings, etc.	2,300.00	
Badges	576.35	
		38,122.02
ACCOUNTS RECEIVABLE:		
Members for past dues	11,365.75	
Members for entrance fees	425.00	
		11,790.75
Miscellaneous	289.54	
Subscriptions	26.98	
For advertising	2,007.25	
		2,323.77
Total Assets		$644,937.70

* The Farmers' Loan and Trust Co. deposit account includes $17.35 of the Mailloux Fund.

† $75.00 of the Weaver donation.

LIABILITIES AND SURPLUS.

FUNDS:		
Land, Building and Endowment Fund	$13,348.31	
Carnegie (Library) Fund	3,168.19	
General Library Fund	244.15	
Compounded Membership Fund	5,600.00	
Mailloux Fund	1,017.35	
International Electrical Congress of St. Louis, 1904, Library Fund:		
Principal	2,146.12	
Unexpended Income	107.22	
		$25,631.34
Reserve fund (U. S. Govt. Bonds)		8,320.00
United Engineering Society (for cost of land)		81,000.00
Total liabilities		$114,951.34
SURPLUS:		
In cash	$12,243.06	
New York City bonds	12,677.50	
In property and accounts receivable	505,065.80	
		529,986.36
Total liabilities and surplus		$644,937.70

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Receipts and Disbursements for General Purposes for Year Ended April 30, 1908.

Exhibit B.

Receipts.

Entrance Fees	$6,865.00	
Current Dues	45,036.73	
Past Dues	4,088.75	
Advance Dues	182.50	
Students' Dues	3,323.00	
Transfer Fees	460.00	
Badges	2,442.00	
		$62,397.98
Sales, Transactions, etc.	1,240.77	
Subscriptions, Proceedings	1,085.90	
Advertising	7,782.87	
Binding	71.00	
Exchange	20.65	
Interest:		
U. S. Govt. Bonds	240.00	
Bank Balances	770.30	
Royalty	154.40	
		11,365.89
Total		$73,763.87

Disbursements.

Stationery and Printing	$4,559.27	
Postage	3,713.93	
General Expenses	2,460.69	
Meeting Expenses	2,436.60	
Section Meetings	2,749.82	
Badges	2,247.39	
Salaries	12,086.00	
Dedication	1,064.71	
Office Furniture	783.12	
		$32,101.53
Proceedings—Transactions:		
Printing	13,324.23	
Salary	2,392.00	
Engraving	830.96	
Volumes	4,202.70	
Electrotyping	1,008.66	
Binding	512.60	
Express	793.89	
Advertising Commissions	1,959.02	
		25,024.06
Library:		
Furniture	290.80	
Librarians salaries	1,314.00	
Books and binding	425.49	
Library insurance	60.90	
Moving expenses	643.83	
Miscellaneous	165.96	
		2,900.98
United Engineering Society Assessments		6,533.33
Total		$66,559.90
Excess receipts over disbursements deposited in general fund		7,203.97
		$73,763.87

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Receipts and Donations for Designated Purposes, also Expenditures for Year Ended April 30 1908.

Exhibit C.

Receipts.

Land, Building and Endowment fund, Donations, Interest, etc.	$14,335.85
Carnegie (Library) Fund, Interest	107.43
General Library Fund, Interest	7.15
Compounded Membership Fund	794.79
International Electrical Congress of St. Louis, 1904, Library Fund, on account of Principal	48.56
International Electrical Congress of St. Louis, 1904, Library Fund, on account of Interest	64.68
Mailloux Fund, Interest	30.00
Weaver Donation	100.00
Total	$15,488.46
Excess of Expenditures over Receipts (as below)	82,819.32
	$98,307.78

Expenditures.

Land, Building and Endowment Fund, on account of Mortgage	$90,000.00
Land, Building and Endowment Fund, on account of Interest and Expense	7,133.61
Carnegie (Library) Fund	468.84
Compounded Membership Fund	660.13
Mailloux Fund	20.20
Weaver Donation	25.00
Total Expenditures	$98,307.78

AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS.

Condensed Cash Statement.

Exhibit D.

Cash on deposit April 30th, 1907	$126,149.90	
Secretary's petty cash, April 30th, 1907	500.00	
	$126,649.90	
Receipts for general purposes, Exhibit "B"	73,763.87	
Receipts for designated purposes, Exhibit "C"	15,488.46	
		$215,902.23
Disbursements for general purposes, Exhibit "B"	$66,559.90	
Expenditures for designated purposes, Exhibit "C"	98,307.78	
Expended for New York City 4½% Bonds	12,677.50	
		$177,545.18
Balance on hand, April 30th, 1908		$38,357.05
On deposit for designated purposes, Exhibit "A"	$25,613.99	
On deposit for general purposes, Exhibit "A"	12,243.06	
Secretary's petty cash, Exhibit "A"	500.00	
		$38,357.05

PROPERTY ACQUIRED DURING THE YEAR.

Office Furniture and Fixtures	$783.12
Library Books and Binding	855.34

All outstanding bills due prior to May 1, 1908, have been paid.
Respectfully submitted for the Board of Directors.

RALPH W. POPE, *Secretary.*

New York, May 19, 1908.

REPORT OF THE LIBRARY COMMITTEE.

We beg to submit herewith our report giving the more important features of the growth and operation of the library for the year ending April 30, 1908; with a statement of the present condition of the several library funds and of the expenditures for the year.

The gifts to the Library through its members and others have numbered nearly 300 volumes together with a very large number of pamphlets as yet unlisted and not included in the statistics presented herewith. The total valuation of the books and pamphlets is now placed at $23,406.55 an increase of $855.34 for the year. The complete list of donors with the number of volumes presented by each is as follows:

Donors

Adams, Edward D	11
American Institute of Electrical Engineers	16
Andrews, W. S	4
Audel & Co., Theo	1
Boston Transit Commission	1
Boucher, W. J	9
Bureau of Labor, Connecticut	1
Camp, W. M	1
Chemical Publishing Company	4
Clayton & Craig	1
Crossman, T. E	1
Cushing, H. C., Jr.	1
Department of the Interior	1
Doremus, Dr. Chas. A	1
Drake & Co., Frederick	4
Electrician Printing and Publishing Company, London	6
Engineering News Publishing Company	1
Floy, Henry	1
Fowler, W. H	1
Franklin, Prof. W. S.	1
Gauthier-Villars	1
General Electric Co	1
Hammond, R	1
Henley Publishing Company, N. W	2
Hering, Carl	2
Hobart & Ellis	1
Jenks, W. J	1

KANSAS GAS, WATER, ELECTRIC LIGHT & STREET RAILWAY ASSO.. 1
LIBRARY OF CONGRESS.................................... 1
LINDSAY, C. E.. 1
LOCKWOOD, THOMAS D..................................... 1
MCGRAW PUBLISHING COMPANY..............................39
MACMILLAN COMPANY, THE................................. 7
MAILLOUX, C. O...11
MARTIN, T. C... 1
MORTON, DR. WM. J...................................... 1
MUNICIPAL OWNERSHIP PUBLISHING BUREAU.................. 4
NATIONAL CIVIC FEDERATION.............................. 3
NEW YORK EDISON COMPANY................................ 2
OHIO ELECTRIC LIGHT ASSOCIATION........................ 2
POOLE, CECIL P... 2
PUBLIC SERVICE COMMISSION FOR THE FIRST DISTRICT.......23
RAILROAD GAZETTE....................................... 2
SCIENTIFIC PUBLISHING COMPANY.......................... 2
SCRIBNER'S SONS, CHARLES............................... 1
SHELDON, DR. SAMUEL.................................... 1
SPON AND CHAMBERLAIN................................... 5
U. S. NAVAL INTELLIGENCE OFFICE........................ 6
UNIVERSITY OF ILLINOIS................................. 1
VAIL, J. H... 6
VAN NOSTRAND, D., & CO................................. 2
WOLCOTT, TOWNSEND...................................... 1
WEAVER, W. D...66
WESTERN UNION TELEGRAPH COMPANY........................ 1
WESTINGHOUSE ELECTRIC AND MANUFACTURING COMPANY........ 3
WHITTAKER & CO... 4
WILEY & SONS, JOHN..................................... 6
WYNKOOP, H. S.. 1

The general statistics of the Library and its valuation are shown in the following tabulation:

STATISTICS OF LIBRARY, MAY 1, 1908

Source	Titles	Vol-umes	Pam-phlets	Valuation
Report of May 1, 1907	7649	11186	565	$22,551.21
PURCHASES:				
Mailloux Fund	4	11		20.20
Institute Appropriation	82	84		210.14
Weaver Fund	3	47		25.00
Periodicals	160	253		506.00
GIFTS:				
Edward D. Adams	3	11		39.00
From all other sources	75	236		55.00
Totals including duplicates	7976	11828	565	$23,406.55
Duplicates	308	609		824.74

In the following table are given the figures for the total valuation of the library property including $290.80 expended for new stacks during the past year:

TOTAL VALUATION

Books	$23,406.55
Stacks	1,761.05
Furniture, Catalogues, Cases, etc.	208.28
	$25,375.88

The following tabulations give the state of the five funds from which the Library Committee is entitled to draw:

DONATIONS (GENERAL LIBRARY FUND)

Dr.		Cr.	
Balance May 1, 1907	$237.00		
Interest May 1, 1908	7.15	Unexpended	244.15
	$244.15		$244.15

CARNEGIE FUND

Dr.		Cr.	
Balance May 1, 1907	3529.60	Wheeler Bibliography	468.84
Interest May 1, 1908	107.43	Unexpended	3168.19
	$3637.03		$3637.03

MAILLOUX ENDOWMENT FUND ($1000)

(Proceeds for the maintenance of certain sets of periodical publications)

Dr.		Cr.	
Balance May 1, 1907	$ 7.55	Subscriptions	20.20
Interest May 1, 1908	30.00	Unexpended	17.35
	$37.55		$37.55

INTERNATIONAL ELECTRICAL CONGRESS OF ST. LOUIS, 1904, FUND

(Principal)

Principal May 1, 1907	$2097.56
Additions to Fund	48.56
	$2146.12

(Proceeds)

(Available for the purchase of International Electrical Literature)

Dr.		Cr.	
Interest accrued prior to May 1, 1907	42.54		
Interest to May 1, 1908	64.68		
	$107.22	Unexpended	$107.22

WEAVER FUND

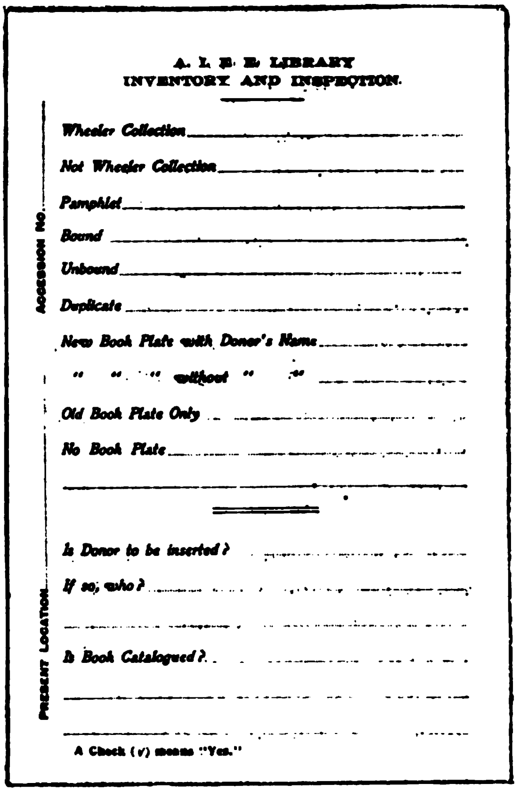
(For early and out of print electrical literature)

Dr.		Cr.	
Amount of Fund	$100.00	Expended	$ 25.00
		Unexpended	75.00
			$100.00

INSTITUTE APPROPRIATION ACCOUNT

Dr.		Cr.	
Appropriation for Maintenance for Year ending October 31, 1907	$3500.00	Insurance	$ 60.90
		Salary, Librarian / " Assistant	1314.00
		Binding	215.35
		Supplies	41.62
		Subscriptions	40.74
		Express and Postage	6.90
		Books	210.14
		Fixtures	290.80
		Miscellaneous	76.70
		Moving	643.83
		Unexpended	599.02
			$3500.00

The above table shows total expenditures of $2900.98 out of the entire appropriation of $3500.00. The salary account is increased nearly $500 by reason of the additional attendance made necessary by keeping the



A. I. E. E. LIBRARY
INVENTORY AND INSPECTION.

ACCESSION NO.

Wheeler Collection

Not Wheeler Collection

Pamphlet

Bound

Unbound

Duplicate

New Book Plate with Donor's Name

" " " without " "

Old Book Plate Only

No Book Plate

PRESENT LOCATION

Is Donor to be inserted?

If so, who?

Is Book Catalogued?

A Check (✓) means "Yes."

Library open evenings until nine o'clock. The binding of many volumes of periodicals left over from previous years in an unbound form has increased this item of expense from $135.00 to $215.00. "Fixtures," which cost $290 this year against $73 last year, include new book stacks and shelving. It should also be pointed out that the entire expense of moving the Library from Liberty Street to the new Engineers' Building, $643.83, is included in the disbursements for this year.

General Inventory and Inspection

It has been found necessary to make a complete inventory and inspection of the books and pamphlets on the shelves, preparatory to making a complete subject catalogue which will include all of the material possessed by the library. This work has progressed continuously for several months by a personal examination of each book and pamphlet, a permanent record being made on cards to show the exact condition of each item with relation to the library records. The card reproduced herewith shows the character of the information obtained by this inventory.

When this work is completed the library will not only be in excellent and most satisfactory and well arranged condition as regards many of the minor details that have not heretofore had proper care, but will also be well prepared for the larger and more important undertaking of producing the card catalogue which shall include all of the electrical literature now on the library shelves. The production of an exhaustive catalogue of this nature is now under consideration and it is planned to include in this catalogue a detailed index to several important sets of electrical periodicals and collections of electrical literature among which may be mentioned the Transactions and Proceedings of the American Institute of Electrical Engineers, the *Journal* of the Institution of Electrical Engineers, London, and the *Transactions* of the International Electrical Congress of St. Louis, 1904.

Important Gifts

Among the gifts deserving of special commendation may be mentioned a fund of $100 given by Mr. W. D. Weaver to be applied to the purchase of early and out of print electrical books. Mr. Carl Hering, a former president of the Institute, has offered to present to the Library a collection of books relating to meetings of various International Electrical Congresses with which he has at various times been associated. The committee has accepted this gift, but the books have not yet been turned over to the library. The New York State Public Service Commission for the First District has very kindly presented to the Library an almost complete set of the reports and records of the Commission and its predecessor, the Rapid Transit Commission of the City of New York. These records include many drawings and plans relating to the construction of the New York City subways. Mr. Edward D. Adams has again met the expenses for the subscription and binding of the *Proceedings* and *Transactions* of the Royal Society of London, and for the additional volumes of the International Catalogue of Scientific Literature. He has also kindly consented to bear the expenses of binding the entire set of volumes of this important set of scientific records. The proceeds of the fund presented by Mr. C. O. Mailloux have again been drawn upon to furnish the yearly continuation of four important periodical sets originally presented by Mr. Mailloux to the Library. Mr. W. S. Andrews has set a worthy example to other members of the Institute by presenting four items of very great historical interest including an original manuscript written by Mr. Thomas A. Edison in 1883, and an original note book compiled by Mr. Andrews from 1881 to 1885 while engaged in designing and installing Edison lighting plants. Mr. J. H. Vail has also presented several important books and pamphlets of historical interest chiefly relating to the early days of electric lighting in the United States.

Completion of Sets

The Library has been very fortunate in being able to complete several important sets which heretofore it possessed in very incomplete form; among these may be mentioned:

American Street Railway Investments; Vols. 1 to 14.
Boston Transit Commission; Annual Reports, Vols. 1 to 13.
New York Public Service Commission for the First District; complete set of reports.
American Bell Telephone Company and American Telephone and Telegraph Company; a complete set of annual reports, 1881 to 1907.
Western Union Telegraph Company; a complete set of annual reports, 1874 to 1907.
General Electric Company; a complete set of annual reports, 1893 to 1908.
New York Edison Company and its predecessors; a complete set of annual reports.
New York State Railroad Commission's Reports; a complete set from 1883 to 1904.
American Street Railway Directory and its successors. This set has been made practically complete.
American Electrical Directory and its successors to date. This set has also been made practically complete.

Early Periodicals

Mention should also be made of a long list of early electrical periodicals recently purchased at a small cost. Nearly all of these periodicals were transient in their nature but all are of great value to the Library from their historical importance. A complete list of the titles is here given:

Telegraphers' Advocate, New York.
Electrical and Street Railway Reporter
Electrical Doings, New York.
Electric Telegraph and Railway Review, London.
Boston Telegram.
Electrical Era, St. Louis.
Electrical Magazine, Washington.
The Wire, New York.
Popular Electric Monthly, Chicago.
Phonogram, New York.
Electrical Literature, Chicago.
Whipple's Electric Reports, Detroit.
Elektron, Boston.
Electricity, Cincinnati.
Electrical Industries Directory of Central Stations, etc., Chicago.
Directory of Electric, Cable and Horse Railways in the U.S. and Canada, New York.
Electrical Progress and Development, Boston.
Electrical Worker.
Street Railway News, New York.
Street Railway and Electrical News, St. Paul and Minneapolis.
Electrical News, Boston.
Electric Railway Advertiser, Boston.
Electricity and Railroading, Boston.
The Telephone, Chicago.
Electrical Northwest.
Electrical Items, New York.
Miscellaneous Bulletins of Electrical World, issued at the National Electric Light Association meetings

A small collection of books has recently been purchased from the library of Mr. J. B. Stearns, one of the early inventors of telegraph apparatus. From this source a number of important items have been obtained not already in the library's collections.

Exchanges

An effort has been made to increase the facilities of the Library by adding to the already large list of exchanges received from all parts of the world. Working in harmony with the other two Engineering Societies whose libraries are located with ours, a complete list of exchanges was made up numbering in all 561 titles. This long list of engineering periodicals has already been added to by the solicitation of other exchanges, and it is hoped at an early date to make the resources of the library in this particular far superior to those of any other engineering collection in the United States.

Recent Books

An attempt has also been made during the past year to increase the facilities of our library by the addition of modern and recently published electrical books;and a large part of our purchases, aside from those made to complete deficiencies in existing sets have been along these lines.

Library of Congress Cards

In the cataloguing of recent books the use has been made of the Library of Congress printed cards. These are obtained from the Librarian of Congress at a nominal cost and when received are practically ready for use in the card catalogue. A sample of these cards is herewith given.

Hutchinson, Rollin William, *jr.*, 1880–

Long-distance electric power transmission, being a treatise on the hydro-electric generation of energy; its transformation, transmission, and distribution, by Rollin W. Hutchinson, jr. ... New York, D. Van Nostrand company, 1907.

v, 345 p. illus., diagrs. 20^{cm}.

Bibliographies at end of most of the chapters.

1. Electricity—Distribution.

Library of Congress Copyright 7-10598

Opening of the Library on Week Day Evenings

Through the joint action of the American Institute of Mining Engineers, the American Society of Mechanical Engineers, and the American Institute of Electrical Engineers, the libraries of all three societies were opened to the public every evening until nine o'clock, except Sundays and legal holidays, beginning November 1, 1907. This has necessitated considerable additional expense for attendance, light, heat, elevator service, etc. The total attendance both day and evening for November, 1907 to April, 1908 is shown in the following table, (the attendance for November not being divided):

	Day	Evening
November 1907 (total)	570	
December, 1907	275	371
January, 1908	526	248
February, 1908	439	241
March, 1908	417	203
April, 1908	403	210

Bibliography of Wheeler Collection

This work has been carried on this year under the direction of a special committee appointed by the Board of Directors so that no report by the Library Committee is called for.

Gifts Desired

The Library Committee solicits from members or others gifts of portraits, photographs, drawings and engineering sketches as well as books and pamphlets, many of which would be of great value to the resources of the library. Many of these can only be obtained through the cooperation of members.

Respectfully submitted,

The Library Committee

Edward Caldwell,
Chairman
Morgan Brooks
Max Loewenthal
F. A. Scheffler
Philip Torchio

INDEX.

PAPERS AND DISCUSSIONS.

INDEX OF AUTHORS.

MEETINGS OF THE INSTITUTE

BUSINESS OF THE INSTITUTE

TRANSACTIONS

OF THE

AMERICAN INSTITUTE

OF

ELECTRICAL ENGINEERS

JUNE 30 TO DECEMBER 31, 1908

VOL. XXVII, PART II

PUBLISHED BY THE
AMERICAN INSTITUTE OF ELECTRICAL ENGINEERS
33 WEST THIRTY-NINTH STREET
NEW YORK, N. Y., U. S A.
1909